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Can Central Bank Interventions Affect the Exchange Rate Volatility? Multivariate GARCH Approach Using Constrained Nonlinear Programming

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Abstract

This study[5] examines the impact of foreign currency market interventions of the Central Bank of Turkey (CBT) in a multivariate GARCH framework. CBT has switched to the floating exchange rate regime since 2001 crisis and announced that the interventions in the foreign exchange markets are aimed at reducing the volatility of the USD/YTL and EUR/YTL. However the literature documents that, foreign exchange interventions lead to an increase in exchange rate volatility. In an attempt to calculate the volatility, we employ a bivariate GARCH estimation with non-linear constrained optimization (NLP) [22] and BEKK [3] on the USD/YTL and EUR/YTL. Our results shed some doubt about the efficiency of these interventions in stabilizing the Turkish Lira market.

Keywords: Time series econometrics, Constrained Nonlinear programming, Multivariate GARCH, FOREX Interventions.

JEL Codes: c32, e58, f31

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1. Introduction

In the period of Bretton Woods exchange rate system, those countries that fixed their currencies to the US dollar, had to intervene in the foreign exchange markets when there were deviations from the target parity. After the collapse of the system in 1973 Central Bank's continued to intervene in the foreign exchange markets and these interventions received considerable attention by academicians and policy makers. During the last 30 years we are witnessing an ongoing debate on the ability of these interventions in influencing both the level and volatility of the exchange rate. Interventions were intended to affect the level of the exchange rate and the common belief was they were effective. This was done to counteract any excessive volatility that may jeopardize financial and economic stability. Although diminished in nature, both emerging and developed economy monetary authorities (sometimes Treasuries) continue to intervene in foreign exchange markets.

Intervention studies mainly concentrated on the level and volatility of the exchange rates for the currencies of developed economies. For example Dominguez [10] investigates the effect of interventions on USD-DM and USD-JPY between 1977-1994 by using univariate GARCH models. He documents that the interventions are positively correlated with the level of volatility. Ballie and Osterberg [4] show that interventions between 1985-1990 had no significant effect on the level and volatility of the USD-DM exchange rate. Kim et.al [20] examines foreign exchange interventions of the Reserve Bank of Australia between 1983-1997. They conclude that large interventions have a stabilising effect in terms of the level and volatility of the exchange rate. Frenkel et.al [13] studies the interventions of the Bank of Japan using the official data between 1993-2000 and find that interventions increases the USD-JPY volatility. Recently Beine et.al. [5] examines the impact of interventions carried out by Federal Reserve, European Central Bank and Bank of Japan over the period 1989-2003. Their results confirm the increased volatility hypothesis caused by interventions particularly for interventions carried out unilaterally[6].

A recent study by Beine et.al.[6] using Realized moments method and Bayesian framework try to examine the effects of interventions on the currency components of the exchange rates. Many of the above studies involving GARCH framework are conducted using univariate models, Yet, Analysis regarding to the interventions of the Central Bank's in developed economies are further complicated by the continuous trading and interaction of these hard currencies all over the world. Such an analysis brings forward concerns of coordinated and uncoordinated interventions by different country Central Bank's, in addition to the particular currency that the intervention is made into i.e. The Bank of Japan's intervention into the Japanese Yen (JPY) against the US dollar (USD) will have implications on the value of Euro. So a study of an intervention to the Euro against USD must take into account the interactions in cross currencies. For a researcher it is not as easy to tackle such a problem since the correlation between all these currencies have to be taken into account in estimating the effect of interventions. Multivariate estimation techniques help to estimate the correlation between cross currency movements and foreign currency interventions. The widely used multivariate setting that researchers employ is the one that is proposed by BEKK. However, Beine et.al. [6] states that due to the technical difficulties in the optimization of the multivariate GARCH models many studies choose to use the univariate representation. The major difficulty in the multivariate case stems from the highly nonlinear and non-convex nature of the resulting optimization problem.

This paper proposes to overcome the difficulty of multivariate estimation by using the nonlinear programming technique proposed in Salih et.al [22]. In the NLP multivariate GARCH approach there is no need to impose artificial restrictions for tractability. GARCH models are

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[6] Research on the topic investigated various dimensions of the interventions and contained both parallel and contradicting results. The type of questions addressed ranged from the type and size of interventions, the length and magnitude of the effect, whether the effect was observed in the first and second moments to signaling effect of interventions. Considerable number of studies indicated that monetary authorities are effective in intervention policies. Please refer to Sarno and Taylor for a detailed survey of this literature

usually presented as unconstrained optimization models in econometrics,(see e.g., Hamilton [17], Gourieroux [16]) with recursive terms whereas they actually fall into the domain of non-convex nonlinearly constrained nonlinear programming. They can be solved by extensions of Newton or quasi-Newton methods that take into account the recursive nature of terms defining the objective function. We solve the problem with nonlinearly constrained non-convex program using the AMPL modeling language (Fourer et al. [14]), and the state-of-the-art optimization packages available through the recently developed NEOS[7] interface at the Argonne National Laboratory.

