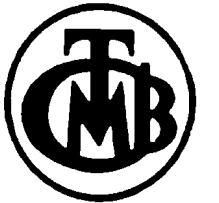


**Exchange Rates and Fundamentals: Is  
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# Exchange Rates and Fundamentals: Is there a Role for Nonlinearities in Real Time?

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**Abstract:** We examine the out-of-sample predictive power of real time linear monetary models with possible nonlinear adjustment in forecast errors for the GBP/USD exchange rates. Real time revisions of U.K. and U.S. monetary aggregates and output are significant; therefore the use of final data on fundamentals in forecasting exchange rates may yield misleading inferences. By studying recursive out-of-sample forecast errors we claim that in several instances, real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas the adjustment towards fundamentals driven equilibrium values may take a discrete or smooth nonlinear form. Revisions in fundamentals, particularly in the US and UK monetary aggregates and real output, seem to matter mainly for short term forecastability of exchange rates. We find short term forecastability in the form of discrete nonlinear adjustment in some real time vintages. We also document long term forecastability in the form of a smooth nonlinear adjustment towards fundamentals determined equilibrium values of exchange rates.

**JEL Classification:** F31, F37

**Keywords:** monetary model, exchange rates, nonlinear adjustment, real time, unit roots, forecasting, forecast consistency

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## 1. Introduction

Ever since Meese and Rogoff's (1983) work on the out-of-sample forecast comparison of monetary model variants and a naïve random walk model, a consensus view has emerged that monetary models are largely unsuccessful in forecasting exchange rates, at least in the short term. This literature casts doubts about the suitability of economic models based on fundamentals in forecasting exchange rates (see Cheung and Chinn, (2001) or Marsh et al. (2004) for evidence based on surveys).

The work of Mark (1995) revived interest in monetary models by focusing on long term predictability of exchange rates. From this perspective, models based on fundamentals are essentially valid in the long run. That means there is a tendency in the exchange rates to adjust to their long term values as suggested by the fundamentals. With the use of nonparametric bootstrapping methods, he was able to show that monetary models with linear mean reversion are of better use in predicting exchange rates in long horizons rather than short horizons. He found some out-of-sample predictability for Japanese Yen, German Mark and Swiss Frank exchange rates vis-à-vis US Dollar at 12 and 16 quarters forecast horizons.

Mark's (1995) work has been subject to criticism on several grounds. Firstly, Berkowitz and Giorgianni (2001) argue that distribution of the bootstrap test statistic as implemented by Mark depends on the assumption of cointegration between the fundamentals and exchange rates. Given that Mark assumes cointegration between fundamentals and exchange rates to generate bootstrap critical values, if fundamentals and exchange rates are not cointegrated in actual data, critical values and therefore inference from the test would be incorrect. Berkowitz and Giorgianni report very weak evidence of cointegration in the data. Kilian (1999) findings corroborate this view. He finds that even if there is cointegration between fundamentals and exchange rates, mean reversion in forecast errors are very slow. Secondly, data generating process and assumed mean reversion has been criticized. Since the work of Neftçi (1984), it has been

increasingly popular to test for nonlinearities and structural instabilities in economic time series. Enders and Granger (1998) show that if nonlinearities are prevalent under the alternative of stationarity, linear tests for unit roots suffer from a lack of power. Not surprisingly, Kilian and Taylor (2003) show that if there is evidence of a nonlinear mean reversion standard tests of long-horizon predictability of exchange rates are invalidated. Finally, Faust, Rogers and Wright (2003) argue that data on fundamentals are subject to continuous revisions. They show that Mark's linear adjustment results are mainly the outcome of a certain window of vintages of the real time dataset and therefore not generally valid.

Failure of linear versions of monetary models to predict exchange rates even in the long run led a number of researchers to explore the nonlinear data generation process in the long term adjustment of exchange rates towards their equilibrium value given by the fundamentals. In this view, fundamentals based models with an appropriately modelled nonlinear mean reversion will be useful in forecasting exchange rates at least in the long term. The recent work by Balke and Fomby (1997), Taylor and Peel (2000), Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003) provide evidence of nonlinear adjustments of exchange rates.

Prominent explanations for nonlinear adjustment in exchange rates are related to the existence of transaction costs and heterogeneous beliefs/players. In the case of transaction costs, financial agents are assumed to be rational. Transaction costs in the financial markets create a band within which exchange rates do not respond to small deviations from the long term equilibrium. For large deviations, however, there is a tendency to revert to the fundamental equilibrium to exploit the profitable arbitrage activity. In this view, the speed of mean reversion towards the equilibrium increases in deviations from the fundamental equilibrium calling for nonlinearity in the exchange rate adjustment. The

heterogeneity argument is motivated by the existence of heterogeneous agents using different information sets.<sup>1</sup>

Even though observed real time data on exchange rates and interest rates are valid at all time periods, monetary aggregates, output and prices are subject to regular revisions. Given that finance professionals and policymakers possess only real time data at the time of the forecast and are unable to perfectly predict future data revisions of the macroeconomic fundamentals, they will likely form their exchange rates expectations based on the data publicly available at the time when the forecast are made. An econometric study that implements the monetary model based on revised data may therefore yield incorrect inference if time series properties are significantly altered after revisions. Several authors find that revisions to preliminary GDP data are large and in general far from being predictable.<sup>2</sup>

In this paper, we extend the real time critique of Faust et al. (2003) to capture the dynamic nonlinear adjustment towards fundamental equilibrium values of GBP/USD exchange rates. As there is no consensus in the literature about the likely form of the nonlinear adjustment, we study two different models. For instance, suppose that the transaction cost argument is valid. If these costs are uniformly distributed among financial agents, one can expect a sharp correction in the exchange rate towards the value dictated by the fundamentals, once the uniform transaction costs band is reached. In this case, threshold autoregressive model (TAR) appears to be more suitable form of a

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<sup>1</sup> De Grauwe and Dewachter (1993) make a distinction between chartists and fundamentalists and Kilian and Taylor (2003) between noise traders and rational speculators. Kilian and Taylor (2003) argue that agents cannot form a consensus view over the underlying fundamental equilibrium if the deviations are small. In that case, we can expect to observe random walk behaviour of exchange rates at values close to long term equilibrium value. As the deviations from the long term equilibrium value are getting large, rational speculators will take a stronger position and prevail. Eventually the mean reversion occurs towards the unobserved long term equilibrium value of exchange rates. The nonlinear adjustment is apparent.

<sup>2</sup> Recently, importance of real time data in macroeconomic evaluations has been addressed by several authors. Debate essentially concentrates about the nature of the revisions, i.e. whether these are news or noise. For instance, Mankiw et al. (1984) find that U.S. money data revisions reduce noise. Faust et al. (2005) examine G-7 countries' output forecasts and find that Italy, Japan & U.K. output revisions are forecastable in real time whereas US output revisions are not.

nonlinear model to account for the discrete adjustment in exchange rates (Tong, 1990).<sup>3</sup> Alternatively, if transaction costs are not uniformly distributed -therefore there exists a continuum of thresholds- the smooth nonlinear adjustment might be expected. In this case, the exchange rate behaviour is possibly more appropriately modelled in the form of an exponential smooth nonlinear adjustment (ESTAR) as suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994).

A common approach to evaluate forecasting performance of alternative models is to compare the root mean squared errors (RMSEs) obtained from an out-of-sample forecasting exercise. This is often complemented with a series of Diebold-Mariano tests to obtain statistical evidence on the performance of alternative models. In this paper we also utilize the *forecast consistency* argument as developed by Cheung and Chinn (1998) next to standard forecast assessment methods. The forecast consistency argument imposes no assumption on the long term properties of out-of-sample forecast error time series; rather it tests long term properties of the series at hand, therefore is more general. Secondly, with this evaluation, out-of-sample forecast errors need not be serially uncorrelated. As Cheung and Chinn show this can happen when the model is correctly specified, however, fundamentals data may be subject to measurement errors. Finally, due to measurement errors, the assumption of unitary elasticity of the coefficients on the right hand side of the exchange rate equation may be violated, even if forecasts are optimal projections. As measurement errors are the main focus of real time dataset discussions, we find the forecast consistency approach the most relevant method for our purposes. Long term properties of forecast error series is also at the core of the work by Kilian and Taylor (2003). This means that we can test long-term equilibrium relationship between fundamental based exchange rates and actual exchange rates via a battery of linear and nonlinear integration/cointegration tests.

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<sup>3</sup> TAR model seems to fit well with the observed exchange rate behaviour such as volatility and jumps in the short run (see Coakley and Fuertes (2001)).

