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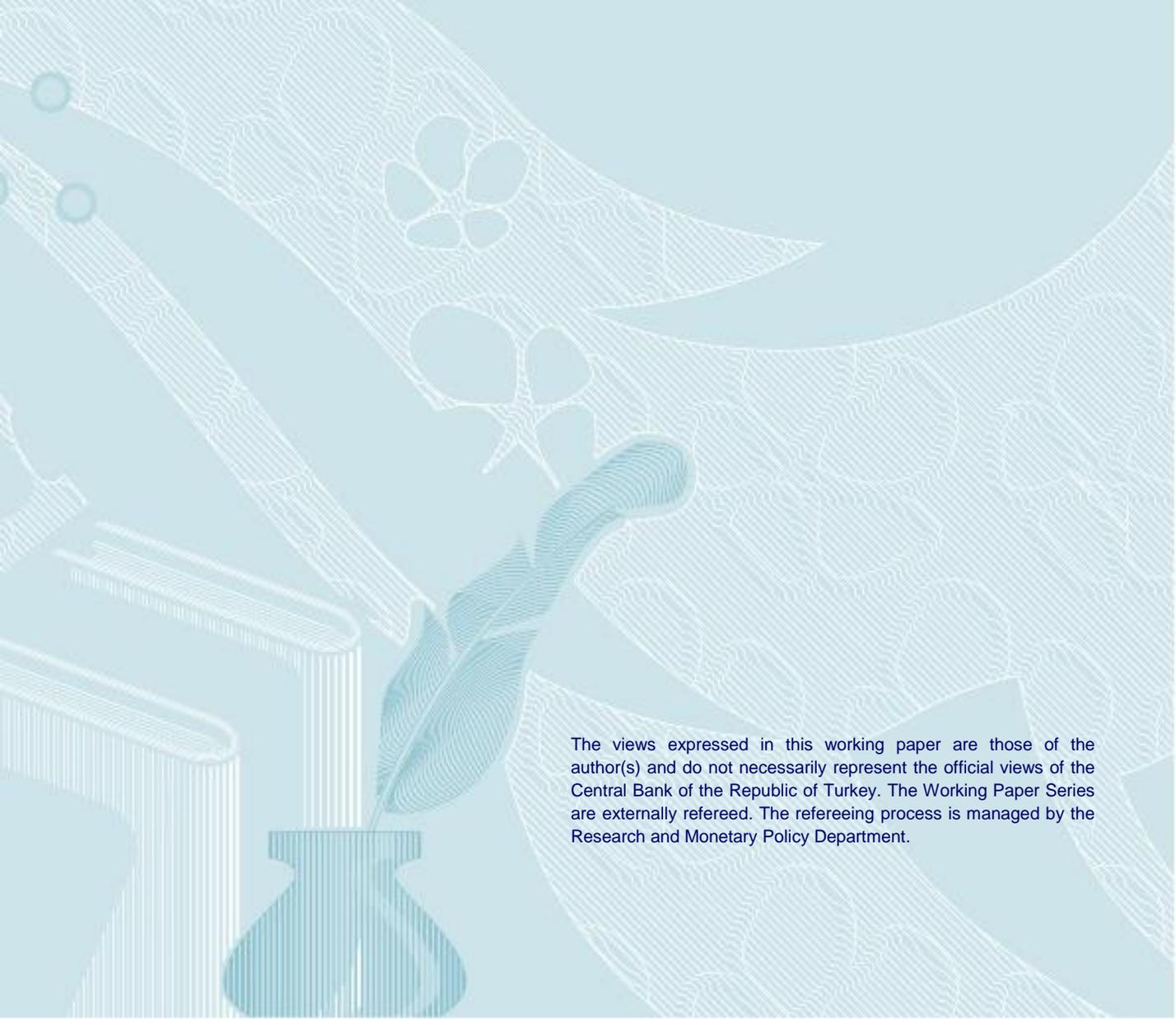
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# Imported Intermediate Goods and Product Innovation: Evidence from India

Murat Şeker Daniel Rodriguez-Delgado and Mehmet Fatih Ulu\*

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## Abstract

In this study, we build a structural model of multi-product firms that illustrates how access to new foreign intermediate goods contributes to product innovation. We establish a stochastic dynamic model of firm evolution allowing firms to be heterogeneous in their efficiency levels. Through introducing importing decision to this dynamic framework, we show that the effects of importing intermediate goods are twofold: i) it increases the revenues per each product created and ii) through the knowledge spillovers obtained from importing, firms become more likely to introduce new varieties. Calibration of the model to Indian data shows that the model can successfully explain the dynamics of product evolution and other moments related to importing and product distribution. Finally the comparison of autarky with trade equilibrium shows how liberalizing trade increases innovation performances and product growth.

JEL Classification: F12, F13, L11, O31

Keywords: Firm dynamics, heterogeneous firms, innovation, endogenous product scope, importing intermediate goods, trade liberalization, Indian manufacturing sector.

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# 1 Introduction

An important role of international trade is the exposure of firms to new goods. The purpose of this study is to build an analytical framework to illustrate the potential gains in an economy that international trade can generate as new products are introduced resulting from the access to new foreign intermediate goods. We illustrate this mechanism through a trade liberalization episode. We show how aggregate innovation rates as well as individual firm's dynamics change after the liberalization period.

The process through which international trade expands the set of traded goods has been the subject of recent studies. Kehoe and Ruhl (2009) find that trade liberalization episodes are characterized by a significant increase in the traded volume of goods not previously traded. They further document that such role of the extensive margin of trade seems proper of structural changes in the economy (e.g. trade liberalization episodes) and absent from higher frequency events like business cycles. Arkolakis (2010) derives a similar result within his model of heterogeneous firms and costly access to trade. In our framework, trade brings access to new intermediate goods previously unavailable to firms, provided they find it profitable to pay the fixed costs associated with international trade.

At the firm level, Goldberg et al. (2010b) establish a robust relationship between access to new foreign intermediate inputs and the introduction of new products by domestic firms. Using data for India, they find that 31 percent of the expansion of products by firms could be explained by declining input tariffs. Similar to their empirical specification, at the center of our theoretical framework is a set of heterogeneous multi-product firms that endogenously decide on investing in the development of new product varieties. In our model access to new intermediate inputs increase firms' revenues of existing products and they also improve firms' innovation technology.

Klette and Kortum (2004), Lentz and Mortensen (2008), Luttmer (2011), and Şeker(2012) have studied firm size and industry dynamics where multi-product firms could innovate. Bernard Redding, and Schott (2010, 2011) explained several attributes of multi-product firms at the detail of firm-product level. The findings of Goldberg et al. (2010b) and their other accompanying work have highlighted the link between imported intermediates, and firms product scope. In this paper, we fill the missing link by explaining the interaction between innovation, and im-

ported intermediates for multi-product firms. Imported intermediates boost innovative effort through two channels in our setup. The first one is more standard love of variety in production technology<sup>1</sup>. The second and more novel channel is the spillover effect of imported varieties on firm innovative activities.

The concept of innovation used throughout the paper is one of horizontal product innovation; that is, an innovation or discovery consists of the knowledge required to manufacture a new final good that does not displace existing ones. This type of framework is a natural complement to models of vertical innovation where the “creative destruction” process makes previous products obsolete. Horizontal innovation models have been extensively used in growth theory, where Romer (1990) and Grossman and Helpman (1991) represent seminal studies. Furthermore, it has been applied in trade theory to explain wage inequality, cross-country productivity differences among other topics (see Gancia and Zilibotti (2005)). Among disaggregated firm-level models, Luttmer (2011) and Şeker (2012) are relevant references. Both of these papers follow from Klette and Kortum (2004) which present a stylized model that explains the regularities in firm and industry evolution. In their model, an establishment is defined as a collection of products and each product evolves independently. Every product owned by an establishment can give rise to a new product as a result of a stochastic innovation process or can be lost to a competitor. This birth and death process of the products is the source of firm evolution. Through this model of innovation, they explain various stylized facts that relate R&D, productivity, and growth. The extension introduced by Şeker (2012) allows exogenous heterogeneity in firms’ efficiency levels which provides a better fit to the data on explaining firm size distribution as well as correlations between size, age, exit and firm growth<sup>2</sup>. Our basic framework, which draws extensively from Şeker (2012), is meant to illustrate the role of international trade in the introduction of new products. It is also relevant to mention that in contrast to frameworks of learning externalities, ours is one in which firms fully internalize the dynamic consequences of their innovation efforts.