Studies related to the interventions by Central Banks of emerging economies are rather few. The effect of this intervention in emerging economies is more pronounced since the domestic currency liquidity is contained within that country. In other words, a large share of trade for the local currencies takes place within working hours of the local Banking system. In this regard, movements in the domestic foreign currency market would not affect the rest of the world but the rest of the world would affect domestic market. This is an advantage to disregard endogeneity problem but requires to control for external developments. Herrera and Özbay [16] and Akinci et al [1], [2] are the first to examine the foreign currency intervention in Turkey, the country of interest in this study, within a univariate framework. The former study employs TOBIT and CLAD methodologies to find a reaction function for the Central Bank of Turkey (CBT) by modeling deviations from normal distribution and changing variances. Their result indicates that interventions increase volatility. In the latter two, event studies are used that differentiated both purchase and sale interventions. They find that purchase interventions reduce volatility whereas sale interventions increase volatility. However, they lack statistical support for the result. In this study we, too, will investigate the effect of foreign currency interventions by CBT. We are motivated largely by being the first two employ an unconstrained non-linear programming problem to solve a Multivariate GARCH setting into a Central Bank foreign currency intervention exercise. Secondly, the availability of foreign currency intervention data, especially for an emerging market like Turkey has added extra convenience. And, lastly, CBT organizes intervention in four different forms (three more ways that the literature examined) where details of the form and data for the forms are also readily available. Therefore, we will have a larger domain to see the impact of CBT interventions on the foreign currency market.

Concentrating on widely traded currencies and availability of data motivate the efforts for looking just into direct purchase and sale interventions. However, in the case of Turkey, there are complementary or substituting methods of currency interventions. CBT interventions in the foreign currency markets assume 4 forms in the post 2001 floating exchange rate regime. 1) Direct purchase and sale interventions; these are unannounced in nature where CBT purchase or sell Turkish Lira in return for US dollars. 2) Purchases through previously announced auctions, 3) Altering the foreign currency lending rates of the Central Bank[8], and 4) CBT announcements. There are two motives for direct interventions. First one is to control or rather decrease volatility. As CBT has declared the commitment to the flexible exchange rate regime, interventions have not been directed towards a possible change in the level of USD/YTL or EUR/YTL. The second one is to accumulate enough reserves for the foreign debt payments of the treasury and other foreign currency denominated obligations of the CBT.

We document that both direct interventions and auctions affect the volatility of the USD/YTL exchange rate. However, changes in the foreign currency lending rates do not have any impact.[9] The paper is organized as follows. Next section will give a brief review about the intervention methods of CBT. Section 3 presents the estimation methodology with NLP multivariate GARCH and compares NLP with BEKK and VECH models. Section 4 will discuss empirical results and lastly, section 5 will conclude.

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[7] http://www-neos.mcs.anl.gov; see Cyzik et al. [9]

[8] Central Bank still holds the lender of the last resort position by charging the highest interest rates that is lent to the Banking sector, although the market is virtually open, naturally it is not preferred by the Banking sector. However, still act as a signaling device on how the CBT perceives the exchange rate movements.

[9] Although in the cases of both auctions and lending rate interventions BEKK challenges the results of NLP, the likelihood function is maximized on NLP than BEKK.

2. Motives and Data on CBT Interventions

Starting from 2002, CBT publicly announced that there are two motives for direct interventions. First one is to control or rather decrease volatility. And the second is to accumulate enough reserves for the foreign debt payments of the treasury and other foreign currency denominated obligations of the CBT. Under floating exchange rate regime, since its inception in February 2001, Central Bank gave up the option to control inflation through exchange rate adjustments therefore had to move in line with its commitment to full floating exchange rate regime. As CBT has been determined about the flexible exchange rate regime, interventions have not been directed towards a possible change in the level of USD/YTL or EUR/YTL. Announcements, now, are concentrated on the second moment rather than the level of the exchange rate to stabilize the foreign exchange market. The immediate implication of the switch in the exchange rate regime on financial markets was high levels of volatility in the exchange rate. This has concerned the central bank due to its possible implications on volatility in financial markets and therefore on real economy and price stability.[10] Due to commitment to price stability Central Bank conducted direct interventions to reduce volatility. Later on, announcements were modified to include the term "expected" volatility" rather than "volatility" per se when empirically one does not observe any current change in the volatility right before CBT interventions.[11]