We first find that, while real output and broad monetary aggregates data is subject to significant revisions, price levels are rarely revised. Output and monetary aggregates revisions contain both news and noise components. Second, we compare the performance of linear models with linear adjustments in the out-of-sample forecast errors with the naïve random walk model, as standard in the literature. We confirm the vast literature that the linear monetary models with linear long term adjustment perform very poorly. Thirdly, we account for nonlinear adjustment in out-of-sample forecast errors in linear models. We implement TAR and ESTAR nonlinear adjustment processes in the out-of-sample forecast errors. We find some evidence of nonlinear mean reversion in out-of-sample forecast errors over 1 to 16 quarters forecast horizons. More specifically, a discrete form of the nonlinear mean reversion is observed in shorter term out-of-sample forecast errors; whereas a smooth (exponential) form of the mean reversion is observed in longer term out-of-sample forecast errors. An implementation of the TAR unit root test suggest that up to 25% of the real time vintages exhibit a discrete form nonlinear mean reversion within 1 quarter forecast horizon and up to 44% of the vintages exhibit a discrete form nonlinear mean reversion within 2 quarters forecast horizon, when we take into account revisions in monetary aggregates and real output. Fourthly, we do not detect a discrete form of the nonlinear mean reversion in longer term out-of-sample forecast errors. Fifthly, an implementation of the ESTAR nonlinear unit root test developed by Kapetanios, Shin and Snell (2003) show that there is indeed some evidence of a nonlinear smooth mean reversion in long term out-of-sample forecast errors when real time price level is used as fundamentals.<sup>4</sup> In about half of real time estimations a nonlinear mean reversion occurs within 16 quarters forecast horizon at 5% significance level. Similarly, in about 20% of estimations a nonlinear mean reversion occurs within 4 quarters forecast horizon at 5% significance level.<sup>5</sup> Finally, we find very little evidence of a smooth nonlinear mean reversion in alternative monetary models, where revisions in monetary aggregates and real output are taken into account.

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<sup>4</sup> This amounts to estimating a real exchange rate equation.

<sup>5</sup> For applications of the Kapetanios et al. (2003) test in the context of real exchange rates see, for instance, Choratreas and Kapetanios (2004).



We thus claim at several instances that, i) real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas the adjustment towards fundamentals driven equilibrium values may take a discrete or smooth nonlinear form, ii) revisions in fundamentals data matter for the short term forecastability of exchange rates (We lend support on the importance of real time data analysis by Faust et al. (2003)), iii) there is some short term forecastability of exchange rates in the form of a discrete nonlinear adjustment, iv) there is some long term forecastability in the form of a smooth nonlinear adjustment when hardly revised real time price level data is used. (We lend support on the importance of smooth nonlinear adjustment in ‘real’ exchange rates by Taylor and Peel (2000), Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003)). We, thus, claim that an accurate description of the exchange rate behaviour has to take into account both real time datasets and possible nonlinear adjustments.

The paper is organised as follows. Section 2 provides a discussion on the importance of data revisions and the real time dataset we use in the paper. Section 3 presents results for two possible nonlinear models (TAR and ESTAR) and discuss related non-linear unit root tests. Finally, Section 4 concludes.

## 2. Real Time Datasets and Data Revisions

We define the final value of a variable as follows:

$$x_t^f = x_t^{t+1} + r_t^f$$

where  $x_t^{t+1}$  denote a statistical agency’s initial announcement (at  $t+1$ ) of a variable that was realized at time  $t$ ,  $x_t^f$  denote the final or true value of the same variable, and  $r_t^f$  is the final revision which can potentially be never observed.

We have quarterly real time *vintages* of the U.K. and U.S. monetary aggregates, real output and price level data as of period *1973Q3* onwards. Exchange rates are never revised. We use quarterly end of period GBP/USD exchange rates made available by the IMF/IFS. We use *1973Q3* to *1984Q4* vintages to construct the *first* operational real time



dataset for the *1985Q1* vintage. This means we have 16 real time datasets for the *1985Q1* - *1989Q1* period (one dataset for each quarter; excluding *1988Q3*). Our shortest dataset corresponds to the first dataset (*1985Q1*) and contains 57 quarterly data points. Our longest dataset corresponds to the latest dataset (*1989Q1*), and contains 74 quarterly data points.

There are a few further details about the construction of real time datasets. Firstly, published statistics provided in economic bulletins cover only a limited time period (up to 16 quarters). We therefore conclude that *only* published data was within the reach of financial agents. In other words, we rule out privileged accesses to revised official data.<sup>6</sup> This allows us to extend the data backward with the data published in previous bulletins. Secondly, we use end of period £M3 for the UK and the quarterly average of M2 for the US. As this data is published consistently for the specified time period, we assume that finance professionals made use of this real time data.<sup>7</sup> Further source details are provided in Appendix A.

Insert Table 1 about here

In Table 1 we report descriptive statistics on the size of revisions for each individual data point over 16 datasets. We also report mean revisions after one, two, four, eight or sixteen quarters after the initial announcement and mean standard deviations in revisions.<sup>8</sup>

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<sup>6</sup> For example January 1986 issue of Economic Trends provides statistics from January 1981 to December 1985, which amounts to 16 data points. We assumed that there is no revision in data covering before January 1981. Therefore, we can extend the data backward until 1970Q3 (i.e. till the first available vintage).

<sup>7</sup> We restrict sample period due to data limitations in the UK monetary aggregates. Office for National Statistics (formerly known as Central Statistical Office) in the UK published £M3 data continuously up to August 1989 (under the name of M3 after August 1987, whereas the old M3 is renamed as M3c after this date). After August 1989, UK statistical agency ceased to publish M3 and M3c, and publish M4c data which is a redefinition of M4 introduced in May 1987. Given that UK joined Exchange Rate Mechanism between 1990 and 1992, where the monetary policy was effectively delegated to the German Bundesbank, we prefer to use M3 data.

<sup>8</sup> We only report changes vis-à-vis the first announcement. See Table notes.

Typically, well behaved revisions have three characteristics.<sup>9</sup> First, revisions are expected to be mean zero, i.e. the initial announcement of the statistical agency is an unbiased estimate of the final value. Secondly, the variance of the final revision should be small compared to the variance of the final value. Thirdly, the final revision should be unpredictable given the information set at the time of the initial announcement.

In our case, data revisions are not well behaved. First, quarter to quarter data revisions are frequent, large and volatile for monetary aggregates and for real output. Revisions have a non-zero mean. Price level revisions are less frequent and small. Secondly, revisions are continuous. Data revisions are quite sizeable even after 16 quarters of the initial announcement. Finally, revisions contain both news and noise component, implying there is substantial scope for forecast improvement by taking into account future revisions.<sup>10</sup>

### 3. Forecasting Exchange Rates

#### 3.1. Monetary Models of Exchange Rates

We assume that financial agents use real time data in forming their exchange rate forecasts. We use superscripts to indicate the date of data announcement and subscripts to indicate the time announced data refers to. The real time *fundamental value* of the log exchange rate at time  $t$  based on the initial announcement of fundamentals at time  $t+1$  ( $f_t^{t+1}$ ) is predicted by a simple nested monetary model that takes the following form:

$$f_t^{t+1} = \alpha_1(m_t^{t+1,US} - m_t^{t+1,UK}) + \alpha_2(y_t^{t+1,US} - y_t^{t+1,UK}) + \alpha_3(p_t^{t+1,US} - p_t^{t+1,UK}) \quad (1)$$

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<sup>9</sup> See for instance Aruoba (2005).

<sup>10</sup> Mincer-Zarnowitz forecast efficiency test results are available from authors upon request. These findings are in line with Mankiw et al. (1984) who find that U.S. money data revisions reduce noise, while Faust et al. (2005) examine G-7 countries' output forecasts and find that Italy, Japan & U.K. output revisions are forecastable in real time whereas US output revisions are not.

where  $m_t^{t+1}$ ,  $y_t^{t+1}$  and  $p_t^{t+1}$  are the logs of money aggregates, output and price levels at time  $t$ , based on the initial announcements at time  $t+1$ . Equation (1) describes a parsimonious relationship between macroeconomic fundamentals and the exchange rate.<sup>11</sup> We will consider three models where the *MY Model* imposes  $\alpha_3=0$ , the *PY Model* imposes  $\alpha_1=0$ , and finally the *P Model* (the real exchange rate model) imposes  $\alpha_1 = \alpha_2=0$ . These three monetary models form fundamentals basis for rational agents in forming their expectations about the future evolution of the exchange rates.

The standard practice in the literature is to restrict parameters on fundamentals equal to 1. Here, we allow financial agents to pursue a slightly more sophisticated statistical strategy. We assume that rational agents have access to a simple OLS estimation technology. Agents estimate a monetary model with real time information about fundamentals up to period  $t-1$  and update their forecasts as new information arrives. This parameter updating mechanism ensures that the coefficients of fundamentals reflect optimal projections on past exchange rates and therefore need not necessarily be equal to 1. They use this information about the coefficient estimates in making their exchange rate forecast for  $t+k$ ,  $k$  being the forecast horizon.<sup>12</sup> This parameter updating mechanism is in line with the forecast consistency argument à la Cheung and Chinn (1998) discussed earlier.

### 3.2. Calculation of the Out-of-sample Forecast Errors

Denote  $\tilde{z}_{t+k}^{t+1}$  as the out-of-sample forecast error for forecast horizon  $t+k$ , where forecasts are based on the real time information available at  $t+1$ . Equation (2) gives the exchange rate forecast error formulation:

$$\tilde{z}_{t+k}^{t+1} = f_{t+k}^{t+1} - s_{t+k} \quad (2)$$

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<sup>11</sup> For versions of the monetary model with microfoundations see, for instance, Lucas (1982) or Stockman (1987).

<sup>12</sup> Rolling regression coefficient estimates are available from authors upon request.

where  $s_{t+k}$  refers to the actual exchange rate at time  $t$  and  $f_{t+k}^{t+1}$  refers to the real time forecast of the fundamental value of the exchange rate based on information available at  $t+1$  (real time). Therefore, the difference between the fundamental value of the exchange rate and the actual exchange rate gives the forecast error.