In our framework international trade affects both the marginal benefit and the marginal cost of innovation efforts. As in Şeker (2012), each firm decides how much resources to invest

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<sup>1</sup>Kasahara and Lapham (2013), Halpern et al (2006), Gopinath and Neiman (2013) are among several others that have highlighted this channel.

<sup>2</sup>Lentz and Mortensen (2008) present a model that also introduces heterogeneity in firms’ innovation capacities. See Şeker (2012) for a comparison of both models.

in innovation, where a higher investment increases the probability of successfully introducing a new product and consequently enjoying a new stream of revenue. In the model, higher variety of intermediate goods benefits a given firm with larger returns to scale (productivity) both in its existing products as well as in any new product. Within the empirical literature, Kasahara and Rodrigue (2008) find a robust and significant increase in productivity among Chilean firms that import intermediate goods. Halpern et al. (2011) find that among Hungarian firms, such gain comes mostly from increased variety of intermediates and to a lesser extent from quality improvements.

In our model exposure to foreign intermediate goods reduces, *ceteris paribus*, the cost of innovation. As firms learn from the knowledge embodied in such goods, they could obtain a higher probability of success investing the same amount of resources. Empirical support for this assumption can be found in Goldberg et al. (2010b). They find that: 1) firms that faced stronger input tariff reductions were found, *ceteris paribus*, more likely to introduce new products; 2) such firms were more likely to invest in R&D; and 3) the main channel of these effects was through new varieties of intermediate goods and not through the trade expansion of previously traded ones.

A central element of our analysis is the heterogeneity among firms. Besides their exogenous component of productivity level, firms also differ in their portfolio of goods currently in production. A seminal work in the area of heterogeneous firms and trade is Melitz (2003). In his framework, more productive firms are self-selected into export markets as only they find it profitable to pay the fixed costs associated with exporting. In our study, we focus on the importing side of trade rather than exporting. However the mechanism is quite similar to the one presented in Melitz (2003). Firms face fixed costs of importing. Only the efficient ones can compensate these costs and import goods. Our analysis also highlights that trade liberalization can have a redistribution effect, where some firms could end worse-off (lower profits) after the trade liberalization episode even if they import intermediate goods. We further document a similar redistribution of innovation efforts toward most productive firms away from the least productive. The reallocation of the non-reproducible factor of production, labor, is central for this redistribution effect.

In the model, firms own multiple products and their evolution is roughly determined by the

sum of the evolution of each of their products. In this respect, the model complements several existing models explaining product scope. Bernard, Redding, and Schott (2010,2011) provide empirical evidence on how multi-product producers dominate total production in the U.S. economy<sup>3</sup>. Contribution of firms' product margin towards output growth significantly exceeds the contribution of entry and exit. They also construct a static model of multi-product firms and analyze their behavior during trade liberalization. They introduce two margins (intensive and extensive) of growth, and these margins are positively correlated with each other. However, their model lacks a dynamic framework of firm evolution. The novelty of our study is its ability to analytically explain the evolution of firm's products through a structural model which we test through fitting it to the data on product dynamics and how this relates to importing decision.

The remainder of the paper is organized as follows. Section 2 describes the model economy in two main parts: 1) selection of the mix of intermediate goods and labor, as well as firm's decision whether to import intermediate goods from abroad; 2) the selection of the optimal level of innovation. Section 3 describes the data source and section 4 presents the quantitative model and the calibration exercise. Section 5 concludes. We further present in the appendix, detailed derivations of some of the key equilibrium conditions of the model.

## 2 Model

Recent empirical evidence presented in Goldberg et al. (2009, 2010b) show that imported intermediate goods lead to higher innovation rates of new products and hence faster growth. In this study, we present a structural model of firm evolution that can explain this evidence as well as other stylized facts on the dynamics of multi-product firms. Following Klette and Kortum (2004) and their extensions by Lentz and Mortensen (2008) and Şeker (2012) we introduce a model of firm growth that accounts for the heterogeneity in producers' efficiency levels as well as entry and exit decisions. In introducing the heterogeneity in efficiency levels, we follow Melitz (2003). There is a continuum of final good producers each of which produce a different variety. Unlike Melitz (2003), here firms produce multiple products. The dynamic nature of the model with heterogeneous firms allows us to capture the differences in the evolution of importers and

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<sup>3</sup>Some other studies on multi-product firms are Nocke and Yeaple (2006), Eckel and Neary (2010), and Baldwin and Gu (2009).

non-importers and contribution of importing to new product creation and consequently the aggregate growth.

The model is presented in a general equilibrium framework. First, we discuss the demand side of the economy. Then following Kasahara and Lapham (2013), we introduce the static profit maximization problem of producers. We incorporate this static problem into a dynamic framework to discuss the growth patterns of firms distinguishing the differences between importers and non-importers.

## 2.1 Consumers

Consider an economy with  $N + 1$  identical countries. In a continuous time setup, each country consists of a representative consumer with an intertemporal utility function given as

$$U_t = \int_t^\infty e^{-\rho(\tau-t)} \ln C_\tau d\tau,$$

where  $\rho$  is the discount rate and  $C_t$  is the aggregate consumption of the composite good at time  $t$ . Instantaneous utility obtained from consumption at time  $t$  is  $\ln C_t$ . The consumer is free to borrow or lend at the interest rate  $r_t$ . Aggregate expenditure at time  $t$  is  $E_t = P_t C_t$  where  $P_t$  is the price of the composite good. The optimization problem of the consumer yields  $\frac{\dot{E}}{E} = r_t - \rho$ . We let total expenditure be the numeraire and set it to a constant for every period.

In each country there is a mass of available products each of which is indexed by  $j$ . The total mass of products is represented by  $J$ . Consumers have a taste for variety and consume  $y_t(j)$  units of variety  $j$ . Goods are substitutes with elasticity of substitution  $\sigma > 1$ . The composite good is determined by the following constant elasticity of substitution aggregator.

$$C_t = \left( \int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

Composite good producer makes zero profit. Solution of this problem yields

$$y_t(j) = C_t \left( \frac{p_t(j)}{P_t} \right)^{-\sigma}.$$

Price of the composite good  $P_t$  can be found as follows

$$\begin{aligned}
 P_t C_t &= \int p_t(j) y_t(j) dj \\
 P_t &= \left( \int_{j \in J} p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.
 \end{aligned} \tag{2}$$

## 2.2 Intermediate Goods Producers

In the intermediate goods sector, firms are perfectly competitive. All firms have identical linear technologies and have the same productivities which are set as one. Labor is the only input used in the production. There is a unit continuum of domestic intermediate goods produced within a country. There is also free entry in this sector. Each intermediate good producer is a price taker hence price of these products equals to marginal cost which is the wage rate  $w$ . Interaction among the intermediate goods producers (and between intermediate and final goods producers, as explained below) happens through the labor market as they compete for the same resource.

## 2.3 Final Goods Producers: Import Decision

In this section, we solve the producer's decision problem regarding its importing status. Producers are characterized by their efficiency level  $\varphi$ , the number of products in their portfolio  $n$  and by their importing status:  $m$  if they decide to import some of their intermediate goods or  $h$  if they decide not to import intermediate goods. There are two main decisions a given firm takes, 1) whether to import or not, and 2) the amount of resources to invest in innovation, having as its ultimate goal to maximize the present discounted value of profits,  $rV_\varphi(n)$ . We assume that firms decide whether to import when they introduce their first product and that such decision is permanent. This assumption is analogous to deciding to import in every period for every product variety produced.

