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Cengiz TUNÇ
Denis PELLETIER

Address:
Central Bank of the Republic of Turkey
Head Office
Research and Monetary Policy Department
İstiklal Caddesi No: 10
Ulus, 06100 Ankara, Turkey

Phone:
+90 312 507 54 02

Facsimile:
+90 312 507 57 33

Endogenous Life-Cycle Housing Investment and Portfolio Allocation*

Denis Pelletier

North Carolina State University

and

Cengiz Tunc

Central Bank of the Republic of Turkey

Abstract

This paper develops a life-cycle portfolio allocation model to address the effects of housing investment on the portfolio allocation of households. The model employs a comprehensive housing investment structure, Epstein-Zin recursive preferences and a stock market entry cost. Furthermore, rather than resorting to calibration we estimate the value of the relative risk aversion and elasticity of intertemporal substitution. The model shows that housing investment has a strong crowding out effect on investment in risky assets throughout the life-cycle and predicts that homeowners are, by and large, wealthier than renters and invest more in risky assets than renters.

JEL classification: G0, G11, G12, D10

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1 Introduction

Low stock market participation rates and moderate equity holdings for stock market participants are two important empirical observations in US data. For instance, the 2007 Survey of Consumer Finance (SCF) shows that only 55.3% of US households have direct or indirect holdings of risky assets.[1] Furthermore, data from the Panel Study of Income Dynamics (PSID) for the 1968-2007 period show that the median household direct risky asset holdings and indirect risky asset holdings are zero. Despite this fact, theoretical models with the assumption of the historically prevailing equity premium predict that almost 100% of households should hold risky assets as part of their financial portfolio. This gap between theoretical predictions and empirical observations still poses a great challenge to life-cycle models.

While the primary investment asset for US households is investment in owner-occupied housing, it is generally ignored in portfolio allocation models. The 2003-2007 PSID data show that about two-thirds of US households are homeowners. It is quite logical that a typical household has a higher priority to invest in housing in order to have an owner-occupied house than investing in the stock market. This fact is probably the main reason for the low stock market participation observed in the data. Cocco (2004) showed that due to the investment in housing, younger and poorer households have less wealth to invest in the stock market. In other words, housing investment crowds out stock holding for young and low income people.

The focus of our research is to incorporate housing investment into a life-cycle asset allocation model to provide an explanation for these two empirical observations: low stock market participation rates and moderate equity holdings for stock market participants. Specifically, we investigate the effects of housing investment on portfolio allocation of households in a life-cycle model through the construction of a structural model. This involves solving and simulating the life-cycle asset allocation model and estimating some crucial parameters.

Housing is different from other financial assets because it serves dual benefits. First, it is a durable

[1] Risky assets include tax-deferred accounts, directly held stocks, directly held pooled investment funds, bonds, and managed investment accounts or equity in a trust or annuity. Tax-deferred retirement accounts consist of both personally established individual retirement accounts (IRAs) and job-based 401(k) accounts. For detailed information see Bucks, Kennickell, Mach, and Moore (2009).

consumption good from which owners obtain utility. Second, it also serves as an investment asset that enables owners to hold home equity. Contrary to liquid financial assets such as bonds and stocks, housing investment is illiquid and often highly leveraged. While housing plays an important role in portfolio allocation, it is largely unexplored in the literature because of the difficulty of dealing with various frictions associated with the housing market, such as homeowner/renter distinction, mortgage payment, liquidation cost, moving decision, etc.

Beside housing, our model incorporates some key features in order to better explain the asset allocation profiles of households. The first feature is the use of Epstein-Zin (EZ) preferences, see Epstein and Zin (1989), where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS). The main drawback of the commonly used additive utility functions such as the constant relative risk aversion (CRRA) utility function is that RRA, which gives information about how agents deal with uncertainty across possible states of the world, is the inverse of EIS, which is mere time preference. In other words, CRRA utility imposes two different roles on the same parameter. However, EZ-type preferences provide the flexibility to disentangle RRA from EIS. The second feature is that households need to pay a fixed entry cost the first time they decide to invest in the stock market (*i.e.*, buy risky assets). Stock market entry cost is widely accepted in the literature even though too little investigation has been done on its magnitude.[2] A computationally easy way of introducing an entry cost is considering it as a fixed proportion of annual labor income as in Gomes and Michaelides (2007), Guvenen (2009b), Guo (2004), and Alan (2006). This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, and acquiring and evaluating information about the stock market plus the opportunity cost of time. The third feature is that labor supply is inelastic and households receive uninsured labor income in each period. We use PSID data from 2003-2007 to realistically calibrate the life-cycle labor income process of households. Finally, we incorporate a bequest motive in our model which assumes that households bequest all their financial investments and investments made in housing to their inheritors in the terminal period.

Households wealth allocation throughout the life-cycle has been analyzed in the portfolio allocation literature since the pioneering studies of Merton (1969) and Samuelson (1969). Although it has

[2]For example see Alan (2006) for a detailed analysis of the entry cost.

been ignored by a huge body of the portfolio allocation literature over a very long period of time, there is a gradually growing literature that treats housing as an important determinant of portfolio allocation. Leung (2004) provides a comprehensive literature review of housing and asset pricing. Grossman and Laroque (1990) develop an asset allocation model where infinitely lived households derive utility from a single indivisible durable consumption good. They argue that an adjustment cost for illiquid assets could answer the equity premium puzzle. Flavin and Nakagawa (2008) extend the Grossman and Laroque (1990) model by including both durable and nondurable consumption goods into the utility function. Using a continuous time framework, the paper compares a housing model to a habit persistence model and finds that while both deliver many of the same implications, empirical tests using household level data strongly favor the housing model. Longstaff (2009) studies the implications of illiquid assets in a continuous time asset pricing exchange economy with heterogeneous agents. Villaverde and Krueger (2010) present a general equilibrium model of life-cycle asset allocation to demonstrate the effects of consumer durable goods on consumption and asset allocations. Piazzesi, Schneider, and Tuzel (2007) consider a consumption based asset pricing model where housing is explicitly incorporated into the model both as an asset and as a consumption good. Their paper focuses on the effects of housing-consumption asset pricing models on the predictability of the return on stocks.

The closest articles to our research are Hu (2005), Cocco (2004) and Yao and Zhang (2005). Hu (2005) develops a standard life-cycle portfolio allocation model that provides the flexibility for households to endogenously decide whether to be a renter or a homeowner. The model employs a CRRA utility function with two sets of preference parameters (renters and homeowners) and considers only five time periods where each period corresponds to either 10 or 15 years. Although the paper obtains low levels of investment in the stock market relative to standard models with no housing, the investment in the stock market is still significantly higher than the empirically observed values. Furthermore, the results of this paper shows that renters hold more risky assets in their portfolio than homeowners. This result actually is in contradiction with the data because homeowners are, by and large, wealthier than renters and people usually want to become homeowners before investing in risky assets. Cocco (2004) analyzes portfolio choice in the presence of housing in a standard life cycle model with a CRRA utility function. Each time period in this model corresponds to five years. The

model in that paper assumes that all agents are homeowners. There is no endogenous decision to be a renter or homeowner. The paper concludes that (1) house price risk crowds out stock holding, (2) households have a relatively low stock market participation rate compared to standard models with no housing, and (3) younger and poorer investors have limited financial wealth to invest in the stock market. However it is less successful at matching the share of wealth invested in stocks conditional on participation with predicted values much higher than those observed in the data. The last paper similar to ours is Yao and Zhang (2005). The model in this paper is similar to Cocco (2004) except it allows households to decide between renting or owning a house. Results show that renters invest a higher portion of their financial wealth in the stock market than homeowners. Furthermore, the share of wealth invested in the stock market is greatly higher than empirical observations.

Our research differs from these studies in several dimensions. First, we develop a comprehensive life-cycle portfolio allocation model that incorporates all features introduced by different studies in the portfolio allocation literature. Among these are housing as an investment and a durable consumption good, an endogenous decision on being a renter or homeowner, a stock market entry cost and an endogenous decision on stock market investment, EZ-type preferences, and a bequest motive. Second, the life-cycle portfolio allocation papers that incorporate housing into the model generally calibrate the parameters of the model. Instead, we estimate two crucial parameters: the relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS). To our knowledge, this paper is the first one in the life-cycle housing-portfolio allocation literature to estimate parameters instead of resorting to calibration.

This comprehensive model is able to explain the aforementioned empirical observations stated above. More importantly, the model is able to explain them by successfully estimating suitable relative risk aversion and elasticity of intertemporal substitution parameters. We obtain a low stock market participation rate as well as moderate equity holding among the stock market participants. Hence, housing investment, incorporated into a life-cycle asset allocation model has strong crowding out effects on investment in the risky asset. The effects are more significant for young and middle-aged households.

The other results are as follows. The model over-estimates the life-cycle homeownership rates to some extent. While PSID data show that about three-quarters of US households are homeowners

after age of 35, the model predicts this rate to be at or above 80%. The model is also able to explain that homeowners, by and large, have more investment in the risky asset than renters. Similarly, the stock market participation rates among homeowners are significantly higher than the participation rates among renters.