Further motives in purchasing foreign exchange emerges due to CBT's responsibility as the agent of the Turkish Treasury and projections to reduce the stock of workers remittances on behalf of the CBT.[12] Central Bank has to accumulate reserves to meet the upcoming demand for foreign debt payments and help recently aimed reduction on the level of workers remittances. Naturally, reserves can also be used to insure against excessive volatility. The motivations towards reducing exchange rate volatility to avoid hampering price stability and motivate reserve accumulation, Central Bank began to "unannounced" foreign currency interventions to counteract volatility, and starting from April 2002 Central bank initiated foreign exchange purchase auctions.[13] There were several interruptions to auctions during times of financial turmoil's where the economy observes capital outflows. However, they were all transitory and the accumulation of reserves continued.

2.1 Direct Purchase (Sale) Interventions

The immediate intervention method that is considered is the (un)announced direct purchase and sale interventions. The general approach to such interventions is an unannounced purchase (sale) that is aimed at the actual or expected volatility in foreign exchange markets. If there is a sharp depreciation (appreciation) Central Bank intervene with direct sales (purchases).[14]

Economic stability program that has been implemented since the financial crisis of 2001, Central Bank gaining instrumental independence, reforming of the banking system, mono-party political stability along with the developments in the European Union (EU) accession process all contributed to the falling risk premiums and increased productivity. Moreover, long-term capital inflows were motivated by the ongoing privatizations. With gained stability, investors began to

[10] It is common to see high levels of volatility at the start of floating exchange rate regimes when markets begin to understand the dynamics of the system.
[11] The change in the language came in the second half of 2005.
[12] Turkish citizens who are residents of foreign countries can deposit their foreign earnings at the Central Bank of Turkey. This was motivated with privileged rates in the earlier periods of the inception of the system when there was foreign currency shortage in the country. At the moment the Bank is now determined to diminish the stock of worker's remittances over time and the deposit rates are not as competitive as the market. However, probably due to existing memories of earlier private bank failures, CBT is still considered to be the safe heaven for foreign currency deposits.
[13] After the financial turbulence of May/June 2006, the Bank stopped direct purchase interventions but continued foreign exchange purchases through auctions.
[14] In order to participate in foreign exchange market, Bank's have to purchase and sale 1 million US dollars on the ask and bid rates that they announce.

invest more in local currency assets than in foreign currency assets in their portfolios.[15] This resulted in the increased demand and consequently appreciation of the local currency. In this regard, the acceleration in the appreciation process increased volatility in the foreign exchange market that was counteracted by Central Bank foreign currency purchases. Table 1 displays the details of direct purchase and sale interventions for the 2002 - 2005 period. Numbers in bold presents direct purchases and others direct sales. In this period, Central Bank's net purchases were 16.9 billion dollars, which is equivalently reflected in foreign currency reserve accumulation.[16]

Table 1 - Foreign Currency Purchase and Sale Interventions (Million US dollar)*

Value Date	Amount
11.07.2002	3
02.12.2002	16
24.12.2002	9
12.05.2003	62
21.05.2003	517
09.06.2003	566
18.07.2003	938
10.09.2003	704
25.09.2003	1,442
16.02.2004	1,283
11.05.2004	9
27.01.2005	1,347
09.03.2005	2,361
03.06.2005	2,056
22.07.2005	2,366
04.10.2005	3,271

Source: CBT, Markets Department
*Numbers in bold indicate sale interventions.

In reserve accumulation Central Bank's announcement is in the direction of foreign currency purchases during excess supply of foreign currency. In order to achieve the pair of objectives at hand CBT must possess a strong foreign currency position. CBT is constrained by the announced exchange rate regime. In this regard, the interventions and auctions have to be conducted in a way to avoid affecting the equilibrium value of the currency.[17]

In this period, the confidence towards the sustainability of economic stability, the rapid fall in inflation and the ongoing privatizations all contribute to the influx of foreign capital. The details on inflows are given in Table 2. The impact of capital inflow is a parallel rise in CBT reserves. Total inflows, however, were larger than the CBT direct purchase interventions. In the end that began a period of the local currencies gain against the US dollar.