Our specification of the production function follows from Kasahara and Lapham (2013) which extends Melitz (2003) by incorporating importing decision to the firm's optimization problem. The difference in our model is that here firms produce multiple products. The final goods sector is combined of a continuum of monopolistically competitive firms producing

horizontally differentiated goods. The production of each variety requires employment of labor and intermediate goods which might be either domestically produced or imported. Producers of the final goods are distinguished from each other by their efficiency levels, indexed by  $\varphi > 0$  which is randomly drawn from a continuous cumulative distribution  $F(\varphi)$ . As in Melitz (2003) higher efficiency level means producing a symmetric variety at a lower marginal cost. We assume that efficiency levels exogenously grow at rate  $g$  for all firms. Solution of the monopolistic competition model yields revenue  $r(\varphi)$  and profit  $\pi(\varphi)$  from each product  $j$  as follows<sup>4</sup>

$$\begin{aligned} r(\varphi) &= p(\varphi)y(\varphi) = E\left(\frac{p(\varphi)}{P}\right)^{1-\sigma} \quad \text{for } \forall j \in J \\ \pi(\varphi) &= \frac{r(\varphi)}{\sigma}. \end{aligned} \quad (3)$$

Each product is produced by a single producer, hence we can write the equilibrium levels of revenue and profit from each product as a function of firm's efficiency level  $\varphi$ .

Firms employ labor and intermediate goods in the production of final goods. We follow the macro-growth and trade literature and assume that there is increasing returns to variety in intermediate goods. All intermediate goods enter symmetrically in the production function, implying that none of them is intrinsically better or worse than any other. Similar approach has been used in Ethier (1982) and Romer (1987) among many others<sup>5</sup>. This specification is consistent with the empirical findings in Amiti and Konings (2008), Halpern et al. (2011), and Kasahara and Rodrigue (2008) which associate the use of foreign intermediate goods with higher productivities.

Importing firms incur a fixed cost  $f_m$  and iceberg transport cost  $\tau > 1$ . Let  $I \in \{0, 1\}$  refer to the import status of the firm. Production function for final goods is given as

$$y(\varphi, I) = \varphi l^\alpha \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj' \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}, \quad (4)$$

where  $l$  shows the labor,  $q_d(j)$  shows the domestic and  $q_f(j)$  shows the imported intermediate goods employed in production and  $I$  is equal to one if the firm imports. The output elasticity

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<sup>4</sup>Since in this section we only solve for firm's static optimization problem, we exclude the time subscript  $t$  from the variables for brevity.

<sup>5</sup>See Kasahara and Lapham (2013) for a discussion of the benefits of using this specification.

of labor in production is represented by  $0 < \alpha < 1$  and the elasticity of substitution between intermediate inputs is  $\gamma > 1$ .  $N$  is the number of partner countries that are traded with.

The solution of the final good producer's static optimization problem is given in the appendix. In the solution we get  $q_d(j) = q_d$ ,  $q_f(j) = q_f$  for  $\forall j$  and  $q_f = \tau^{-\gamma} q_d$ . This finding simplifies the production function to

$$y(\varphi, I) = \varphi \left(1 + N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}} l^\alpha [q_d + IN\tau q_f]^{1-\alpha}.$$

Here  $\varphi \left(1 + N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}}$  could be interpreted as a total factor productivity term. Price of a final good which only uses domestic intermediate goods is  $p^h(\varphi)$  for a  $\varphi$ -type producer. This price is found as

$$p^h(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \varphi}.$$

If imported intermediates are used in production of the good, then the price is  $p^m(\varphi) = p^h(\varphi) / (1 + N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}$ . Since  $(1 + N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}} > 1$ ,  $p^m(\varphi) < p^h(\varphi)$  for  $\forall \varphi$ . Once the prices are determined, it is straightforward to find revenues gained by importing and non-importing firms. Define  $r^h(\varphi)$  as the per-product revenue generated by a  $\varphi$ -type firm that does not import which is calculated as

$$r^h(\varphi) = \left(\frac{p(\varphi)}{P}\right)^{1-\sigma} E = \left(\frac{\sigma}{\sigma - 1} \frac{w}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \varphi P}\right)^{1-\sigma} E. \quad (5)$$

Similarly, per-product revenue of an importing firm  $r^m(\varphi)$  is

$$r^m(\varphi) = Z_m r^h(\varphi), \quad (6)$$

where  $Z_m = (1 + N\tau_m^{1-\gamma})^{\frac{(1-\alpha)(\sigma-1)}{\gamma-1}} > 1$ . From equations 5 and 6 we see that revenue is proportional to the firm's efficiency level. Defining  $\pi_i^h(\varphi)$  and  $\pi_i^m(\varphi)$  as the profit level of a non-importing and an importing firm respectively (*the latter net of the fixed cost of importing*  $wf_m N$ ) we are now ready to describe the importing decision problem.

As mentioned earlier, besides its effects on productivity, importing intermediates also affects firms' innovation technology so that the decision to import is also of a dynamic nature. Firms basically contrast the fixed costs necessary to import intermediate, with not only the extra

productivity today but also with the benefit of increasing their prospects for introducing new products in the future. That is, both today's and future profits would be affected by the import decision (equations 7 and 8). To illustrate this point, consider the import decision of a firm with one product. In the following equations  $V_\varphi(\mathbf{1})$  represent the value of a firm with one product and efficiency level  $\varphi$  (when characterizing the solution of the model we will also denote such value as  $v(\varphi)$  whenever there is no risk of confusion).

$$rV_\varphi(\mathbf{1}) = \max [rV_\varphi^m(\mathbf{1}), rV_\varphi^h(\mathbf{1})] \quad (7)$$

$$rV_\varphi^j(\mathbf{1}) = \max_{\lambda \geq 0} \left\{ \begin{array}{l} \pi^j(\varphi) - w c^j(\lambda) + \\ \lambda [V_\varphi(\mathbf{2}) - V_\varphi(\mathbf{1})] \\ + \mu [-V_\varphi(\mathbf{1})] \end{array} \right\}, j = m, h \quad (8)$$

We apply numerical methods to compute the cutoff efficiency level for importing  $\varphi_m^*$  so that firms for which  $\varphi \geq \varphi_m^*$  will find it profitable to import.

## 2.4 Final Goods Producers: Innovation Decision

The dynamic optimization problem of the firm follows from Klette and Kortum (2004) and their extension in Şeker (2012). The monopolistic competition in the final goods sector results in each product being produced by a single firm. Firms can produce multiple varieties. The number of products  $n$  determines the portfolio of the producer. This portfolio increases by innovating new product varieties and it decreases by destruction of the existing products<sup>6</sup>. This process determines evolution of the firm.

### 2.4.1 Innovation technology

The innovation production function determines the Poisson rate of arrival of the next product, based on the firm's investment in research and development (R&D)  $R$ , which is measured in labor units, and knowledge capital. The key distinction between importers and non-importers is what represents their knowledge capital  $n$ .

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<sup>6</sup>An alternative way of how innovations materialize in the economy is quality improvements. In Klette and Kortum (2004) and Lentz and Mortensen (2013) innovations arrive in this way.

The innovation production function is strictly increasing and strictly concave in  $R$ . It is strictly increasing in the knowledge capital and homogeneous of degree one in  $R$  and knowledge capital. For non-importing firms knowledge capital is measured by the total knowledge accumulated through past innovations. We use the number of products innovated  $n$  to represent this capital stock. For the importing firms, knowledge can be accumulated through their own past innovations as well as through knowledge embodied in the imported intermediate goods,  $(1 + N)^\zeta n$ .

Grossman and Helpman (1991) discuss several ways in which international knowledge spillover is possible. They argue that residents of a country may find occasions to learn technical information from meeting with foreign counterparts that contributes to their stock of general knowledge. Also the use of differentiated intermediate goods that are not available in the domestic market can increase the insights that local producers gain from inspecting and using these goods. Grossman and Helpman (1991) use the cumulative volume of trade between countries as the international spillover for innovation. We use a function of the total flow of intermediate goods that are imported. We measure the magnitude of the spillover by  $\zeta > 0$ .