The rest of the paper is organized as follows. Section 2 presents the model with all features and defines the optimization problem. The parametrization of the model constitutes Section 3. We discuss estimation of the RRA and EIS parameters in Section 4. Section 5 presents the results of the model, comparative static analysis and comparison of the results of the model with the data. Section 6 provides concluding remarks. The exposition of the solution technique of the life-cycle model is relegated to an Appendix.

2 Model

2.1 Household Preferences

This model is a discrete time life-cycle model where each period corresponds to one year. As a general convention in the life-cycle literature each year is actually the real age of a household minus 19. We assume that households live for at most T periods. The probability that a household is alive at age t conditional on being alive at age $t-1$ is q_t.

In each period, a household derives utility from a constant elasticity of substitution (CES) utility function with nondurable consumption goods, C, and housing investment (or consumption), H. Preferences are in the form of Epstein-Zin, where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS):

$$V_t = \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[q_{t+1} V_{t+1}^{1-\gamma} + (1-q_{t+1}) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{1}$$

$$\theta = \frac{1-\gamma}{1-1/\psi} \tag{2}$$

$$u\left(C_t, H_t\right) = \left[C_t^v + H_t^v\right]^{\frac{1}{v}}, \tag{3}$$

where β is the time discount factor, γ is the RRA parameter, and ψ is the EIS parameter. The intratemporal elasticity of substitution between nondurable consumption goods and housing investment

is $1/(1-\upsilon)$. W_{t+1} is the total wealth that a household would bequest if it passes away at age t. Note that contrary to work such as Hu (2005), we do not have to assume households with different preferences to obtain homeowners and renters.

2.2 Labor Income Process

Following the standard specifications in the life-cycle literature, labor income depends on the age-specific profiles of households with a shock that has a persistent component. During their working life households supply labor inelastically in each period and receive stochastic labor income Y_{it}. Define $y_{it} = \log(Y_{it})$ where y_{it} has the following process:

$$y_{it} = f(t, Z_{it}) + u_{it} + \varepsilon_{it}^{l_1}, \tag{4}$$

$$f(t, Z) = \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 marital\_status + \beta_5 educ, \tag{5}$$

where Y_{it} is labor income received by household i at age t, $f(t, Z_{it})$ is a deterministic function of age t and household characteristics Z_{it} (education, marital status, and gender). The shock to the log of labor income u_{it} follows an AR(1) process:

$$u_{it} = \phi u_{i,t-1} + \varepsilon_{it}^{l_2}, \tag{6}$$

where $\varepsilon_{it}^{l_1}$ and $\varepsilon_{it}^{l_2}$ are $i.i.d.$ random shock with distribution $N\left(0, \sigma_{l_1}^2\right)$ and $N\left(0, \sigma_{l_2}^2\right)$ respectively.[3]

For simplicity, we assume that the retirement age is deterministic. Households work until period K where K corresponds to an age of 65 $(K = 46)$. During the retirement periods $(t > K)$, households receive a constant and deterministic labor income $Y_{it} = \xi Y_{iK}$ where $0 < \xi < 1$ (a fraction of their income during the last year before retiring)[4].

2.3 Housing Investment

Households enter the market as renters in the first period. From the second period on, they endogeneously decide to buy a house and become homeowners or stay in their current house as renters.

[3]For similar labor income processes, see for example Campbell, Cocco, Gomes, and Maenhout (2001) and Guvenen (2009a).

[4]During the whole retirement period u_{it} becomes u_{iK}.

A typical homeowner at any time can endogeneously decide either (1) to stay in his current house, or (2) to sell his current house and buy a new one, or (3) to sell his house and become a renter. Similarly, a renter can decide either (1) to stay in current rented house, or (2) to rent a new house, or (3) to buy a house and become a homeowner. Buying a house's requires paying a proportion d of the house market value as down payment and financing the rest through a mortgage. To capture the illiquidity of housing investment, households incur a liquidation cost equal to a fraction κ of the house market value. Homeowners pay an amount equal to a proportion δ of the house's market value for maintenance and depreciation expenses in each period. Renters, on the other hand, don't pay for maintenance and depreciation costs; they only pay an annual rent which is equal to a proportion α_R of the house's market value. House-related expenses are calculated as a proportion of the house's market value in the literature because it is easy in this way to incorporate them into the model and reduces the computational burden.

Per unit price of housing is denoted by P_t^h, such that a house of size H_i has price $P_t^h H_i$ at time t. Define p_t^h as the log of house price, $p_t^h = \ln(P_t^h)$, and assume that it follows the following stochastic process:

$$\Delta p_t^h = \mu_h + \varepsilon_t^h, \qquad \varepsilon_t^h \sim N(0, \sigma_h^2). \tag{7}$$

where he average growth rate of house prices is μ_h.

When a household buys a house of size H_i at time t, it pays the down payment which is equal to $dP_t^h H_i$ and finances the rest through a mortgage with a fixed rate r^m. Denote RM_t as the amount of mortgage debt a household has at age t. When buying a house, households can borrow up to the house value minus the down payment:

$$RM_t \leq (1-d)\, P_t^h H_i. \tag{8}$$

2.4 Financial Assets and Wealth Accumulation

There are only two financial assets that households can invest in: a risky asset and a riskless asset. Return on investment in the risky asset follows the following stochastic process as in Campbell, Cocco, Gomes, and Maenhout (2001), Cocco (2004), and others:

$$R_{t+1}^s - R^b = \mu_s + \varepsilon_{t+1}^s, \tag{9}$$

where R^s_{t+1} is the gross return on the risky asset at time $t+1$, R^b is the constant gross return on the riskless asset, μ_s is the excess return from the risky asset over the riskless one (equity premium) and ε^s_{t+1} is an $i.i.d.$ random shock with $N\left(0, \sigma^2_s\right)$. In order to invest in the risky asset, households are required to pay a one-time fixed entry cost. This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, the cost of acquiring and evaluating information about the risky asset, and the opportunity cost of time. Alan (2006) estimates this cost as approximately 2 percent of annualized labor income, whereas Gomes and Michaelides (2007) calibrates this fixed cost at 6 percent of annualized labor income. Unlike the risky asset, there is no cost for investing in the riskless asset.

2.4.1 Wealth Accumulation of Renters

In this model, there are two types of households: renters and homeowners. Depending on their homeownership status, households have different budget constraints and cash-on-hand structures. The liquid wealth (LW) of a typical household who was renter at age $t-1$ is the sum of investments made at age $t-1$ in both risky (S) and riskless (B) assets:

$$LW_t = R^b B_{t-1} + R^s_t S_{t-1}. \tag{10}$$

We refer to the sum of liquid wealth and labor income as cash-on-hand. At age t, the cash-on-hand is $X_t \equiv LW_t + Y_t$. A typical renter uses this cash-on-hand to decide whether to buy a house, pay the down payment and begin to pay annual mortgage payments or just stay in a rental property and pay an annual rent. He also decides on consumption expenditure, whether to pay the fixed entry cost for investment in the risky asset (if it has not been paid yet) and decides the portfolio composition among the different financial assets:

$$X_t \;=\; C_t + S_t + B_t + FIX_t \alpha_F Y_t + (1 - HR_t)\left[\alpha_R P^h_t H_t\right] + HR_t\left[M_t + dP^h_t H_t\right] \tag{11}$$

where S_t is the total investment made in the risky asset and B_t is the total investment made in the riskless asset at age t. FIX_t is a dummy variable that takes the value of 1 if a renter pays the fixed entry cost at age t, and 0 otherwise. The term $\alpha_F Y_t$ represents the fixed entry cost which is the proportion α_F of the annualized labor income at age t. HR_t is a dummy variable that takes the

value of 1 if a household is homeowner at time t and it is 0 otherwise. M_t is the household's annual mortgage payment at time t.