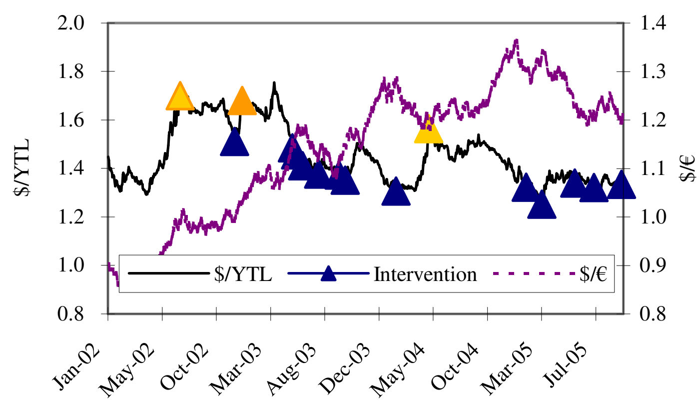
[15] Reverse dollarization process.
[16] CBT began to direct purchase interventions at the 19 billion US dollar level. By January 2007 foreign currency reserve position of the CBT reached the 60 billion US dollar level.
[17] CBT has to be strong both in foreign currency and balance sheet size.

Table 2 - Capital Flows (Billion US dollar)

	2002	2003	2004	2005	2006
1.Capital Inflows (net)	0.6	6.4	23.6	44.0	55.5
Direct Investment (net)	1.0	1.2	2.0	8.7	19.2
Portfoliio Investment (net)	-0.6	2.5	8.0	13.4	7.3
Net Credit Use (Excluding IMF credit)	1.2	2.9	13.8	20.4	26.1
Deposits	0.3	1.4	0.6	0.5	4.6
Other	-1.3	-1.6	-0.8	0.9	-1.8
2. IMF Credits	6.4	-0.1	-3.5	-5.4	-4.5
3. Reserves (- indicates an increase)	-5.6	-3.3	-6.8	-18.2	-16.4
CBT's Direct Interventions	0.0	4.2	1.2	11.4	3.3
CBT's Foreign Currency Purchase Auctions	0.8	5.6	4.1	8.1	3.3

Source: CBT

Graph 1 - Direct Purchase and Sale Interventions against the US dollar, US dollar/Euro Parity, US dollar/New Turkish Lira Parity*



* Light (yellow) triangles represent sale interventions, and dark (blue) ones indicate purchase interventions.

2.2 Foreign Currency Purchase Auctions

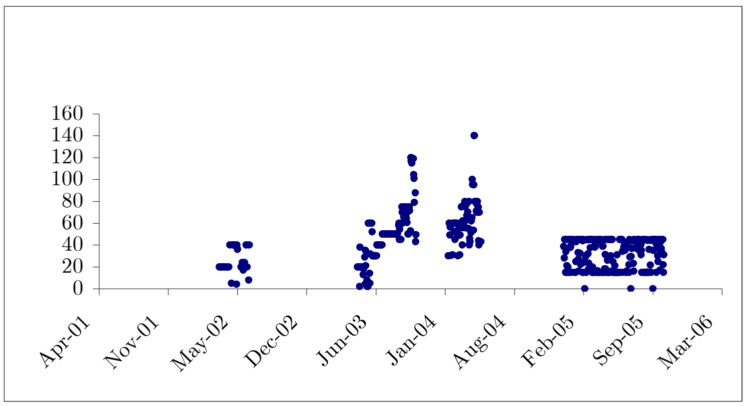
In order to comply with the reserve accumulation policy on 28 March 2002 CBT announced that she would conduct foreign currency purchase auctions as of 1 April 2002. Given rules and quantity of auctions frequently change CBT made a significant foreign currency purchases matching up with direct interventions in amount. In 2002 total purchases reached 800 million US dollar level where as in 2005 this number increased by ten-fold to 8 billion US dollars.

It is argued by the CBT that auctions have minimum impact on the level and the volatility of the foreign currency market since the amount purchased daily is very small given the size of the market and they are pre-announced therefore markets already infuse the effect into the price before the operation takes place.

However, CBT halted auctions four times during the times of foreign exchange shortages in the 2002 – 2005 period. Contrary to those times, during excess foreign currency supply the quantity purchase through markets were increased and reached to 140 million US dollars (daily) at certain times (Graph 2). Moreover, starting from 2005 in order to ensure predictability CBT pre-announced a yearlong program for the purchase amounts in auctions.[18]

There are two distinct time periods in the history of foreign currency auctions. Until 17 July 2003 Central Bank conducted auctions through a "maximum price" rule. Under this rule, the maximum price that CBT will be paying at the end of the day is constrained on the upper side by the average US dollar / New Turkish Lira parity of the first 5 hours of the trading day.[19] CBT conducted the auction one hour before the foreign exchange market closed (starting from 14.30). However, CBT quit operating on this rule after the aforementioned date. This was due to CBT's having to end auctions with less than intended amount during times when the exchange rate is in favor of the foreign currency.[20] The immediate solution to compensate the situation was to add the short amount to next auctions total intended amount.