Goldberg et al. (2010b) provide empirical evidence that motivates the inclusion of imported intermediate goods in the innovation function. They show that higher availability of foreign intermediate products have increased the innovation capacity of firms in the Indian manufacturing sector. They find that reduction of input tariffs led to imports of new varieties in the economy and this has led to an expansion of within-firm product scope by almost 8 percent.

As in Klette and Kortum (2004), innovations arrive at a Poisson rate represented as  $I = \lambda n$ , where  $\lambda$  would be denoted the innovation intensity. The innovation function for an importing firm can be formalized as

$$\begin{aligned} \lambda n &= R^\theta \left( (1 + N)^\zeta n \right)^{1-\theta} \text{ where } 0 < \theta < 1 \text{ and } \zeta > 0 \\ \lambda n &= R^\theta n^{1-\theta} (1 + N)^{\zeta(1-\theta)} \\ R &= \left( \frac{\lambda}{(1 + N)^{\zeta(1-\theta)}} \right)^{\frac{1}{\theta}} n \end{aligned}$$

If we define  $1 + c_1 = \frac{1}{\theta} > 1$  then  $\frac{1-\theta}{\theta} = c_1 > 0$ . We can rewrite R&D investment as

$$R = \frac{\lambda^{1+c_1}}{(1+N)^{c_1}} n = c(\lambda)n$$

This equation captures that higher use of imported intermediate goods reduces the cost of R&D and therefore increases the probability of innovating new varieties for a given level of R&D spending. In the calibration we parametrize this equation for importers as  $wc^m(\lambda)n = wc_0 \frac{\lambda^{1+c_1}}{(1+N)^{c_1}} n$  for  $c_0, c_1 > 0$  where  $c_0$  is a scale factor. The R&D cost for the non-importing firm is same as the cost for importing firm without the spillover term  $wc^h(\lambda)n = wc_0 \lambda^{1+c_1} n$ .

### 2.4.2 Bellman equation

Each producer also faces a Poisson hazard rate  $\mu$  of losing any of its products capturing that products in practice get obsolete. A similar interpretation is presented in Luttmer (2011). He assumes that producer of a differentiated commodity needs a blueprint to produce and these blueprints depreciate in a one-hoss-shay fashion. While each firm takes the value of  $\mu$  as given, its equilibrium value, would be determined by the aggregate creation in the economy which will be discussed below. Firms exit if all of their products are destroyed. There is no re-entering once exit occurs.

Assuming a constant interest rate,  $r$ , the Bellman equation for a producer of efficiency,  $\varphi$ , and number of products,  $n$ , is formulated as follows<sup>7</sup>

$$rV_\varphi^j(\mathbf{n}) = \max_{\lambda \geq 0} \left\{ \begin{array}{l} \pi^j(\varphi)n - wnc^j(\lambda) + \\ n\lambda [V_\varphi^j(\mathbf{n} + \mathbf{1}) - V_\varphi^j(\mathbf{n})] \\ + \mu n [(V_\varphi^j(\mathbf{n} - \mathbf{1}) - V_\varphi^j(\mathbf{n}))] \end{array} \right\}, \text{ for } j = m, h \quad (9)$$

This equation shows that current value of firm is equal to the sum of three terms. The first term on the right hand side shows the current profit net of R&D costs. The other two terms show the net future value of the firm. The second one is the gain in value caused by the innovation of a new variety and the last one is the expected loss associated with a loss of a randomly chosen product<sup>8</sup>.

<sup>7</sup>Constancy of interest rate is obtained from the consumer's optimization problem.

<sup>8</sup>Existence of a solution for a similar dynamic optimization problem under heterogeneous firms is proven in

Since the value function is linear in the number of products, it is possible to obtain an analytical solution to optimization problem. Following Lentz and Mortensen (2008), we conjecture that the value function is

$$V_{\varphi}^j(\mathbf{n}) = \left( \frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi) \right) n \quad (10)$$

where  $\Theta(\varphi)$  is the type conditional continuation value of innovation. Then we incorporate this conjecture into the Bellman equation which simplifies to

$$\begin{aligned} (r + \mu) \Theta^j(\varphi) &= \max_{\lambda \geq 0} \left\{ \lambda \left( \frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi) \right) - wc^j(\lambda) \right\} \\ \Theta^j(\varphi) &= \max_{\lambda \geq 0} \left\{ \frac{\frac{\lambda \pi^j(\varphi)}{r + \mu} - wc^j(\lambda)}{r + \mu - \lambda} \right\}. \end{aligned} \quad (11)$$

In order to have the conjectured value function solve this problem, all variables have to be stationary. In the appendix we show how the economy grows on a balanced growth path. Solving the simplified Bellman equation, we get

$$\frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi) = wc^{j'}(\lambda). \quad (12)$$

We define  $v^j(\varphi) = \frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi)$  as the expected value of a single product for a  $\varphi$ -type producer. It is the sum of the discounted stream of the profits and the innovation option value. Using the value of  $\Theta^j(\varphi)$  from equation 11 and implementing it into equation 12 we get

$$wc^{j'}(\lambda) = \frac{\pi^j(\varphi) - wc^j(\lambda)}{r + \mu - \lambda}. \quad (13)$$

Klette and Kortum (2004) show that the optimal value of  $\lambda$  is an increasing function of profit level. Since the profit level monotonically increases in  $\varphi$ , this result shows firms with high efficiencies are more likely to innovate. Plugging the value of  $c^j(\lambda) = \frac{c_0 \lambda^{1+c_1}}{(1+N)^{c_1}}$  for an importing

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Lentz and Mortensen (2005).

firm into this equation we get,

$$\begin{aligned} \frac{wc_0(1+c_1)\lambda^{c_1}}{(1+N)^{\zeta_{c_1}}} &= \frac{\pi^m(\varphi) - w\frac{c_0\lambda^{1+c_1}}{(1+N)^{\zeta_{c_1}}}}{r+\mu-\lambda} \\ wc_0(1+c_1)\lambda^{c_1} &= \frac{(1+N)^{\zeta_{c_1}}\pi^m(\varphi) - wc_0\lambda^{1+c_1}}{r+\mu-\lambda} \text{ for } \varphi > \varphi_m^*. \end{aligned} \quad (14)$$

Similarly for the non-importing firm the solution is

$$wc_0(1+c_1)\lambda^{c_1} = \frac{\pi^h(\varphi) - wc_0\lambda^{1+c_1}}{r+\mu-\lambda} \text{ for } \varphi \leq \varphi_m^*.$$

A key difference between the solution for importing and non-importing firms is the spillover term which is captured by  $(1+N)^{\zeta_{c_1}} > 1$ . It enters into the equation as a positive multiplier of the profit level. Hence importing gives extra advantage to firms to innovate. To guarantee the existence of a stationary size distribution in equilibrium the condition  $\mu > \lambda(\varphi)$  for all efficiency levels  $\varphi$  must hold. If the innovation rate of a firm exceeds or becomes equal to the aggregate destruction rate, size and age of some firms diverge to infinity which precludes having a stationary size distribution<sup>9</sup>. This condition can be written as an upper bound on the efficiency type distribution which is presented in the appendix.