We obtain real time recursive out-of-sample forecast error series for five different out-of-sample forecast horizons, with  $k=1, 2, 4, 8, 16$ . For each combination of model ( $P, PY, MY$ ), forecast horizon ( $k$ ) and real time datasets (16 datasets), we calculate corresponding forecast error series, where the minimum sample size is set equal to twenty sample points. In other words, we calculate a total of  $3 \times 5 \times 16$  (model x forecast horizon x real time dataset) out-of-sample forecast error series.<sup>13</sup> Gauss programme codes are available upon request.

### **3.3. Monetary Models with Linear Adjustment in Out-of-sample Forecast Errors**

In this section we report standard out-of-sample forecast performance evaluation of linear monetary models with linear adjustment towards the fundamentals determined value of the exchange rates vis-à-vis the naïve random walk model of exchange rates. This assessment is akin to the work by Meese and Rogoff (1983) and Nelson (1995) among others and readily comparable to the work by Faust et al. (2003). We compute out-of-sample forecast errors generated by equations (1) and (2) for each individual dataset, model and forecast horizon combination and compare these w.r.t. the naïve random walk model with the use of the standard Diebold-Mariano test of the forecast accuracy of linear models with a linear adjustment in forecast errors.

Insert Figure 1 about here

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<sup>13</sup>Mark (1995) estimates an error correction specification (ECM) to generate out-of-sample forecast errors. As discussed earlier, we opt for the recursive out-of-sample forecast errors instead of relying on the out-of-sample forecast errors based on the error correction specification due to critiques of Berkowitz and Giorgianni (2001), Kilian (1999) and Kilian and Taylor (2003).

Note that the null hypothesis is equal forecast accuracy, i.e. there is no qualitative difference between the forecasts from two models against the hypothesis that the forecasts are different. Throughout Column 1 and Column 2 in Figure 1, we report  $p$ -values of the Diebold Mariano forecast evaluation. We further report a direct comparison of the RMSEs of the model based evaluation with the RMSEs based on the random walk model in Column 3. We firmly reject the hypothesis of equal forecast accuracy across most of the linear models with a linear adjustment in real time, corroborating the findings of Faust et al. (2003).

### **3.4. Accounting for Nonlinear Adjustment in Out-of-sample Forecast Errors**

Given the recent evidence on nonlinear adjustment in out-of-sample forecast errors (Taylor and Peel (2000), Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003)) we implement nonlinear unit root tests for out-of-sample forecast errors. In the following sections we will allow two types of out-of-sample forecast error adjustment dynamics of exchange rates towards the fundamentals based equilibrium evaluated in real time.

We first analyze an immediate transition threshold analysis proposed by Tong (1990) for which unit root tests are developed by Caner and Hansen (2001). Second model we consider is the more realistic exponential smooth transition threshold dynamics model (ESTAR Model) proposed by Granger and Teräsvirta (1993) for which unit root tests are developed by Kapetanios et al. (2003).

As argued earlier in the introductory section, this evaluation is in line with the forecast consistency argument developed by Cheung and Chinn (1998) that is superior to comparing RMSEs as it imposes no assumption on the long term properties of the out-of-sample forecast error time series. Secondly, with this consistency evaluation, out-of-sample forecast errors need not be serially uncorrelated. As Cheung and Chinn show this can happen when the model is correctly specified, however, fundamentals data may be subject to measurement errors. Finally, due to measurement errors, unitary elasticity of

the coefficients on the right hand side of the equation may be violated, even if forecasts are optimal projections. The forecast consistency argument addresses adequately our concerns about measurement errors in the fundamentals.

**3.4.1. TAR Unit Root Tests (Caner and Hansen (2001)):** We postulate following equation (3) as an appropriate TAR Model:

$$\Delta \tilde{z}_{t+k} = I_t \left[ \theta_1' \tilde{z}_{t+k-1} + \sum_{j=1}^p \gamma_{1j} \Delta \tilde{z}_{t+k-j} \right] + (1-I_t) \left[ \theta_2' \tilde{z}_{t+k-1} + \sum_{j=1}^p \gamma_{2j} \Delta \tilde{z}_{t+k-j} \right] + \zeta_t, \quad (3)$$

where  $\tilde{z}_{t+k-1} = (1 \ t \ \tilde{z}_{t+k-1})$ ,  $\zeta_t$  is an *i.i.d.* error, and  $I_t$  is the indicator function that takes the form:

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} < \lambda \\ 0 & \text{if } y_{t-1} \geq \lambda \end{cases}$$

where  $\lambda$  is a threshold and the variable  $y_t$  is any stationary variable that would determine the change of regime. As in most economic applications we can set  $y_t = \tilde{z}_{t+k} - \tilde{z}_{t+k-m}$ . That is, we assume that  $\tilde{z}$  behaves differently depending on whether past changes in  $\tilde{z}$  have been higher or lower than a certain threshold  $\lambda$ . This is a self-exciting M-TAR model with two regimes as in Enders and Granger (1998). The lag length  $m$  for the changes in  $\tilde{z}$  is determined by the data as is the search for the optimal threshold  $\lambda$ . The parameter vectors  $\theta_1$  and  $\theta_2$  can be partitioned as:

$$\theta_1 = \begin{pmatrix} \mu_1 \\ \delta_1 \\ \rho_1 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \mu_2 \\ \delta_2 \\ \rho_2 \end{pmatrix},$$

where  $\mu_i$  is an intercept,  $\delta_i$  is the parameter of the deterministic trend, and  $\rho_i$  is the autoregressive parameter with  $i = 1, 2$ . In order to search for the optimal threshold  $\lambda$ ,

Caner and Hansen (2001) follow Chan (1993) and find  $\lambda$  as the value of  $\Delta \tilde{z}_{t+k-m}$  that minimises the residual sum of squares of the OLS estimation of (3).<sup>14</sup> In order to test for the existence of asymmetry in the adjustment under both regimes they test the null hypothesis  $H_0 : \theta_1 = \theta_2$  on the OLS estimation of (1), making use of a Wald statistic (W). They propose to choose  $m$  to minimise the residual sum of squares of (3). Given that the Wald test of asymmetry is a monotonic function of the residual variance,  $m$  is chosen as the value which maximizes the Wald test of asymmetry.

The unit root hypothesis involves testing for  $H_0: \rho_1 = \rho_2 = 0$ . There are two possible alternatives:  $H_1: \rho_1 < 0$  and  $\rho_2 < 0$  and

$$H_2 : \begin{cases} \rho_1 < 0 \text{ and } \rho_2 = 0 \\ \text{or} \\ \rho_1 = 0 \text{ and } \rho_2 < 0 \end{cases}$$

The first alternative corresponds to the stationary case, whilst the second implies stationarity in only one of the regimes, which implies overall non-stationarity but a different behaviour from the classic unit-root. Caner and Hansen (2001) develop asymptotic theory for the distribution of this unit-root test. However, for finite samples they recommend the use of bootstrapping. As the distribution of the test statistic will depend on whether or not a threshold effect exists,  $p$ -values obtained through the bootstrap are not unique. Monte Carlo experiments show that this unit root test has substantial power gains against the linear ADF test as threshold effects become larger. In order to discriminate between the two alternatives in  $H_2$ , Caner and Hansen (2001) recommend another Wald statistic ( $RI$ ) which is constructed as the sum of the squared values of the individual one sided  $t$ -statistics for  $\rho_1$  and  $\rho_2$ .<sup>15</sup>

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<sup>14</sup> In practice, outliers are eliminated by trimming the series for the highest and lowest values of  $\Delta y_{t-m}$ .

<sup>15</sup>  $RI$  is the one sided Wald test for a unit root, whereas they also propose a two-sided Wald test,  $R2$ .



The economic interpretation of this model would be that, for certain macroeconomic variables, positive and negative shocks – or shocks above or below a certain threshold – may have different effects on the mean and speed of convergence of the data.<sup>16</sup>

In testing for the unit root we treat the threshold as unidentified, in which case the bootstrap is based on a linear *AR* model.<sup>17</sup> This test is implemented by choosing the estimated delay parameter *m* that minimizes the residual variance.<sup>18</sup> We report the Wald statistic ( $W_T$ ) for the threshold effect (for nonlinearity), threshold unit root bootstrap p-values (for nonstationarity), and corresponding *t* statistics to distinguish between rejection of unit roots and nonstationarity for each series of out-of-sample forecast errors obtained from 16 real time datasets.<sup>19</sup>

Table 2 about here

First, in Table 2, Columns 1 throughout 4, we report the fraction of the datasets we can reject the linearity in out-of-sample forecast errors for alternative monetary models under alternative deterministic specifications as regards the trend and the constant. It appears that in a significant fraction of the series we can not rule out the hypothesis of linearity. We can reject the hypothesis of linearity in the case of MY model (with trend) up to 36% of the 1-quarter ahead, up to 44% (without trend) of the 2-quarters ahead forecast error series estimated. In the case of PY model for up to 27% of the 1 or 2-quarters ahead forecast error series and in the case of P model for up to 50% of the 1 or 2-quarters ahead forecast error series we can reject the hypothesis of linearity at 10% significance level when the constant and a trend is included in the estimation. It seems that it is more likely

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<sup>16</sup> See the seminal work of Balke and Fomby (1997) for the analysis of cointegration relations subject to TAR adjustment dynamics. In their case, the threshold is determined by the size of the lagged error correction mechanism.

