TÜRKİYE CUMHURİYET MERKEZ BANKASI

WORKING PAPER NO: 13/25

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July 2013

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Abstract

We present and study the properties of a sticky information exchange rate model where consumers and producers update their information sets infrequently. We find that introducing inattentive consumers has important implications. Through a mechanism resembling the limited participation models, we can address the exchange rate volatility for reasonable values of risk aversion. We observe more persistence in output, consumption and employment which brings us closer to the data. Impulse responses to monetary shocks are hump shaped, consistent with the empirical evidence. Forecast errors of inattentive consumers provide a channel to reduce the correlation of relative consumption and real exchange rate. However, we find that decline in the correlation is quantitatively small.

*E-mail: Fatih.Ekinci@tcmb.gov.tr. I am very grateful to Mark Aguiar, Mark Bils, Yongsung Chang and Alan Stockman for their support and encouragement. I would also like to thank seminar participants at the Central Bank of Turkey, Federal Reserve Board of Governors, Magyar Nemzeti Bank, Sabancı University, T.O.B.B. University, University of Rochester, 2012 Computing in Economics and Finance conference by Society of Computational Economics and 2012 European Economic Association meetings for helpful discussions and comments. The views expressed in this paper are those of the author and do not necessarily represent the official views of the Central Bank of the Republic of Turkey.

1 Introduction

Empirical evidence indicates that nominal and real exchange rates have been excessively volatile relative to major economic aggregates during the post-Bretton Woods period[1]. This paper presents a two country model with the assumption of infrequent information updating for consumers and producers. We show that sticky information on the consumer side provides a new mechanism to generate volatile exchange rates.

Most common approach in the literature to address the exchange rate volatility is proposed by Chari, Kehoe and McGrattan (2002). They exploit the positive and strict link between the ratio of marginal utilities of consumption and the real exchange rate that characterizes economies with complete markets. If risk aversion is sufficiently high, the variability of the ratio of home to foreign consumption observed in the data can correspond to large equilibrium movements in the real exchange rate. Corsetti, Dedola and Leduc (2008) label this strategy as "Risk Aversion Approach". However, the necessary amount of risk aversion required to address real exchange rate volatility is on the high end of business cycle calibrations[2].

This paper proposes a new approach to address exchange rate volatility. We present and study the properties of a sticky information exchange rate model where consumers and pro-

[1]We use data for the U.S. Dollar and a synthetic aggregate of the Euro-zone to quantify exchange rate volatility. Similar patterns have been consistently uncovered between the U.S. and other major OECD countries. See Chari, Kehoe and McGrattan (2002).

[2]Chari, Kehoe and McGrattan (2002) set the degree of risk aversion as 5, which corresponds to an elasticity of intertemporal substitution (EIS) of 0.2. Guvenen (2006) provides a comprehensive discussion on estimates of EIS, and the implications of EIS for real interest rates and consumption. Following a simple calculation through the Euler equation, a lower bound for the real interest rate can be calculated as the product of risk aversion and the growth rate of consumption. In the U.S. data, annual growth rate of consumption is around 2 percent. If risk aversion is set to 5, this implies a 10 percent lower bound for the annual real interest rate. This result is known as the "Risk-free Rate Puzzle". Furthermore, an upper bound for risk aversion is critical for calculations regarding the welfare costs of business cycles. By using consumption data, Lucas (2003) calculates an upper bound of 2.5 for risk aversion.

ducers update their information sets infrequently[3]. Similar to an environment with limited participation models, exchange rates are linked to the marginal utilities of attentive consumers who updated their information set in the current period. When a shock alters the supply side in this economy, consumption plans of inattentive consumers remain unchanged as they remain unaware of this information. The goods market is cleared by the demand response of attentive consumers who are able to update their consumption plans. As the fraction of attentive consumers decreases, their response needs to increase to clear the market. As a result, the consumption of attentive consumers is more volatile than aggregate consumption, and gets more volatile as we decrease the frequency of information updating for consumers. Since the real exchange rate is determined by the marginal utilities of attentive consumers, we observe higher volatility in real exchange rates. With an average information updating duration of 4 quarters, real exchange rates generated by the model are as volatile as in the data for a risk aversion[4] of 2.

When we look at frictions on the producer side[5] assuming attentive consumers, we observe that the sticky information model is virtually identical to the sticky price model. Introducing inattentiveness to the consumer side brings the model in line with the data by (i) increasing

[3]Microfoundations of sticky information models rely on the *inattentiveness* framework proposed by Reis (2006a) and Reis (2006b). Agents are subject to an information processing and updating cost, therefore they optimally choose the duration between the updates in this setup. Once they update their information set, they learn all shocks and all variables up to that date. Sticky information models assume that information updating is exogenous. Micro evidence of inattentiveness is based on the data reported in public and professional forecaster surveys. Carroll (2003) shows that public expectations follow the forecasters' expectations with a lag. Mankiw, Reis and Wolfers (2004) report that cross-section volatility of expectations is higher when the economy is hit by a large shock, consistent with inattentiveness. This assumption distinguishes our work from assuming hand to mouth consumers as in Kollmann (2012).

[4]Trabant and Uhlig (2010) refer to a value of 2 as a consensus in macro literature.

[5]Differences regarding the correlations of output and inflation, the speed of price response to monetary shocks are out of scope for this study. We concentrate on the moments which describe the international business cycles. Inflation dynamics under different assumptions on the producer side have been studied extensively for closed economy models. Mankiw and Reis (2002) show that inflation response to monetary shocks is delayed with sticky information models when monetary policy is described by a money growth rule. Keen (2007) shows that sticky information models do not generate this delayed response when monetary policy is described by an interest rate rule. Our result is consistent with that finding.

the volatility of exchange rates, (ii) generating hump-shaped impulse responses[6] for quantities to a monetary shock, therefore increasing persistence and (iii) reducing the correlation between relative consumption[7] and real exchange rates. Forecast errors of inattentive consumers provide a channel to reduce the Backus-Smith correlation[8].

Organization of the paper is as follows. First, we introduce our model in a nested framework, where we distinguish a standard sticky price model and the proposed sticky information model. Next, we present results regarding the "Risk Aversion Approach"[9] by using sticky price and sticky information models with attentive consumers. We proceed by giving the results with inattentive consumers, discussing the mechanism that generates more exchange rate volatility and checking the robustness of our volatility amplification result. Then, we compare alternative models by reporting a set of business cycle moments. Final section concludes.

[6]See Kim (2001) and Landry (2009) for VAR evidence regarding the impulse responses to a monetary shock.

[7]Notice that the real exchange rate is related to the consumption of *attentive* consumers, not the aggregate consumption in this framework. However, we observe that size of the decline in the correlation is quantitatively small. Considering the simple structure of the model, this channel needs to be further investigated.

[8]Theoretical models produce large and positive correlations between the real exchange rate and relative consumption, as the real exchange rate is tightly linked to the ratio of marginal utilities of consumption. Standard theory implies that consumption is higher wherever it is cheaper, in stark contrast with the data. Real exchange rates in the data appreciate when domestic consumption is higher than foreign consumption, leading to a low and often negative correlation between real exchange rates and relative consumption. Therefore, consumption is higher where it is more expensive. See Backus-Smith (1993) and Chari, Kehoe and McGrattan (2002).

[9]Regarding the "Elasticity Approach" mentioned by Corsetti, Dedola and Leduc (2008), we observe the price-quantity volatility trade-off with our no-frictions model, e.g. attentive consumers, attentive producers and a flexible price setting environment. Since we develop a framework with nominal rigidities, we compare our mechanism with the "Risk Aversion Approach" of Chari, Kehoe and McGrattan (2002). See Backus, Kehoe and Kydland (1995) and Corsetti, Dedola and Leduc (2008) for further discussion.

2 Model

We start by describing the economy where consumers update their information set every period. That is, consumers are assumed to be *attentive*. Then, we describe the economy with inattentive consumers. For producers, we summarize the price-setting problem[10] under two alternative assumptions: the first setup features sticky prices (infrequent *price* updating), while the second assumes sticky information (infrequent *information* updating). Our benchmark model features inattentive consumers and inattentive producers (IC-IP model). Alternative models are also introduced for comparison. We can summarize underlying assumptions as follows: (i) Attentive consumers and sticky prices (AC-SP model) and (ii) Attentive consumers and inattentive producers (AC-IP model). We assume an incomplete markets setup through this paper. This assumption is necessary when the consumers are inattentive, because consumers would be able to insure themselves against the risk of having outdated information with state contingent assets. In this case, model would collapse into a representative agent setup. We compare incomplete market models for all specifications[11].

2.1 Households

2.1.1 Environment

The world economy consists of two countries, home and foreign[12], each specialized in the production of a composite traded good. Households maximize lifetime utility,

$$\max E_t \sum_{s=0}^{\infty} \beta^s U\left(C_{t+s}, N_{t+s}\right)$$

[10]We assume time-dependent price/information updating.

[11]Earlier versions of this paper report results of complete markets model with attentive consumers. We find that results are similar between the complete markets setup and bond economy. See Baxter and Crucini (1995) for a detailed disccussion on the implications of alternative asset market specifications in open economy models.

[12]Countries are assumed to be of equal size.

subject to a sequence of budget constraints, which is expressed in domestic currency units as

$$W_t N_t + \Pi_t + B_t + e_t D_t \geq P_t C_t + v_{t,t+1} B_{t+1} + e_t \ q_{t,t+1} \ D_{t+1} + e_t \ \Theta_b(D_{t+1}) \tag{1}$$

where W_t is the nominal wage rate and N_t is the labor supply. P_t represents the price index for home country and C_t is the composite[13] consumption good. Π_t is the profits of domestic[14] intermediate goods producers. B_t is the amount of nominal bonds held by domestic consumers between time t and t+1, and $v_{t,t+1}$ is the time t price of the bond which pays one unit of home currency at time t+1. Home and foreign households can trade discount bonds[15] denominated in foreign currency, and e_t is the nominal exchange rate. D_t is the amount of foreign currency nominal bonds held by domestic consumers between time t and t+1, and $q_{t,t+1}$ is the price of the foreign currency bond. Domestic country consumers incur a convex cost of holding foreign currency bonds which is given as $\Theta_b(D) = \frac{\theta_b}{2} D^2$. Budget constraint for the foreign consumer is given by

$$W_t^* N_t^* + \Pi_t^* + D_t^* \geq P_t^* C_t^* + q_{t,t+1} D_{t+1}^* \tag{2}$$

Foreign country variables are denoted with an asterisk, the amount of one period nominal bonds denominated in foreign currency[16] held by foreign consumers is D_{t+1}^*. Decision variables for the households are bond holdings and labor supply.

[13] Home and foreign goods are aggregated by a constant elasticity of substitution index following Armington (1969).

[14] Domestic firms are assumed to be owned by home consumers.

[15] We allow for international trade only for the foreign currency bonds.

[16] Notice that foreign country budget constraint is expressed in foreign currency units.

2.1.2 Composite Consumption Index

Consumption preferences are described by the following composite index of domestic and imported bundles of goods:

$$C_t \equiv \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. Weight of imported goods in the consumption basket[17] is determined by γ. Each consumption bundle $C_{H,t}$ and $C_{F,t}$ is composed of imperfectly substitutable varieties, with elasticity of substitution $\nu > 1$. Optimal allocation of expenditure between each variety of goods yields,

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} C_{H,t}; \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\nu} C_{F,t}.$$

where each variety is indexed by i, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$ and $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$. Optimal expenditure allocation on home and foreign goods gives,

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \quad C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t.$$

where $P_t \equiv \left[(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the CPI index. We can express the log-linearized[18] inflation dynamics as follows,

$$\hat{\pi}_t = (1-\gamma)\hat{\pi}_{H,t} + \gamma\hat{\pi}_{F,t} \tag{4}$$

where hat notation represents the log-deviations from steady state.

[17] For the foreign country, goods produced at home country are the import goods. Therefore, γ determines the share of home goods in the foreign consumption basket.

[18] Log-linearization is around the zero-inflation steady state, assuming symmetry across home and foreign countries.

2.1.3 Optimality Conditions

We denote the marginal utility of consumption by λ^c and the marginal disutility of labor as λ^n. We obtain the foreign country gross nominal interest rate from the first order conditions of home and foreign consumers for foreign currency bond holdings,

$$R_t^{*-1} \equiv q_{t,t+1} = \beta E_t \left[\frac{\lambda_{t+1}^{c*}}{\lambda_t^{c*}} \frac{P_t^*}{P_{t+1}^*} \right] = \beta E_t \left[\frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{P_t}{P_{t+1}} \frac{e_{t+1}}{e_t} \right] - \Theta_b(D_{t+1})' \tag{5}$$

Using the first order condition of home currency bond holdings, gross nominal interest rate for home country is defined as follows,

$$R_t^{-1} \equiv v_{t,t+1} = \beta E_t \left[\frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{P_t}{P_{t+1}} \right] \tag{6}$$

No-arbitrage condition for foreign currency bonds, definitions of the interest rates and log-linearizing the results gives us the uncovered interest parity condition,

$$\hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{e}_{t+1} - \theta_b \tilde{D}_{t+1} \tag{7}$$

where tilde variables correspond to level deviations from the steady state. We define the real exchange rate as $rer_t \equiv e_t \frac{P_t^*}{P_t}$, and we can express the real exchange rate dynamics in terms of marginal utilities as follows,

$$E_t \Delta r\hat{e}r_{t+1} = E_t \Delta \hat{\lambda}_{t+1}^* - E_t \Delta \hat{\lambda}_{t+1} + \theta_b \tilde{D}_{t+1} \tag{8}$$

Labor supply is determined by the static first order condition, which sets the real wages equal to the marginal rate of substitution between consumption and leisure,

$$\frac{W_t}{P_t} = -\frac{\lambda_t^n}{\lambda_t^c} \tag{9}$$

2.2 Consumers with Sticky Information

In this section we describe the decision making process of the household under inattentiveness assumption. Household solves a two-step problem. Allocating the best bundle of varieties for the composite consumption good is the *intra-temporal* decision, and consumption amount of the composite good for each period is the *inter-temporal* decision. We assume that household is composed of a shopper-planner pair. The shopper makes the intra-temporal decision by observing the relative prices of varieties. The planner solves the inter-temporal problem. Every period, the planner observes[19] the amount of domestic and foreign bonds, $B_{t,j}$ and $D_{t,j}$. Here, the second index is the number of periods by which the information set is outdated. We assume that consumers sign an insurance contract so that they all start each period with the same wealth, $B_{t,j} = B_t$ and $D_{t,j} = D_t$. The payments from this contract are $\tau_{t,j}$. This way, we do not have to track[20] the wealth distribution. If she knows all variables up to date t, the probability of updating her information set at date t+1 is $1 - \delta$. We can state the problem of the attentive consumer as follows

$$V(B_t, D_t) = \max_{\{C_{t+i,i}, W_{t+i,i}\}} \sum_{i=0}^{\infty} \beta^i \delta^i U(C_{t+i,i}, N_{t+i,i}) + \beta(1-\delta) \sum_{i=0}^{\infty} \beta^i \delta^i V(B_{t+i+1}, D_{t+i+1}) \quad (10)$$

First term is the expected discounted utility if the planner never updates her information set again. Second term is the sum of continuation values over all possible future dates at which planner may update the information, where probability of receiving an information update i periods later is $(1-\delta)\delta^i$. Attentive planner decides on a consumption plan for each period without any information update. For the labor market, a wage rate for future periods is posted along with the assumption that each household is a monopolistic supplier

[19]We assume that shopper and planner does not share any information.