## 2.5 Entrant's Problem

From a constant potential pool of entrants, successful ones enter the economy as a result of an innovation of a new variety. Firms discover their efficiency types immediately after they enter. Entrants face the same innovation cost function as incumbent firms and they innovate at rate  $\lambda_e$ . Entry rate is determined by the free entry condition given as

$$\underbrace{wc'(\lambda_e)}_{\text{marginal cost of innovation}} = \underbrace{\int v(\varphi)\phi(\varphi)d\varphi}_{\text{net gain from innovation}}. \quad (15)$$

Here  $\phi(\cdot)$  is type distribution for entrants and  $v(\varphi)$  is the value of a single product for a  $\varphi$ -type producer. Entry rate  $\eta$  is the product of mass of potential entrants  $M_e$  and innovation rate of

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<sup>9</sup>The condition needed to guarantee  $\mu > \lambda(\varphi)$  for  $\forall \varphi$  is  $wc'(\mu) > \frac{\pi-wc(\mu)}{r}$  which follows from equation 13.

entrants  $\lambda_e, \eta = M_e \lambda_e$ . From incumbent firm's optimization problem we had  $w c'(\lambda(\varphi)) = v(\varphi)$ . Incorporating this result in equation 15 we can solve for the innovation rate of entrants as a function of innovation rates of incumbent firms

$$\begin{aligned} w c_0 (1 + c_1) \lambda_e^{c_1} &= \int w c'(\lambda(\varphi)) \phi(\varphi) d\varphi \\ \lambda_e &= \left[ \int \lambda(\varphi)^{c_1} \phi(\varphi) d\varphi \right]^{1/c_1} \\ &= \left[ \int_0^{\varphi_m^*} \lambda(\varphi)^{c_1} \phi(\varphi) d\varphi + \int_{\varphi_m^*}^{\infty} \frac{\lambda(\varphi)^{c_1}}{(1+N)^{\zeta c_1}} \phi(\varphi) d\varphi \right]^{1/c_1}. \end{aligned}$$

This result shows that keeping all else constant, higher knowledge spillover from trade decrease the innovation rate of entrants, hence reduces the entry rate.

## 2.6 General Equilibrium

Firms evolve as a result of a birth and death process of products. We define  $M_n(\varphi)$  as total mass of  $\varphi$ -type firms with  $n$  products.  $M(\varphi) = \sum_{n=1}^{\infty} M_n(\varphi)$  is total mass of  $\varphi$ -type firms. Then  $\delta(\varphi) = M(\varphi)/M$  is defined as the steady state type distribution where  $M$  is the total mass of firms. Mass of products produced by  $\varphi$ -type firms is represented as  $\Lambda(\varphi) = \sum_{n=1}^{\infty} n M_n(\varphi)$ . In steady state, the rate of product destruction  $\mu$  should be equal to the sum of the product creation rates of entrants and incumbent firms

$$\mu = \eta + \int \lambda(\varphi) \Lambda(\varphi) \delta(\varphi) d\varphi. \quad (16)$$

In equilibrium, total mass of products produced by  $\varphi$ -type firms is found as<sup>10</sup>

$$\Lambda(\varphi) = \frac{\eta \phi(\varphi)}{\mu - \lambda(\varphi)}. \quad (17)$$

In the appendix, we also derive the entry type distribution  $\phi(\cdot)$  as a function of the steady state type distribution  $\delta(\cdot)$ . The final equilibrium condition is labor market clearing condition.

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<sup>10</sup>For completeness of the model, derivation of this equation is reproduced in the appendix.

## 2.7 Labor Market Equilibrium

There is a fixed measure of workers in the economy which is denoted as  $L$ . Labor is allocated across four activities for every  $\varphi$ -type incumbent firm: final good production  $l_f$ , intermediate good production  $l_i$ , R&D investment  $c(\lambda)$ , and fixed cost of importing  $Nf_m$ . There is also a part of the labor force allocated to research and development for potential entrants  $c(\lambda_e)$ . The labor market clearing condition is stated as

$$L = \int_0^{\infty} (l_f(\varphi) + l_i(\varphi) + c(\lambda(\varphi)) + I(\varphi)Nf_m)\Lambda(\varphi)\delta(\varphi)d\varphi + M_e c(\lambda_e) \quad (18)$$

where  $I(\varphi) = 1$  for  $\varphi > \varphi_m^*$ . Given these equilibrium conditions, a stationary equilibrium for this economy consists of aggregate destruction rate  $\mu$ , wage rate  $w$ , and interest rate  $r$  such that for given values of  $(\mu, w, r)$ , *i*) any  $\varphi$ -type incumbent producer chooses the optimal innovation rate  $\lambda(\varphi)$ , decides on whether to import or not, and solves equation 13 to maximize its value, *ii*) potential entrants choose  $\lambda_e$  in solving equation 15 and break even in expectation, *iii*) representative consumer maximizes utility subject to budget constraint, *iv*) labor market clears as in equation 18, and *v*) equations 16 and 17 hold. The equilibrium level of aggregate price index decreases with rate  $g$  as it is derived in the appendix.

In our setup  $L_i(\varphi)$  is increasing in firm productivity  $\varphi$  regardless of the import decision of the firm. Labor employed in R&D activity  $c(\lambda(\varphi))$  is also monotonically increasing in  $\varphi$ <sup>11</sup>.

## 3 Data

The data used for the analysis is obtained from the Prowess database which is collected by Center for Monitoring the Indian Economy (CMIE). The data is constructed as a panel of relatively large firms spanning the period from 1989 to 1997. These firms account for 60% to 70% of the all economic activity in India. We restrict our analysis to manufacturing firms. Prowess records detailed product-level information at firm level and it enables us to track firm's

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<sup>11</sup>In Lentz and Mortensen (2008), since the producing any intermediate input requires labor and capital in fixed proportions more profitable firms not necessarily employ relatively more labor. In their setup labor demand for production of per product is decreasing in product profitability whereas labor demand for innovation increases in product profitability and dominant one of the two forces determines the direction of net impact on labor demand.

product mix over time. This information is available for 2927 firms which correspond to 85% of the manufacturing firms and 90% of total output in Prowess. The data on product mix of firms allows us to test the model’s ability to explain product evolution of firms.

Definition of a product is based on the CMIE’s internal product classification which is based on the Harmonized System and National Industry Classification. This data on product-mix of firms has been used in several studies by Goldberg et al (2009, 2010a, 2010b). Goldberg et al. (2010a) in an unpublished appendix explain data cleaning process in obtaining the product level data. We follow their methodology in getting this information. They define 1886 products linked to 108 four-digit NIC industries. In the data multi-product firms account for almost half of the firms and they account for 80% of the output.

## 4 Quantitative Model and Calibration

The model is calibrated to match the data from Indian manufacturing sector for 1989-1997 time period. The novel contribution of the model is explaining the product evolution of firms and how this evolution is related to importing intermediate goods. To test the model’s ability to explain the data we choose eight data moments as presented in Table 1. The moments that relate to product distribution help identify efficiency-type distribution and innovation cost parameters. First two moments are the mean and standard deviation of product distribution<sup>12</sup>. Since the model is capable of explaining firm evolution, we include three moments that relate to firm dynamics: mean and standard deviation of product growth distribution for surviving firms and percentage of firms that do not change their product scope in a year. Data show that 90% of firms are in-active in the sense that they do not add or subtract products in a given year<sup>13</sup>. In order to highlight the model’s capacity in explaining the significance of multi-product firms in economic activity, we include the contribution of their sales to aggregate output (82%). All these data moments are obtained from the Prowess database and represent average values for 1989-1997<sup>14</sup>.

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<sup>12</sup>In the data maximum number of products obtained by any firm is 35. In the calibration we use this value as the maximum number of products that any firm can produce.

<sup>13</sup>Using five year averages of firm activity instead of one year, Goldberg et al. (2010a) show that the share of in-active firms are 72%.

<sup>14</sup>India went through a balance of payments crisis at the end of 1991. We exclude this year in computation of the moments.

Table 1: Data Moments

Definition	Value
<i>Product distribution moments</i>	
Average number of products	1.9
Standard deviation of number of products	1.7
<i>Firm dynamics moments</i>	
Average of product growth (conditional on survival)	1.9%
Standard deviation of product growth (conditional on survival)	0.17
Fraction of firms with no net annual change in products	90%
Contribution of multi-product firms to total sales	82%
<i>Imported intermediate moments</i>	
Fraction of firms importing intermediates	19%
Import intensity (Cost of foreign inputs/Sales)	19%

Two moments that relate to the decision of importing identify fixed cost of importing as well as the elasticity of substitution between intermediate inputs. The Prowess database does not include any information about importing. Hence these moments are obtained from Annual Survey of Industries for 2001-2002 period. In India during this period roughly 19% of the firms participate in import activity. We also compute average import intensity of the importing firms as 19%. For each firm, import intensity is computed as the share of total foreign input costs to total sales.