<sup>17</sup> The alternative is to treat the threshold as identified, and to base the bootstrap on simulations from a unit root TAR process. Caner and Hansen (2001) show Monte-Carlo evidence that suggests the unidentified threshold bootstrap test suffers from less size distortion than the identified threshold test or a test based on the asymptotic critical values for possible threshold nonlinearities.

<sup>18</sup> Caner and Hansen (2001) point out that as the Wald test  $W_T$  is a monotonic function of the residual variance, this is tantamount to choosing *m* as the value that maximizes  $W_T$ .

<sup>19</sup> Bootstrap p-values are calculated using the unidentified threshold bootstrap as described in Section 5.3 in Caner and Hansen (2001).

that the hypothesis of linearity is rejected against the TAR alternative when we look at shorter out-of-sample forecast horizons. In the same table we report the fraction of series for which we can reject the hypothesis of unit root by looking at the Wald statistic and individual  $t$ -statistics. The results are broadly consistent with linearity tests. The hypothesis of unit root is rejected in a substantial fraction of the shorter horizon forecast errors. As the out-of-sample forecast horizon becomes longer (8 to 16 quarters) there are very few forecast error series obtained under alternative monetary model specifications for which we can reject the hypothesis of unit root. Finally,  $t$ -statistics indicate that even if one can reject the hypothesis of a unit root in a number of series, this result does not indicate that we can reject the hypothesis of nonstationarity. Indeed,  $t1$  and  $t2$  tests jointly taken into account indicate that it is almost impossible to rule out nonstationarity if the data generating process is assumed to be of TAR type.

For the sake of completeness we report detailed results for each individual dataset in Figures 2 and 3 and in Table 4.

Insert Table 4 and Figures 2 to 3 about here

We report bootstrap  $p$ -values for the unit root tests and  $t$ -values for the nonlinearity tests. In Figure 2 we plot  $p$ -values for the threshold effect, bootstrap  $p$ -values for the unit root tests and  $t1$  and  $t2$  tests for nonstationarity for each  $k$ -quarters ahead forecast error series (dataset) estimated with the TAR model without trend. Similarly, in Figure 3 we plot  $p$ -values for the threshold effect, bootstrap  $p$ -values for the unit root tests and  $t1$  and  $t2$  tests for nonstationarity for each  $k$ -quarters ahead forecast error series (dataset) estimated with the TAR model with trend. Horizontal axis represents estimated datasets starting in *1985Q1* and end in *1989Q1* (excluding *1988Q3* as mentioned before).

In Figure 2 (TAR models estimated without trend) we confirm that the results for individual datasets for the linearity test mainly coincide with the tests for unit roots. In the case of MY model, tests for nonlinearity and unit roots reject the null hypothesis for most of the 1 and 2 quarters ahead forecast errors. Specifically, datasets for which both

linearity and unit roots are rejected are between *1985Q3* and *1988Q2*. *P*-values for both *t*-tests indicate that even for those series for which we could reject both linearity and unit root, we are unable to do so for the assumption of nonstationarity. Other monetary models do rather poorly in both linearity and unit root tests at 5% significance level.

Figure 3 (TAR models estimated with trend) reports that, with the exception of P model for which we are able to reject the null of linearity and unit roots at 1 and 2 quarters ahead forecast errors for datasets ranging from *1986Q3* to *1987Q3*, we are unable to detect TAR form of nonlinear mean reversion in most of the alternative model/dataset combination.

In sum, TAR unit root tests results suggest that real time monetary aggregates and real output provide quite valuable information about short term forecastability of exchange rates for *1985Q3* - *1988Q2* datasets.

As a next step we assess the implications of another nonlinear dynamic adjustment specification in the out-of-sample forecast errors. ESTAR model is considered to be more plausible type of nonlinear dynamic adjustment process for exchange rates in the long term.

**3.4.2. ESTAR Unit Root Tests** (*Kapetanios, Shin and Snel (2003)*): ESTAR model has been very popular recently. As argued earlier transaction cost arguments or existence of heterogeneous traders/beliefs in the financial markets may trigger a smooth asymmetric adjustment of the exchange rate towards its linear fundamental equilibrium. As discussed in Granger and Teräsvirta (1993) in general and Taylor and Peel (2001) and Kilian and Taylor (2003) for the monetary exchange rate models, we postulate a smooth transition autoregressive model of the form:

$$\Delta \tilde{z}_{t+k} = \rho_1 \tilde{z}_{t+k-1} + \rho_2 \tilde{z}_{t+k-1} G(y_t; \phi, \lambda) + \varepsilon_t, \quad (4)$$

where  $G$  is a transition function,  $\varepsilon_t$  is an *i.i.d.*( $0, \sigma^2$ ) error,  $y_t$  is a state variable,  $\phi$  is the speed of transition variable, and  $\lambda$  is a threshold. Because of the particularly interesting properties of ESTAR models for economic applications, Kapetanios et al (2003), focus on tests for a unit root when the DGP follows an ESTAR process under the alternative. When we set the state variable as,  $y_t = \tilde{z}_{t+k-d}$  it represents a self-exciting ESTAR model. In this case (4) becomes:

$$\Delta\tilde{z}_{t+k} = \rho_1\tilde{z}_{t+k-1} + \rho_2\tilde{z}_{t+k-1}[1 - \exp(-\phi(\tilde{z}_{t+k-d} - \lambda)^2)] + \varepsilon_t$$

Transition function  $[1 - \exp(-\phi(\tilde{z}_{t+k-d} - \lambda)^2)]$  determines the degree of nonlinearity as a function of the speed of adjustment coefficient  $\phi$ . In line with most of the literature we set the delay parameter  $d$  equal to 1. (See for instance Teräsvirta (1994), or Taylor, Peel and Sarno (2001)).

As Kapetanios et al. (2003) assume that  $\tilde{z}_{t+k}$  is a mean-zero stochastic process, one can set  $\lambda = 0$ . This makes  $G = 1 - \exp\{-\phi\tilde{z}_{t+k-1}^2\}$ . As  $\tilde{z}_{t+k-1} \rightarrow \pm\infty$ ,  $G \rightarrow 1$ , and as  $\tilde{z}_{t+k-1}$  gets close to zero,  $G \rightarrow 0$ . Hence, the process shows three regimes, a middle regime when  $\tilde{z}_{t+k-1}$  is close to zero and two symmetric outer regimes when  $\tilde{z}_{t+k-1}$  becomes large (either positive or negative). The smoothness of the transition between these regimes depends on the parameter  $\phi$ .

Kapetanios et al. (2003) further impose the assumption that  $\rho_1 = 0$ . This assumption can be justified on the grounds of transaction costs arguments or heterogeneity in beliefs as discussed earlier. The variable displays a mean reverting behaviour towards an attractor when it is sufficiently far away from it, but a random walk representation in the neighbourhood of the attractor. In this case, we have that:

$$\Delta\tilde{z}_{t+k} = \rho_2\tilde{z}_{t+k-1}[1 - \exp(-\phi\tilde{z}_{t+k-1}^2)] + \varepsilon_t \quad (5)$$

And the test for the *joint* null hypothesis of linearity and a unit root can be achieved by testing  $H_0: \phi = 0$  against  $H_1: \phi > 0$ . Using a first order Taylor series approximation to (5), one can obtain:

$$\Delta \tilde{z}_{t+k} = \varphi \tilde{z}_{t+k-1}^3 + error \quad (6)$$

The unit root test is based on the t-statistic for the null  $\varphi = 0$  against the alternative  $\varphi < 0$  from the OLS estimate of  $\varphi$  ( $\hat{\varphi}$ ). The asymptotic distribution of this test ( $t_{NL}$ ) is non-standard and Kapetanios et al. (2003) derive it and provide asymptotic critical values. We refer for the asymptotic critical values of the  $t_{NL}$  to Kapetanios et al.(2003) Table 1.

When the process  $\tilde{z}_{t+k}$  is not mean zero, they propose the use of transformations of the data. For the case of a non-zero mean, i.e.  $x_t = \mu + \tilde{z}_{t+k}$ , they propose the use of demeaned data  $\tilde{z}_{t+k}^* = x_{t+k} - \bar{x}$ , where  $\bar{x}$  is the sample mean. For the case of a non-zero mean and a non-zero deterministic trend, i.e.  $x_{t+k} = \mu + \delta t + \tilde{z}_{t+k}$  they propose the use of the demeaned and de-trended data  $\tilde{z}_{t+k}^* = x_{t+k} - \hat{\mu} - \hat{\delta}t$ , where  $\hat{\mu}$  and  $\hat{\delta}$  are the OLS estimators of  $\mu$  and  $\delta$ . This procedure allows carrying out the test using (6) with the de-meaned/de-trended data.<sup>20</sup> In line with the suggestion of Kapetanios et al. (2003) we append to equation (6) one or four autoregressive lags based on Akaike Information Criteria.

Insert Table 3 about here

We implement the ESTAR joint linearity and unit root test for 16 available datasets in real time. In Table 3 we report the percentage of datasets for which we can reject the

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<sup>20</sup> Note that this does not ensure a zero mean in the regression, as  $y_{t,j}^3$  may have a mean that is different from zero. Alternative demeaning would involve full demeaning the left and right hand side of the equation (6). Although this would not affect the distribution of the statistic under the null, it will affect the test results. In the empirical application we use both types of demeaning. We denote Kapetanios et al demeaning exercise as ESTAR1- $t_{NL}$ , and full demeaning at the estimation as ESTAR2- $t_{NL}$ . We will tabulate both set of results.

hypothesis of a unit root together with the linearity; therefore conclude in favour of nonlinear mean reversion.