[20]We can track the wealth distribution by assuming a staggered information updating setup. To avoid the computational burden, we assume this transfer scheme following Mankiw and Reis (2006).

of a specific labor variety. Sequence of budget constraints is given by

$$
\begin{aligned}
W_{t+i,i}N_{t+i,i} &+ B_{t+i} + e_{t+i}D_{t+i} + \Pi_{t+i} \geq P_{t+i}C_{t+i,i} + v_{t+i,t+i+1}B_{t+i+1} \\
&+ q_{t+i,t+i+1}e_{t+i}D_{t+i+1} + e_{t+i}\theta_b(D_{t+i+1}) + \tau_{t+i,i} \qquad (11)
\end{aligned}
$$

Information frictions on the consumer side implies that the price of domestic currency bond is determined by the marginal utility of attentive consumers[21],

$$
R_t^{-1} \equiv v_{t,t+1} = \beta E_t \left[\frac{\lambda^c_{t+1,0}}{\lambda^c_{t,0}} \frac{P_t}{P_{t+1}} \right] \qquad (12)
$$

Defining the real interest rate as $rr_t \equiv R_t \frac{P_t}{P_{t+1}}$ and log-linearization around the steady state gives the following optimality conditions

$$
\begin{aligned}
\hat{\lambda}^c_{t,0} &= E_t \left[\hat{\lambda}^c_{t+1,0} + \hat{rr}_t \right] \qquad (13) \\
\hat{\lambda}^c_{t,j} &= E_{t-j} \hat{\lambda}^c_{t,0} \qquad (14)
\end{aligned}
$$

Aggregate consumption is obtained by using the weights of information cohorts in the population, $\hat{c}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{c}_{t,j}$. In this economy, the no-arbitrage condition for the foreign currency bonds include marginal utilities of the attentive consumers,

$$
q_{t,t+1} = \beta E_t \left[\frac{\lambda^{c*}_{t+1,0}}{\lambda^{c*}_{t,0}} \frac{P^*_t}{P^*_{t+1}} \right] = \beta E_t \left[\frac{\lambda^c_{t+1,0}}{\lambda^c_{t,0}} \frac{P_t}{P_{t+1}} \frac{e_{t+1}}{e_t} \right] - \Theta'_b(D_{t+1}) \qquad (15)
$$

Log-linearizing this equation and using the real exchange rate definition, the real exchange rate dynamics is given by

$$
E_t \Delta \hat{rer}_{t+1} = E_t \Delta \hat{\lambda}^{c*}_{t+1,0} - E_t \Delta \hat{\lambda}^c_{t+1,0} + \theta_b \tilde{D}_{t+1} \qquad (16)
$$

[21]Details are provided in the Appendix.

2.3 Labor Market with Sticky Information

We assume that each household is a monopolistic supplier of a specific labor variety. The demand for the labor variety h is given by $N_{t,h} = \left(\frac{W_{t,h}}{W_t}\right)^{-\chi} N_t$, where χ is the elasticity of substitution between labor varieties. The planner posts a nominal wage rate using the available information. Using results from the consumption decision and plugging in the demand for labor variety, we obtain the following condition for wage posting in the case of attentive consumers,

$$\frac{W_{t,0}}{P_t} = -\mu_\chi \frac{\lambda^n_{t,0}}{\lambda^c_{t,0}} \tag{17}$$

where $\mu_\chi = \frac{\chi}{\chi-1}$ is the markup over the marginal rate of substitution between consumption and leisure. Agents who have outdated information post wages by forecasting the decision of attentive agents

$$\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0} \tag{18}$$

The aggregate nominal wage rate is given by $\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{w}_{t,j}$.

2.4 Producers

Intermediate goods are produced by labor. Production function[22] for the domestic producer of variety i is given by $Y_H(i) = A_t N_t(i)^\zeta$. Demand from the domestic country for the variety produced by firm i is given by $Y_H(i) = \left\{\frac{P_H(i)}{P_H}\right\}^{-\nu} Y_H$. We assume that firms set prices in buyers' currencies to maximize their expected profits.

[22] Aggregate productivity follows an AR(1) process and denoted as A.

2.4.1 Producers Under Sticky Price Assumption

Producers are attentive, they update their information set every period. They update their *prices* when they receive a Calvo signal. The probability of updating their prices is $1-\theta$, while prices stay constant with probability θ. They set prices in the local currencies for domestic and foreign country to maximize their expected profits:

$$\max_{P_H(i),P_H^*(i)} \sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k} \left\{ P_{H,t}(i) Y_{H,t+k}(i) - W_{t+k} \left(\frac{Y_{H,t+k}(i)}{A_t} \right)^{\frac{1}{\zeta}} \right\} \right]$$
$$+ \sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k} \left\{ e_{t+k} P_{H,t}^*(i) Y_{H,t+k}^*(i) - W_{t+k} \left(\frac{Y_{H,t+k}^*(i)}{A_t} \right)^{\frac{1}{\zeta}} \right\} \right].$$

Using the demand for the variety, the first order condition for home prices of locally produced goods is

$$\sum_{k=0}^{\infty} \theta^k E_t \left[v_{t,t+k} Y_{H,t+k}(i) \left\{ P_{H,t}(i) - \frac{\nu}{\nu-1} \frac{W_{t+k}}{\zeta A_{t+k}^{\frac{1}{\zeta}}} Y_{H,t+k}^{\frac{1-\zeta}{\zeta}} \right\} \right] = 0 \tag{19}$$

Imposing symmetry, log-linearizing and rearranging, we can express the final result as a sticky price Philips curve relation between the real marginal cost and inflation,

$$\hat{\pi}_{H,t} = \kappa \ \Omega \ \hat{mc}_{H,t} + \beta E_t \hat{\pi}_{H,t+1} \tag{20}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$, $\Omega \equiv \frac{\zeta}{\zeta+(1-\zeta)\nu}$ and real marginal cost is $\hat{mc}_{H,t} \equiv \hat{W}_t - \hat{P}_{H,t} + \frac{1-\zeta}{\zeta}\hat{Y}_{H,t} - \frac{1}{\zeta}\hat{A}_t$. Import inflation for the foreign country is given by

$$\hat{\pi}_{H,t}^* = \kappa \ \Omega \ \left[\hat{mc}_{H,t}^* - \hat{e}_t \right] + \beta E_t \left[\hat{\pi}_{H,t+1}^* \right] \tag{21}$$

where real marginal cost is defined as $\hat{mc}_{H,t}^* \equiv \hat{W}_t - \hat{P}_{H,t}^* + \frac{1-\zeta}{\zeta}\hat{Y}_{H,t}^* - \frac{1}{\zeta}\hat{A}_t$.

2.4.2 Producers Under Sticky Information Assumption

Firms update their *information set* with probability $1-\theta$ each period. They proceed using their outdated information with probability θ. The firm which sets the price at time t according to the information received j periods ago solves the following static problem

$$\max_{P_H(j),P_H^*(j)} E_{t-j}\left[P_{H,t}(j)Y_{H,t}(j) - W_t\left(\frac{Y_{H,t}(j)}{A_t}\right)^{\frac{1}{\zeta}}\right] + E_{t-j}\left[e_t P^*_{H,t}(j)Y^*_{H,t}(j) - W_t\left(\frac{Y^*_{H,t}(j)}{A_t}\right)^{\frac{1}{\zeta}}\right].$$

In this case, home country inflation for domestic goods is obtained by aggregating information cohorts across firms. Final expression which gives the home inflation rate of domestic goods is,

$$\hat{\pi}_{H,t} = \frac{1-\theta}{\theta}\Omega\hat{mc}_{H,t} + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\left[\Omega\Delta\hat{mc}_{H,t} + \hat{\pi}_{H,t}\right]\right) \tag{22}$$

Import inflation in the foreign country is

$$\hat{\pi}^*_{H,t} = \frac{1-\theta}{\theta}\Omega\left(\hat{mc}^*_{H,t} - \hat{e}_t\right) + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\left[\Omega\left(\Delta\hat{mc}^*_{H,t} - \Delta\hat{e}_t\right) + \hat{\pi}^*_{H,t}\right]\right) \tag{23}$$

Regarding the inflation dynamics, we observe a forward looking relation with sticky prices. Current inflation is a function of the expectation of future inflation rate. On the other hand, we observe that inflation is a function of lagged expectations of current inflation with the sticky information assumption. We discuss the implications of the different price setting mechanisms by assuming attentive consumers and different price setting mechanisms in our results section.

2.5 Monetary Policy and Market Clearing

We close the model by defining the monetary policy rule and imposing the market clearing condition. Interest rates follow a Taylor rule with a stochastic component

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t\right] + \epsilon_{R,t} \tag{24}$$

Market clearing condition for domestic goods is given by

$$Y_t = C_{H,t} + C^*_{H,t} \tag{25}$$

We define the relative prices of imported goods as $rp_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and $rp^*_t \equiv \frac{P^*_{H,t}}{P^*_{F,t}}$. Using the optimal allocation from the demand functions, market clearing condition for domestic goods can be expressed in log-linearized form as follows

$$\hat{y}_t = (1-\gamma)\hat{c}_t + \gamma\hat{c}^*_t + \gamma(1-\gamma)\eta \; \hat{rp}_t - \gamma(1-\gamma)\eta \; \hat{rp}^*_t \tag{26}$$

2.6 Parametrization and Calibration Strategy

We log-linearize the system around the zero-inflation steady state, which yields a system of second order difference equations[23] in the case of sticky price models. These systems can be solved by standard methods outlined in Klein (2000). Sticky information models include the lagged expectations of variables. We can write our models in the following form:

$$AE_tY_{t+1} + B_0Y_t + \sum_{i=1}^{I} B_iE_{t-i}Y_t + CY_{t-1} + GW_t = 0 \tag{27}$$

[23] A summary of sticky price model (assuming attentive consumers, AC-SP model) and sticky information model (featuring inattentive consumers and producers, IC-IP model) are provided in the Appendix.

where Y_t is vector of endogenous variables and W_t is vector of exogenous variables with a law of motion $W_t = NW_{t-1} + \epsilon_t$. The solution is in the form of $Y_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j}$. We can manipulate this structure by plugging the solution into the system and truncating at a large number of lags. This reduces the model to a block tridiagonal structure which can be easily solved[24]. We report the theoretical moments[25] of the models studied in this paper.

Our choice of the parameter values is summarized in Table 1. We assume a utility function of the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \xi \frac{N_t^{1+1/\phi}}{1+1/\phi} \tag{28}$$

Notice that utility is separable between consumption and leisure.

For the preference parameters, we assume a discount factor $\beta = 0.995$, which implies an annual real return of 2 percent at steady state[26]. The curvature parameter of the utility function (σ) determines the degree of risk aversion. We set this parameter as 2 for our benchmark calibration. Regarding the home bias in the consumption basket, γ is set to 0.06 following Chari, Kehoe and McGrattan (2002).

The Frisch elasticity of labor supply is determined by ϕ. Following the evidence[27] from micro-econometric studies, we use $\phi = 0.5$ for our exercises. Elasticity of substitution between home and foreign goods (η) is set to 1.5, following Backus, Kehoe and Kydland (1994)

[24] Earlier literature introduced lagged expectations as new variables to the endogenous state vector. This approach increases the computational burden, and accuracy depends on the number of lags included. Meyer-Gohde (2010) provides a new solution method for this class of models. A summary of the solution method is provided in the Appendix.

[25] We apply a two-sided filter following King and Rebelo (1993) to obtain the HP-filtered theoretical moments.

[26] Steady state labor supply is determined by ξ, log-linearized solution does not depend on this parameter.

[27] See Chetty et al. (2011).

and Chari, Kehoe and McGrattan (2002).

The elasticity of substitution across the varieties of goods, ν, is set to 10. This is consistent with a price markup of 11 percent as documented in the U.S. data by Basu (1996). The elasticity of substitution among labor varieties is set to 10, following Mankiw and Reis (2006). We set the degree of price/information stickiness for the producers to $\theta = 0.75$. This implies an average duration of 4 quarters for price/information updating.

We follow Chari, Kehoe and McGrattan (2002) to describe our exogenous productivity processes. Assuming symmetry across countries, we set the persistence and standard deviation of the productivity shocks as $\rho_A = 0.95$, and $\sigma_A = 0.7$ percent respectively. Cross-country correlation of productivity shocks is set to 0.25. For the monetary policy rule, we use $\rho_R = 0.85$, $\psi_\pi = 1.5$ and $\psi_y = 0.5$. We check the sensitivity of real exchange rate volatility using alternative Taylor rule parameters.