2.4.2 Wealth Accumulation of Homeowners

The liquid wealth structure of a typical homeowner at age t is the same as the liquid wealth structure of a renter at time t:

$$LW_t = R^b B_{t-1} + R_t^s S_{t-1}. \tag{12}$$

Similarly, the cash-on-hand of homeowners at age t is the sum of liquid wealth and labor income $X_t \equiv LW_t + Y_t$. A typical homeowner decides in every period whether to stay in his current house or sell the current house and buy another or sell the current house and move to a rental property. Once he sells his house, he would use his cash-on-hand plus the proceeds from selling the house to either buy a new house (upgrade or downgrade) by paying the down payment or move to a rental property and pay rent. He would then decide on consumption expenditure and investment in financial assets using the remaining cash-on-hand. On the other hand, if he decides to stay in the current house, then he would pay the annual mortgage payment and use the remaining cash-on-hand for consumption expenditure and investments in financial assets. Furthermore, households pay for the maintenance and depreciation expense which is equal to proportion δ of the house's market value. It follows that the budget constraint of households takes the following form:

$$\begin{aligned} X_t &= C_t + S_t + B_t + FIX_t \alpha_F Y_t + HR_t \left(1 - MS_t\right) \left[M_t + \delta P_t^h H_{t-1}\right] + \\ & HR_t MS_t \left[dP_t^h H_t + M_t + \delta P_t^h H_{t-1} - \left((1-\kappa) P_t^h H_{t-1} - RM_t\right)\right] + \\ & \left(1 - HR_t\right) MS_t \left[\alpha_R P_t^h H_t - \left((1-\kappa) P_t^h H_{t-1} - RM_t\right)\right], \end{aligned} \tag{13}$$

where MS_t is a dummy variable that takes the value of 1 if a homeowner moves to a new house or rental property and 0 if he stays in his current house. The term $(1-\kappa) P_t^h H_{t-1}$ denotes the amount a homeowner receives if he sells his house, adjusted for the liquidation cost. RM_t is the total mortgage debt a homeowner has at time t and M_t is the annual mortgage payment, which consists of both the annual interest payment MI_t and the annual principal payment MP_t. The mortgage debt and annual mortgage payments have the following processes In each period, MI_t is calculated from remaining

mortgage balance:

$$M_t = MP_t + MI_t, \tag{14}$$

$$RM_t = RM_{t-1} - MP_t. \tag{15}$$

The mortgage interest payments are set such that the mortgage is paid back over a period of 25 years, given the mortgage rate r^m.

Households are assumed to satisfy the non-negativity constraints on consumption and housing investment and short-sale constraints on the risky and the riskless asset. They cannot have negative amounts of consumption and housing investment. Furthermore, households cannot borrow at the riskless rate in order to invest in the risky asset.

$$C_t \geq 0, S_t \geq 0,\ B_t \geq 0, H_t \geq 0,\ \forall t \tag{16}$$

Furthermore, we define the correlations between the shocks to returns on investment in the risky asset and the persistent shocks to labor income as ρ_{sl}, the shocks to returns on investment in the risky asset and the shocks to returns on housing prices as ρ_{sh}, and finally the shocks to returns on housing investment and the persistent shocks to labor income as ρ_{hl}.

It is common in the recent life-cycle portfolio allocation models to incorporate a bequest motive in order to match the skewness of the wealth distribution.[5] Conditional on being alive at age $t-1$, a household could pass away with probability $1-q_t$. We assume that households bequest their total liquid wealth and all investments made in housing to their inheritors at any period they pass away:

$$W_t = R^b B_{t-1} + R_t^s S_{t-1} + P_t^h H_{t-1} - RM_t. \tag{17}$$

2.5 Optimization Problem

Before defining the optimization problem and the value function, we list the state and the control variables in a compact form. The state variables are age (t), liquid wealth (LW_t), risky asset participation status ($IFIX_t = 1$ if the fixed cost has been paid, $= 0$ otherwise), homeown-

[5]For example see Laitner (2002), Gomes and Michaelides (2005), Gourinchas and Parker (2002), and Yao and Zhang (2005).

ership status ($O_t = 1$ if homeowner, $= 0$ otherwise), the size of house owned from the previous period (H_{t-1}), and the lagged labor income shock. We denote the state variables by Ω, where $\Omega_t = \{t, LW_t, IFIX_t, O_t, H_{t-1}, u_{i,t-1}\}$. The control variables are consumption (C_t), the size of house a household choose at age t (H_t), the risky asset participation decision (FIX_t), and the share of financial investment in the risky asset (s_t). We denote the control variables by Ψ, where $\Psi_t = \{C_t, H_t, FIX_t, s_t\}$. Given the stochastic labor income, stochastic house price evolution and stochastic risky asset returns, the household's optimization problem is then

$$V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[q_t V_{t+1}(\Omega_{t+1})^{1-\gamma} + (1-q_t) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{18}$$

subject to dynamics, restrictions and budget constraints in equations (4) to (17).

3 Parameterization

Two crucial parameters (RRA and EIS) will be estimated but we must calibrate the remaining parameters of the model. Tables 1 and 2 display the list of parameters that are calibrated and their values. Each period in the model corresponds to one year. Households are assumed to enter the market at age of 20 and live for at most 80 additional years.

The age-dependent labor income process is based on PSID data from 1968-2007. In order to obtain a random sample of US households, we dropped families that were part of the Survey of Economic Opportunities.[6] We treat retirement period labor income as a constant fraction of labor income of the last working period as in Gomes and Michaelides (2005) and Cocco (2004). The permanent component of labor income, equation (5), is a function of households' age and age squared, as well as dummy variables for education (1 if the head of the household has at least 12 years of education, 0 if not), marital status (1 if married and 0 if not), and gender (1 if head of household is male and 0 if female). We define labor income widely enough to account for endogenous ways of self insurance against labor income shocks. Labor income includes total labor income, unemployment

[6]The original PSID sample was drawn from two independent sources: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally-representative sample of roughly 3000 families from all states. Since the SEO is over-sample of low income families, it isn't a random sample of the US population.

compensation, social security, total transfers, child support, and other welfare of both the head of the family and his spouse if present. The shock to log-labor income follows an AR(1) process plus noise. As in Guvenen (2009a), we set the persistence parameter ϕ to 0.75 and as in Gourinchas and Parker (2002), and Gomes and Michaelides (2005), we set the standard deviation of $\varepsilon_{it}^{l_1}$ and $\varepsilon_{it}^{l_2}$ at 10% and 15% respectively.

We calibrate several parameters related to housing. Yao and Zhang (2005) and Hu (2005) set the required down payment rate at 20% of the house value while Cocco (2004) sets it at 15%. We set the down payment rate as 20% of the house market value. To be consistent with previous studies, we set the annual rental rate of houses at 7% of the market value of the rental property, and annual maintenance and depreciation cost at 1% of the house's market value.[7] We set $v = 0.2$ so that the intratemporal elasticity of substitution between nondurable consumption and housing, $1/(1-v)$, is 1.25 as in Piazzesi, Schneider, and Tuzel (2007).

We next discuss the value of the parameters related to housing prices in equation (7). One approach could be as in Cocco (2004) who uses self-assessed house values from the PSID data from 1970-1992 to construct a house price index. He calculates an average annual growth of 1.6% for house prices. However, one should be skeptical about the house prices in the PSID data for at least two reasons. First, the house prices are self-assessed values. Second, while earlier PSID surveys were conducted every year, later surveys are conducted every two years. Thus, a house price index obtained from PSID data may not give sufficient information about the most recent years. To avoid these issues we instead estimate (7) using the Case-Shiller index. To illustrate the difference between the Case-Shiller index and the PSID house price data, we calculated a house price index using PSID data from all waves (i.e. 1968-2007). Figure 1 shows the growth rate for both indices. The index obtained from PSID shows sharp fluctuations after the mid 1980s. The annual average real growth rate of housing prices is 1.9% for the Case-Shiller index with a standard deviation of 5.72%. However the PSID data produce annual growth rate of 1.6% with a standard deviation of 8.33%. We use the estimates from the Case-Shiller index and set the real growth rate of house price as 1.9% with a standard deviation

[7] According to Housing Statistics of the United States, in 2000 the median price specified housing units in the United States is $119,600 and monthly median rent is $602. Then a rough calculation shows an annual rent of 6.1% of house value, close to 7% used in this paper.

of 5.72%. Accordingly, we set $\mu_h = 0.019$ and $\sigma_h = 0.0572$.

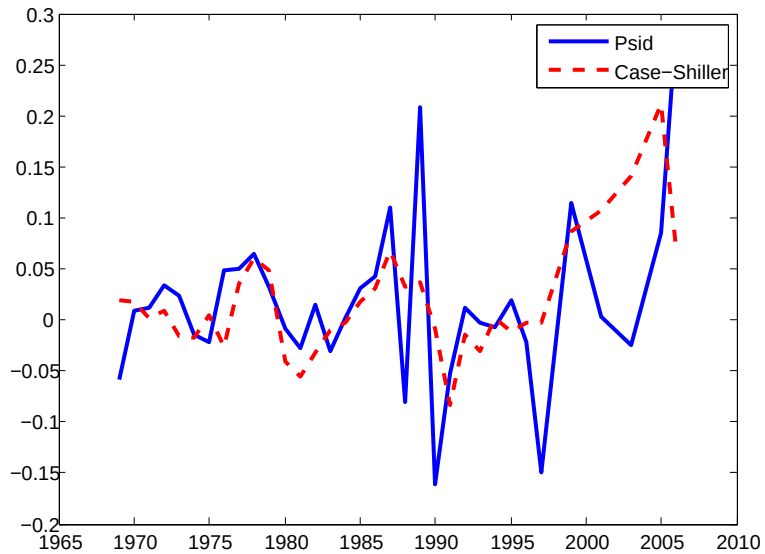

Figure 1: Growht Rate of House Price Indices

We set the time discount factor β to 0.95. The real risk free rate R^b is 3%. We set the annual mean return on the risky asset at 7.5% which means a 4.5% equity premium μ_s. The standard deviation of shocks to the return on the risky asset is 20%.