Graph 2 - FX Purchase Auctions (Million US dollars)



Source: The Central Bank of Turkey

2.3 Foreign Currency Lending Rates

In Turkey, Banks are allowed to borrow foreign exchange from the CBT. The maturity on such debt is short – term and limited to one week. Rates are given in the following table for the period of analysis. Following the lender of last resort policy, the lending rates offered by the CBT is always higher than the market rate, in this regards, we do not observe a trade between the CBT and the Banks. However, it is argued that the rate serves as a signaling device in the market, therefore can be considered as a foreign currency market intervention.

Table 3 - FX Lending Rates

	Rate (%)
01.04.2002	20
02.12.2002	12
20.03.2003	8
24.03.2003	6
05.02.2004	10
11.05.2004	6
20.12.2004	10

Source: CBT, Markets Department

[18] On 22/12/2004 and 22/12/2005 CBT announced the auction program for the year proceeding.
[19] The average of the Banks' "on the hour" foreign exchange rates.
[20] If the exchange rate is falling during the day then the average of 5-hour rates fall below the traded rates during the auction period. Therefore, Bank's find it profitable to sell foreign currency in exchange for local currency to the CBT.

As evident from Table 3, in the analysis period, CBT changed foreign currency lending rates eight times in total.[21]

3 Model Specifications

GARCH family of models has been a major tool in predicting volatility since their introduction in 1982 by Engle [11]. To describe the multivariate GARCH representations that are employed in this study we will adopt the following notation. Following autoregressive process for USD/YTL and EUR/YTL exchange rate returns are assumed, which explains the behavior of the random variable in terms of its past values as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_m Y_{t-m} + \varepsilon_t$$

where $\varepsilon = E(\varepsilon_t)$ is a weak white noise satisfying the martingale difference sequence condition:

$$E(\varepsilon_t \left| \varepsilon_{t-1} \right.) = 0$$

where the notation $E(.)$ denotes mathematical expectation and $\varepsilon_{t-1} = \left\{ \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \right\}$ represents the vector of past values. When the error term ε_t is a multivariate process of dimension n, for all $t = 1, \ldots T$ we have $Y_t \in \mathbb{R}^n$ and $\varepsilon_t \in \mathbb{R}^n$ with components Y_{lt} and ε_{lt}, $l = 1, \ldots, n$, respectively. We denote the components of the $n \times n$ conditional variance-covariance matrix $H_t = E(\varepsilon_t \varepsilon_t^T \left| \varepsilon_{t-1} \right.)$ by h_{klt}.

3.1 VECH and Diagonal VECH Model

Following Kraft and Engle [21] and Bollerslev, Engle, and Wooldridge [7], VECH model can be formulated as follows:

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-1} \varepsilon'_{t-i}) + \sum_{j=1}^{v} B_j vech(H_{t-j})$$

where *vech(.)* is the operator that stacks the lower triangle and diagonal elements of an $N \times N$ matrix to a $N(N + 1)/2 \times 1$ vector. The VECH model is pretty intuitive and easy to understand, and estimates the covariances as a geometrically declining weighted average of past cross products of the error terms. The major weakness of this model is the number of parameters to be estimated. For example for the simplest *vech(1,1)* model $N(N + 1)(N(N + 1) + 1)/2$ number of parameters must be estimated , which can be difficult in practice. Moreover, there is no guarantee for a positive definite covariance matrix without imposing additional restrictions.

Bollerslev, Engle, and Wooldridge [7] proposed the Diagonal VECH model where A_i and B_i are assumed to be diagonal matrices. For GARCH(1,1) process the entries of the H_t can be written as

$$h_{ij,t} = c_{ij} + a_{ij} \varepsilon_{i,t-1} \varepsilon_{j,t-1} + b_{ij} h_{ij,t-1}$$

and in matrix notation it can be characterized as follows:

$$H_t = C + A \odot \left(\varepsilon_{t-1} \varepsilon'_{t-1} \right) + B \odot H_{t-1}$$

[21] The second dimension of CBT interventions is to conduct sterilization operations in accord with foreign currency interventions. In this regard, if CBT is buying foreign currency, the local currency injected in the market is sterilized through Open Market reverse Repos. This prevents the secondary or chain effects of the foreign currency interventions in money markets.

where $\odot$ represents the Hadamard products. In D-Vech specification the number of parameters to be estimated reduces to $N(N + 5)/2$. Despite the decreased number of parameters, restrictions on semi-definiteness on C, A, B and initial matrix H_0 still remain.