A quick inspection of Table 3 suggests that, first, several out-of-sample forecast error series over different horizons obtained from estimations of 16 datasets do not reveal much nonlinear mean reversion at shorter horizons. In the case of PY and MY models with or without trend we do not detect significant ESTAR type mean reversion in short term forecast errors (1 to 4-quarters ahead). Only in the case of the P model at 4-quarters forecast horizon we find some exchange rate predictability (18.8% of the series at 5% significance level).

In the long term (16-quarters), however, in the case of MY model without trend, about 18.8% of the forecast errors series seem to exhibit ESTAR form of mean reversion at 5% significance level. When we implement the same test for the P model (real exchange rate model) we find that about half of the forecast error series within 16-quarters are mean reverting at 5% significance level. This corroborates to some extent the findings of Kilian and Taylor (2003) and Taylor et al. (2001) as regards the smooth nonlinear mean reversion of real exchange rates; in our case valid for about half of the real exchange rate models using real time datasets.

Insert Figure 4 and Table 5 about here

Next we report the performance of individual datasets with the ESTAR specification. In Figure 4 we plot the  $t_{NL}$ -statistics for each individual series estimated. Note again that horizontal axis represents datasets that start in *1985Q1* and end in *1989Q1* (excluding August *1988Q3*). As we observe, P model is useful in forecasting exchange rates at 4-quarters forecast horizon in *1985Q1* to *1985Q3* datasets and at 16-quarters forecast

horizon in *1985Q2*, *1985Q3* and *1986Q1* to *1987Q2* datasets, whereas MY model is useful in forecasting *1985Q1*, *1985Q2* and *1989Q1* datasets.<sup>21</sup>

#### **4. Conclusions**

In this paper we examine the real time out-of-sample predictive power of fundamentals based linear monetary exchange models with nonlinear adjustments in forecast errors. We extend the analysis of Faust et al. (2003) in the direction of nonlinear mean reversion and Kilian and Taylor (2003) in the direction of accounting for real time revisions in datasets of fundamentals. We utilize the forecast consistency argument next to standard forecast performance evaluation methods.

We claim that in several instances, real time fundamental equilibrium values of exchange rates may be determined in a linear fashion, whereas adjustment towards the fundamentals driven equilibrium values may take a discrete or smooth nonlinear form. Revisions in fundamentals, particularly in the US and UK monetary aggregates and real output, seem to matter mainly for short term forecastability of exchange rates. Our evidence suggests that in some real time datasets even short term forecastability can be found in the form of discrete nonlinear adjustment. On the other hand, long term forecastability may be present in the form of smooth nonlinear adjustment towards fundamentals determined equilibrium value of exchange rates. Remarkably, long term forecastability appears to be much less affected by data revisions.

There is a clear potential to extend the model to capture even more realistic data learning processes in financial markets. In this paper, we focused on the out-of-sample forecasting performance of monetary models based on separate datasets, as is the case in the literature. An obvious alternative would be to calculate forecast errors by allowing financial agents to discard old data and estimate exchange rates (based on fundamentals)

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<sup>21</sup> We have also implemented the same series of tests with the use of period average instead of end of period GBP/USD exchange rates. The percentages of vintages that exhibit nonlinear mean reversion under various forecast horizons are much higher in this case. Results are available upon request.



based on entirely new dataset each quarter. This requires a large number of vintages to conduct meaningful statistical analysis; therefore we leave this highly promising avenue for future research.

## References:

- Aruoba, B. (2005), Data Revisions are not Well-behaved, mimeo. University of Maryland.
- Balke, N. S. and Fomby, T.B. (1997) Threshold Cointegration, *International Economic Review*, **38**, pp. 627-45.
- Berkowitz, J., and Giorgianni, L., (2001), Long-horizon Exchange Rate Predictability?, *Review of Economics and Statistics*, **83**, pp.81-91.
- Caner, H., and Hansen, B., (2001), Threshold Autoregression with a Unit Root, *Econometrica*, **69**, pp. 1555-96.
- Chen, J., and Mark, N. (1995) Alternative Long Horizon Exchange Rate Predictors, *International Journal of Finance & Economics*, **1**, pp. 229-250.
- Cheung, Y.W. and Chinn, M.D. (1998), Integration, Cointegration and the Forecast Consistency of Structural Exchange Rate Models, *Journal of International Money and Finance*, **17**, pp. 813-830.
- Cheung, Y.W. and Chinn, M.D., (2001) Currency Traders and Exchange Rate Dynamics: A Survey of the US Market, *Journal of International Money and Finance*, **20**, pp. 439-471.
- Chortareas, G. and Kapetanios, G. (2004), The Yen Real Exchange Rate May Be Stationary After All: Evidence from Non-linear Unit-roots Tests, *Oxford Bulletin of Economics and Statistics*, **66**, pp. 113-121.
- Coakley, J. and Fuertes, A.M. (2001), Border Costs and Real Exchange Rate Dynamics in Europe, *Journal of Policy Modeling*, **23**, pp.669-76.
- De Grauwe, P., and Dewachter, H., (1993), Chaos in the Dornbusch Model: The Role of Fundamentalists and Chartists, *Open Economies Review*, **4**, pp. 351-379
- Diebold, F. and Mariano, R. (1995) Comparing Predictive Accuracy, *Journal of Business and Economic Statistics*, **13**, pp. 253-63.
- Enders, W. and Granger, C.W.J., (1998), Unit-Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates, *Journal of Business & Economic Statistics*, **16**, pp. 304-311.
- Faust, J., Rogers J.H. and Wright, J., (2005), News and Noise in G-7 GDP Announcements, *Journal of Money, Credit, and Banking*, **37**, pp. 403-19.
- Faust, J., Rogers J.H. and Wright, J., (2003), Exchange Rate Forecasting: The Errors We've Really Made, *Journal of International Economics*, **60**, pp. 35-59.
- Granger, C.W.J. and Teräsvirta, T., (1993), *Modelling Nonlinear Economic Relationships*, Oxford University Press, Oxford.
- Kapetanios, G., Shin, Y., and Snell, A., (2003), Testing for a Unit Root in the Nonlinear STAR framework, *Journal of Econometrics*, **112**, pp. 359-79.
- Kilian, L., (1999), Exchange Rates and Monetary Fundamentals: What Do We Learn from Long-Horizon Regressions?, *Journal of Applied Econometrics*, **14**, pp. 491-510.
- Kilian, L., and Taylor, M. P., (2003), Why is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?, *Journal of International Economics*, **60**, pp. 85-107.

- Lucas, R.E., (1982), Interest Rates and Currency Prices in a Two-country World, *Journal of Monetary Economics*, **10**, pp. 335-59.
- Mankiw, N.G. and Shapiro, M.D., (1987), News or Noise? An Analysis of GNP Revisions, NBER Working Paper No. 1339, Cambridge MA.
- Mankiw, N.G., Runkle, D.E. and Shapiro, M.D., (1984), Are Preliminary Announcements of the Money Stock Rational Forecasts? *Journal of Monetary Economics* **14**, pp. 15-27.
- Mark, N., (1995), Exchange Rates and Fundamentals: Evidence on Long Horizon Predictability, *American Economic Review*, **85**, pp.201-218.
- Marsh, I., Cheung, Y.W., and Chinn, M.D., (2004), How Do UK-based Foreign Exchange Dealers Think their Market Operates?, *International Journal of Finance and Economics*, **9**, pp 289-306.
- Michael, P., Nobay, A.R. and Peel, D.A., (1997), Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: an Empirical Investigation, *Journal of Political Economy*, **105**, pp. 862–879.
- Neely, C, and Sarno, L., (2002), How Well do Monetary Fundamentals Forecast Exchange Rates?, *Review*, Federal Reserve Bank of St. Louis, issue Sep, pp. 51-74.
- Neftçi, S.N., (1984), Are Economic Time Series Asymmetric over the Business Cycle?, *Journal of Political Economy*, **92**, pages 307-28.
- Sercu, P., Uppal, R. and Van Hulle, C., (1995), The Exchange Rate in the Presence of Transactions Costs: Implications for Tests of Purchasing Power Parity. *Journal of Finance*, **50** 4, pp. 1309–1319.
- Stockman, A., (1987), The Equilibrium Approach to Exchange Rates, *Economic Review*, Federal Reserve Bank of Richmond, pp. 12-31.
- Taylor, M.P., (1995), The Economics of Exchange Rates, *Journal of Economic Literature*, **33(1)**, pp. 13-47.
- Taylor, M.P. and Peel, D.A., (2000), Nonlinear Adjustment, Long-run Equilibrium and Exchange Rate Fundamentals, *Journal of International Money and Finance*, **19**, pp. 33-53.
- Taylor, M.P., Peel, D.A., and Sarno, L., (2001), Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution To the Purchasing Power Parity Puzzles, *International Economic Review*, **42**, pp. 1015-1042.
- Teräsvirta, T., (1994), Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models, *Journal of the American Statistical Association* **89**, pp. 208–218.
- Tong, H., (1990), *Nonlinear Time Series: A Dynamical System Approach* Clarendon Press, Oxford.