There is no consensus in the literature on the size of correlations between shocks to the returns on the risky asset, persistent shocks to labor income, and housing prices. Flavin and Yamashita (2002) use PSID data and find that there is almost no correlation between return on investment in the risky asset and the rate of growth of housing prices. Although the authors find that the city-level Case-Shiller index conflicts with PSID results, they claim that the PSID data are nationwide while city level data aren't and so set $\rho_{sh} = 0$. Cocco (2004), Hu (2005), and Yao and Zhang (2005) also assume ρ_{sh} is 0. Furthermore, the correlation between persistent shocks to labor income and shocks to the return on investment in the risky asset ρ_{sl} is set to zero in Cocco (2004), Hu (2005), and Yao and Zhang (2005). As for the correlation between persistent shocks to labor income and shocks to the return on housing investment ρ_{hl}, Cocco (2004) sets it to 0.553, Hu (2005) sets it to 0 and finally Yao and Zhang (2005) use 0.2.

Table 1: Baseline Parameter Values

Description	Parameter	Value
Time discount factor	β	0.95
Gross return on the riskless asset	R^b	1.03
Equity premium	μ_s	0.045
Liquidation cost	κ	0.10
Intratemporal elasticity of substitution	ν	0.2
Rental rate	α_R	0.07
Mortgage rate	r^m	0.04
Fixed entry cost	α_F	0.05
Down payment	d	0.20
Depreciation and maintenance	δ	0.01
Average growth rate on housing prices	μ_h	0.019
Std. of persistent shock to labor income	σ_{l_2}	0.15
Std. of temporary shock to labor income	σ_{l_1}	0.10
Std. of shocks to return on housing inv.	σ_h	0.057
Std. of shocks to return on risky asset inv.	σ_s	0.20
Retirement income factor	ξ	0.66
Persistence parameter of labor income shocks	ϕ	0.75

Table 2: Correlations Between Shocks from Different Datasets

		PSID[a]	NIPA[b]	Case-Shiller[c]
ρ_{sh}	(risky asset and housing)	0.019	0.186	0.042
ρ_{sl}	(risky asset and labor income)	0.024	0.096	
ρ_{hl}	(housing and labor income)	0.059	0.074	

[a] We use PSID data from 1999 to 2007 for all correlations in this column.

[b] We obtained labor Income from National Income and Products Accounts's (NIPA) Personal Income and Outlays. We choose S&P500 for return on investment in the risky assets. Finally, we use Census Bureau's Average Price of Houses Sold for housing investment. All datasets cover the period from 1968 to 2007.

[c] We obtained housing related data from Case-Shiller Index and risky assets investment data from S&P500 Index.

We use different datasets in order to find good and reliable values for these correlations. Table 2 displays the value of these correlations for each dataset. We can estimate all three correlations with PSID data. We can also estimate these correlations with labor income from National Income and Product Account (NIPA), returns on the S&P500 as the risky asset and data from the Census Bureau's average price of houses sold to obtain a return on housing investment. Furthermore, we computed the correlation between the growth rate of the Case-Shiller index and the S&P500 index to get an additional estimate of ρ_{sh}. Our judgement is that the results with the NIPA data are the most trustworthy estimates and accordingly set $\rho_{sh} = 0.186$, $\rho_{sl} = 0.096$ and $\rho_{hl} = 0.074$.

In general, the mortgage rate is higher than the constant return on the riskless asset because mortgages bear a long-term interest rate risk and a default risk. Since our model doesn't have interest rate risk and default risk we set the interest rate on mortgage loans close to the return on the riskless asset at 4%. Households pay their mortgage debt over 25 years and the mortgage payment variables (MP_t, MI_t) are computed accordingly. The one-time fixed-entry cost for investment in the risky asset is set at 5% of annual labor income. Finally, we parameterized the conditional survival probabilities q_t from

the mortality tables produced by the National Center for Health Statistics.[8]

There is no analytical solution to this problem. Therefore we use numerical approximation based on the value function iteration, starting from the last period T and move backward in time. The details are given in the Appendix.

4 Estimation

Because solving numerically our structural model is time consuming we estimate only two parameters: the relative risk aversion γ and the elasticity of intertemporal substitution ψ. The remaining parameters are calibrated to the values discussed above. These two parameters play an important role in obtaining life-cycle asset allocation profiles; different values lead to different life-cycle allocations. Furthermore, there is no clear consensus in the literature on the exact values of these parameters. For instance, Vissing-Jorgensen (2002) suggests that limited asset market participation is important for the estimation of the elasticity of intertemporal substitution. She estimates the EIS for stockholders to be around 0.3-0.4 and for bond holders around 0.8-1. On the other hand, Guvenen (2003) uses an EIS for stockholders close to 1 while for nonstockholders close to 0.1. Hall (1988) finds that the EIS is unlikely to be much higher than 0.1. However, Hansen and Singleton (1982) and Vissing-Jorgensen and Attanasio (2003) estimate the EIS as greater than 1. They also estimate relative risk aversion between 5-10. In a life-cycle portfolio allocation model, Gomes and Michaelides (2005) uses different values of RRA and EIS and conclude that life-cycle portfolio allocations are very sensitive to the values of RRA and EIS. Because these two parameters play critical roles and because there is no consensus in the literature on the exact values for them, it is important to estimate these parameters.

We estimate our model using a minimum distance estimator. See for example Greene (2008, Chapter 15). Let us denote by $w_{i,t}$ the value taken by variable i at age t over the life-cycle. Let us also denote by $g_{i,t}(\gamma, \psi)$ the predicted value generated by the model for variable i at age t for a given value

[8] http://www.cdc.gov/nchs/data/nvsr/nvsr58/nvsr58_21.pdf, Table 1.

of the parameters γ and ψ.[9] A consistent estimator of the RRA and EIS parameters is then given by

$$(\hat{\gamma}, \hat{\psi}) = \arg\min_{\gamma,\psi} \sum_{i=1}^{N} \sum_{t=1}^{K} \left(w_{i,t} - g_{i,t}(\gamma, \psi)\right)^2, \tag{19}$$

where N is the number of variables used in the minimum distance estimation.

This estimation procedure is similar to the impulse-response matching approach where the estimates are chosen by minimizing the distance between the empirical impulse-response function and the impulse-response function generated by a model (see Inoue and Kilian (2012) and Hall, Inoue, Nason, and Rossi (2012) and the numerous references therein). Although similar, it is more simple because we directly match the data ($w_{i,t}$) and the prediction from the model. We do not have to deal with the complications associated to the estimation of the empirical impulse-response function.

The life-cycle data we use for the estimation are consumption, investment in the risky asset, investment in the risky asset as a share of total financial assets, and homeownership status. Consumption and risky asset investment data are normalized by average labor income over the life-time. The PSID data are too disaggregated to obtain good life-cycle consumption data. Hence, we use the Consumer Expenditure Survey (CEX) of 2008 to get life-cycle consumption data for an average US household. For the other variables, we use the waves of PSID data that contain information on the financial wealth of households (*i.e.* the 1984, 1989, 1994, 1999, 2001, 2003, 2005 and 2007 waves). We exclude the Survey of Economic Opportunity (SEO) sample from the data to have a random sample of the US population.

Total investment in risky assets in the PSID data refers to the sum of shares of stocks in publicly or privately held corporations including those held in mutual funds, investment trusts and Individual Retirement Accounts (IRAs). Total financial assets refers to total risky assets plus bonds in publicly or privately held corporations, money in saving and checking accounts, money market funds, certificates of deposits, government saving bonds, Treasury bills, other savings or assets, such as cash value in a life insurance policy and valuable collections for investment purposes. All values are adjusted to their current corresponding values using the CPI.

Data used for the estimation include only households that report all demographic information (i.e. age, gender, marital status, education background) and necessary data for the estimation. We average-

[9] We simulate a very large number of life-cycle paths (50,000) and use the average as the predicted value.

out the data for each age and further smooth-out the lifetime profiles through kernel regression. Kernel regression, as a non-parametric regression method, is appropriate when the relation between the dependent and the independent variables is clearly not linear. Define $\bar{y}_t^a$ as the age-dependent averaged data for any one of the variables, then the goal of doing a kernel regression is to estimate the following functional form,

$$\bar{y}_t^a = m(x_t) + e_t, \tag{20}$$

where $m(x_t)$ isn't specified and $x_t = t$. To estimate $m(x_t)$ we use a kernel $K_h(x)$ constructed from the normal distribution,

$$K_h(x) = \frac{1}{h\sqrt{2\pi}} e^{-\frac{x^2}{2h^2}}, \tag{21}$$

which yield the following kernel regression estimator:

$$\widehat{m}_h(x_o) = \frac{\sum_{t=1}^{T} K_h\left(x_t - x_0\right) \bar{y}_t^a}{\sum_{t=1}^{T} K_h\left(x_t - x_0\right)}. \tag{22}$$

We set the bandwidth equal to $h = 0.2\hat{\sigma}_x$ where $\hat{\sigma}_x$ is the sample standard deviation of x_t. We perform this kernel regression separately for each series. The resulting predicted values are used as the observed data $w_{i,t}$ when performing the minimum distance estimation. Figure 2 displays the raw data and the kernel-smoothed data for the estimation.