3.2 BEKK Model

Engle and Kroner [12] suggested a new model to eliminate the hard restrictions imposed by VECH model on positive definiteness of H_t. BEKK with an exogenous dummy variable in the conditional variance-covariance matrix can be characterized by the following equation as implemented by S-Plus:

$$H_t = C'C + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + DZ_tD'$$

where A, B and C are N x N matrices with C symmetric and positive definite while Z_t is a diagonal matrix with exogenous dummy on the diagonal and D is the coefficient matrix. While the model makes progress on restrictions of H_t, it increases the number of parameters to be estimated. From a numerical optimization point of view, the BEKK model also increases the nonlinearity of the constraints by utilizing a higher-order polynomial representation.

3.3 Constrained Nonlinear Programming Model

For multivariate GARCH optimization problem, Salih et al. [22] proposed a new representation, which transforms the general multivariate GARCH problem into a nonlinearly constrained nonconvex program as follows

$$\max -\frac{TN}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t)$$

subject to

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-1}\varepsilon_{t-1}') + \sum_{j=1}^{p} B_j vech(H_{t-j}).$$

The above mathematical program is the most general multivariate GARCH specification model, from which simplified specifications were obtained by imposing certain restrictions on matrices A_i and B_j .

In the particular NLP approach we parameterize the H_t as $L_t D_t L_t'$, $t=1,..T$ where L_t is a unit triangular matrix and D_t is a diagonal matrix. Therefore, H_t yields:

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} d_{1t} & d_{1t}l_{21t} \\ d_{1t}l_{21t} & d_{1t}l_{21t}^2 + d_{2t} \end{pmatrix}$$

As we will employ an intervention dummy in the variance equation for our estimation purposes a $GARCH(1,1)$ process with dummy variables in the conditional variance, NLP representation of $vech(H_t)$ takes the following form:

$$\begin{bmatrix} d_{1t} \\ d_{1t} l_{21t} \\ d_{1t} l_{21t} + d_{2t} \end{bmatrix} = \begin{bmatrix} c_{11} + a_{11}\varepsilon_{1,t-1}^2 + a_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{13}\varepsilon_{2,t-1}^2 + b_{11}d_{1,t-1} \\ +b_{12}d_{1,t-1}l_{21,t-1} + b_{13}\left(d_{1,t-1}l_{l,t-1}^2 + d_{2,t-1}\right) + \gamma_1 z_{1t} \\ c_{12} + a_{12}\varepsilon_{1,t-1}^2 + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_3\varepsilon_{2,t-1}^2 + b_{12}d_{1,t-1} \\ +b_{22}d_{1,t-1}l_{21,t-1} + b_{23}\left(d_{1,t-1}l_{l,t-1}^2 + d_{2,t-1}\right) + \gamma_2 z_{1t} \\ c_{22} + a_{13}\varepsilon_{1,t-1}^2 + a_{\backslash 23}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{33}\varepsilon_{2,t-1}^2 + b_{13}d_{1,t-1} \\ +b_{23}d_{1,t-1}l_{21,t-1} + b_{33}\left(d_{1,t-1}l_{l,t-1}^2 + d_{2,t-1}\right) + \gamma_3 z_{1t} \end{bmatrix}$$

In this model, positive definiteness of H_t can be satisfied with $D_{ii,t} > 0$, t = 1, .., T .

Gaussian-maximum likelihood estimation is used in the estimation process for two reasons. First, it is easy to implement and second, following Weiss [24], Bollerslev and Wooldridge [8], when the normality assumption is violated but the first two conditional moments are specified, under suitable regularity conditions QMLE estimates of *L(θ)* will be asymptotically normal and consistent. Robust standard errors of Bollerslev and Wooldridge [8] for the MLEs that use Gaussian maximum likelihood are calculated. Following Kawakatsu [19], robust BW asymptotic covariance matrix for the MLEs can be written as:

$$V(\theta) = \frac{1}{n}\left(\frac{1}{n}\sum_{i=1}^{n}\Im_t\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{\partial l_t}{\partial\theta}\frac{\partial l_t}{\partial\theta'}\right)\frac{1}{n}\left(\frac{1}{n}\sum_{i=1}^{n}\Im_t\right)$$

where Fischer information matrix is:

$$\Im_t = (\nabla\varepsilon_t)' H_t^{-1}(\nabla\varepsilon_t) + \frac{1}{2}(\nabla H_t)'\left(H_t^{-1} \otimes H_t^{-1}\right)(\nabla H_t)$$

4. Estimation and Empirical Results

In this section we test the impact of CBT interventions on the volatility and the correlation of the USD/YTL and EUR/YTL exchange rates using the popular bivariate BEKK GARCH estimation and the bivariate NLP GARCH with intervention dummy in the variance-covariance matrix. Data used in the estimations is the daily log returns of the two exchange rates calculated from the exchange rate levels supplied by the CBT recorded at 15:30 local time. [22]The data set covers the period from 4.4.2002 to 6.10.2005. Although we have six data points per day, we do not have the exact timing of the intervention and interventions continue through out the day we choose to use the closing data points in our analysis. Bivariate BEKK GARCH estimation is performed using S-PLUS GARCH module implementing the BHHH algorithm (see the text [18] for a discussion of the BHHH algorithm), and bivariate NLP GARCH is estimated with SNOPT software.