We assume that exogenous efficiency type distribution is lognormal ( $\varphi$ -LN  $(\mu_\varphi, \sigma_\varphi)$ ) where  $\mu_\varphi$  and  $\sigma_\varphi$  are the mean and standard deviation of this distribution<sup>15</sup>. The parameter vector that is chosen for identification is  $\Delta_1 = \{c_0, c_1, \zeta, \mu_\varphi, \sigma_\varphi, f_m, \alpha, \gamma\}$ . Innovation cost has three parameters: scaling factor  $c_0$ , the convexity of the cost  $c_1$ , and spillover parameter from imports  $\zeta$ . There are two parameters from the production function which are the labor share of production  $\alpha$  and the elasticity of substitution between intermediate products  $\gamma$ . We estimate  $\gamma$  analytically from the model to match average import intensity of firms. Cost of imported intermediates for a product is  $w\tau Nq_f$ . Dividing this value by the revenue obtained from a product

<sup>15</sup>There is no particular reason why log normal distribution is used here. The same distribution is also used in Şeker (2012).

which is derived in equation 6, we get import intensity  $Int = \frac{\sigma-1}{\sigma} (1 - \alpha) \tau^{1-\gamma} \frac{N}{(1+N\tau^{1-\gamma})}$ . This equation allows us to get the value of  $\gamma$  once the other parameters are determined. The final parameter to calibrate is the fixed cost of importing  $f_m$ .

We combine several methods to determine the rest of the parameters  $\Delta_2 = \{g, N, r, \sigma, \tau, M_e\}$ . In the model there are two components of growth: growth of efficiency levels which is exogenously set and evolution of products which is endogenously determined. The data allow us to compute the contribution of each part to the aggregate growth. Average growth rate of sales per product (the intensive margin) over the sample period is 8.9%. This value corresponds to the value of  $g$ . Number of trading partner countries  $N$  is set to one as a normalization. Similarly mass of potential entrants  $M_e$  is set to unity. As in Lentz and Mortensen (2008) real interest rate is set as 5%. Elasticity of substitution between final goods  $\sigma$ , is set as 2.8 which is the median value of elasticities calculated by Broda and Weinstein (2006). Tariff rate on intermediate inputs is computed as 24% which yields the iceberg cost as 1.24 following Khandelwal and Topalova (2011) who compute input tariffs by running industry-level tariffs through India's input-output table for 1993-1994 period.

Solution of the model requires lengthy fixed point iterations for each choice of model parameter candidates. To avoid these iterations and in the spirit of Lentz and Mortensen (2008) we set the wage level at 200 and set total labor supply  $L$  that would satisfy the labor market clearing condition in equation 18. In the simulation results the average ratio of wage bill to sales is about 10 percent, slightly higher than the 7 percent found for India by Khandelwal and Topalova (2011)<sup>16</sup>.

To solve the model, we simulate a panel of 10,000 firms for two periods to obtain the moment values. Then we seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. Since the model is highly nonlinear, we use down-hill simplex method (amoeba) for optimization. The steps to computationally solve the equilibrium and calibrate the model are described in the appendix.

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<sup>16</sup>In the calibration exercise we do not match any data moment like total sales, productivity, or total compensation that relies on the scale of the economy. Thus the wage value used in estimation is not critical for the values of simulated moments chosen for calibration. Aggregate expenditure is normalized to 10,000.

## 4.1 Estimation Results

The parameter estimates are presented in Table 2. The estimated parameter values are broadly consistent with those obtained from several recent studies. Kasahara and Lapham (2013) use the same production function for final goods in estimating trade premia of importing and exporting firms using data from Chilean manufacturing sector. Their estimates of the fixed importing costs vary between 0.4 and 0.8 across three industries (wearing apparel, plastic products, and structural metal). The share of labor in their production function is around 0.26. Our estimate of elasticity of substitution between intermediate goods differs from their estimates of 4.5-5. The convexity of the innovation cost  $c_1$  is comparable to the estimate obtained in Lentz and Mortensen (2008) which fit their model to Danish firms. They find the value of this parameter as 3.73<sup>17</sup>. Similarly Şeker (2012) estimate this parameter for five Chilean manufacturing industries with values varying between 3.8 and 5.7.

The calibration results are presented in Table 3. The model performs reasonably well in matching the data. It over-estimates the contribution of multi-product firms to total sales. Percentage of importers is also matched quite well. Import intensity is exactly matched as it was derived analytically from the model. In addition to these targeted moments, the model performs well in capturing several other data moments. Two of these un-targeted moments are presented at the bottom of the table. Goldberg et al. (2010a) show that approximately 8.5% to 11.5 % of the variation in output across firms can be attributed to the variation in extensive margin which is defined as the number of products firms produce. This statistics could be produced by the model. Number of products at steady state has a logarithmic distribution which is derived in Klette and Kortum (2004). Using the formulas for the mean and variance of this distribution, we can decompose the total variation in total sales into its components of intensive and extensive margins. Total sales is  $\bar{E} = \int E[r(\varphi) n|\varphi] \delta(\varphi) d\varphi$  where  $\delta(\varphi)$  was defined as the efficiency type distribution of firms at steady state,  $r(\varphi)$  is revenue per product derived in equation 3 and  $E[\cdot]$  is the expectation operator. Total variation in size is determined as

$$Var_{Total} = \underbrace{\int Var[r(\varphi) n|\varphi] \delta(\varphi) d\varphi}_{Var_{Within}} + \underbrace{\int (E[r(\varphi) n|\varphi] - \bar{E})^2 \delta(\varphi) d\varphi}_{Var_{Between}} \quad (19)$$

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<sup>17</sup>The parameter  $c_0$  is a scaling factor and our estimate is likely to differ from Lentz and Mortensen (2008).

Table 2: Parameter Values

<b>Calibrated Parameters</b>		
Parameter	Description	Value
$c_0, c_1, \zeta$	Innovation cost parameters	1490.7***, 3.27***, 1.07***
$\mu_\varphi, \sigma_\varphi$	Efficiency distribution	3.03***, 2.87***
$f_m$	Fixed cost of importing	0.08***
$\alpha$	Labor share in production	0.39***
$\gamma$	Elasticity of subst. btw. int. goods	1.26***
<b>Other Parameters</b>		
$r$	Real interest rate	0.05
$g$	Growth rate of efficiency levels	0.089
$\tau$	Iceberg costs of importing	1.24
$N$	Number of trading partner countries	1
$M_e$	Potential pool of entrants	1
$\sigma$	Elasticity of substitution between varieties	2.8

Notes:\*\*\* means  $p < 0.01$ . While calculating the standard errors for parameters we follow the path described in Gourieroux & Monfort (2002) .

Using the estimated parameter values, total variation decomposition shows that almost 12% of the variation in output is attributed to the variation in the extensive margin. Another moment is decomposition of aggregate growth. Following Goldberg et al. (2010a) we decompose aggregate annual change in output of continuing firms into the changes in product mix (extensive margin), and changes in output of existing products (intensive margin). The contribution of extensive margin to aggregate growth is 15% in the data where it is 11.8% in the model. Close accordance with the data in these moments relies on a novel feature of the model. The model incorporates two forces that generate persistent differences in firm performance. In his review of models on firm evolution, Sutton (1997) lists these forces as: (i) intrinsic efficiency differences that are determined before entering the economy (ii) differences that are generated through idiosyncratic innovations that accumulate through the life of the establishment. Both forces have drawn great

attention in the literature<sup>18</sup>. Our model incorporates both types of forces which allows it to produce rich firm dynamics.

Table 3: Model Fit to the Data

<b>Targeted Moments</b>	Data	Model
Average number of products	1.95	1.13
Standard deviation of number of products	1.7	1.2
Average of product growth (conditional on survival)	1.9%	1.2%
Standard deviation of product growth (conditional on survival)	0.17	0.18
Fraction of firms with no net change in products	90%	94%
Contribution of multi-product firms to total sales	82%	92%
Fraction of firms importing intermediates	19%	17%
Import intensity (Cost of foreign inputs/Sales)	19%	19%
<b>Other Moments</b>		
Variation of output wrt extensive margin	8.5-11.5%	12%
Between share of annual aggregate growth	15%	11.8%

Finally we look at the model’s ability to explain product distribution of multi-product firms. The model slightly underestimates the product distribution in mean and standard deviation of the product distribution when all firms are included. Restricting the comparison to multi-product firms we get the results presented in Table 4. The product distribution that emerges from the simulation has a longer right tail than the one observed in the data. Although mean, median, and some of the percentiles are quiet comparable, the 99<sup>th</sup> percentile is much higher in the model.