We choose the standard deviation of the monetary shocks so that the volatility of output is the same in the model as in the U.S. data[28] for each specification. We set the cross-country correlation of monetary shocks as 0.5 and assume the shock is symmetric for the rest of the world, i.e. the standard deviation of the foreign country monetary shock is the same.

To pin down the degree of information stickiness on the consumer side, we carry out an exercise with consumption growth following Mankiw and Reis (2006). If consumption[29] follows a random walk, then the variance of growth rate from t to t+2 should be twice as the variance of the growth rate from t to t+1. However, in the US data, we observe

[28]Details of data sources are described in the Appendix.
[29]Transformed by taking the logarithm of the data.

that $\left(2 \times \frac{Var(c_t - c_{t-1})}{Var(c_t - c_{t-2})}\right)$ is equal to 0.73, which means consumption adjusts gradually[30] to the shocks driving the economy. Furthermore, if consumption follows a random walk, the autocorrelation of consumption growth should be 0. Positive autocorrelation observed in the data indicates a gradual adjustment as well. We calibrate our sticky price and sticky information models to match output volatility as described above. Results are reported in Table 2. We find that the variance ratio is greater than 1 for our sticky price model and sticky information model with attentive consumers and we observe that autocorrelation of consumption growth is negative for both models. Information stickiness on the consumer side helps us to bring the model closer to the data for these two moments. Mankiw and Reis (2006) and Reis (2009) report estimation results for US and Europe, for closed economy models. The range for δ in these studies is between 0.64 and 0.92. We report results for $\delta = 0.5$ and $\delta = 0.75$ for our exercises, and we set $\delta = 0.75$ for our benchmark[31] calibration.

Looking at our alternative models, setting $\delta = 0$ is equivalent to assuming attentive consumers. For the sticky price model (AC-SP), we always assume attentive consumers. Benchmark model with inattentive consumers and producers (IC-IP) collapses to the model with attentive consumers and inattentive producers (AC-IP) when δ is set to 0.

3 Results

We start with numerical results of "Risk Aversion Approach" to address real exchange rate volatility and explain the underlying mechanism. We assume attentive consumers for this exercise. Then, we present the new approach proposed in this paper by introducing inatten-

[30] Mankiw and Reis (2006) use $\frac{std(c_t - c_{t-1})}{std(c_t - c_{t-4})}$ as a calibration target.

[31] Average duration of information updating is given by $\frac{1}{1-\delta}$. Setting $\delta = 0.75$ is consistent with the findings of Carroll (2003). He estimates a model of information diffusion using public and forecaster survey data, and reports that public expectations follow forecasters' expectations with a one year lag.

tive consumers. We show that our results are robust to alternative specifications regarding monetary policy rules, production technology, elasticity of substitution between home and foreign goods, degree of nominal rigidities and share of import goods in consumption. Results suggest that exchange rate volatility becomes closer to the data under all alternative specifications. We also present the results with habit formation and attentive consumers to emphasize the distinction from assuming inattentiveness on the consumer side. We show that real exchange rate volatility declines as we increase the level of habit formation.

Next, we report business cycle statistics for alternative models. We show that different forms of rigidity in price setting behaviour produce similar results regarding international business cycles. Then, we discuss the business cycle statistics of our benchmark sticky information model with information frictions on both consumer and producer side.

3.1 Exchange Rate Volatility with Attentive Consumers

First, we derive the relationship between real exchange rate volatility and the level of risk aversion for models with attentive consumers. This exercise helps to understand the dynamics of the "Risk Aversion Approach" *á* la Chari, Kehoe and McGrattan (2002). Real exchange rate dynamics are driven by the growth rates[32] of the marginal utilities of home and foreign consumers

$$E_t \Delta r\hat{e}r_{t+1} = E_t \Delta \hat{\lambda}^*_{t+1} - E_t \Delta \hat{\lambda}_{t+1} + \theta_b \tilde{D}_{t+1} \qquad (29)$$

We define an auxiliary variable[33] to obtain a closed form expression for real exchange

[32]Effect of net foreign asset position arising from the portfolio adjustment cost assumption is very small. This assumption ensures the stationarity of the real exchange rates. Chari, Kehoe and McGrattan (2002) impose borrowing constraints.

[33]We replace the consumption growth rates with the levels in equation 29 and drop the net foreign asset variable to simplify the analysis. This variable corresponds to the real exchange rates when we assume

rate volatility: $\widetilde{rer}_t^{levels} = \hat{\lambda}_t^{c*} - \hat{\lambda}_t^{c}$. With separable utility, $\hat{\lambda}_t^{c} = -\sigma \hat{c}_t$, we can express the auxiliary variable for the real exchange rate in terms of relative consumption, $\widetilde{rer}_t^{levels} = \sigma(\hat{c}_t - \hat{c}_t^*)$. Dividing by the variance of output, expanding the relative consumption variance and imposing symmetry gives

$$\frac{std(\widetilde{rer}^{levels})}{std(y)} = RISK\ AVERSION \times \sqrt{2\left(1 - corr(\hat{c}, \hat{c}^*)\right)}\frac{std(\hat{c})}{std(y)} \tag{30}$$

This relation shows a direct link between the level of risk aversion (parametrized as σ) and real exchange rate volatility. We report the theoretical moments of the model[34] in Table 3. We observe that cross country consumption correlation and volatility of consumption are not the main driving forces when we change the level of risk aversion. We also observe that volatility of relative consumption declines as we increase the degree of risk aversion and we need to set risk aversion parameter as 5 to match the real exchange rate volatility with the data. This result does not change whether we impose a sticky price or sticky information structure for the producer side.

3.2 Real Exchange Rate Volatility with Inattentive Consumers

When we have sticky information on the consumer side, the real exchange rate is determined by an asset pricing condition based on the marginal utilities of attentive consumers. Defining $\widetilde{rer}_t^{levels,0} = \hat{\lambda}_{t,0}^{c*} - \hat{\lambda}_{t,0}^{c}$ and following similar steps to the case with attentive consumers yields

$$\frac{std(\widetilde{rer}^{levels,0})}{std(y)} = RISK\ AVERSION \times \sqrt{2\left(1 - corr(\hat{c}_0, \hat{c}_0^*)\right)}\frac{std(\hat{c}_0)}{std(y)} \tag{31}$$

complete markets.

[34]Auxiliary variable (rer^{levels}) defined to obtain closed form expressions about the real exchange rate volatility. We report the moments of actual real exchange rates and other variables after calibrating the models.

This equation links the volatility of real exchange rate with the attentive consumer's consumption. Aggregate consumption response is a weighted average of the responses from all information cohorts, $\hat{c}_t^{agg} = (1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{c}_{t,i}$. We can express the consumption response of an agent who updated her information set i periods ago as her expectation of the long rate conditioning on the available information, that is $\hat{c}_{t,i} = -\frac{1}{\sigma}E_{t-i}\hat{lr}_t$. The long rate is defined as $\hat{lr}_t = \sum_{j=0}^{\infty}\hat{rr}_{t+j}$.

We start by establishing that the volatility of attentive consumers' consumption is at least as high as aggregate consumption. For any moving average process x_t, $\text{var}(x_t) > \text{var}(E_{t-j}x_t)$ when $j > 0$. Since we can express our model solution as a moving average process, then $\text{var}(\hat{lr}_t) > \text{var}(E_{t-j}\hat{lr}_t)$ for $j > 0$. It is easy to show that $\text{var}(\hat{c}_0) > \text{var}(\hat{c}_j)$ for $j > 0$. It follows that aggregate consumption is less volatile than the consumption of attentive consumers, $\text{var}(\hat{c}_0) > \text{var}(\hat{c}^{agg})$ for $\delta > 0$. We can also analytically show that volatility of attentive consumers' consumption increases as we increase the degree of information stickiness on the consumer side. Numerical results for varying degrees of information stickiness on the consumer side are reported in Table 4.

To understand the intuition, we plot the impulse response to a one unit negative innovation[35] which decreases home interest rate in Figure 1. Output and aggregate consumption move very closely. On impact, only the consumers who updated their information set in the current period have this shock in their information set. Therefore, aggregate consumption response is a fraction of the attentive consumer's response. The consumption plans of inattentive consumers remain the same since they do not have information on that. Supply response to clear the goods market is relatively small compared to the case where all consumers are attentive. As the fraction of attentive consumers goes down, output response necessary to

[35] 1 unit negative shock to Taylor rule, ϵ_R.

clear the markets decreases. Consequently, attentive consumers' consumption is more volatile than the aggregate consumption and output. Since the real exchange rate is determined by the marginal utilities of attentive consumers, we observe higher volatility in real exchange rates.

3.3 Calibration Results and Impulse Responses

We focus on the business cycle moments and international transmission of monetary shocks in this section. To understand the effect of imposing different frictions on the producer side, we compare the sticky price model and the sticky information model with attentive consumers. Next, we discuss the business cycle properties of the benchmark model with inattentive consumers and present the results under a lower degree of import elasticity.

3.3.1 Attentive Consumers

Table 5 reports business cycle moments for alternative models. Comparing models with attentive consumers, we observe that the form of the friction on the producer side has a small effect on the moments[36] generated by the model.

For models with attentive consumers, we observe that consumption and employment are more volatile[37] in the model compared to the data. Net exports are less volatile than the data, but we should note that the volatility of net exports is sensitive to the degree of home bias. Models with attentive consumers generate less persistence in quantities and prices compared to the data. Models with attentive consumers capture the fact that cross country consumption correlation is lower than output correlation. The real exchange rate and rel-

[36] See Keen (2007) for a comparison of sticky price and sticky information models with a closed economy setup.

[37] For simplicity, we abstract from capital accumulation. Chari, Kehoe and McGrattan (2002) target consumption volatility by changing an investment adjustment cost parameter.

ative consumption exhibit perfect correlation contradicting the data, widely referred as the Backus-Smith puzzle.

Since monetary shocks play the dominant role in determining the dynamics of our model, we focus on the impulse responses to a home monetary shock[38] to understand the effect of introducing inattentive consumers. Figure 2 plots the impulse response functions for the sticky information model with attentive consumers and our benchmark model with inattentive consumers and producers.

When we look at the model with attentive consumers, we observe that home consumption increases following a decline in the interest rate. Due to increased demand from home consumers, domestic and foreign output increases, and inflation rises in both countries. The foreign interest rate increases via feedback from the monetary authority to increased output and inflation. Foreign consumption decreases as a result of the increase in the interest rate. Transmission of a monetary shock is negative in consumption and positive in output. This helps to explain the fact that cross country output correlation is higher than that of consumption in the data. As the shock dissipates, quantities and real exchange rates return to their steady state values monotonically. Therefore, our model with attentive consumers generates low persistence in quantities. Real exchange rate persistence is also low since it is tightly linked to relative consumption in this model.

[38] Direction of the impulse responses to a productivity shock remains same across the models for key variables. When home productivity increases, prices of home goods decrease. This leads to a rise in demand for home goods, which raises home and foreign consumption. Home consumption increases less than home output. By the decline in home inflation, the home interest rate decreases. Since demand shifts away from foreign goods, foreign output and inflation decrease. By the monetary policy rule, foreign interest rate goes down. The increase in home (attentive) consumption is greater than foreign (attentive) consumption. We observe hump shaped impulse responses, due to the negative comovement between output and inflation combined with the feedback from the interest rate rule. See Steinsson (2008) for a more comprehensive discussion of real shocks.

3.3.2 Inattentive Consumers

We observe that nominal and real exchange rate volatility[39] is magnified with inattentive consumers and the persistence of quantities and prices becomes closer to the data[40]. Since real exchange rates are determined by the attentive consumer's consumption instead of aggregate consumption, inattentiveness on the consumer side provides a channel for a lower Backus-Smith correlation. We use the auxiliary variable for real exchange rates again to obtain a closed form expression: $\widetilde{rer}_t^{levels,0} \equiv \sigma \times (\hat{c}_{t,0} - \hat{c}^*_{t,0})$. We aggregate consumption from information cohorts and define forecast errors on the real exchange rate movements as $\widetilde{f}_{t,j} = \widetilde{rer}_t^{levels,0} - E_{t-j}\widetilde{rer}_t^{levels,0}$. We obtain the following relation which links the auxiliary variable for the real exchange rates and relative consumption,

$$\widetilde{rer}_t^{levels,0} = RISK \quad AVERSION \times (\hat{c}_t - \hat{c}^*_t) + (1-\delta)\sum_{j=1}^{\infty} \delta^j \widetilde{f}_{j,t}$$

This expression shows that the correlation of real exchange rates and relative consumption depends on the size of forecast errors made by the agents who have outdated information. However, calibration results show the size of the decline is quantitatively small. This channel needs to be further investigated. Inattentive consumer models perform less well on some issues compared to the models with attentive consumers. The cross country consumption correlation is higher than that of output, and we obtain procyclical net exports.

To understand the dynamics, we look at the impulse responses from the benchmark sticky information model in Figure 2 to compare with the sticky information model with attentive consumers. Since we previously investigated the results on exchange rate volatility, we skip the distinction between aggregate consumption and consumption of the attentive consumer

[39]We refer to the volatilities relative to the output volatility.

[40]Results are reported in Table 5.

here. Demand from home consumers increases gradually in this case. Consumers react to the monetary shock as they update their information set. Therefore, the decline in home output and consumption is not as fast as in the full information case. These dynamics help us to get more persistence in quantities, moving the model closer to the data. We also observe that the gradual adjustment of home demand changes the nature of the transmission dynamics for a monetary shock. The direction of inflation response in foreign country changes with inattentive consumers. A larger nominal exchange rate depreciation (with respect to the inflation rates compared to the model with attentive consumers) creates a decline in imported goods inflation in the foreign country. The decline in the inflation is reflected in interest rates, which leads to a positive consumption response as opposed to the negative one for the case with attentive consumers. Weak demand response also leads to a decline in the consumption of import goods in the home country since foreign goods became more expensive for home consumers due to the depreciation. This leads to a positive net exports response at home country with inattentive consumers.