5 Results

5.1 Model with Housing Investment

The EZ recursive preferences provides us with the flexibility of estimating RRA different than the inverse of the EIS. The estimated value for RRA ($\hat{\gamma}$) and EIS ($\hat{\psi}$) are respectively 0.76 (0.55) and 0.80 (0.23), with the standard errors between the parentheses. The estimate for risk aversion is less than unity as in Hansen and Singleton (1982) and Epstein and Zin (1991). Contrary to Hall (1988) and Campbell and Mankiw (1990), the estimate for EIS, $\hat{\psi}$, is significantly positive though not greater than one. It is in line with the findings of Hahm (1998) 0.3-0.8, Vissing-Jorgensen (2002) 0.3-1. As

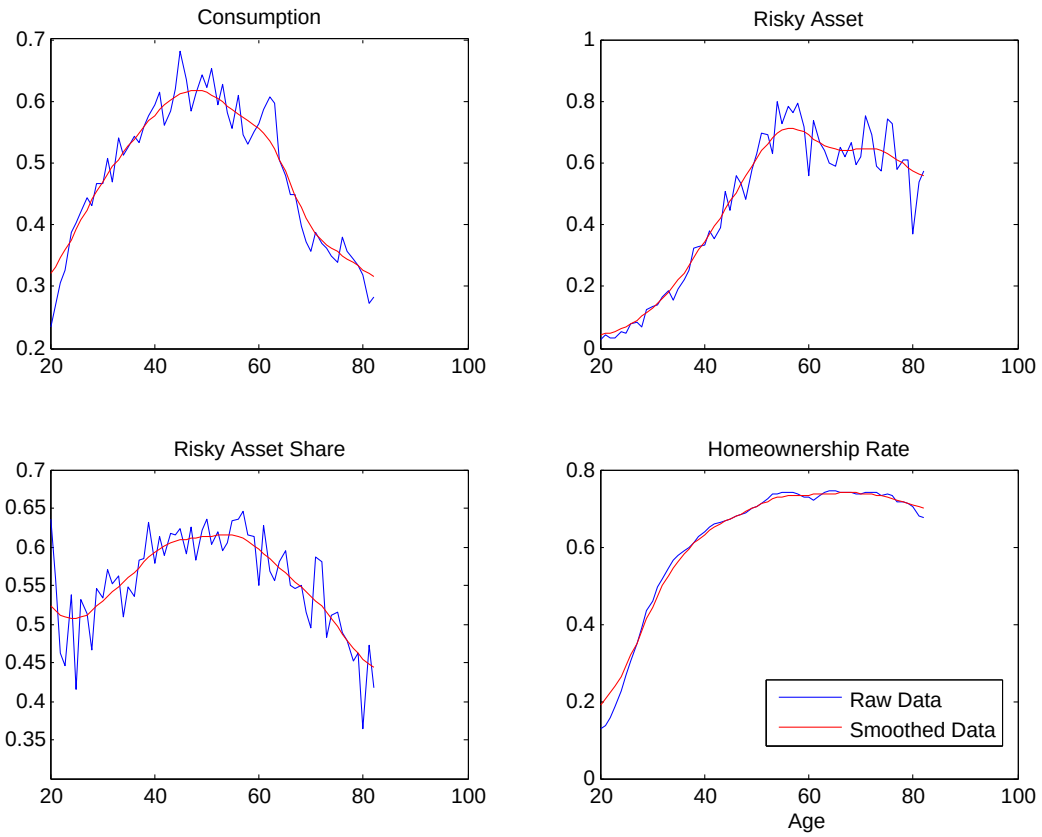

Figure 2: Raw and Smoothed Data for the Estimation

Table 3: Life-Cycle Profiles - Baseline Model

Age	20-35	36-50	51-65	65-80	81+
Risky Asset	0.398	0.612	0.671	0.425	0.488
Risky Asset / Financial Asset	0.420	0.588	0.605	0.445	0.440
Risky Asset Participation Rate	0.514	0.726	0.753	0.549	0.534
Homeownership Rate	0.277	0.803	0.891	0.940	0.957

the estimate for EIS is relatively high and the real interest rate is also high, households have a strong motive for saving. Similar to Epstein and Zin (1991), we find that households prefer late resolution of uncertainity as $\hat{\gamma} < 1/\hat{\psi}$.

Having estimated these parameters within acceptable intervals, we use the corresponding optimal solution to simulate the corresponding life-cycle consumption, housing investment and portfolio allocation profiles of households. Table 3 shows life-cycle portfolio allocation and homeownership status of households for the model with baseline parameters (Tables 1 and 2). The first row of the table shows the risky asset investment normalized average labor income over the life time. The second row displays the share of risky asset investment in overall financial assets. The risky asset participation rate in the third row displays the proportion of households actively investing in the risky assets. Finally, the last row shows the proportion of the population that is homeowner.

The first two rows of Table 3 show that households gradually increase both risky asset investment and its share of total financial assets during the working period. During the first 15 years this share is about one third of total financial assets. It reaches to about 60 percent during the next thirty years. After the retirement, households reduce their risky asset investment because it would be difficult to compensate any big negative shock to risky asset during the retirement period. The third row of Table 3 shows the proportion of the population that has paid the fixed entry cost and is actively investing in the risky asset. During the first fifteen years, about 50% of the population actively invest in the risky asset. This rate increases further to above 70% for the next thirty years. Similar to the risky asset share,

the participation rate decreases during the retirement period. These results shows that the model is able to generate a moderate level of risky asset market participation rate and moderate level of risky asset holding. These results is striking in the sense that the portfolio allocation literature in incomplete markets has shown that given the historically prevailing equity premium, the models predict that households always invest nearly all of their financial assets in the risky asset. For example see Cocco (2004), Hu (2005), Campbell, Cocco, Gomes, and Maenhout (2001), Cocco, Gomes, and Maenhout (2004), Gomes and Michaelides (2005), Polkovnichenko (2007), and Heaton and Lucas (2000). The predictions of the model are closer to the empirical evidence presented in the next section.[10]

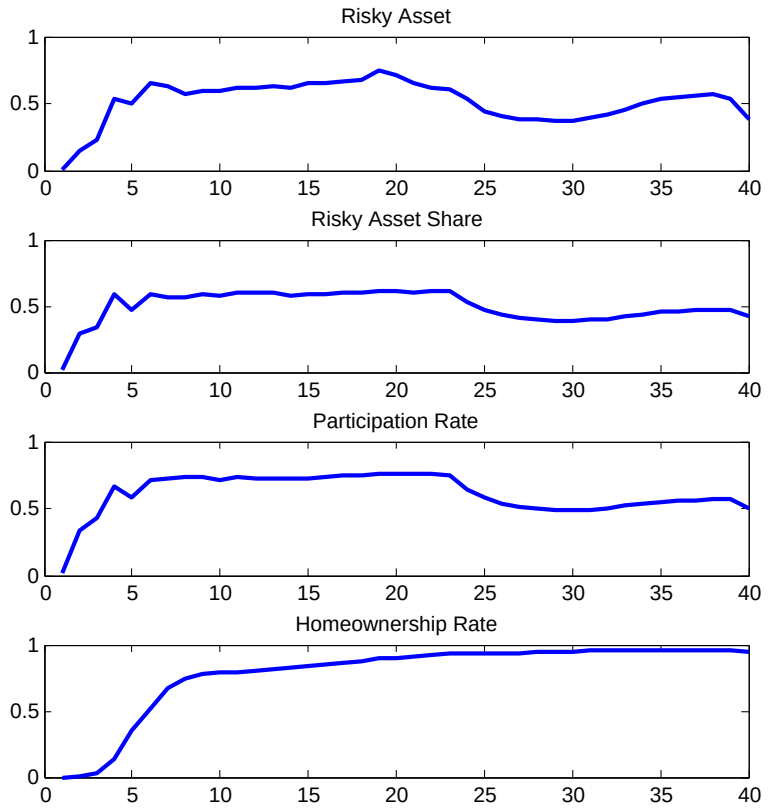

Figure 3: Life-Cycle Profiles - Baseline Model

[10]The possibility of the household borrowing for a mortgage while still holding positive amounts of bonds could seem counterintuitive. However, we see in real world simultaneous debt and positive amount of bond holding. Furthermore, Cocco (2004), Hu (2005), and Hurst and Willen (2007) find similar results.

One can argue that it would be optimal to invest the remaining wealth in the risky asset after households make housing investment and consumption decisions. However, there are some reasons that keep households away from investing all remaining wealth only in the risky asset. One reason is precautionary saving which leads young households to invest in the riskless asset in order to hedge against both labor income shocks and shocks to the return on the risky asset. This is especially true when a positive correlation exists between labor income shocks and shocks to the return on the risky asset. Finally, a positive correlation between shocks to the return on the risky asset and shocks to the return on housing investment encourages households to have some investment in the riskless asset as a hedge against negative shocks to the returns on these investments.