Before we move onto the dummy variable approach we provide the following graph 3 that depicts the impact of direct purchase and sale interventions on the volatility of the YTL/USD exchange rate. Here the circles (diamonds) indicate the volatility estimates from BEKK methodology and squares (triangles) present the NLP method. Circles and squares are purchase interventions; diamonds and triangles are sale interventions. The ratios are calculated by dividing average volatility estimate of 10 day after to before the intervention date. Out of 16 interventions BEKK indicates that only 4 of them do not move the volatility whereas in NLP this is indicated only twice. This provides us evidence that CBT direct interventions affect the volatility however the direction does not necessarily be the same in all. Both BEKK and NLP indicates that in 5 of 16

[22] For GARCH diagnosis, autocorrelation functions and Ljung-Box statistics have been checked. The data can be supplied upon request.

interventions volatility increased, and in 7 of them with BEKK and 9 of them with NLP volatility decreased. This indicates that CBT interventions are effective and they are more towards reducing volatility than increasing it.

Graph 3 – Volatility change before and after interventions

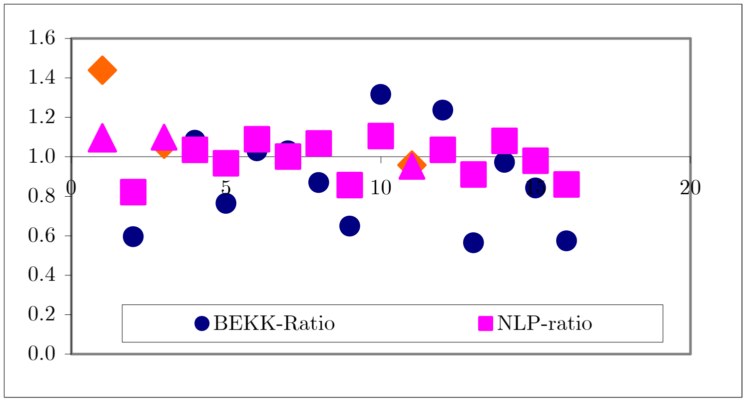


Table 4 reports the coefficients, standard errors, and the log-likelihood values for both bivariate NLP and BEKK GARCH specifications with intervention dummy included in the variance covariance matrix. Standard errors are the robust standard errors of Bollerslev and Wooldridge [8].

It is important to realize that the coefficients do not have an intuitive interpretation for both the Constrained NLP and the BEKK. However, log-likelihood values show that Constrained NLP brings a substantial improvement over the BEKK representation in the solution of the multivariate GARCH formulation. For completeness we also report the AIC and SIC statistics.

Although both of the models provide a solution to the same multivariate GARCH estimation problem, one can say that Constrained NLP estimation is superior to BEKK model based on the AIC and SIC tests.

An examination of coefficients reveals that for bivariate GARCH(1,1) BEKK with direct intervention dummy in the variance-covariance matrix all the coefficients except a_{11}, a_{21}, a_{22} and b_{21} are statistically significant at the 5 percent level (Columns 1 & 2). Moreover the dummy variables of the volatility equations of USD/YTL and EUR/YTL are positive and statistically significant at the 5 percent level. This means if the CBT is using BEKK model to predict volatility then direct interventions have a tendency to increase the volatility rather than decreasing it. The coefficients of the GARCH (1,1) NLP with dummy variable in the variance-covariance matrix are statistically significant at the 5 percent level with the exception of the b_{33} and the coefficients of the dummy variables. Therefore from this exercise one can infer that interventions do not have any effect on neither the volatility nor the correlation of these two exchange rates.

In order to provide statistical evidence we then bring dummy variables for CBT interventions into both volatility and correlation estimates. We have two simultaneously estimated equations, one is the YTL/USD and the other is the YTL/EUR parity. In order to control for the effect of international developments on the exchange rate we should include the volatility estimate of the EUR/USD parity. This will help us to extract the effect of intervention from international parity differentials. In the NLP approach since we estimate both volatility equations simultaneously our coefficients do not suffer a bias. We present the results for estimating the effect of interventions on Table 4's columns 3 through 6. We also differentiate between the types of interventions. Both BEKK and NLP indicate that direct interventions reduce volatility. Since NLP has a larger log-likelihood we believe that BEKK overestimates the effect of interventions. On auction interventions BEKK estimates are insignificant and NLP indicates an increase in the YTL/USD volatility. And finally on changing the USD lending rates NLP indicates that CBT is ineffective on the market where as BEKK contradicts the result and indicate a decrease in the volatility.