Introducing inattentive consumers generates a positive transmission in consumption and a negative transmission in output in response to a monetary shock, therefore cross-country consumption correlation is higher than that of output. This result is sensitive to the elasticity of substitution between home and foreign goods. When we calibrate our benchmark model for a lower import elasticity (by setting $\eta = 0.5$), we obtain counter-cyclical net exports, and cross country correlation of output is higher than that of consumption. Results from using a lower elasticity in the benchmark model are reported in Table 6. Aside from parametrization, abstracting from capital is also an important influence on our results. Countercyclical trade fluctutations reflect in large part on the dynamics of capital formation: expansions are associated with investment booms financed by borrowing from international capital markets. Since we assume labor is the only production input, moments of net exports are hard to capture with our model. Overall, our results show that introducing inattentive consumers to a standard open economy model fits the data better in many dimensions. However, we observe that some extensions to this baseline model can improve the results further.

3.4 Sensitivity Analysis

We report the volatility of real exchange rates under alternative specificiations for varying degrees of information stickiness on the consumer side in Table 7. We observe that real exchange rate volatility increases as the inertia in the monetary policy rule increases. Real exchange rate is more volatile with passive monetary policy (lower ψ_π), and this effect is more pronounced at the lower levels of consumer information stickiness. Lower interest rate response to output increases the real exchange rate volatility for all levels of consumer information stickiness substantially. However, we find that real exchange rate volatility is amplified as the degree of information stickiness increases and this result is robust to different monetary policy rules. Although there are some changes in the other business cycle properties, we observe that this volatility amplification result survives under different specifications regarding the degree of information stickiness on the producer side, the production function, the level of import share in preferences and the level of import elasticity.

Finally, we introduce external habit formation into our utility function to emphasize the difference from sticky information assumption on the consumer side. Marginal utility of consumption in this case is given by $\hat{\lambda}^c = -\sigma(\hat{c}_t - h\hat{c}_{t-1})$. We report the results for varying degrees of habit formation. As the degree of habit increases, we observe that marginal utilities become less volatile. Numerical results[41] are reported in Table 8, showing that habit formation reduces the volatility of real exchange rates.

[41] We can also show that analytically.

4 Conclusion

We present and study the properties of a model which imposes infrequent information updating for consumers and producers. Comparing a sticky price and sticky information model with attentive consumers, we find that the form of frictions on the producer side has a small affect for the moments which describe the international business cycles. On the other hand, imposing sticky information on the consumer side provides a new mechanism to address the exchange rate volatility without setting the degree of risk aversion too high.

Introducing inattentive consumers exhibit a similar mechanism to the limited participation models of asset pricing literature. In this framework, exchange rates are linked to the relative consumption of attentive consumers who updated their information set in the current period. Their consumption is more volatile than aggregate consumption because inattentive consumers cannot adjust their consumption plans to the current shocks. As the fraction of attentive consumers falls, we observe more volatility in their consumption. This increases the volatility of marginal utilities, resulting in more volatile exchange rates. Setting the degree of risk aversion at a consensus value, where the intertemporal elasticity of substitution is 0.5, an average duration of 4 quarters between information updates can account for the exchange rate volatility observed in the data.

Sticky information on the consumer side brings the model closer to the data in other dimensions as well. We observe hump shaped impulse responses to monetary shocks, which increases the persistence of output, consumption and employment. We also see a small decline in the correlation of relative consumption and real exchange rates due to the forecast errors of inattentive consumers.

Possible extensions to improve the fit of the model are introducing capital into the production function and having non-tradable goods in the consumption basket. Furthermore, imposing staggered information updating and solving for the level of net foreign assets can allow us to examine the implications for current account dynamics. These extensions might be useful to improve the model's performance, and reinvestigate some policy questions in the field of international finance.

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Table 1: **Benchmark Parameter Values**

Description	Parameter	Value
Risk Aversion	σ	2
Frisch Elasticity	ϕ	0.5
Discount Factor	β	0.995
Elasticity of Substitution		
goods	ν	10
labor varieties	χ	10
home and foreign	η	1.5
Import Share	γ	0.06
Price/Information Stickiness		
producers	θ	0.75
consumers	δ	0.75
Portfolio Adjustment Cost	θ_b	0.001
Monetary Policy Rule		
inertia	ρ_R	0.85
inflation	ψ_π	1.5
output	ψ_y	0.5
corr$(\epsilon_R, \epsilon_{R^*})$		0.5
Productivity Process		
persistence	ρ_A	0.95
st.dev.	σ_A	0.7
corr$(\epsilon_A, \epsilon_{A^*})$		0.25

Notes: Countries are assumed to be symmetric in terms of parameters and exogenous processes. The standard deviation of monetary shock is set to target output volatility.

Table 2: **Sticky Information: Consumers**

	Data	AC-SP	AC-IP	IC-IP	IC-IP
Degree of Consumer Information Stickiness (δ)	–	0	0	0.5	0.75
$2\frac{Var(c_t-c_{t-1})}{Var(c_t-c_{t-2})}$	0.73	1.19	1.20	0.83	0.72
$\rho(\Delta c_t\ )$	0.36	-0.16	-0.17	0.21	0.39

Notes: We report the unfiltered ratio of variances for consumption growth and the autocorrelation of consumption growth for different models. Second column is the sticky price model with attentive consumers (AC-SP), and others are results from the benchmark sticky information (featuring inattentive consumers and producers, IC-IP) model for varying degrees of stickiness on the consumer side. Average duration of information updating is $\frac{1}{1-\delta}$. All models are calibrated to match HP-filtered US output volatility by changing the standard deviation of the monetary shock.

Table 3: **Risk Aversion Approach**

	Data	AC-SP Model			AC-IP Model		
Risk Aversion	–	1	3	5	1	3	5
std($r\hat{e}r$)	4.47	1.51	2.97	4.72	1.55	3.10	4.86
std($\hat{c}$)	0.84	1.02	1.05	1.06	1.03	1.06	1.07
std($\hat{c} - \hat{c}^*$)	0.75	1.49	1.00	0.96	1.52	1.04	0.99
corr($\hat{c}, \hat{c}^*$)	0.47	0.40	0.43	0.43	0.36	0.40	0.41

Notes: Sticky price (AC-SP) and sticky information (AC-IP) models with attentive consumers calibrated to match HP-filtered US output volatility by changing the standard deviation of the monetary shock. All series are logged and HP-filtered. Standard deviations are normalized by dividing the output volatility. We report volatility of real exchange rates, consumption, relative consumption and cross country consumption correlation for varying degrees of risk aversion.

Table 4: **Real Exchange Rate Volatility with Inattentive Consumers**

	Data	AC-IP	IC-IP	IC-IP
Cons. Info. Stickiness(δ)	–	0	0.5	0.75
std($\hat{rer}$)	4.47	2.32	3.14	4.64
std($\hat{c}^{agg}$)	0.84	1.05	1.00	0.98
std($\hat{c} - \hat{c}^{*}$)	0.75	1.16	1.04	0.96
std($\hat{c}_0$)	–	1.05	1.54	2.40
corr($\hat{c}_0$,$\hat{c}_0^{*}$)	–	0.39	0.47	0.52

Notes: Benchmark sticky information model (IC-IP, with inattentive consumers and producers) is calibrated to match the standard deviation of HP-filtered US output for varying degrees of information stickiness on the consumer side (δ). Average duration of information updating is $\frac{1}{1-\delta}$. The degree of information stickiness on the producer side (θ) is set to 0.75 and the level of risk aversion is 2. All volatilities are normalized by dividing the output volatility. All series are logged and HP-filtered. Standard deviations of real exchange rates, aggregate consumption, relative consumption, consumption of attentive consumers and cross country consumption correlation (attentive consumers) are reported.

Table 5: **Selected Business Cycle Moments with Inattentive Consumers**

	Data	Benchmark	Attentive Consumers	
		IC-IP	AC-SP	AC-IP
Consumer Info. Stickiness(δ)	–	0.75	0	0
Output Volatility	1.68	1.68	1.68	1.68
Volatilities (Relative to GDP)				
Consumption	0.84	0.98	1.04	1.05
Employment	0.77	1.75	1.66	1.73
Real Exchange Rate	4.47	4.64	2.22	2.32
Nominal Exchange Rate	4.67	4.70	2.52	2.65
Net Exports	0.24	0.05	0.11	0.16
Autocorrelations				
Output	0.89	0.84	0.54	0.53
Consumption	0.90	0.84	0.54	0.53
Employment	0.93	0.81	0.54	0.56
Real Exchange Rate	0.84	0.59	0.54	0.55
Nominal Exchange Rate	0.85	0.61	0.59	0.59
Net Exports	0.86	0.94	0.76	0.82
Correlations				
cross-country				
Output	0.60	0.48	0.53	0.54
Consumption	0.47	0.52	0.43	0.39
Employment	0.55	0.41	0.47	0.46
Real Exchange Rate and				
Nominal Exchange Rate	0.99	0.99	0.87	0.91
Relative Consumption	-0.15	0.87	1.00	1.00
Output and Net Exports	-0.55	0.32	-0.27	-0.25

Notes: All series are logged and HP-filtered. IC-IP model is the benchmark sticky information model with inattentive consumers and producers. AC-SP model is the sticky price model with attentive consumers. AC-IP model features attentive consumers, and inattentive producers.

Table 6: **Selected Business Cycle Moments with Lower Import Elasticity**

	Data	IC-IP	IC-IP
Import Elasticity (η)	–	1.5	0.5
Output Volatility	1.68	1.68	1.68
Volatilities (Relative to GDP)			
Consumption	0.84	0.98	1.01
Employment	0.77	1.75	1.74
Real Exchange Rate	4.47	4.64	4.88
Nominal Exchange Rate	4.67	4.70	4.60
Net Exports	0.24	0.05	0.03
Autocorrelations			
Output	0.89	0.84	0.84
Consumption	0.90	0.84	0.84
Employment	0.93	0.81	0.81
Real Exchange Rate	0.84	0.59	0.60
Nominal Exchange Rate	0.85	0.61	0.58
Net Exports	0.86	0.94	0.80
Correlations			
cross-country			
Output	0.60	0.48	0.54
Consumption	0.47	0.52	0.49
Employment	0.55	0.41	0.46
Real Exchange Rate and			
Nominal Exchange Rate	0.99	0.99	0.99
Relative Consumption	-0.15	0.87	0.87
Output and Net Exports	-0.55	0.32	-0.39

Notes: All series are logged and HP-filtered. IC-IP model is the benchmark sticky information model with inattentive consumers and producers.

Table 7: **Real Exchange Rate Volatility Under Alternative Specifications**

	$\delta = 0$	$\delta = 0.5$	$\delta = 0.75$
Benchmark Model	2.32	3.14	4.64
Monetary Policy			
$\rho_R = 0.8$	2.29	3.06	4.48
$\rho_R = 0.9$	2.96	3.97	5.65
ψ_π=1.25	2.42	3.27	4.68
ψ_π=1.75	2.18	3.07	4.61
ψ_y=0	2.78	4.06	6.16
ψ_y=1	2.23	3.00	4.47
CRS Production Function ($\zeta = 1$)	2.12	2.81	3.93
Lower Import Elasticity ($\eta = 0.5$)	2.17	3.16	4.88
Lower Rigidity on Producers ($\theta = 0.5$)	2.18	2.97	4.14
Higher Import Share ($\gamma = 0.12$)	2.59	3.25	4.53

Notes: Standard deviation of real exchange rates (relative to output) under alternative specifications are reported. Benchmark sticky information model (IC-IP, with inattentive consumers and producers) is calibrated to match the standard deviation of HP-filtered US output for varying degrees of information stickiness on the consumer side (δ). Average duration of information updating is $\frac{1}{1-\delta}$. For the benchmark model; (i) monetary policy parameters are $\rho_R = 0.85$, ψ_π=1.5, ψ_y=0.5, (ii) production function is decreasing returns to scale with $\zeta = 2/3$, (iii) import elasticity (η) is 1.5, (iv) degree of information stickiness on the producer side (θ) is 0.75, and (v) import share (γ) is 0.06.

Table 8: **Real Exchange Rate Volatility with Habit Formation**

	h=0	h=0.5	h=0.75
AC-SP model	2.22	1.72	1.33
AC-IP model	2.32	1.82	1.59

Notes: Standard deviation of real exchange rates (relative to output) under varying degrees of habit formation are reported. Models are calibrated to match the standard deviation of HP-filtered US output. The degree of information/price stickiness on the producer side(θ) is set to 0.75. Consumers are assumed to be attentive. AC-SP model introduces sticky prices and the AC-IP model features inattentive producers. Risk aversion is set to 2.