The theoretical literature on portfolio allocation in the presence of nontradable labor income predicts that the portfolio shares invested in the risky asset decreases over life. On the other hand, the empirical literature has found that the portfolio shares invested in risky asset increase over the lifetime. There is also some mixed evidence that the shares decrease slightly in later life. The second row of Table 3 shows that the share of the portfolio invested in the risky asset increases until the 60s and slightly decreases for the rest of the lifetime.

The baseline model finally shows that households have a strong preference for homeownership. While during the first fifteen years, about 27% of the population owns a house, the homeownership rate jumps to 80% during the next fifteen years. This rate further increases to 89% for the 51-65 age interval. For the retirement period, the model predicts that more than 90% of the population are homeowners. As we will see in the subsequent sections, this rate is always less than 80% in the PSID data. Hence the model slightly over-estimates the homeownership rate relative to the empirical evidence for the retirement period. One reason for this over-estimation is the fact that the model doesn't include the case where older people are selling their house and moving because of health reasons.

We next investigate the life-cycle profiles of homeowners and renters. The model assumes that all households are renters in the initial period. From the second period on in each period households endogeneously make a homeownership decision. For instance, a homeowner at age t has three options regarding homeownership: (1) stay in the current house, (2) move to a new house, (3) move to a rental property. Similarly, a renter decides to either stay in the current rental property or move to a bigger

or smaller rental property or buy a house and become a homeowner. Table 4 presents the life-cycle profiles for both homeowners and renters. Note that a typical household can be a renter for some periods and a homeowner for some other periods. The homeowners part of the table shows the life-cycle profiles for households when they are homeowners. On the other hand, the renter part of the table shows the life-cycle profiles for households when they are renters. For example, assume that a household was a renter for the first 15 years, and endogeneously became a homeowner for the next 30 years, and then again endogeneously became a renter for the rest of his life. Then the life-cycle profile for the first 15 years will be used for obtaining renters' life-cycle profiles, the next 30 years will be used to for obtaining an average homeowner's life-cycle profile, and finally remaining life-cycle profiles will again be used for obtaining an average renter's life-cycle profile.

The first two rows of both homeowners and renters sections of Table 4 shows that homeowners invest more in risky asset and hold a significantly higher share of their investments in risky assets than renters with the gap between homeowners renters decreasing over their retirement periods. We observe a similar outcome for the risky asset participation rate in the last rows. The proportion of homeowners who participate in risky asset investment are significantly higher than those for renters. Renters have a quite low participation rate for especially working period. Furthermore, the gap between the homeowners' and renters' participation rate also shows a decreasing trend during the retirement period. The overall results show that the model is able to replicate the observed life-cycle portfolio allocation differences between renters and homeowners.

5.2 Model Without Housing Investment

Next, we compare the aforementioned model to the same model with no housing investment (NHI) in order to analyze and measure the effects of housing investment on portfolio allocation. First, it is worth making clear the main features of the NHI model. It includes all features of the main model except those related to housing investment: there is no housing investment, house price risk, mortgage payments and homeownership status. All other parameter values are the same, including RRA and EIS. Households make decisions on consumption expenditure and on investment in financial markets (riskless and risky assets). Table 5 shows the life-cycle portfolio allocation profiles of households for

Table 4: Life-Cycle Profiles - Homeowners vs Renters

	23-35	36-50	51-65	66-80	81+
Homeowners					
Risky Asset	0.580	0.710	0.723	0.416	0.486
Risky Asset / Financial Asset	0.684	0.685	0.658	0.437	0.444
Risky Asset Participation Rate	0.787	0.834	0.812	0.539	0.538
Renters					
Risky Asset	0.408	0.223	0.207	0.316	0.650
Risky Asset / Financial Asset	0.403	0.192	0.161	0.323	0.418
Risky Asset Participation Rate	0.499	0.275	0.247	0.421	0.503

Table 5: Life-Cycle Profiles - No Housing Investment

Age	20-35	36-50	51-65	65-80	81+
Risky Asset	1.055	3.231	4.054	3.819	2.167
Risky Asset / Financial Asset	0.758	0.977	0.948	0.950	984
Risky Asset Participation Rate	0.771	1	1	1	1

the NHI model.

There are significant changes in life-cycle wealth allocation profiles between these two models. It is clear that housing investment has a strong crowding out effect on investment in the risky asset during the life-cycle. For instance, during the first 15 years, due to housing investment, the portfolio share of the risky asset is only 42% while the same portfolio share is about 76% in the absence of housing investment. Similarly, the NHI model predicts that for households after their 35's, the risky asset share stays above 90%, while the baseline model predicts that the risky asset constitutes at most 75% of financial investment. The gap between the prediction of the models on the share of the risky asset overtime shows an increasing trend.

A similar crowding out effect is observed in the risky asset participation rate. The participation rate in the NHI model shows that after the first 15 years, all households have investment in the risky asset whereas the highest participation rate in the baseline model reaches 75% in 51-65 age interval. Figure 2 visually depicts the crowding out effect of housing investment on risky asset investment and households participation rate during the life-cycle.

One main reason for observing a long lasting crowding out effect of housing investment is the duration of mortgage debt. Most of the households become homeowners and begin to pay their mortgage debt between the ages of 20-30. They pay back the mortgage over 25 years. When reaching ages around 50, a majority of households are homeowners with less mortgage debt. They can then make investments in financial assets without much concern about their housing investment because of very low or no mortgage debt. However, we also should take into account the fact that some

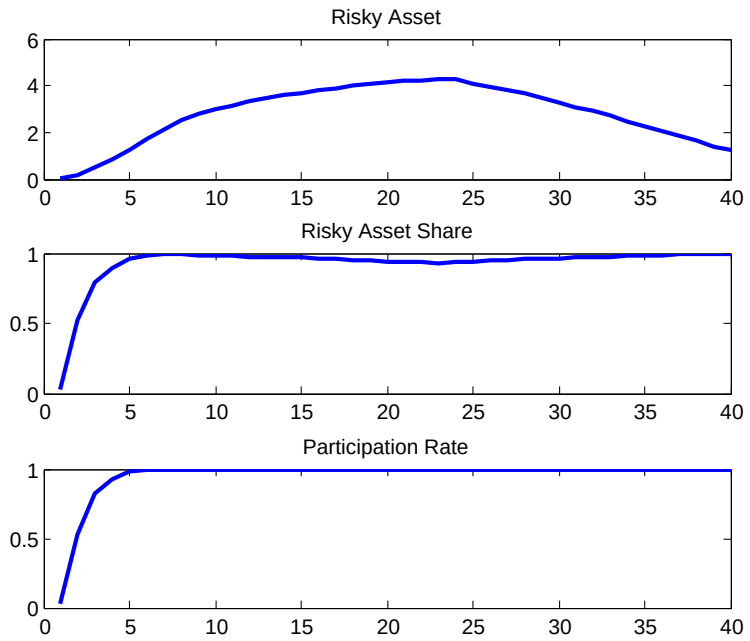

Figure 4: Life-Cycle Profiles - No Housing Model

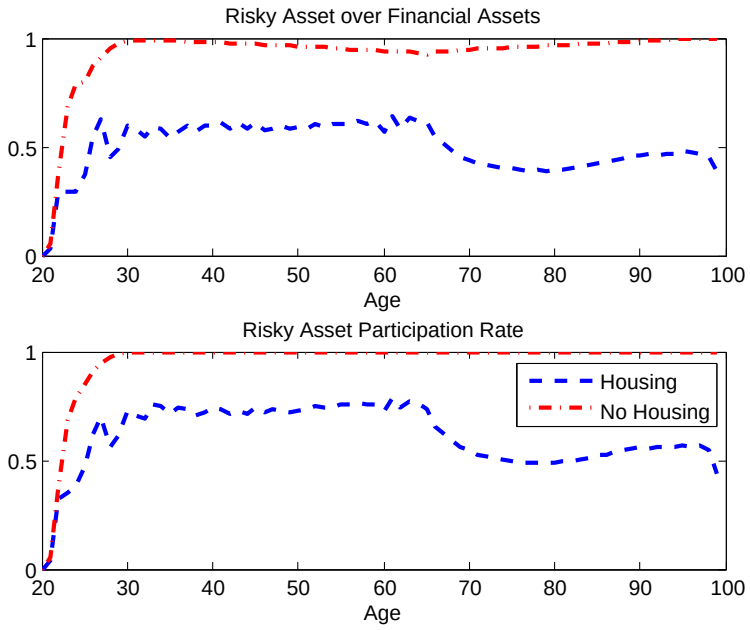

Figure 5: Life-Cycle Risky Asset Investment and Participation Rate

households may want to sell their relatively small house buy a larger house. Hence, the negative effects of mortgage debts on financial asset investment could last to extended periods.

5.3 Comparison of the Data and the Model with Housing

In this part of the paper, we investigate the extent to which the portfolio allocation profiles in the model match the data. We use all PSID waves that contain information about households financial wealth. These waves are 1984, 1989, 1994, 1999, 2001, 2003, 2005, and 2007. The data collection process and the definition of each variable is explained in detail in the discussion covering the estimation of the model.