## Appendix A: Real Time Dataset

	Variable	Definition	Base Year Changes	Source
UK	Real Output (GDP)	Seasonally adjusted	Sep 1983, Sep 1988	Economic Trends and Economic Trends Annual Supplement <a href="http://www.bankofengland.co.uk/statistics/gdpdatabase/">http://www.bankofengland.co.uk/statistics/gdpdatabase/</a>
	Money Supply (£M3)	Seasonally adjusted, end of period		Economic Trends and Economic Trends Annual Supplement For details in revisions see <a href="http://www.bankofengland.co.uk/statistics/ms/articles/art2jul03.doc">http://www.bankofengland.co.uk/statistics/ms/articles/art2jul03.doc</a>
	Prices	Retail Price Index	Nov 1987, Nov 1988	Economic Trends
US	Real Output (GNP)	Seasonally Adjusted, fixed-weight	1986Q1	Federal Reserve Bank of Philadelphia <a href="http://www.phil.frb.org/econ/forecast/reaindex.html">http://www.phil.frb.org/econ/forecast/reaindex.html</a>
	Money Supply (M2)	Seasonally Adjusted, quarterly average of monthly data		Federal Reserve Bank of Philadelphia
	Prices	Consumer Price Index	May 1983	IMF/IFS
<b>Exchange Rate</b>	GBP/USD exchange rate	End of period		IMF/IFS



Table 2: TAR Estimation: Percentage of the vintages that reject the null hypothesis of unit root

	linearity				unit root								# of vintages used						
	Wald test for threshold effect				One-sided Wald test for unit root				t1 test for unit root				t2 test for unit root				constant	constant and trend	
	constant		constant and trend		constant		constant and trend		constant		constant and trend		constant		constant and trend				
	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%	10%	
<b>MY Model</b>																			
1-quarter	25%	69%	36%	64%	31%	81%	0%	0%	94%	94%	0%	9%	0%	0%	0%	0%	16	11	
2-quarters	44%	75%	36%	64%	63%	94%	0%	0%	94%	94%	0%	9%	0%	0%	0%	0%	16	11	
4-quarters	0%	0%	0%	0%	0%	0%	0%	0%	6%	38%	0%	9%	0%	0%	0%	0%	16	11	
8-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	16	10	
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	9	2	
<b>PY Model</b>																			
1-quarter	0%	0%	0%	27%	0%	0%	0%	0%	25%	38%	64%	91%	0%	0%	0%	0%	16	11	
2-quarters	0%	0%	0%	27%	0%	0%	0%	0%	6%	25%	36%	91%	0%	0%	0%	0%	16	11	
4-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	16	10	
8-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	16	4	
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	9	1	
<b>P Model</b>																			
1-quarter	0%	0%	0%	50%	0%	0%	0%	50%	6%	63%	50%	60%	0%	0%	0%	0%	16	10	
2-quarters	0%	0%	0%	50%	0%	0%	0%	50%	0%	0%	50%	60%	0%	0%	0%	0%	16	10	
4-quarters	0%	0%	0%	13%	0%	0%	0%	13%	0%	0%	0%	50%	0%	0%	0%	0%	16	8	
8-quarters	0%	0%	0%	0%	0%	0%	0%	17%	0%	0%	0%	33%	0%	0%	0%	0%	16	6	
16-quarters	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	30%	0%	0%	10	1	

**Table 3. ESTAR Estimation: Percentage of the vintages that reject the null hypothesis of unit root**

	ESTAR1-t <sub>NL</sub>				ESTAR2-t <sub>NL</sub>				
	Constant		Constant and Trend		Constant		Constant and Trend		
	5%	10%	5%	10%	5%	10%	5%	10%	
<b>MY Model</b>									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8-quarters	0.0	6.3	0.0	0.0	0.0	0.0	6.3	18.8	18.8
16-quarters	18.8	18.8	12.5	18.8	12.5	12.5	12.5	12.5	12.5
<b>PY Model</b>									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	0.0	0.0	0.0	6.3	0.0	0.0	6.3	25.0	25.0
8-quarters	0.0	0.0	0.0	18.8	0.0	0.0	6.3	18.8	18.8
16-quarters	0.0	0.0	0.0	6.3	0.0	0.0	0.0	6.3	6.3
<b>P Model</b>									
1-quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2-quarters	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4-quarters	18.8	25.0	0.0	0.0	12.5	12.5	0.0	0.0	0.0
8-quarters	0.0	6.3	0.0	0.0	0.0	6.3	0.0	0.0	0.0
16-quarters	50.0	50.0	50.0	50.0	6.3	6.3	0.0	12.5	12.5



Table4. TAR Unit Root Test: Vintages that reject the null hypothesis of unit root

**MY Model**

Vintage / Forec. Hor.	Wald test for Linearity					1-Sided Wald Test for Unit Root					t1 test for unit root					t2 test for Unit Root																		
	Constant					Constant and Trend					Constant					Constant and Trend					Constant					Constant and Trend								
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8
1985Q1					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X				
1985Q2					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X				
1985Q3	**	**			X	X	X	X	X	X	**	**	*		X	X	X	X	X	X					X	X	X	X	X	X				
1985Q4	**	**			X	X	X	X	X	X	**	**	*		X	X	X	X	X	X					X	X	X	X	X	X				
1986Q1	*	**			X	X	X	X	X	X	**	**	*		X	X	X	X	X	X					X	X	X	X	X	X				
1986Q2	**	**			X	**	**		X	X	**	**	**		X			X	X					X			X	X						
1986Q3	*	**			X	**	**		X	*	**			X			X	**	**					X			X							
1986Q4	*	**				**	**		X	*	**			X	**	**		*	*					X						X				
1987Q1	**	**				**	**		X	**	**			X	**	**			*				X							X				
1987Q2	*	*				*	*		X	*	**			X	**	**							X							X				
1987Q3	*	*				*	*		X	*	**			X	**	**							X							X				
1987Q4	*	*				*	*		X	*	**			X	**	**							X							X				
1988Q1	*	*							X	*	*			X	**	**							X							X				
1988Q2		*							X	*	*			X	**	**							X							X				
1988Q4									*	*				**	**	*																		
1989Q1																																		

**PY Model**

Vintage / Forec. Hor.	Wald test for Linearity					1-Sided Wald Test for Unit Root					t1 test for unit root					t2 test for Unit Root																		
	Constant					Constant and Trend					Constant					Constant and Trend					Constant					Constant and Trend								
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8
1985Q1					X	X	X	X	X	X				**	**	*				X	X	X	X	X				X	X	X	X	X		
1985Q2					X	X	X	X	X	X				*	*					X	X	X	X	X				X	X	X	X	X		
1985Q3					X	X	X	X	X	X				*						X	X	X	X	X				X	X	X	X	X		
1985Q4					X	X	X	X	X	X				*						X	X	X	X	X				X	X	X	X	X		
1986Q1					X	X	X	X	X	X				*						X	X	X	X	X				X	X	X	X	X		
1986Q2					X	*	*	X	X	X				X			X	**	**	X	X	X				X			X	X	X	X		
1986Q3					X	*	*	X	X	X				X			X	**	**	*	X	X				X			X	X	X	X		
1986Q4						*	*	X	X	X				X	X	**	*			**	**	*	X	X				X	X	X	X	X		
1987Q1								X	X	X				X	X	**				**	**	*	X	X				X	X	X	X	X		
1987Q2								X	X	X				X	X	*				**	*	*	X	X				X	X	X	X	X		
1987Q3								X	X	X				X	X	**				**	*	*	X	X				X	X	X	X	X		
1987Q4								X	X	X				X	X					**	*	*	X	X				X	X	X	X	X		
1988Q1								X	X	X				X	X					*	*		X					X	X	X	X	X		
1988Q2								X	X	X				X	X					*	*		X					X	X	X	X	X		
1988Q4								X	X	X				X	X					*	*		X					X	X	X	X	X		
1989Q1																																		

\*\* denotes 5% and \* denotes 10 % significance level , X denotes a vintage that is not used due to data limitations

Table4 (Continued). TAR Unit Root Test: Vintages that reject the null hypothesis of unit root

Vintage / Forec. Hor.	P Model																																																	
	Wald test for Linearity										1-Sided Wald Test for Unit Root										t1 test for unit root										t2 test for Unit Root																			
	Constant					Constant and Trend					Constant					Constant and Trend					Constant					Constant and Trend					Constant					Constant and Trend														
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16										
1985Q1					X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1985Q2					X	X	X	X	X	X					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1985Q3					X	X	X	X	X	X					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1985Q4					X	X	X	X	X	X					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1986Q1					X	X	X	X	X	X					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1986Q2					X	X	X	X	X	X					X	X	X	X	X	X	*				X	X	X	X	X	X					X	X	X	X	X	X					X	X	X	X	X	X
1986Q3						*	*	X	X	X					*	*	X	X	X	X	*				*	*	X	X	X	X					*	*	X	X	X	X					*	*	X	X	X	X
1986Q4					*	*	X	X	X	X					*	*	X	X	X	X	*				*	*	X	X	X	X					*	*	X	X	X	X					*	*	X	X	X	X
1987Q1					*	*	*	X	X	X					*	*	*	X	X	X	*				*	*	*	X	X	X					*	*	*	X	X	X					*	*	*	X	X	X
1987Q2					*	*		X	X	X					*	*		X	X	X	*				*	*		X	X	X					*	*		X	X	X					*	*		X	X	X
1987Q3					*	*			X	X					*	*		*	X	**					*	*		*	X	**					*	*		*	X	**					*	*		*	X	**
1987Q4									X	X									X	X									X	X									X	X									X	X
1988Q1									X	X									X	X									X	X									X	X									X	X
1988Q2									X	X									X	X									X	X									X	X									X	X
1988Q4									X	X									X	X									X	X									X	X									X	X
1989Q1									X	X									X	X									X	X									X	X									X	X