Figure 1: Inattentive Consumers: Impulse Response to Home Monetary Shock

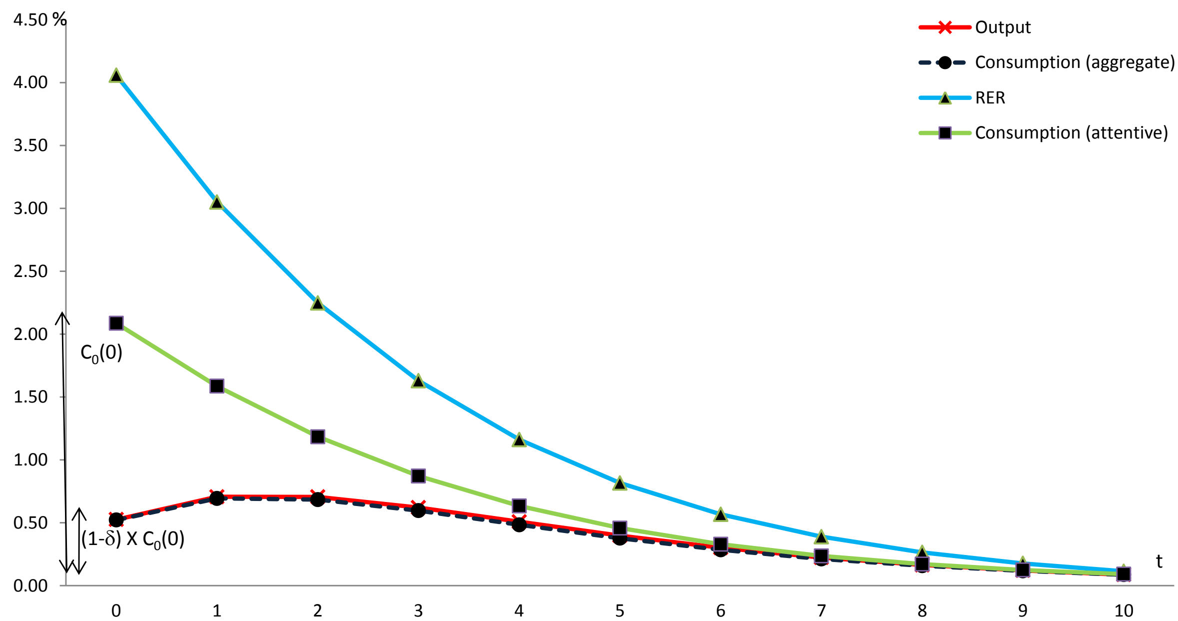
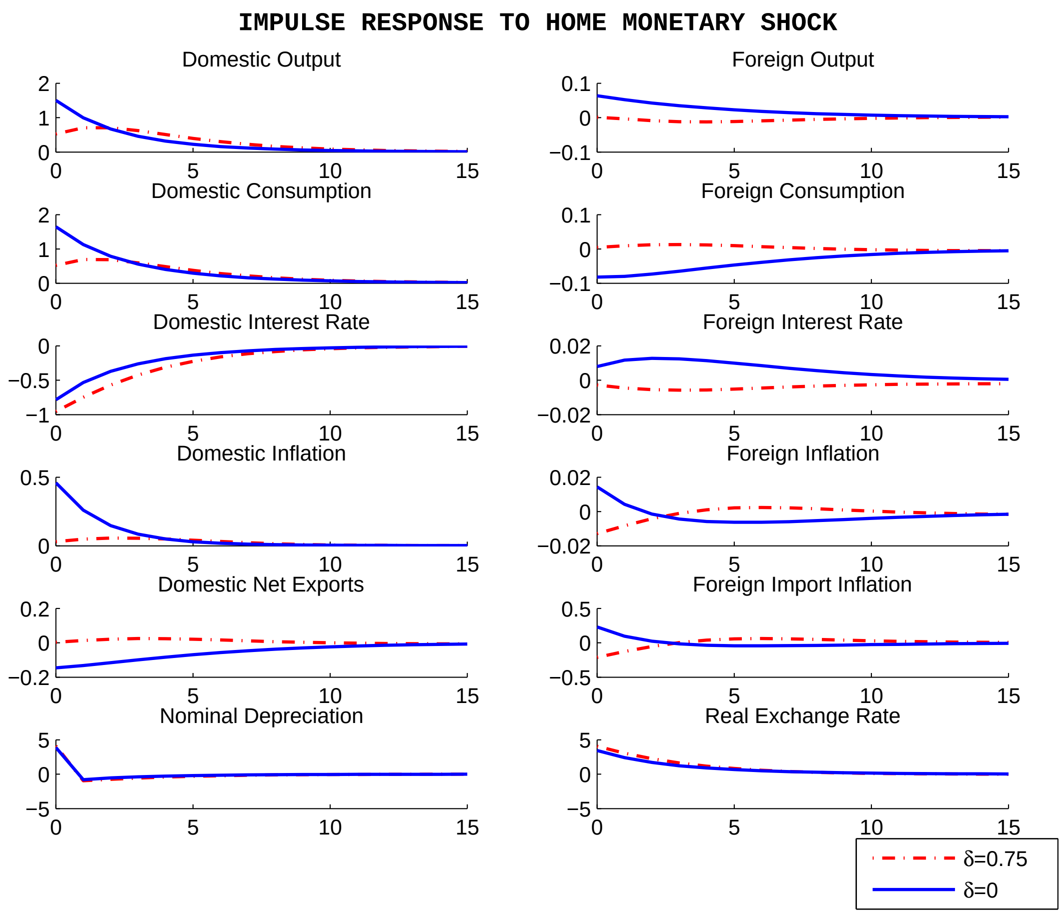


Figure 2: Sticky Information Model with Inattentive Consumers



Appendix A : Data

Data is quarterly. Our sample period is between 1973Q1 and 2009Q4. Data sources are the FRED2 database from the Federal Reserve Bank of St. Louis, Area Wide Model (AWM) of the European Central Bank, OECD Economic Outlook and International Financial Statistics (IFS) by the IMF. All series are logged and HP-filtered. The ratio of net exports to GDP is filtered without using a log transformation.

US Output Real GDP series is obtained from GDPC96-Fred2.

Euro Area Output YER-AWM series is used for real output.

US Price Index Quarterly series based on Consumer Price Index for All Urban Consumers CPIAUCSL-Fred2. Monthly series are converted to quarterly by arithmetic averaging.

Euro Area Price Index Based on harmonized index, HICP-AWM.

US Consumption Real consumption series is obtained from PCECC96-Fred2.

Euro Area Consumption PCR-AWM series is used for real consumption.

US Employment . CE16OV-Fred2 series for civilian employment.

Euro Area Employment LNN-AWM series for employment.

Exchange Rates Prior to 1999, fixed conversion rates between the national currency units and the Euro are weighted[42] by real GDP shares. Data source is the IFS database, and the calculation gives an artificial bilateral exchange rate. The Euro-Dollar exchange rate is used after 1999. Prior to 1999, we define the nominal exchange rate as $E_t \equiv \Pi_{i=1}^{n}(f_i E_{i,t})^{w_i}$. We calculate the real exchange rate as $RER_t = \frac{E_t P_{EU}}{P_{US}}$.

US Net Exports Ratio of difference between exports (EXPGSC96) and imports (IMPGSC96) to real GDP.

[42] The weights are Austria=0.03, Belgium=0.036, Finland=0.017, France=0.201, Germany=0.283, Greece=0.025, Ireland=0.015, Italy=0.195, Luxembourg=0.003, Netherland=0.06, Portugal=0.024, Spain=0.111.

Appendix B : Sticky Information Derivations

Consumer's Problem

A household is composed of a shopper-planner pair. The shopper chooses the optimal bundle of varieties and does not share the information about relative prices with the planner. The planner solves an intertemporal problem to allocate total expenditure. Planners proceed with outdated information with probability δ every period. All planners are identical aside from the period of their last information update. The assumption of an insurance scheme implies that $B_{t,j} = B_t$ and $D_{t,j} = D_t$, the same for all planners. The second subindex refers to the number of periods which planner could not update the expectations. The planner chooses the stream of consumption until the next information update. Planner's dynamic program is

$$V(B_t, D_t) = \max_{\{C_{t+i,i}, W_{t+i,i}\}} \sum_{i=0}^{\infty} \beta^i \delta^i U(C_{t+i,i}, N_{t+i,i}) + \beta(1-\delta) \sum_{i=0}^{\infty} \beta^i \delta^i V(B_{t+i+1}, D_{t+i+1})$$

subject to a sequence of budget constraints,

$$\begin{aligned} W_{t+i,i} N_{t+i,i} \quad &+ \quad B_{t+i} + e_{t+i} D_{t+i} + \Pi_{t+i} \geq P_{t+i} C_{t+i,i} + v_{t+i,t+i+1} B_{t+i+1} \\ &+ \quad q_{t+i,t+i+1} e_{t+i} D_{t+i+1} + e_{t+i} \theta_b (D_{t+i+1}) + \tau_{t+i,i} \end{aligned}$$

where $C_{t+i,i}$ is the planned consumption amount in case of not updating the information set for i periods. P_{t+i} represents the price index for home country. $N_{t+i,i}$ is the labor supply and $W_{t+i,i}$ is the nominal wage rate posted by the planner. Π_{t+i} is the profits of domestic intermediate goods producers. $B_{t+i,i}$ is the amount of nominal bonds held by domestic consumers between time t+i and t+i+1, and $v_{t+i,t+i+1}$ is the time t+i price of the bond which pays one unit of home currency at time t+i+1. $D_{t+i,i}$ is the amount of foreign currency nominal bonds held by domestic consumers between time t+i and t+i+1, and $q_{t+i,t+i+1}$ is the time t+i price of these bonds which pay one unit of foreign currency at time t+i+1. $\Theta_b(D) = \frac{\theta_b}{2}D^2$ is the cost of holding foreign currency bonds for home consumers.Domestic currency bonds are not internationally traded, and they are at zero net supply in equilibrium. First term in the objective function is the expected discounted utility if the planner never updates information again. Second term is the sum of continuation values over all possible dates at which the planner may update the information, which occurs with probability $(1-\delta)\delta^i$. Envelope condition is

$$V_1(B_t, D_t) = \frac{\lambda_{t,0}^c}{P_t}$$

First order condition is

$$\beta^i \delta^i \lambda_{t+i,i}^c = \beta(1-\delta) \sum_{k=i}^{\infty} \beta^k \delta^k E_t \left[V_1(B_{t+k+1}, D_{t+k+1}) \frac{P_{t+i}}{v_{t+i,t+k+1}} \right]$$

Using the first order condition for attentive consumers at period t, and first order condition for t+1 consumption plan with period t information and rearranging gives,

$$\lambda_{t+1,1}^c = E_t \lambda_{t+1,0}^c$$

In general, we have $\lambda_{t,j}^c = E_{t-j}\lambda_{t,0}^c$. Rearranging period t attentive consumer optimality condition, combining with the first order condition of attentive consumer at t+1, iterating envelope condition one period forward, we obtain the Euler equation

$$\lambda_{t,0}^c = \beta E_t \left[\lambda_{t+1,0}^c \frac{P_t}{P_{t+1}} \frac{1}{v_{t,t+1}} \right]$$

First order condition with respect to foreign currency bond holdings is

$$\beta^i \delta^i \lambda^c_{t+i,i} = \beta(1-\delta) \sum_{k=i}^{\infty} \beta^k \delta^k E_t \left[V_1(B_{t+k+1}, D_{t+k+1}) \frac{P_{t+i}}{\widetilde{q}_{t+i,t+k+1} e_{t+i}} \right]$$

where $\widetilde{q}_{t+i,t+k+1} \equiv \prod_{s=i}^{k} [q_{t+s,t+s+1} + \Theta'_b(D_{t+s+1})]$. Envelope condition is $V_2(B_t, D_t) = \frac{e_t \lambda^c_{t,0}}{P_t}$. Repeating similar steps, we obtain

$$q_{t,t+1} + \Theta'_b(D_{t+1}) \quad = \quad \beta E_t \left[\frac{\lambda^c_{t+1,0}}{\lambda^c_{t,0}} \frac{P_t}{P_{t+1}} \frac{e_{t+1}}{e_t} \right]$$

Foreign country consumer solves the following problem

$$V(D^*_t) = \max_{\left\{C^*_{t+i,i}, W^*_{t+i,i}\right\}} \sum_{i=0}^{\infty} \beta^i \delta^i U(C^*_{t+i,i}, N^*_{t+i,i}) + \beta(1-\delta) \sum_{i=0}^{\infty} \beta^i \delta^i V(D^*_{t+i+1})$$

subject to,

$$W^*_{t+i,i} N^*_{t+i,i} + D^*_{t+i} + \Pi^*_{t+i} \geq P^*_{t+i} C^*_{t+i,i} + q_{t+i,t+i+1} D^*_{t+i+1} + \tau^*_{t+i,i}$$

we obtain

$$\begin{aligned}
\lambda^{c*}_{t,j} \quad &= \quad E_{t-j} \lambda^{c*}_{t,0} \\
\lambda^{c*}_{t,0} \quad &= \quad \beta E_t \left[\lambda^{c*}_{t+1,0} \frac{P^*_t}{P^*_{t+1}} \frac{1}{q_{t,t+1}} \right]
\end{aligned}$$

We define the gross nominal interest rates as $R_t \equiv \frac{1}{v_{t,t+1}}$ and $R^*_t \equiv \frac{1}{q_{t,t+1}}$. For the foreign currency bonds, we have the following no-arbitrage condition

$$q_{t,t+1} = \beta E_t \left[\frac{\lambda^{c*}_{t+1,0}}{\lambda^{c*}_{t,0}} \frac{P^*_t}{P^*_{t+1}} \right] = \beta E_t \left[\frac{\lambda^c_{t+1,0}}{\lambda^c_{t,0}} \frac{P_t}{P_{t+1}} \frac{e_{t+1}}{e_t} \right] - \Theta'_b(D_{t+1})$$

Log-linearization of this no-arbitrage condition, using the interest rate definitions and functional form of portfolio adjustment cost, we obtain the uncovered interest parity,

$$\hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{e}_{t+1} - \theta_b \tilde{D}_{t+1}$$

where tilde variables correspond to level deviations. To obtain the aggregate consumption, we can summarize the results for the home country in the log-linearized form as follows,

$$\begin{aligned}
\hat{rr}_t &= \hat{R}_t - E_t \hat{\pi}_{t+1} \\
\hat{\lambda}^c_{t,0} &= \hat{\lambda}^c_{t+1,0} + \hat{rr}_t \\
\hat{\lambda}^c_{t,j} &= E_{t-j} \hat{\lambda}^c_{t,0}
\end{aligned}$$