Table 4 - Intervention

	Direct Interventions		Auctions		Fx Lending Rates	
Coefficients	**NLP**	**BEKK**	**NLP**	**BEKK**	**NLP**	**BEKK**
c11	-0.0774956 (0.002702)	0.056077 (0.030668)	-0.0563264 (0.0018)	0.090796 (0.019302)	-0.0238705 (0.027615)	0.037166 (0.06304)
c12	0.581901 (0.00022)	0.093839 (0.051614)	0.564281 (0.0007)	0.073824 (0.017610)	0.165057 (0.001005)	0.045142 (0.07393)
c22	0.0743941 (0.00248)	0.026260 (0.120731)	0.0686881 (0.0010)	0.052870 (0.015715)	0.0153932 (0.027371)	0.042663 (0.03153)
a11	0.756483 (0.031286)	0.361745 (0.051894)	0.716681 (0.0381)	0.404083 (0.052850)	0.798523 (0.586096)	0.389735 (0.05771)
a12	-1.17045 (0.005685)	0.131480 (0.044503)	-1.09474 (0.0009)	0.102462 (0.043269)	-1.27735 (0.105849)	0.091454 (0.04512)
a21		0.251157 (0.056137)		0.321550 (0.053624)		0.291271 (0.05812)
a13	0.545669 (0.020357)		0.508265 (0.0197)		0.608612 (0.426304)	
a22	2.18297 (0.007652)	0.248066 (0.047288)	2.04635 (0.0068)	0.183873 (0.045371)	2.42707 (0.206491)	0.211162 (0.04616)
a23	-0.941957 (0.002657)		-0.878755 (0.0031)		-1.06099 (0.075999)	
a33	0.482918 (0.01327)		0.463137 (0.0407)		0.555276 (0.461696)	
b11	-0.506034 (0.246324)	0.956003 (0.052675)	-0.448261 (3.6886)	0.906646 (0.053560)	-0.620059 (2.462765)	0.928455 (0.06936)
b12	2.5135 (0.025380)	-0.099070 (0.049208)	2.30542 (0.0669)	-0.067264 (0.046203)	2.93069 (0.723708)	-0.082698 (0.06270)
b21		-0.017801 (0.055891)		-0.048707 (0.053781)		-0.050771 (0.07196)
b13	-0.372988 (0.154026)		-0.408548 (2.4487)		-0.507111 (2.372384)	
b22	-8.06461 (0.019569)	0.880968 (0.052908)	-7.77113 (0.0129)	0.908152 (0.046226)	-8.9828 (3.419834)	0.896189 (0.06694)
b23	0.885539 (0.014569)		0.931281 (0.0708)		1.28239 (0.480121)	
b33	-0.0552726 (0.11444)		-0.0201902 (1.7803)		-0.027392 (3.395203)	
Dummy h11	-0.0114676 (0.004471)	-0.312610 (0.072444)	0.00424481 (0.0001)	-0.001284 (0.025686)	-0.602694 (1.329229)	-0.261362 (0.04857)
Dummy h12	0.0336278 (0.008563)		0.00448457 (0.0008)		3.74262 (0.075436)	
Dummy h22	-0.00818026 (0.006470)	-0.355085 (0.078061)	-0.0015301 (0.0010)	-0.039456 (0.025373)	0.193991 (2.475306)	-0.292997 (0.05623)
Log-Likelihood	602.6	-336.3	599.2	-341.1	622.1	-341.7
AIC						
SIC						

5 Conclusions

This study analyzes the effect of direct foreign exchange market interventions carried by CBT during 2002-2005 by employing the NLP multivariate GARCH formulation of Salih et.al. [22] and BEKK multivariate GARCH representation of Baba et.al [1]. First we conclude that the direct interventions serve the purpose of decreasing predicted volatility in the foreign exchange market as announced by the CBT. Second NLP brings a substantial improvement over the BEKK representation in the solution of the bivariate GARCH (1,1) with dummy variables in the variance-covariance matrix, in terms of log-likelihood, AIC and SIC criteria. Moreover, we observe that auctions increase the volatility whereas change in the lending rates does not have any effect on it.

As documented in Salih et.al [22] this paper also confirms that the simplifications in the estimation of the multivariate GARCH model in the interest of solvability are unnecessary from an optimization point of view with the current state-of-the-art in optimization technology. However, there is certainly need for further research to ascertain the comparative advantage of the Constrained NLP approach, especially from a forecasting accuracy point of view.

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