Table 6 displays the life-cycle portfolio shares in the risky asset, the risky asset participation rate, and the homeownership rate obtained from the PSID data. The model slightly over-estimate the risky asset investment in levels but it closely match the risky asset share over the life-cycle. It underestimate the risky asset share only about 10% for the first 15 years, but the estimates are very close for the rest of the life-cycle. In many previous studies of life-cycle portfolio allocation models, the portfolio shares invested in the risky asset were always significantly higher than the empirical evidence. For example, see Campbell, Cocco, Gomes, and Maenhout (2001), Cocco (2004), Cocco, Gomes, and Maenhout (2004), Gomes and Michaelides (2007), Hu (2005), Yao and Zhang (2005) among many. The gap between the predictions of life-cycle models and the empirical evidence is called the "portfolio allocation puzzle" which is a kind of flip side of the "equity premium puzzle" of Mehra and Prescott (1985). Comparing the results in Table 3 with the results in Table 6 we can see that the model is doing a successful job in addressing this puzzle.

Furthermore, the model predicts moderate levels of the participation rate during the life-cycle. The participation rate is higher than the empirical counterpart but the shape of the participation rate is similar to the empirical counterpart. On the other hand, from Tunc (2012) we know that the participation rates in the Survey of Consumer Finances (SCF) data are much higher than in the PSID data. The participation rates for the five age groups used through out this paper are 0.57, 0.67, 0.75, 0.74, and 0.68. These rates don't differ much from the model predictions.

When compared to the PSID data, the model slightly overestimates life-cycle homeownership

Table 6: Life-Cycle Profiles - PSID Data

Age	20-35	36-50	51-65	66-80
Risky Asset	0.1023	0.3985	0.6788	0.6320
Risky Asset / Financial Asset	0.5227	0.5944	0.6027	0.5182
Risky Asset Participation Rate	0.2689	0.4020	0.4631	0.4405
Homeownership Rate	0.3570	0.6426	0.7285	0.7314

rates. The homeownership rate during the first 15 years is 28% in the baseline model while it is about 35% in the data. This rate increases dramatically for the next 15 years to 80% in the model. The gap between the model predictions and the empirical evidence on homeownership rate stays above 20% for the rest of the life-cycle.

5.4 Comparative Statics

In this section we discuss the effects of some of the parameters of our model on the life-cycle portfolio allocations. Specifically, we investigate the effects of house price risk, the size of the down payment and the size of the entry cost. We also compare the life-cycle wealth allocation profiles of homeowners and renters in order to see if these profiles are different.

The baseline model assumes that house prices are affected by stochastic shocks. In order to analyze the effects of these shocks on households' portfolio allocation decisions, we set the variance of these shocks to zero. Thus, the growth rate of the house price is constant at 1.9% per year. Table 7 shows the life-cycle portfolio allocation and homeownership rate of households when there is no house price risk. Comparing Table 3 to Table 7, we see that a riskless constant return on housing investment leads households to invest more in housing. Furthermore, a safer housing investment leads households to hold a relatively more aggressive position in the risky asset. Households increase risky asset investment both in levels and in shares of financial assets. Finally, not surprisingly the homeownership rates are generally lower in the baseline case than in the case with no house price

Table 7: Life-Cycle Profiles - No House Price Risk

	20-35	36-50	51-65	65-80	81+
Risky Asset	0.412	0.800	0.789	0.664	366
Risky Asset / Financial Asset	0.523	0.650	0.698	0.530	368
Risky Asset Participation Rate	0.583	0.797	0.866	0.714	601
Homeownership Rate	0.293	0.960	0.969	0.981	920

risk. Housing is now a more appealing asset because households receive the same expected return without any uncertainty.

Next, we investigate the effects of the size of the down payment rate on the life-cycle wealth allocation profiles of households. Table 8 displays life-cycle profiles of households for different rates of down payment. The homeownership decision is affected by the size of the down payment for the whole life-cycle but especially for the first 15 years. As the size of the down payment decrease from 30% to 0%, more households become homeowner for all age groups. This is especially true for the youngest groups.

The effects of the size of the down payment rate on risky asset investment and on the participation rate are mixed. Households generally reduce their risky asset investment and risky asset participation rate when the down payment rate decreases from 30% to 10%. However, they increase the share of risky asset and the participation rate when the down payment rate is reduced from 10% to 0%.

The baseline case assumes that households pay a one-time fixed entry cost the first time they decide to invest in the risky asset. Similar to Alan (2006), Gomes and Michaelides (2007), Guvenen (2009b), and Guo (2004), this cost is a proportion of households annual labor income. In this part, we want to analyze the impact of this entry cost on households financial investment decisions. Table 9 displays the life-cycle risky asset share, fixed cost payment rate, and risky asset participation rate for different levels of entry cost. The size of the entry cost has an impact on households risky asset share and their participation rate for the first 15 years only. We don't observe any sizable change in

Table 8: Life-Cycle Profiles for Different down payment Rates

	d	20-35	36-50	51-65	66-80	81+
Risky Asset	0.3	0.404	1.022	1.077	0.640	0.595
	0.2	0.398	0.612	0.671	0.425	0.488
	0.1	0.255	1.013	1.569	0.637	0.521
	0	0.339	1.877	2.522	1.939	1.190
Risky Asset/Financial Asset	0.3	0.369	0.665	0.709	0.501	0.481
	0.2	0.420	0.588	0.605	0.445	0.440
	0.1	0.303	0.465	0.534	0.292	0.271
	0	0.256	0.47	0.572	0.346	0.322
Risky Asset Participation Rate	0.3	0.444	0.775	0.828	0.601	0.568
	0.2	0.514	0.726	0.753	0.549	0.534
	0.1	0.400	0.577	0.650	0.389	0.348
	0	0.322	0.566	0.649	0.412	0.374
Homeownership Rate	0.3	0.186	0.759	0.868	0.929	0.967
	0.2	0.277	0.803	0.891	0.940	0.957
	0.1	0.419	0.897	0.979	0.980	0.976
	0	0.728	0.999	1.000	0.997	0.999

households risky asset investment, participation rate for the remaining life-cycle. Furthermore, there is no impact of this cost on households homeownership decision. The main reason for this conclusion is probably the assumption that this cost is incurred only once. Hence, we don't expect to see large or life-time impact of this cost on households financial investment decision.

5.5 Results for Additive Utility

We next investigate the impact of using EZ's recursive utility instead of the more traditional additive utility functions. In the Epstein-Zin preferences, if we set $\gamma = 1/\psi$ (the relative risk aversion being equal to the inverse of the elasticity of intertemporal substitution), then it reduces to a CRRA utility function. We can first test the hypothesis $H_0 : \gamma = 1/\psi$ against $H_1 : \gamma \neq 1/\psi$ to see if our data is informative enough to be able to reject CRRA utility. We can do so by computing a t statistic, $t = (\hat{\gamma}\hat{\psi} - 1)/se(\hat{\gamma}\hat{\psi})$, where the standard error $se(\hat{\gamma}\hat{\psi})$ is computed with the delta method. We find that $se(\hat{\gamma}\hat{\psi}) = 0.1731$ and $t = -2.264$. Using a 5% significance level, we can reject $H_0 : \gamma = 1/\psi$ and the assumption of CRRA utility.

Below, we report lifetime profiles for four natural cases of the CRRA utility function. In the first case, we re-estimate the parameters of the model with CRRA restriction. The estimated parameters in this case are $\gamma = 1.06$ and $\psi = 0.9434$ respectively. The aim of this case is to see the contribution of disentangling the risk aversion from the inverse of intertemporal substitution. In other words, we want to see the contribution of having EZ-type recursive preferences on households life-cycle portfolio allocation. In the second case, we set the risk aversion parameter γ as the inverse of the EIS parameter, ψ, estimated in EZ case. Since the estimate of the EIS parameter is 0.8, the risk aversion parameter $\gamma = 1/0.8 = 1.25$. In this case, we want to see what would happen if we keep ψ constant at its estimated value and have a larger γ through imposing the CRRA restriction. In the third case, however, we set the EIS parameter, ψ as the inverse of γ estimated in EZ case. In this case, $\gamma = 0.76$ and $\psi = 1/0.76$. The third case keeps γ constant at its estimated value and imposes a relatively high value to ψ to see the effects of having a larger intertemporal substitution parameter. In the final case γ is set to 10. A reasonably high value, which is consistent with many papers in the literature such as in Bansal and Yaron (2004) where γ is 9.5 and in Vissing-Jorgensen and Attanasio (2003) where γ is