Using the functional form for marginal utility, $\lambda_{t,0} = -\sigma \hat{c}_{t,0}$ and iterating forward gives,

$$\begin{aligned}
\hat{c}_{t,0} &= E_t \left[\hat{c}_{t+1,0} - \frac{1}{\sigma} \hat{rr}_t \right] \\
\hat{c}_{t+1,0} &= E_{t+1} \left[\hat{c}_{t+2,0} - \frac{1}{\sigma} \hat{rr}_{t+1} \right] \\
\hat{c}_{t,j} &= E_{t-j} \hat{c}_{t,0} = E_{t-j} \left[\hat{c}_{t+2,0} - \frac{1}{\sigma} (\hat{rr}_t + \hat{rr}_{t+1}) \right] \\
\hat{c}_{t,j} &= E_{t-j} \left[\hat{c}_{t+T,0} - \frac{1}{\sigma} \sum_{i=0}^{T} \hat{rr}_{t+i} \right]
\end{aligned}$$

Next, we take the limit as $T \to \infty$, and define the long interest rate $\hat{lr}_t = \sum_{i=0}^{T} \hat{rr}_{t+i}$. As time elapses to infinity all become aware of past news so $lim_{i \to \infty} E_t \hat{rr}_{t+i} = 0$. Moreover, since the probability of remaining inattentive falls exponentially with the length of the horizon, we approach this limit fast enough to ensure that the sum in the second term converges. As for the first term, $lim_{i \to \infty} E_t(\hat{c}_{t+i,0}) = 0$. The shocks in the economy die out in the long run, so consumption is expected to be at the steady state level in the limit. Long interest rate

($\hat{lr}_t \equiv E_t \sum_{i=0}^{\infty} \hat{rr}_{t+i}$) can be defined recursively as follows

$$\begin{aligned} E_t \hat{lr}_{t+1} &= E_t \sum_{i=0}^{\infty} \hat{rr}_{t+1+i} \\ \hat{lr}_t &= \hat{rr}_t + E_t \hat{lr}_{t+1} \end{aligned}$$

Consumption Euler equation can be written as

$$\hat{c}_{t,j} = -\frac{1}{\sigma} E_{t-j} \left[\hat{lr}_t \right]$$

We can write the aggregate consumption as $\hat{c}_t^{agg} = \sum_{j=0}^{\infty} (1-\delta)\delta^j \hat{c}_{t,j}$,

$$\hat{c}_t^{agg} = -\frac{1}{\sigma} \sum_{j=0}^{\infty} (1-\delta)\delta^j E_{t-j} \left[\hat{lr}_t \right]$$

Foreign country aggregate consumption is calculated in the similar fashion. Next, we use the home consumers' budget constraint to obtain a law of motion for the foreign currency bond holdings, $\tilde{D}$.

$$\begin{aligned} W_{t+i,i} N_{t+i,i} &+ B_{t+i} + e_{t+i} D_{t+i} + \Pi_{t+i} \geq P_{t+i} C_{t+i,i} + v_{t+i,t+i+1} B_{t+i+1} \\ &+ q_{t+i,t+i+1} e_{t+i} D_{t+i+1} + e_{t+i} \theta_b (D_{t+i+1}) + \tau_{t+i,i} \end{aligned}$$

We aggregate time t budget constraint to lose the transfer payments, $\sum_{i=0}^{\infty} (1-\delta)\delta^i \tau_{t,i} = 0$. Summing up all information cohorts at time by using their population weights,

$$\begin{aligned} \sum_{i=0}^{\infty} (1-\delta)\delta^i W_{t,i} N_{t,i} &+ B_t + e_t D_t + \Pi_t = P_t \sum_{i=0}^{\infty} (1-\delta)\delta^i C_{t,i} + v_{t,t+1} B_{t+1} \\ &+ q_{t,t+1} e_t D_{t+1} + e_t \theta_b (D_{t+1}) \end{aligned}$$

We can express the profits of the firm as follows,

$$\Pi_t = P_{H,t}C_{H,t} + e_t P^*_{H,t}C^*_{H,t} - \sum_{i=0}^{\infty}(1-\delta)\delta^i W_{t,i}N_{t,i}$$

Plugging this into the budget constraint, log-linearization, using the demand functions, and taking the first differences, we get the following expression for the law of motion for the growth rates of net foreign assets

$$\begin{aligned} \beta\Delta\tilde{D}_{t+1} &= \Delta\tilde{D}_t + (1-\gamma)(1-\eta)\hat{\pi}_{H,t} + \gamma(\Delta\hat{C}^*_t - \Delta\hat{C}_t) + [(1-\gamma)\eta - 1]\hat{\pi}_t + \gamma\Delta\hat{e}_t \\ &+ \gamma(1-\eta)\hat{\pi}^*_{H,t} + \gamma\eta\hat{\pi}^*_t \end{aligned}$$

Labor Market

Labor market features workers as the supplier of a specific variety of labour and firms, indexed by i, have a hiring department purchasing a continuum of varieties of workers, indexed by k, in the amount $N_{t,i}(k)$ at the price $W_{t,k}$. Firms combine these varieties into the labor input $N_{t,i}$ according to a Dixit-Stiglitz aggregator. Purchasing department's problem is as follows

$$\begin{aligned} &\min_{\{N_{t,i}(j)\}_{j\in[0,1]}} \int_0^1 W_{t,k}N_{t,i}(k)dk \\ s.t. \quad &N_{t,i} = \left[\int_0^1 N_{t,i}(k)^{\frac{\chi-1}{\chi}}dk\right]^{\frac{\chi}{\chi-1}} \end{aligned}$$

solution to this problem is $N_{t,i}(k) = N_{t,i}(\frac{W_{t,k}}{W_t})^{-\chi}$ where W_t is the static wage index $W_t = \left[\int_0^1 W_{t,k}^{1-\chi}dk\right]^{\frac{1}{1-\chi}}$. Aggregation over the firms gives the demand for labor variety k

$$\int_0^1 N_{t,i}(k)dk = \left(\frac{W_{t,k}}{W_t}\right)^{-\chi}\int_0^1 N_{t,i}di$$

Plugging in the labour demand gives $N_{t+i,i} = \left(\frac{W_{t+i,i}}{W_{t+i}}\right)^{-\chi} N_{t+i}$. Problem for wage posting is

$$\begin{aligned} V(B_t, D_t) &= \max_{\{C_{t+i,i}, W_{t+i,i}\}} \sum_{i=0}^{\infty} \beta^i \delta^i U\left(C_{t+i,i}, \left(\frac{W_{t+i,i}}{W_{t+i}}\right)^{-\chi} N_{t+i}\right) \\ &+ \beta(1-\delta) \sum_{i=0}^{\infty} \beta^i \delta^i V(B_{t+i+1}, D_{t+i+1}) \end{aligned}$$

subject to a sequence of budget constraints,

$$\begin{aligned} W_{t+i,i} \left(\frac{W_{t+i,i}}{W_{t+i}}\right)^{-\chi} N_{t+i} &+ B_{t+i} + e_{t+i} D_{t+i} + \Pi_{t+i} \geq P_{t+i} C_{t+i,i} + \upsilon_{t+i,t+i+1} B_{t+i+1} \\ &+ q_{t+i,t+i+1} e_{t+i} D_{t+i+1} + e_{t+i} \theta_b(D_{t+i+1}) + \tau_{t+i,i} \end{aligned}$$

First order condition is

$$\chi \frac{N_{t+i,i}}{W_{t+i,i}} \lambda^n_{t+i,i} = (1-\chi)\beta(1-\delta) \sum_{k=i}^{\infty} \beta^k \delta^k E_t \left[V_1(B_{t+k+1}, D_{t+k+1}) \frac{N_{t+i,i}}{\upsilon_{t+i,t+k+1}}\right]$$

Using the first order condition with respect to the consumption plan,

$$W_{t+i,i} = \frac{\chi}{\chi - 1} E_t \left[P_{t+i} \frac{-\lambda^n_{t+i,i}}{\lambda^c_{t+i,i}}\right]$$

Following similar steps with the consumption plan, we can show that

$$W_{t,j} = E_{t-j} W_{t,0}$$

Using the functional form of utility, $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \xi \frac{N_t^{1+1/\phi}}{1+1/\phi}$, which gives $\lambda^n_t = -\xi N_t^{\frac{1}{\phi}}$, we obtain

$$\lambda^c_{t,0} = \xi \frac{\chi}{\chi - 1} N_{t,0}^{1/\phi} \frac{P_t}{W_{t,0}}$$

Defining $R_t \equiv \frac{1}{v_{t,t+1}}$, $rr_t \equiv R_t \frac{P_t}{P_{t+1}}$ and using Euler equation, $\lambda^c_{t,0} = \beta E_t \left[\lambda^c_{t+1,0} rr_t\right]$, we obtain

$$\frac{P_t N_{t,0}^{1/\phi}}{W_{t,0}} = \beta E_t \left[\frac{P_{t+1} N_{t+1,0}^{1/\phi}}{W_{t+1,0}} rr_t\right]$$

We can also see that an inattentive worker sets wages so that she mirrors the attentive worker, optimality conditions in the linearized form becomes:

$$\frac{1}{\phi}\hat{N}_{t,0} + \hat{p}_t - \hat{w}_{t,0} = E_t \left[\frac{1}{\phi}\hat{N}_{t+1,0} + \hat{p}_{t+1} - \hat{w}_{t+1,0} + \hat{rr}_t\right]$$

The workers who have outdated information post the wages as follows,

$$\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0}$$

Combining these equations, iterating forward yields

$$\begin{aligned}
\hat{w}_{t,j} = E_{t-j}\hat{w}_{t,0} &= E_{t-j}\left[\frac{1}{\phi}\hat{N}_{t,0} + \hat{p}_t - \frac{1}{\phi}\hat{N}_{t+1,0} - \hat{p}_{t+1} + \hat{w}_{t+1,0} - \hat{rr}_t\right] \\
E_{t-j}\hat{w}_{t+1,j} &= E_{t-j}\left[\frac{1}{\phi}\hat{N}_{t+1,0} + \hat{p}_{t+1} - \frac{1}{\phi}\hat{N}_{t+2,0} - \hat{p}_{t+2} + \hat{w}_{t+2,0} - \hat{rr}_{t+1}\right] \\
\hat{w}_{t,j} &= E_{t-j}\left[\hat{p}_t + \frac{1}{\phi}\hat{N}_{t,0} - \sum_{i=0}^{T}\hat{rr}_{t+i} + \left(\hat{w}_{t+T,0} - \hat{p}_{t+T} - \frac{1}{\phi}\hat{N}_{t+T,0}\right)\right]
\end{aligned}$$

using the definition of long rate, $\hat{lr}_t = \sum_{i=0}^{T}\hat{rr}_{t+i}$, and taking the limit as $T \to \infty$,

$$\hat{w}_{t,j} = E_{t-j}\left[\hat{p}_t + \frac{1}{\phi}\hat{N}_{t,0} - \hat{lr}_t\right]$$

Using labor demand from firms $\hat{N}_{t,0} = \chi\left(\hat{w}_t - \hat{w}_{t,0}\right) + \hat{N}_t$, and $E_{t-j}\hat{w}_{t,0} = w_{t,j}$,

$$\hat{w}_{t,j} = E_{t-j}\left[\hat{p}_t + \frac{\chi}{\phi}\left(\hat{w}_t - \hat{w}_{t,0}\right) + \frac{1}{\phi}\hat{N}_t - \hat{lr}_t\right]$$

Aggregating to find the wage rate, $\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j \hat{w}_{t,j}$,

$$\hat{w}_t = \sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-j}\left[\hat{p}_t + \frac{\chi}{\phi+\chi}(w_t - p_t) + \frac{1}{\phi+\chi}\hat{N}_t - \frac{\phi}{\phi+\chi}\hat{lr}_t\right]$$

Taking the first difference and working on the algebra gives the real wage equation as follows,

$$(\hat{w}_t - \hat{p}_t) = \frac{\delta(\phi+\chi)}{\phi+\delta\chi}(\hat{w}_{t-1} - \hat{p}_{t-1}) + \frac{(1-\delta)(\phi+\chi)}{\phi+\delta\chi}\left[\frac{1}{\phi+\chi}\hat{N}_t - \frac{\phi}{\phi+\chi}\hat{lr}_t\right] - \frac{\delta(\phi+\chi)}{\phi+\delta\chi}\hat{\pi}_t$$
$$+\frac{\delta(\phi+\chi)}{\phi+\delta\chi}\sum_{j=0}^{\infty}(1-\delta)\delta^j E_{t-1-j}\left[\hat{\pi}_t + \frac{\chi}{\phi+\chi}\Delta(w_t - p_t) + \frac{1}{\phi+\chi}\Delta\hat{N}_t - \frac{\phi}{\phi+\chi}\Delta\hat{lr}_t\right]$$

Firms

Firms are committed to producing as much as necessary to clear the market. Intermediate goods are produced by labor input, $Y_H(i) = A_t N_t(i)^\zeta$. Nominal marginal cost is found by solving a cost minimization problem,

$$\min_{N_t} W_t N_t + MC_t\left(Y_t - A_t N_t^\zeta\right)$$

which gives $MC_t = \frac{W_t}{\zeta Y_t/N_t} = \frac{W_t}{\zeta A_t^{\frac{1}{\zeta}}} Y_t^{\frac{1-\zeta}{\zeta}}$. For each producer, we can express the marginal costs as follows,

$$MC_{H,t} = \frac{W_t}{\zeta A_t^{\frac{1}{\zeta}}} Y_{H,t}^{\frac{1-\zeta}{\zeta}}$$

We define relative prices $\hat{rp}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and $\hat{rp}_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*}$. Real marginal cost of home producers for the domestic country market is given by

$$\hat{mc}_{H,t} = \quad \hat{W}_t - \hat{P}_t + \gamma\hat{rp}_t + \tfrac{1-\zeta}{\zeta}\hat{Y}_{H,t} - \tfrac{1}{\zeta}\hat{A}_t$$

Plugging the demand functions into the marginal cost expressions, we obtain

$$\hat{mc}_{H,t} = \quad \hat{W}_t - \hat{P}_t + \tfrac{1-\zeta}{\zeta}(1-\gamma)\hat{C}_t + [\gamma + \tfrac{1-\zeta}{\zeta}\eta(1-\gamma)\gamma]\hat{rp}_t - \tfrac{1}{\zeta}\hat{A}_t$$