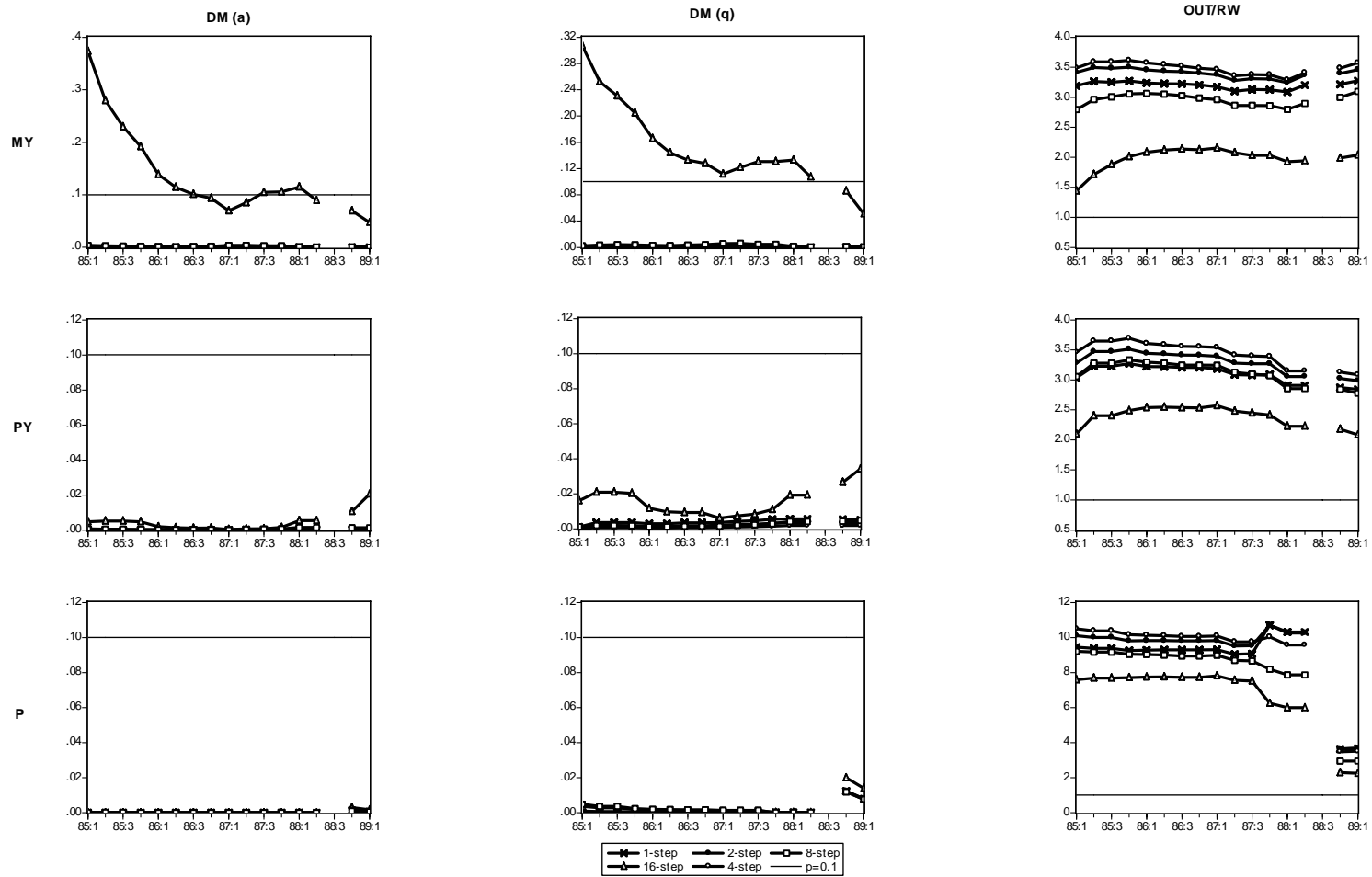
\*\* denotes 5% and \* denotes 10 % significance level , X denotes a vintage that is not used due to data limitations

Table 5. ESTAR Unit Root Tests: Vintages that reject the null hypothesis of unit root

Vintage / Forec. Hor.	MY Model															PY Model														
	ESTAR1-t <sub>NL</sub>					ESTAR2-t <sub>NL</sub>					ESTAR1-t <sub>NL</sub>					ESTAR2-t <sub>NL</sub>														
	Constant		Constant and Trend			Constant		Constant and Trend			Constant		Constant and Trend			Constant		Constant and Trend												
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16
1985Q1					**					**					**															
1985Q2					**					**					**															
1985Q3																														
1985Q4																														
1986Q1																														
1986Q2																														
1986Q3																														
1986Q4																														
1987Q1																														
1987Q2																														
1987Q3																														
1987Q4																														
1988Q1																														
1988Q2																														
1988Q4					*																									
1989Q1					**					**					**															
Vintage / Forec. Hor.	P Model																													
	ESTAR1-t <sub>NL</sub>					ESTAR2-t <sub>NL</sub>																								
	Constant		Constant and Trend			Constant		Constant and Trend																						
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16															
1985Q1					**					**					**															
1985Q2					**					**					**															
1985Q3					**					**					**															
1985Q4															**															
1986Q1					**					**																				
1986Q2					**					**																				
1986Q3					**					**																				
1986Q4					**					**																				
1987Q1					**					**																				
1987Q2					**					**																				
1987Q3					*					*					*															
1987Q4					*					*					*															
1988Q1																														
1988Q2																														
1988Q4																														
1989Q1																														

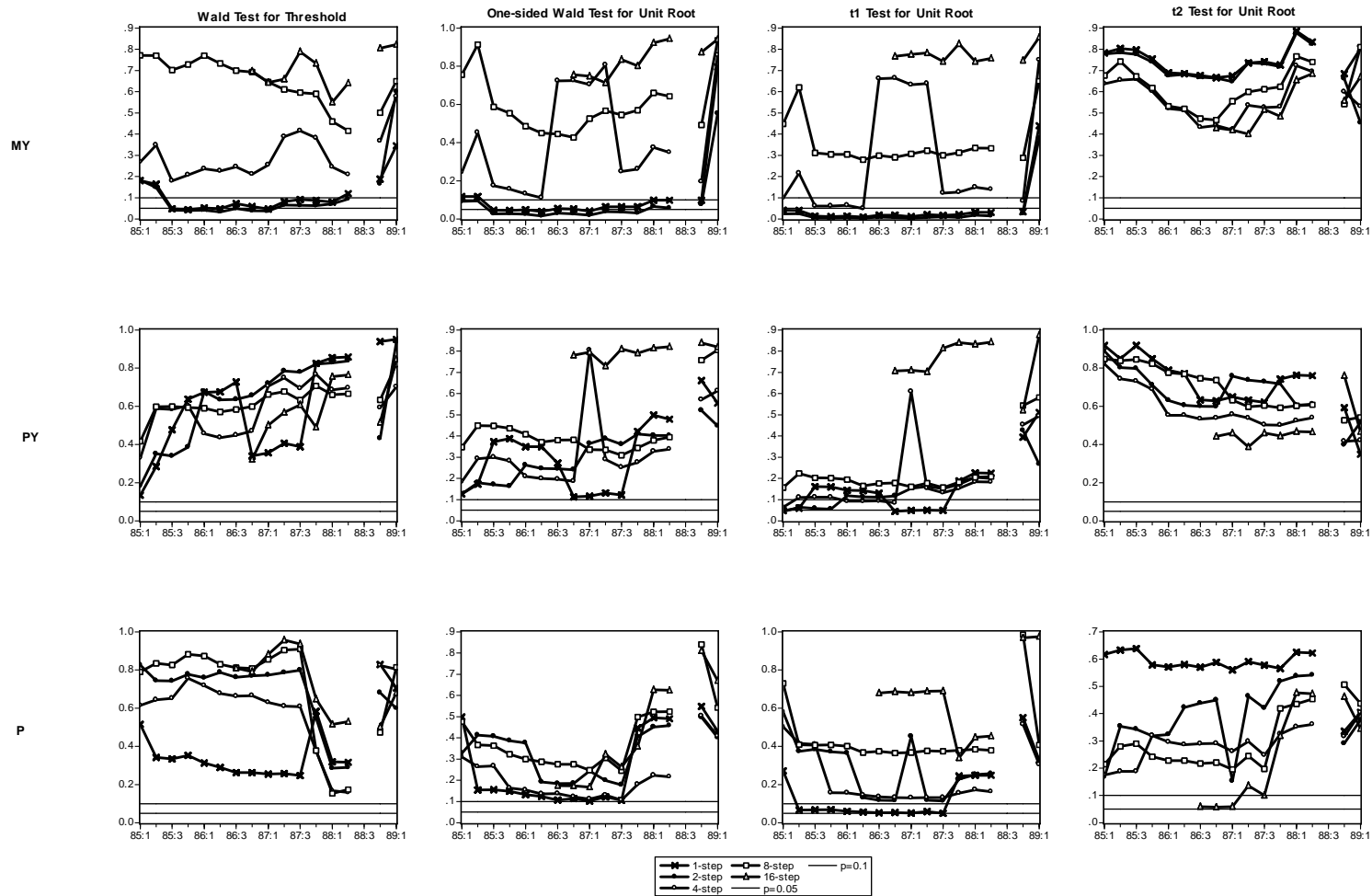
\*\* denotes 5% and \* denotes 10 % significance level