## 4.2 Autarky versus Trade

Goldberg et al. (2010a, b) show the significant contribution of trade liberalization to innovation performance of Indian firms. In order to show the success of these policy reforms in the model, we take an extreme case and compare autarky with free trade equilibrium. In order to make

<sup>18</sup>For a review and comparison of both types of models see Klette and Raknerud (2002) and Şeker(2012).

Table 4: Product Distribution of Multi-Product Firms

	Percentiles					
	Mean	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>	90 <sup>th</sup>	99 <sup>th</sup>
Data	3.1	2	2	3.2	5	10
Model	3.9	2	2	3	7	28

Notes: Single-product firms are exclude in this table.

this comparison, we let the wage rate be determined by the labor market clearing condition. We randomly draw 10,000 efficiency values and use the same set of firms in both autarky and trade.

First, we compare the profit levels of importing firms in autarky and trade. We define  $\pi^m(\varphi)$ ,  $P^m$ ,  $w^m$  as profit, aggregate price, and equilibrium wage in trade equilibrium. Replacing superscripts to  $a$  refers to the same variables in autarky. Profit levels  $\pi^m(\varphi)$  and  $\pi^a(\varphi)$  are calculated as

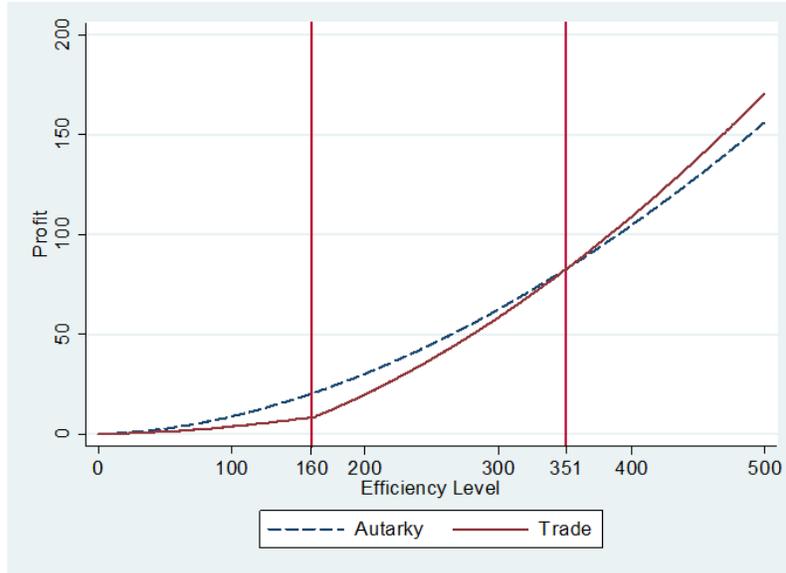
$$\begin{aligned}\pi^m(\varphi) &= \frac{E}{\sigma} \left[ \frac{\sigma-1}{\sigma} \frac{P^m}{w^m} \alpha^\alpha (1-\alpha)^{1-\alpha} (1 + N\tau_m^{1-\gamma})^{\frac{(1-\alpha)}{\gamma-1}} \varphi \right]^{\sigma-1} - w^m N f_m \\ \pi^a(\varphi) &= \frac{E}{\sigma} \left[ \frac{\sigma-1}{\sigma} \frac{P^a}{w^a} \alpha^\alpha (1-\alpha)^{1-\alpha} \varphi \right]^{\sigma-1}.\end{aligned}$$

For an importing firm, the comparison of these profit levels yield

$$\begin{aligned}\pi^a(\varphi) &< \pi^m(\varphi) - w^m f_m, \text{ iff} \\ w^m f_m &< \frac{E}{\sigma} \left[ \frac{\sigma-1}{\sigma} \alpha^\alpha (1-\alpha)^{1-\alpha} \varphi \right]^{\sigma-1} \left[ \left\{ (1 + N\tau_m^{1-\gamma})^{\frac{(1-\alpha)}{\gamma-1}} \frac{P^m}{w^m} \right\}^{\sigma-1} - \left\{ \frac{P^a}{w^a} \right\}^{\sigma-1} \right].\end{aligned}$$

Recall that the cutoff efficiency level for the decision of importing  $\varphi_m^*$  was determined through equation 7. As long as  $D = \left[ \left\{ (1 + N\tau_m^{1-\gamma})^{\frac{(1-\alpha)}{\gamma-1}} \frac{P^m}{w^m} \right\}^{\sigma-1} - \left\{ \frac{P^a}{w^a} \right\}^{\sigma-1} \right] > 0$  in the equation above, there is an efficiency level  $\hat{\varphi} > \varphi_m^*$  such that all firms with efficiency levels greater than  $\hat{\varphi}$  are going to gain higher profits in trade. Unlike the setup in Melitz (2003) it is not analytically possible to prove that  $D > 0$ . Hence we simulate the model to compare the autarky and trade equilibrium.

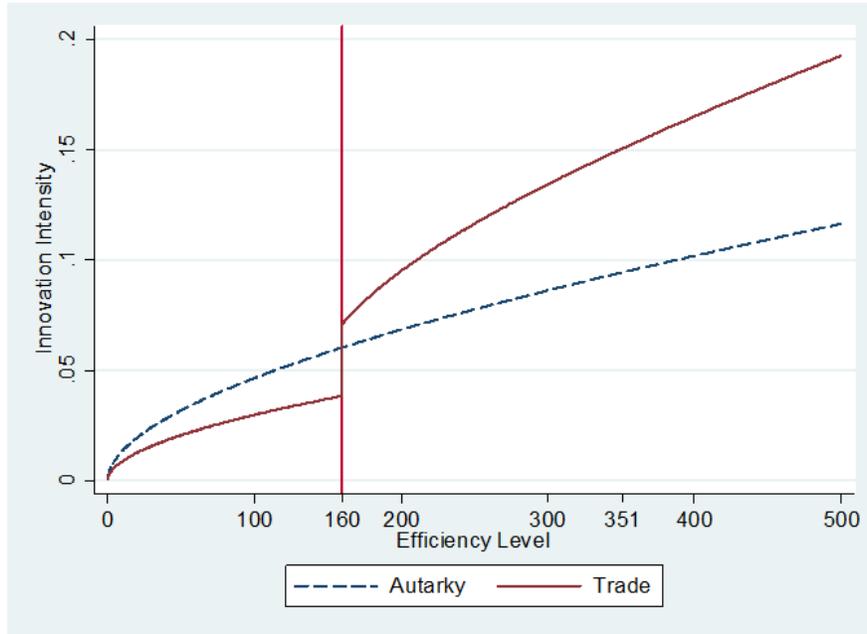
Figure 1: Profit levels ( $\pi$ ) versus efficiency levels( $\varphi$ )



Using the parameter values obtained from the calibration exercise, we compute average profit level  $\pi(\varphi)$  per product and innovation rates  $\lambda(\varphi)$  for each efficiency level  $\varphi$ . Figure 1 shows the relationship between efficiency levels and profit levels. Cutoff efficiency level  $\varphi_m^*$  is 160. The graph shows that not all importers gain from trade in profit due to higher costs of production. This finding is in accordance with Melitz (2003). He shows that among exporting firms only the most efficient ones gain from trade ( $\varphi \geq \hat{\varphi}$ ). In our simulation this threshold value of efficiency levels is  $\hat{\varphi} \cong 351$ . Trade induces reallocation of resources toward the more efficient firms through competing for labor.