Firms update their expectations with probability $1-\theta$ probability each period. They proceed with the outdated information with probability θ. The firm which sets the price at time t according to the information received j periods ago solves the following problem

$$\max_{P_H(j),P_H^*(j)} E_{t-j}\left[P_{H,t}(j)Y_{H,t}(j) - W_t\left(\frac{Y_{H,t}(j)}{A_t}\right)^{\frac{1}{\zeta}}\right] + E_{t-j}\left[e_t P_{H,t}^*(j)Y_{H,t}^*(j) - W_t\left(\frac{Y_{H,t}^*(j)}{A_t}\right)^{\frac{1}{\zeta}}\right]$$

Plugging in the demand functions, taking the first order condition for home prices of locally produced goods, we obtain

$$E_{t-j}\left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\nu} Y_{H,t}\right] = \frac{\nu}{\zeta(\nu-1)}E_{t-j}\left[\frac{1}{P_{H,t}(j)}\frac{W_t}{A_t^{\frac{1}{\zeta}}}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\nu}{\zeta}} Y_{H,t}^{\frac{1}{\zeta}}\right]$$

Log-linearization around symmetric steady state gives

$$\hat{P}_{H,t}(j) = E_{t-j}\left[\hat{W}_t + \frac{1-\zeta}{\zeta}\hat{Y}_{H,t} - \frac{1}{\zeta}\hat{A}_t\right]$$

Using the marginal cost definition,

$$\hat{P}_{H,t}(j) = E_{t-j}\left[\hat{MC}_{H,t}\right]$$

Nominal marginal cost of producing at time t, for goods whose price was set at time t (or conditioning on time t information set) is given by

$$MC_{H,t,t} = \frac{W_t}{\zeta A_t^{\frac{1}{\zeta}}} Y_{H,t}^{\frac{1-\zeta}{\zeta}}$$

We can express the marginal cost which t-j information cohort faces as a function of average marginal cost at time t as follows

$$MC_{H,t-j,t} = MC_{H,t}\left(\frac{Y_{H,t-j,t}}{Y_{H,t}}\right)^{\frac{1-\zeta}{\zeta}} = MC_{H,t}\left(\frac{P_{H,t-j,t}}{P_{H,t}}\right)^{-\nu\frac{1-\zeta}{\zeta}}$$

We can log-linearize as follows,

$$\hat{MC}_{H,t-j,t} = \hat{MC}_{H,t} - \nu\frac{1-\zeta}{\zeta}\left(\hat{P}_{H,t-j,t} - \hat{P}_{H,t}\right)$$

Using $\hat{P}_{H,t-j,t} = E_{t-j}\left[\hat{MC}_{H,t}\right] = \hat{MC}_{H,t-j,t}$,

$$\hat{P}_{H,t-j,t} = \Omega\hat{MC}_{H,t} + \frac{\nu(1-\zeta)}{\zeta+\nu(1-\zeta)}\hat{P}_{H,t}$$

where $\Omega \equiv \frac{\zeta}{\zeta+\nu(1-\zeta)}$. We have a continuum of firms. The fraction which updates information in any given period is $1-\theta$. The fraction of firms which updated their information j periods ago is $(1-\theta)\theta^j$. We can write the price index as $\hat{P}_{H,t} = (1-\theta)\left(\sum_{j=0}^{\infty}\theta^j\hat{P}_{H,t-j,t}\right)$. Defining the real marginal cost as $mc_{H,t} \equiv \frac{MC_{H,t}}{P_{H,t}}$,

$$\hat{P}_{H,t} = (1-\theta)\left(\sum_{j=0}^{\infty}\theta^j E_{t-j}\left[\Omega\hat{mc}_{H,t} + \hat{P}_{H,t}\right]\right)$$

We obtain the following by expanding this expression,

$$\hat{P}_{H,t} = \frac{1-\theta}{\theta}\Omega\hat{mc}_{H,t} + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^j E_{t-j}\left[\Omega\hat{mc}_{H,t} + \hat{P}_{H,t}\right]\right)$$

Collecting terms, taking the lag and rearranging gives the sticky information Philips curve for inflation

$$\hat{\pi}_{H,t} = \frac{1-\theta}{\theta}\Omega \hat{mc}_{H,t} + \frac{1-\theta}{\theta}\left(\sum_{j=1}^{\infty}\theta^{j} E_{t-j}\left[\Omega\Delta\hat{mc}_{H,t} + \hat{\pi}_{H,t}\right]\right)$$

Import inflation for home goods is derived in a similar fashion.

If there are sticky prices, firms receive a Calvo signal to readjust their prices with probability $1-\theta$, so their price stays constant with probability θ. They set the price in the local currency to maximize their expected profits,

$$\max_{P_H(i),P_H^*(i)} \sum_{k=0}^{\infty}\theta^k E_t\left[v_{t,t+k}\left\{P_{H,t}(i)Y_{H,t+k}(i) - W_{t+k}\left(\frac{Y_{H,t+k}(i)}{A_t}\right)^{\frac{1}{\zeta}}\right\}\right]$$
$$+\sum_{k=0}^{\infty}\theta^k E_t\left[v_{t,t+k}\left\{e_{t+k}P_{H,t}^*(i)Y_{H,t+k}^*(i) - W_{t+k}\left(\frac{Y_{H,t+k}^*(i)}{A_t}\right)^{\frac{1}{\zeta}}\right\}\right]$$

Following similar steps, we obtain the sticky price Philips curve,

$$\hat{\pi}_{H,t} = (1-\theta\beta)\frac{1-\theta}{\theta}\Omega\hat{mc}_{H,t} + \beta E_t\hat{\pi}_{H,t+1}$$

Appendix C : Summary of Linearized Models

We summarize the log-linearized system of equations for two models in this appendix. The first one is labeled as the sticky price model which refers to the model where agents have full information, but producers can update their *prices* when they receive a Calvo signal. The second model is the benchmark sticky information model, where consumers and producers update their *information set* with a Calvo signal. Model with attentive consumers and inattentive producers (AC-IP) is the special case of sticky information model where $\delta = 0$.

Sticky Price Model

Consumption Euler equations are

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = -E_t\hat{\lambda}^c_{t+1} + \hat{\lambda}^c_t \tag{A.1}$$

$$\hat{R}^*_t - E_t\hat{\pi}^*_{t+1} = -E_t\hat{\lambda}^{c*}_{t+1} + \hat{\lambda}^{c*}_t \tag{A.2}$$

Uncovered interest parity[43] is given as

$$\hat{R}_t - \hat{R}^*_t = E_t\Delta\hat{e}_{t+1} - \theta_b\tilde{D}_{t+1} \tag{A.3}$$

Evolution of net foreign assets is given by

$$\begin{aligned} \beta\Delta\tilde{D}_{t+1} &= \Delta\tilde{D}_t + (1-\gamma)(1-\eta)\hat{\pi}_{H,t} + \gamma(\Delta\hat{C}^*_t - \Delta\hat{C}_t) \\ &+ [(1-\gamma)\eta - 1]\hat{\pi}_t + \gamma\Delta\hat{e}_t + \gamma(1-\eta)\hat{\pi}^*_{H,t} + \gamma\eta\hat{\pi}^*_t \qquad \text{(A.4)} \\ \Delta\tilde{D}_{t+1} &= \tilde{D}_{t+1} - \tilde{D}_t \qquad \text{(A.5)} \end{aligned}$$

[43] In the case of complete markets, net foreign asset level ($\tilde{D}$) and growth ($\Delta\tilde{D}$) are no longer state variables. Real exchange rate is determined by the following risk sharing equation instead of the uncovered interest parity : $r\hat{e}r_t = \hat{\lambda}^{c*}_t - \hat{\lambda}^c_t$.

real wages are given by

$$\hat{W}_t - \hat{P}_t = \hat{\lambda}_t^n - \hat{\lambda}_t^c \tag{A.6}$$

$$\hat{W}_t^* - \hat{P}_t^* = \hat{\lambda}_t^{n*} - \hat{\lambda}_t^{c*} \tag{A.7}$$

relative PPP condition,

$$\Delta\hat{e}_t = \hat{\pi}_t - \hat{\pi}_t^* + r\hat{e}r_t - r\hat{e}r_{t-1} \tag{A.8}$$

Monetary policy

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t\right] + \epsilon_{R,t} \tag{A.9}$$

$$\hat{R}_t^* = \rho_R \hat{R}_{t-1}^* + (1-\rho_R)\left[\psi_\pi \hat{\pi}_t^* + \psi_y \hat{y}_t^*\right] + \epsilon_{R^*,t} \tag{A.10}$$

Production functions;

$$\hat{y}_t = \hat{A}_t + \zeta \hat{N}_t \tag{A.11}$$

$$\hat{y}_t^* = \hat{A}_t^* + \zeta \hat{N}_t^* \tag{A.12}$$

Exogenous shocks to productivity

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_{A,t} \tag{A.13}$$

$$\hat{A}_t^* = \rho_A^* \hat{A}_{t-1}^* + \epsilon_{A^*,t} \tag{A.14}$$

Goods market clearing;

$$\hat{y}_t = (1-\gamma)\hat{c}_t + \gamma\hat{c}_t^* + \eta\gamma(1-\gamma)\ \hat{rp}_t - \eta\gamma(1-\gamma)\ \hat{rp}_t^* \tag{A.15}$$

$$\hat{y}^*_t = \gamma \hat{c}_t + (1-\gamma)\hat{c}^*_t - \eta\gamma(1-\gamma)\ \hat{rp}_t + \eta\gamma(1-\gamma)\ \hat{rp}^*_t \tag{A.16}$$

Relative prices of import goods with respect to locally produced goods are given by

$$\hat{rp}_t = \hat{rp}_{t-1} + \hat{\pi}_{F,t} - \hat{\pi}_{H,t} \tag{A.17}$$

$$\hat{rp}^*_t = \hat{rp}^*_{t-1} + \hat{\pi}^*_{H,t} - \hat{\pi}^*_{F,t} \tag{A.18}$$

Inflation indices are

$$\hat{\pi}_t = (1-\gamma)\hat{\pi}_{H,t} + \gamma\hat{\pi}_{F,t} \tag{A.19}$$

$$\hat{\pi}^*_t = \gamma\hat{\pi}^*_{H,t} + (1-\gamma)\hat{\pi}^*_{F,t} \tag{A.20}$$

definition of law of one price gap is given by

$$\hat{lop}_t = \hat{rer}_t - (1-\gamma)\hat{rp}_t - \gamma\hat{rp}^*_t \tag{A.21}$$

$$\hat{lop}^*_t = -\hat{rer}_t - \gamma\hat{rp}_t - (1-\gamma)\hat{rp}^*_t \tag{A.22}$$

Auxiliary variables for price setting are defined as follows

$$\hat{p}^1_t = \hat{W}_t - \hat{P}_t + \frac{1-\zeta}{\zeta}(1-\gamma)\hat{C}_t + [\gamma + \frac{1-\zeta}{\zeta}\eta(1-\gamma)\gamma]\hat{rp}_t - \frac{1}{\zeta}\hat{A}_t \tag{A.23}$$

$$\hat{p}^2_t = \hat{W}^*_t - \hat{P}^*_t + \frac{1-\zeta}{\zeta}(1-\gamma)\hat{C}^*_t + [\gamma + \frac{1-\zeta}{\zeta}\eta(1-\gamma)\gamma]\hat{rp}^*_t - \frac{1}{\zeta}\hat{A}^*_t \tag{A.24}$$

$$\hat{p}^3_t = \hat{W}_t - \hat{P}_t + \frac{1-\zeta}{\zeta}\gamma\hat{C}^*_t + \gamma\hat{rp}_t - \frac{1-\zeta}{\zeta}\eta\gamma(1-\gamma)\hat{rp}^*_t - \frac{1}{\zeta}\hat{A}_t + \hat{lop}^*_t \tag{A.25}$$

$$\hat{p}^4_t = \hat{W}^*_t - \hat{P}^*_t + \frac{1-\zeta}{\zeta}\gamma\hat{C}_t + \gamma\hat{rp}^*_t - \frac{1-\zeta}{\zeta}\eta\gamma(1-\gamma)\hat{rp}_t - \frac{1}{\zeta}\hat{A}^*_t + \hat{lop}_t \tag{A.26}$$

Home inflation on locally produced goods is

$$\hat{\pi}_{H,t} = \kappa \hat{p}_t^1 + \beta E_t \hat{\pi}_{H,t+1} \tag{A.27}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta} \frac{\zeta}{\zeta+(1-\zeta)\nu}$. Foreign inflation on locally produced goods is

$$\hat{\pi}^*_{F,t} = \kappa p_t^2 + \beta E_t \hat{\pi}^*_{F,t+1} \tag{A.28}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta} \frac{\zeta}{\zeta+(1-\zeta)\nu}$. Import goods price setting equations are

$$\hat{\pi}^*_{H,t} = \kappa \hat{p}_t^3 + \beta E_t \hat{\pi}^*_{H,t+1} \tag{A.29}$$

$$\hat{\pi}_{F,t} = \kappa \hat{p}_t^4 + \beta E_t \hat{\pi}_{F,t+1} \tag{A.30}$$

linearized net exports to output ratio is given by

$$\hat{nx}_t = \hat{y}_t - \hat{c}_t \tag{A.31}$$

$$\hat{nx}^*_t = \hat{y}^*_t - \hat{c}^*_t \tag{A.32}$$

terms of trade definition is

$$\hat{tot}_t = \hat{rp}_t + \hat{lop}^*_t \tag{A.33}$$

$$\hat{tot}^*_t = \hat{rp}^*_t + \hat{lop}_t \tag{A.34}$$

Marginal utilities from consumption are given as

$$\hat{\lambda}^c_t = -\sigma \hat{C}_t \tag{A.35}$$

$$\hat{\lambda}^{c*}_t = -\sigma \hat{C}^*_t \tag{A.36}$$

Marginal disutilities from labor are given as

$$\hat{\lambda}_t^n = \frac{1}{\phi}\hat{N}_t \tag{A.37}$$

$$\hat{\lambda}_t^{n*} = \frac{1}{\phi}\hat{N}_t^* \tag{A.38}$$

Vector of state variables are

$$x_{38*1} \equiv (\hat{\lambda}^c, \hat{\lambda}^{c*}, \hat{\lambda}^n, \hat{\lambda}^{n*}, \hat{c}, \hat{c}^*, \Delta\hat{e}, r\hat{e}r, r\hat{p}, r\hat{p}^*, t\hat{o}t, t\hat{o}t^*, \hat{y}, \hat{y}^*, \hat{p}^1, \hat{p}^2, \hat{p}^3, \hat{p}^4, \hat{\pi}, , \hat{\pi}^*,$$

$$\hat{\pi}_H, \hat{\pi}_F, \hat{\pi}_H^*, \hat{\pi}_F^*, l\hat{o}p, l\hat{o}p^*, (\hat{W} - \hat{P}), (\hat{W}^* - \hat{P}^*), \hat{R}, \hat{R}^*, \hat{N}, \hat{N}^*, \hat{A}, \hat{A}^*, n\hat{x}, n\hat{x}^*, \tilde{D}, \Delta\tilde{D})'$$