**Figure 1. Out-of-Sample Forecast Performance of Linear Models vis-à-vis the Random Walk Model**



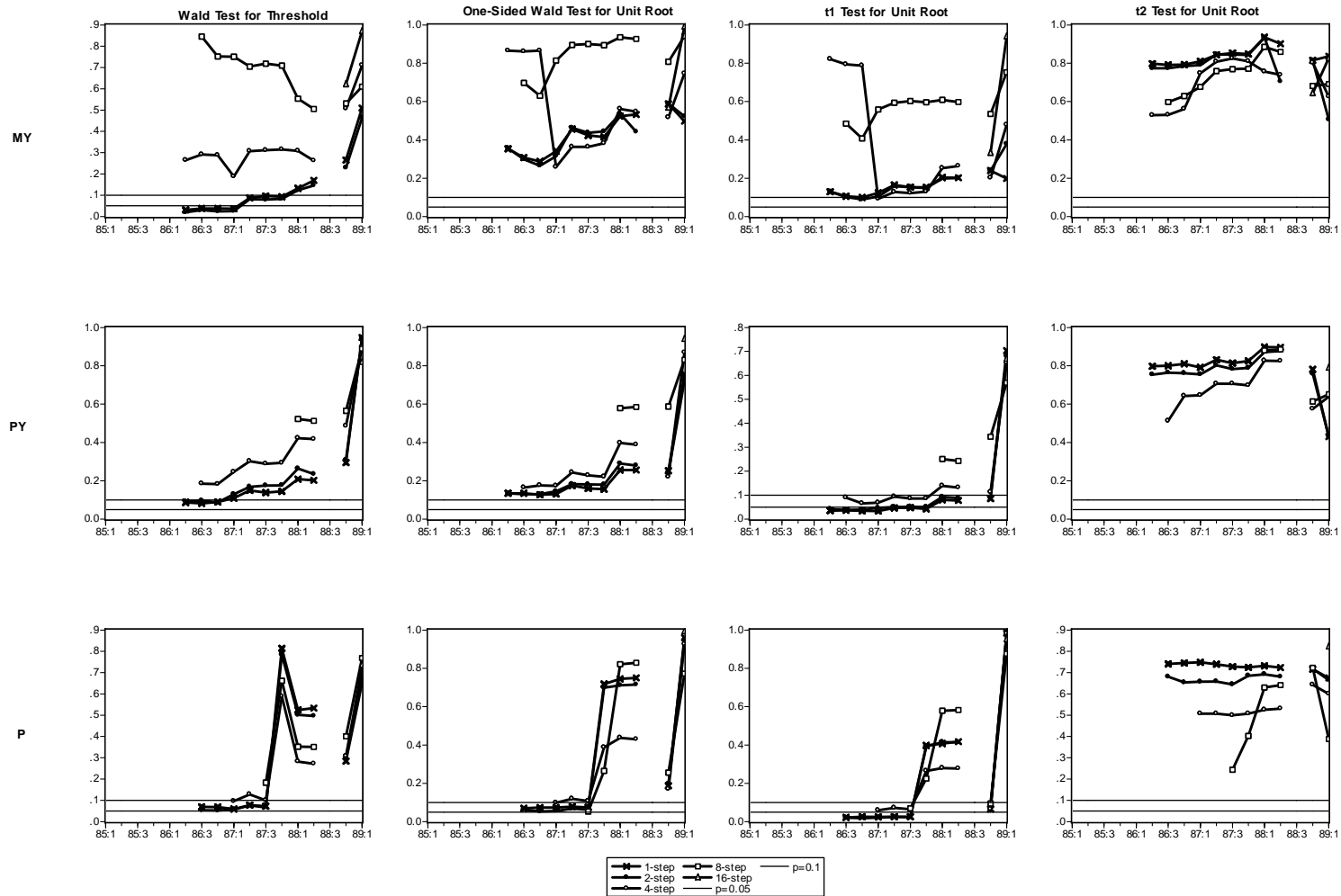
Note: Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing). First two columns present the p-values of the Diebold-Mariano tests. DM(a) denotes the test with absolute value of the forecast errors. DM(q) denotes the test with the square of the forecast errors. Third column presents the ratio of RMSE of the linear model with respect to the Random Walk model.

**Figure 2. Unit Root and Non-Linearity Tests: TAR Model without Trend**



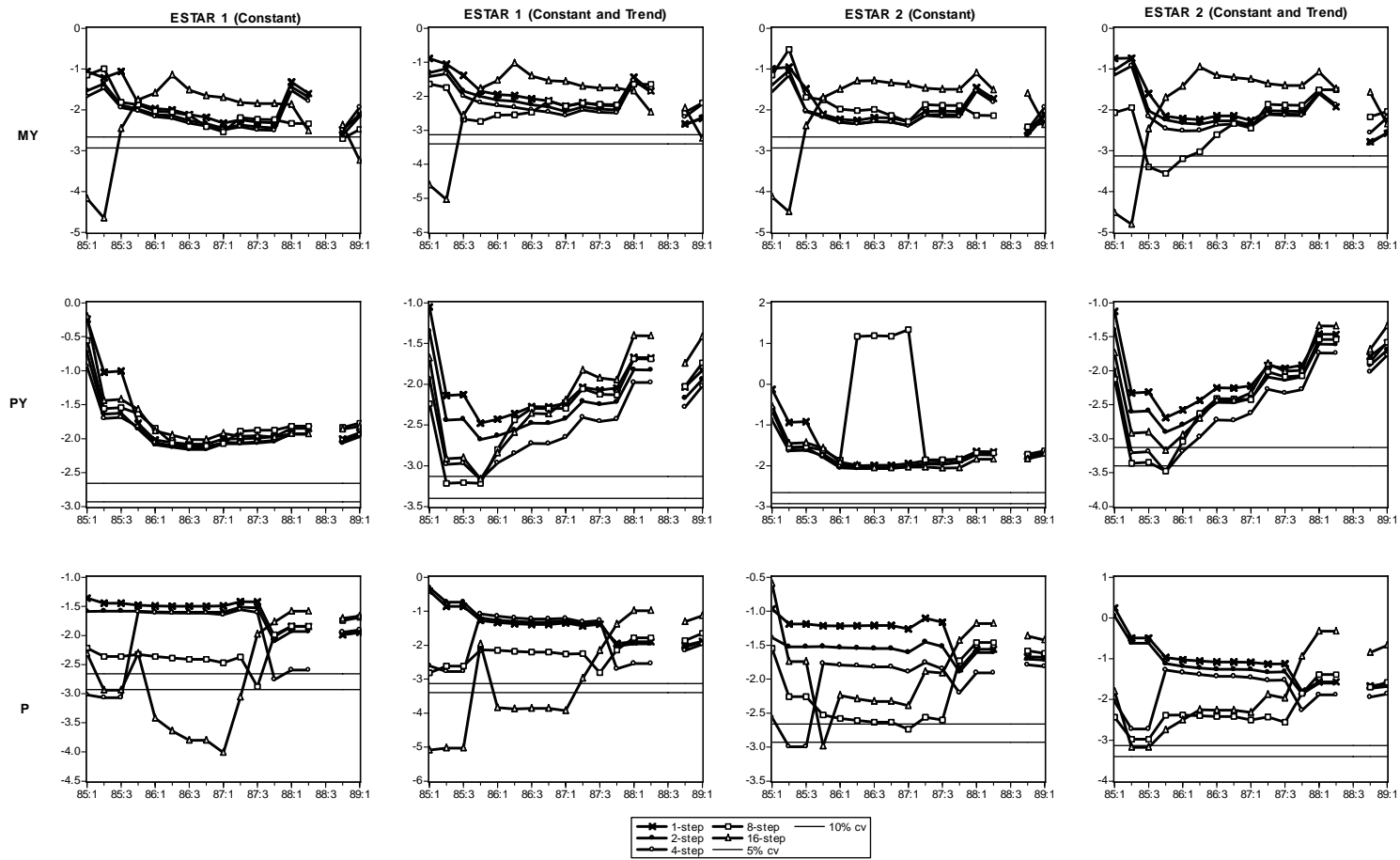
Note: Vertical axis denotes p-values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing).

**Figure 3. Unit Root and Non-Linearity Tests: TAR Model with Trend**



Note: Vertical axis denotes p-values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing).

**Figure 4. Unit Root and Non-Linearity Tests: ESTAR Model**



Notes: Vertical axis denotes  $t_{NL}$  values of estimations from 16 real time dataset. Horizontal axis denotes the real time dataset (note that the third quarter of 1988 vintage is missing). Critical values for the  $t_{NL}$  test statistic at 5% and 10% are given by the straight lines.