Table 9: Entry Cost

	Entry Cost	20-35	36-50	51-65	66-80	81+
Risky Asset	0.15	0.304	0.611	0.674	0.425	0.489
	0.12	0.340	0.611	0.674	0.425	0.491
	0.08	0.382	0.612	0.673	0.426	0.489
	0.04	0.398	0.610	0.671	0.425	0.489
	0.00	0.388	0.590	0.654	0.417	0.479
Risky Asset/Financial Asset	0.15	0.294	0.589	0.607	0.445	0.441
	0.12	0.335	0.588	0.606	0.445	0.442
	0.08	0.398	0.589	0.606	0.445	0.441
	0.04	0.420	0.586	0.604	0.444	0.441
	0.00	0.404	0.566	0.589	0.436	0.431
Risky Asset Participation Rate	0.15	0.360	0.725	0.754	0.548	0.535
	0.12	0.406	0.725	0.754	0.548	0.536
	0.08	0.485	0.727	0.753	0.549	0.535
	0.04	0.512	0.724	0.752	0.548	0.535
	0.00	0.490	0.698	0.733	0.537	0.523
Homeownership Rate	0.15	0.280	0.818	0.894	0.940	0.956
	0.12	0.279	0.813	0.894	0.940	0.957
	0.08	0.276	0.807	0.892	0.940	0.957
	0.04	0.280	0.804	0.891	0.940	0.957
	0.00	0.279	0.807	0.895	0.941	0.957

Table 10: Life-Cycle Profiles - CRRA with $\gamma = 1.06$ and $\psi = 0.9434$

Age	20-35	36-50	51-65	66-80	81 +
Risky Asset	0.360	0.587	0.593	0.397	0.382
Risky Asset / Financial Asset	0.368	0.606	0.615	0.426	0.408
Risky Asset Participation Rate	0.441	0.755	0.781	0.536	0.514
Homeownership Rate	0.274	0.899	0.957	0.929	0.889

between 5 and 10. In this case, we want to see what would happen to households portfolio allocation decision in CRRA utility case with sufficiently high RRA parameter value.

Table 10 displays the life-cycle profiles for the first case where the model re-estimated with the CRRA restriction. Comparing Table 10 to the baseline case in Table 3, it is clear that the life-cycle profiles are not very different. Both in EZ recursive and the CRRA cases households behave very similarly. Households hold similar amounts and shares of risky assets, have very similar participation rates and their homeownership rates don't differ dramatically. The only difference comes from the estimated parameters. While in the formar one $\gamma = 0.76$ and $\psi = 0.80$, in the latter one they are 1.06 and 0.9434 respectively. However, as mentioned above, the beuity of EZ recursive case is that it enables us to disentangle the risk aversion from the elasticity of intertemporal substitution. Hence, although we have similar results under both cases, the flexibility of EZ recursive case is from an economic point of view more meaningful.

Table 11 displays the life-cycle profiles for the case when the RRA, γ, is set as the inverse of the estimated EIS, ψ. The results in this case don't depart as much from the results of the baseline model. For most ages, the share of investments in the risky asset and the fraction of households investing in the risky asset are somewhat lower than for the baseline model. The homeownership rate don't differ much from the baseline case.

However, when the EIS, ψ, is set as the inverse of the RRA, γ, the results are quite different. In this case, ψ above the reasonably high value generally agreed in the lirtature. As shown in Table 12

Table 11: Life-Cycle Profiles - CRRA with $\gamma = 1/\psi$ ($\gamma = 1/0.8$, ψ=0.8)

Age	20-35	36-50	51-65	66-80	81+
Risky Asset	0.332	0.550	0.584	0.373	0.375
Risky Asset / Financial Asset	0.346	0.570	0.582	0.412	0.408
Risky Asset Participation Rate	0.426	0.726	0.746	0.510	0.501
Homeownership Rate	0.247	0.870	0.914	0.902	0.888

Table 12: Life-Cycle Profiles - CRRA with $\psi = 1/\gamma$ ($\gamma = 0.76$, $\psi = 1/0.76$)

Age	20-35	36-50	51-65	66-80	81+
Risky Asset	0.021	0.3012	0.529	0.428	0.395
Risky Asset / Financial Asset	0.019	0.280	0.494	0.324	0.307
Risky Asset Participation Rate	0.024	0.346	0.614	0.396	0.389
Homeownership Rate	0.345	0.981	0.988	0.971	0.954

households have less wealth in risky asset in levels and in total share of financial assets. Similarly, the participation rate is also relatively lower than the baseline case. The homeownership rate, on the other hand, is higher throughout the life-cycle than for the baseline model.

Next, we can see the following changes for the case where the utility function is CRRA with a risk aversion parameter of 10. (Table 13). Compared to the baseline model with EZ preferences, we see an increased fraction of households buying a house in the earlier part of their life. This impact is especially strong for the age 20-35 interval where the homeownership rate goes from 28% for EZ to 80% for CRRA with $\gamma = 10$. Correspondingly, the fraction of households investing in the risky assets is much lower and it's not until their 50's that households with CRRA ($\gamma = 10$) will reach the level of households with EZ preferences (and similar risk aversion). Even for ages where the participation

Table 13: Life-Cycle Profiles - CRRA with $\gamma = 10$, $\psi = 1/10$

Age	20-35	36-50	51-65	66-80	81 +
Risky Asset	0.345	0.751	1.786	2.175	1.278
Risky Asset / Financial Asset	0.106	0.086	0.121	0.321	0.412
Risky Asset Participation Rate	0.283	0.499	0.613	0.839	0.875
Homeownership Rate	0.805	0.930	0.790	0.777	0.934

rate catches up the level obtained with EZ preferences, the shares of investments in the risky assets are all well below the baseline case.

6 Conclusion

In this study, we focus on the effects of housing investment on portfolio allocation in a life-cycle model. The importance of housing investment for households is high since housing constitutes an important part of households' portfolios. In order to fully understand the effects of housing investment on portfolio allocation, we developed a fairly comprehensive life-cycle asset allocation model where many important features of housing investment are taken into account. First, endogeneous homeownership and house size decision, stochastic house price process, annual mortgage interets and principal payments, depreciation and maintenance expenses of houses all are related to housing. Second, we use EZ recursive preferences to disentangle relative risk aversion from the elasticity of intertemporal substitution. Third, households are expected to pay fixed entry cost for the first time to invest in stock market. Fourth, households supply labor inelastically and receive a stochastic labor income. Finally, we introduce a bequest motive structure through which households bequest their wealth to inheritors when thay pass away anytime.

Housing is both an investment asset which enables owners to hold home equity, and a durable consumption good from which households derive utility. We show that housing investment has a

strong crowding out effect on investment in risky assets and this effect is larger for young and middle-aged households. Early in life, households are willing to be homeowners by paying a down payment and annual mortgage payments. This keeps their liquid wealth at low levels so they refrain from paying the fixed entry cost to invest in risky assets. Hence, owner-occupied housing is a substitute for investment in risky assets. Even after accumulating enough wealth to pay the fixed cost and begin investing in risky assets, the portfolio's share of risky assets is still at lower levels than predicted by the model with no housing investment. We also analyze the differences in life-cycle asset allocation between homeowners and renters. The results show that homeowners hold a significantly larger share of their financial wealth in the risky asset. Furthermore, they have higher risky asset participation rates than renters.

There are many features of housing investment that one can analyze. We investigate the effects of some of the features we believe are the most important. We found that, in the absence of house price risk, households invest more in housing, the homeownership rates go up. Similarly the share of investment in the risky assets goes up because the riskless housing investment leads households to be relatively more aggressive in risky asset investment and hence increase the share of risky asset and the participation rate. As expected a decrease in the size of the down-payment increase the homeownership rate significantly. The effects are more prominent particularly for young households.The results also show that a large down payment rate has some mixed effect on investment in the risky asset, on the risky asset participation rate.

Some extensions of the model for future research include focusing more on the real estate side of the model by analyzing the size and the effects of the liquidation cost, introducing an exogenous mandatory moving and selling of houses (job relocation, old age, health issues, etc.), and allowing households to default on their mortgage. Furthermore, estimating the size of the fixed entry cost and introducing more realistic house price dynamics are left for future research.

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Appendix

We begin by discretizing the state space and variables over which the choices are made with equally spaced grids. The density functions of the random variables (i.e. shocks to labor income process, shocks to return on risky asset, and shocks to return on housing investment) are approximated by the Gaussian quadrature method of Tauchen and Hussey (1991).

In period $T+1$, the policy functions are determined by the bequest motive. The value function in this period coincides with the utility function, which is the bequest function. In every period prior to $T+1$, we obtain the utility function for different combinations of housing, consumption, and other state and choice variables. Then the value function for a typical time t is equal to the utility function of that period plus the discounted expected continuation value $(E_t\left[V_{t+1}\right])$. If the continuation value

doesn't lie on the state space grid, we compute the value function using cubic spline interpolation. This backward induction process is iterated from age T to 1.

Once we compute the value function of all the alternatives, we choose the one that maximizes the value function over all choice variables. The optimum policy rules for consumption, housing, and investment in financial assets correspond to ones that maximize the value function. At each point in the state space, the risky asset participation decision is done by comparing the value function conditional on having paid the fixed cost with the value function conditional on having not yet paid the fixed cost. Similarly, the homeownership decision (e.g. house buying or selling decision) is done by comparing the value function conditional on being a renter with the value function conditional on being a homeowner. In both comparisons, adjustments for the payment of the fixed cost of risky asset participation and costs accrued from buying/selling a house (e.g. down payment, annual mortgage payment, liquidation cost etc.) are taken into account respectively.

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