Vector of exogenous variables are

$$\epsilon_{4*1} \equiv (\epsilon_R, \epsilon_{R^*}, \epsilon_A, \epsilon_{A^*})'$$

Sticky Information Model

Common equations with the sticky price model are the UIP condition (equation A.3), law of motion for net foreign assets (equation A.4) and the growth equation for net foreign assets (A.5), relative PPP equation (A.8), monetary policy rules (equations A.9,A.10), production functions (equations A.11,A.12), goods market clearing conditions (equations A.15,A.16), relative prices of import goods (equations A.17,A.18), law of one price gap definitions (equations A.21,A.22), consumer price indices (equations A.19,A.20), definitions of auxiliary variables for price setting (equations A.23,A.24,A.25,A.26), net exports equations (A.31,A.32) and terms of trade definitions (equations A.33,A.34). The remaining equations of the model are described as follows. Definitions of real interest rate and long rates are

$$\hat{rr}_t = \hat{R}_t - E_t\hat{\pi}_{t+1} \tag{A.39}$$

$$\hat{rr}_t^* = \hat{R}_t^* - E_t\hat{\pi}_{t+1}^* \tag{A.40}$$

$$\hat{lr}_t = \hat{rr}_t + E_t\hat{lr}_{t+1} \tag{A.41}$$

$$\hat{lr}_t^* = \hat{rr}_t^* + E_t\hat{lr}_{t+1}^* \tag{A.42}$$

Aggregate consumption for home and foreign countries are given as

$$\hat{c}_t^{agg} = -\frac{1}{\sigma}(1-\delta)\sum_{j=0}^{\infty}\delta^j E_{t-j}\hat{lr}_t \tag{A.43}$$

$$\hat{c}_t^{agg*} = -\frac{1}{\sigma}(1-\delta)\sum_{j=0}^{\infty}\delta^j E_{t-j}\hat{lr}_t^* \tag{A.44}$$

Defining $\Omega \equiv \frac{\zeta}{\zeta+(1-\zeta)\nu}$, auxiliary variables for price setting equations are

$$a\hat{u}x_t^1 = \Omega\Delta\hat{p}_t^1 + \hat{\pi}_{H,t} \tag{A.45}$$

$$a\hat{u}x_t^2 = \Omega\Delta\hat{p}_t^2 + \hat{\pi}_{F,t}^* \tag{A.46}$$

$$a\hat{u}x_t^3 = \Omega\Delta\hat{p}_t^3 + \hat{\pi}_{H,t}^* \tag{A.47}$$

$$a\hat{u}x_t^4 = \Omega\Delta\hat{p}_t^4 + \hat{\pi}_{F,t} \tag{A.48}$$

Home inflation on locally produced goods is

$$\hat{\pi}_{H,t} = \frac{1-\theta}{\theta}\Omega\hat{p}_t^1 + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j}a\hat{u}x_t^1 \tag{A.49}$$

Foreign inflation on locally produced goods is

$$\hat{\pi}_{F,t}^* = \frac{1-\theta}{\theta}\Omega\hat{p}_t^2 + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j}a\hat{u}x_t^2 \tag{A.50}$$

Import goods price setting equations are

$$\hat{\pi}^*_{H,t} = \frac{1-\theta}{\theta}\Omega\hat{p}^3_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j} a\hat{u}x^3_t \tag{A.51}$$

$$\hat{\pi}_{F,t} = \frac{1-\theta}{\theta}\Omega\hat{p}^4_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j} a\hat{u}x^4_t \tag{A.52}$$

Real wage dynamics are given as

$$(\hat{w}_t - \hat{p}_t) = \frac{\delta(\phi+\chi)}{\phi+\delta\chi}(\hat{w}_{t-1} - \hat{p}_{t-1}) + \frac{1-\delta}{\phi+\delta\chi}\hat{N}_t - \frac{\phi(1-\delta)}{\phi+\delta\chi}\hat{lr}_t$$
$$-\frac{\delta(\phi+\chi)}{\phi+\delta\chi}\hat{\pi}_t + (1-\delta)\frac{\phi+\chi}{\phi+\delta\chi}\sum_{j=1}^{\infty}\delta^j E_{t-j} a\hat{u}x_{w1,t} \tag{A.53}$$

$$(\hat{w}^*_t - \hat{p}^*_t) = \frac{\delta(\phi+\chi)}{\phi+\delta\chi}(\hat{w}^*_{t-1} - \hat{p}^*_{t-1}) + \frac{1-\delta}{\phi+\delta\chi}\hat{N}^*_t - \frac{\phi(1-\delta)}{\phi+\delta\chi}\hat{lr}^*_t$$
$$-\frac{\delta(\phi+\chi)}{\phi+\delta\chi}\hat{\pi}^*_t + (1-\delta)\frac{\phi+\chi}{\phi+\delta\chi}\sum_{j=1}^{\infty}\delta^j E_{t-j} a\hat{u}x_{w2,t} \tag{A.54}$$

Auxiliary variables for wage equations are

$$a\hat{u}x_{w1,t} = \left[\hat{\pi}_t + \frac{\chi}{\phi+\chi}\Delta(w_t - p_t) + \frac{1}{\phi+\chi}\Delta\hat{N}_t - \frac{\phi}{\phi+\chi}\Delta\hat{lr}_t\right] \tag{A.55}$$

$$a\hat{u}x_{w2,t} = \left[\hat{\pi}^*_t + \frac{\chi}{\phi+\chi}\Delta(w^*_t - p^*_t) + \frac{1}{\phi+\chi}\Delta\hat{N}^*_t - \frac{\phi}{\phi+\chi}\Delta\hat{lr}^*_t\right] \tag{A.56}$$

Vector of state variables are

$$x_{42*1} \equiv (\hat{rr}, \hat{rr}^*, \hat{lr}, \hat{lr}^*, \hat{c}, \hat{c}^*, \Delta\hat{e}, \hat{rer}, \hat{rp}, \hat{rp}^*, \hat{tot}, \hat{tot}^*, \hat{y}, \hat{y}^*, \hat{p}^1, \hat{p}^2, \hat{p}^3, \hat{p}^4, a\hat{u}x^1, a\hat{u}x^2, a\hat{u}x^3, a\hat{u}x^4,$$
$$\hat{\pi}, \hat{\pi}^*, \hat{\pi}_H, \hat{\pi}_F, \hat{\pi}^*_H, \hat{\pi}^*_F, \hat{lop}, \hat{lop}^*, (\hat{W} - \hat{P}), (\hat{W}^* - \hat{P}^*), \hat{R}, \hat{R}^*, \hat{N}, \hat{N}^*, \hat{nx}, \hat{nx}^*, \Delta\tilde{D}, \tilde{D}, a\hat{u}x_{w1}, a\hat{u}x_{w2})'$$

Vector of exogenous variables are

$$\epsilon_{4*1} \equiv (\epsilon_R, \epsilon_{R^*}, \epsilon_A, \epsilon_{A^*})'$$

Appendix D : Solution of the Models with Lagged Expectations

Sticky price models can be written as second order difference equations and solved by standard methods outlined in Klein (2000). This appendix closely follows Meyer-Gohde (2010). Consider the following model with lagged expectations

$$AE_tY_{t+1} + B_0Y_t + \sum_{i=1}^{I} B_iE_{t-i}Y_t + CY_{t-1} + GW_t = 0$$

where Y_t is $n \times 1$ vector of endogenous variables, and W_t is $k \times 1$ vector of exogenous variables with a law of motion $W_t = NW_{t-1} + \epsilon_t$, or alternatively, with the moving average representation $W_t = \sum_{j=0}^{\infty} N^j \epsilon_{t-j}$. The solution is of the form of $Y_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j}$ with coefficients Θ_j $(n \times k)$. The one period ahead realization is $Y_{t+1} = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t+1-j}$. Taking expectations yields

$$E_tY_{t+1} = E_t \sum_{j=0}^{\infty} \Theta_j \epsilon_{t+1-j} = \sum_{j=1}^{\infty} \Theta_j \epsilon_{t+1-j} = \sum_{j=0}^{\infty} \Theta_{j+1} \epsilon_{t-j}$$

Similarly, $Y_{t-1} = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-1-j}$. Regarding the past expectations, notice that when $i = 0$: $Y_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j}$, and when $i = r$: $E_{t-r}Y_t = \sum_{j=r}^{\infty} \Theta_j \epsilon_{t-j}$. Expanding the expression for the past expectations,

$$\sum_{i=0}^{I} B_iE_{t-i}Y_t = B_0Y_t + B_1E_{t-1}Y_t + B_2E_{t-2}Y_t + B_3E_{t-3}Y_t + ... + B_IE_{t-I}Y_t$$
$$= B_0 \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j} + B_1 \sum_{j=1}^{\infty} \Theta_j \epsilon_{t-j} + B_2 \sum_{j=2}^{\infty} \Theta_j \epsilon_{t-j} + B_3 \sum_{j=3}^{\infty} \Theta_j \epsilon_{t-j} + ... + B_I \sum_{j=I}^{\infty} \Theta_j \epsilon_{t-j}$$
$$= B_0\Theta_0\epsilon_t + (B_0 + B_1)\Theta_1\epsilon_{t-1} + (B_0 + B_1 + B_2)\Theta_2\epsilon_{t-2} + ... + \left(\sum_{j=0}^{I} B_j\right) \sum_{k=j+1}^{\infty} \Theta_k \epsilon_{t-k}$$

Defining $\tilde{B}_j \equiv \left(\sum_{i=0}^{min(I,j)} B_i\right)$, we can write the lagged expectations as follows

$$\sum_{i=0}^{I} B_i E_{t-i} Y_t = \sum_{j=0}^{\infty} \tilde{B}_j \Theta_j \epsilon_{t-j}$$

Plugging the MA representation, the system in terms of the MA coefficients is

$$A \sum_{j=0}^{\infty} \Theta_{j+1} \epsilon_{t-j} + \sum_{j=0}^{\infty} \tilde{B}_j \Theta_j \epsilon_{t-j} + C \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-1-j} + G \sum_{j=0}^{\infty} N^j \epsilon_{t-j} = 0$$

We need to solve for the MA coefficient matrices, Θ_0 ... Θ_I, for a large I. These coefficients solve the following system of equations,

$$\begin{aligned}
\left[A\Theta_1 + \tilde{B}_0 \Theta_0 + G\right] \epsilon_t &= 0, \forall \epsilon_t \\
\left[A\Theta_2 + \tilde{B}_1 \Theta_1 + C\Theta_0 + GN\right] \epsilon_{t-1} &= 0 \\
\left[A\Theta_3 + \tilde{B}_2 \Theta_2 + C\Theta_1 + GN^2\right] \epsilon_{t-2} &= 0 \\
&.... \\
\left[A\Theta_{j+1} + \tilde{B}_j \Theta_j + C\Theta_{j-1} + GN^j\right] \epsilon_{t-j} &= 0
\end{aligned}$$

We have I matrix equations with $I+1$ unknowns, Θ_0 .. Θ_I. The coefficients of the recursion are non-varying when $j \geq I$. Therefore, the last equation is obtained by solving a second order difference equation

$$\begin{aligned}
A\Theta_{j+1} + \tilde{B}_I \Theta_j + C\Theta_{j-1} + GN^I x_j = 0 \quad j \geq I \\
x_I = I_k \qquad and \qquad x_{j+1} = x_j
\end{aligned}$$

The dimensions of the matrices are A , $\tilde{B}_I$ and C: $n \times n$, G : $n \times k$, N : $k \times k$ and x: $k \times 1$. The solution is $\Theta_j = \alpha_\theta \Theta_{j-1} + \alpha_N x_j$ and $\Theta_I = \alpha_\theta \Theta_{I-1} + \alpha_N$. In our system, $I \to \infty$, therefore we need to plug in the limiting matrix $\tilde{B}_I$ and take a large enough number of lags. We can write the resulting system of equations in a tridiagonal structure by setting the initial condition as $\theta_{-1} = 0$.

$$\begin{pmatrix} \tilde{B}_0 & A & 0 & 0 & 0 & 0 & 0 & 0 \\ C & \tilde{B}_1 & A & 0 & 0 & 0 & 0 & 0 \\ 0 & C & \tilde{B}_2 & A & 0 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & C & \tilde{B}_{I-1} & A \\ 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_\theta & I_n \end{pmatrix}_{(I+1)n\times(I+1)n} \begin{pmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ .. \\ \theta_{I-1} \\ \theta_I \end{pmatrix}_{(I+1)n\times k} = \begin{pmatrix} -G \\ -GN \\ -GN^2 \\ .. \\ -GN^{I-1} \\ \alpha_N \end{pmatrix}_{(I+1)n\times k}$$

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