Next in figure 2, we look at how innovation rates are affected by trade. In trade equilibrium importing firms innovate at higher rates than they would in autarky. Importing increases firms' profits and the knowledge spillover from using new input varieties reduce the cost of innovation. When both factors are combined we observe a faster innovation rate of importers in trade equilibrium. This finding is in accordance with the empirical evidence presented in Goldberg et al. (2010b). They show that as a result of the tariff liberalization in 1990's, India experienced a surge in imported inputs mostly in inputs that were not available before the liberalization. Higher usage of new input varieties has led to introduction of new products in the domestic market. Another result that can be obtained from the graph is that although some of importers with efficiency levels  $\varphi$  such that  $\varphi_m^* < \varphi < \hat{\varphi}$  observe drops in their per-product profits  $\pi^m(\varphi)$

Figure 2: Innovation rates ( $\lambda$ ) versus efficiency levels ( $\varphi$ )



relative to autarky, they may still gain higher aggregate profits  $\pi^m(\varphi)n$  in trade equilibrium because they are likely to innovate more products when trade is allowed.

Table 5 presents a comparison of autarky and trade equilibrium. In trade equilibrium average innovation rate of incumbent firms increase facilitated by the spillover from the use of imported intermediate goods and increase in profit levels. This leads to an increase in product portfolio of multi-product firms from 2.95 to 3.89. It also increases the average and aggregate innovation rates in the economy. Since prices decrease and wage rate increases in trade, innovation gets costlier which reduces the entry of new firms.

After the trade liberalization, those firms which find importing intermediate varieties optimal are able to reduce marginal production cost of their products, and decrease their price because of the love-of-variety attribute of the production function. These firms will be reducing the aggregate price index. Also, the new entrants are on average more productive than the entrants of autarky regime. These more productive entrants also charge lower prices than the entrants of the autarky regime, and they also reduce the aggregate price index.

As to the firm size distribution, trade liberalization reduces marginal production costs for importer firms and hence make them increase their output per product and also with the spillover effect of imported goods, importer firms increase their innovation effort and in equi-

Table 5: Autarky and Trade Comparison

<b>Moments</b>	<b>Autarky</b>	<b>Trade</b>
Ave # of products (multi-product firms)	2.95	3.89
Ave innovation rate	3.3%	3.5%
Aggregate innovation rate (by incumbents)	19.2%	34.8%
Entry Rate	5.9%	4.6%
Average product growth rate	1.15%	1.18%
Aggregate price level	0.21	0.07
Equilibrium wage	150.5	200

librium they have more products. These two forces allow more productive firms to import and become even bigger firms. At the same time, due to the higher productivity cutoff for entry, the new entrants are on average larger than the new entrants of the autarky regime.

### 4.3 Firm Growth Rate Distribution and Hazard Rate of Exit

Allowing the increasing returns from imported intermediate inputs increases the dispersion of firm growth rates, as well. Due to the total spillover effect of imported inputs on innovative effort, importer firms exert more R&D effort in the “trade” setup. Hence, dispersion of growth rate increases as long as some firms do not import while the others do. As to the exit hazards, the exit hazard of a one-product firm is directly linked to the common rate of creative-destruction  $\mu$ . Since  $M_e$  is normalized to 1, entry rate  $\eta$  is equal to the innovation rate of entrants  $\lambda_e$  which is an increasing function of  $\varphi_m^*$  because of the spillover from trade. Compared to the autarky setup, innovation rate of entrants will be lower when trade is allowed which will be pulling down  $\mu$  (this impact can also be observed in Figure 2). At the same time, the second component of  $\mu$  in Eq. 16, which is the aggregate arrivals of innovations to incumbent firms will be higher due to the availability of more imported intermediate varieties in the domestic market which makes incumbent producers more innovative. Consequently, when the economy is opened the creative-destruction rate  $\mu$  may be higher or lower depending on model parameters.

## 5 Counterfactuals and Some Robustness Checks

To deeper understand the mechanisms at work and sensitivity of the results to different aspects of the model, in this section we conduct a set of counterfactual analysis and robustness checks. Firstly, an interesting question may be measuring the returns to reducing costs to trade. To this end, in this section, we analyze the impact of reducing iceberg trade costs. Secondly, one other novelty of this study is that it includes the spillover effect of imported intermediates on innovation costs. To understand the impact of this channel on the economic activity, we shut down this channel by setting  $\zeta = 0$ . Finally, we study sensitivity of the equilibrium outcomes to demand elasticity specification of the model.

Tables 6-8 exhibit comparison of some important equilibrium outcomes across different equilibriums. We have realized that in some cases although aggregate labor allocated to the same activity is similar across different specifications, allocation of the aggregate labor across firms may differ substantially across the specifications. That's why, we also report the average labor dedicated to different economic activities across firms. Panel B of Table 7 compares average labor employed at a firm for different activities.

Table 6: Some Equilibrium Outcomes in Different Specifications

	$w$	$P$	Agg. Entry	mean( $n$ )	mean( $\lambda$ )	mean( $\pi$ )	Agg. Innov.
Benchmark	1	1	1	1	1	1	1
$\tau = 1.12$	1.024	0.996	0.994	0.997	0.990	1.000	1.000
$\zeta = 1$	1.000	0.994	1.009	0.993	0.967	0.990	0.960
$\sigma = 2.5$	0.994	1.058	1.217	1.003	1.515	1.135	1.150
$\sigma = 3.1$	1.016	0.961	0.815	0.996	0.687	1.016	0.843

Notes: This table shows how some important equilibrium outcomes vary across equilibriums with different specifications. Benchmark results are produced using the parameters in Table 2. Each row presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a column is normalized with the benchmark value of that column. In all of the computed equilibriums aggregate labor supply is the same, and labor markets clear.

Table 7: Labor Reallocation in Counterfactuals

	Panel A: Total Labor					Panel B: Means				
	$l_f$	$l_{R\&D}$	$l_i$	$l_e$	$l_{f_m}$	$l_f$	$l_{R\&D}$	$l_i$	$l_e$	$l_{f_m}$
Benchmark	1	1	1	1	1	1	1	1	1	1
$\tau = 1.12$	0.976	0.970	1.021	0.973	1.000	0.981	0.972	1.025	0.973	0.998
$\zeta=1$	1.000	0.995	1.001	1.039	0.995	0.991	0.998	0.991	1.039	0.999
$\sigma=2.5$	0.939	1.847	0.939	2.318	1.009	0.654	1.812	0.654	2.318	1.135
$\sigma=3.1$	1.037	0.478	1.037	0.418	0.996	1.509	0.491	1.509	0.418	0.901

Notes: Benchmark results are produced using the parameters in Table 2. Each row presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a column is normalized with the benchmark value of that column. In all of the computed equilibriums aggregate labor supply is the same, and labor markets clear. Panel A of the table shows how much total labor allocated for different activities deviates from the benchmark specification. Panel B compares the mean values of labor employed at a firm for different types of activities across different equilibriums.

## 5.1 Reducing Iceberg Costs

We reduce the iceberg cost by half from 1.24 to 1.12. This reduction causes wages to go up by 2.4%.  $l_f$ ,  $l_{R\&D}$  and  $l_e$  declines by 2.4%, 2.3% and 2.7% while  $l_i$  increases by 2.1%. Since imported intermediates are cheaper, more of them are used in final good production and also more labor is demanded for domestic intermediate goods production. This increasing labor demand pulls up the wage rate and in equilibrium labor is switched from production, R&D and firm entry activities to intermediate good production. Mean  $\lambda$  and aggregate innovation declines by around 1%. This reduction in innovation effort is interesting such that when we open the economy up from autarky we observe a boost in innovative effort. However, further reductions in trade costs do not necessarily support innovative effort in the economy. On the other side, the welfare impact of further reductions in trade barriers is positive. In response to reducing  $\tau$  to 1.2 aggregate consumption increases by 0.4%. This finding is a result of the inefficiency inherent in decentralized market equilibrium of the KK(2004) model<sup>19</sup>. Lentz and Mortensen (forthcoming) point out that planner's solution discourages innovation by low ability innovators as well as entry. Persistent firm types, monopolist innovators, creative destruction of new innovations contribute to inefficiency of the decentralized equilibrium.

<sup>19</sup>Lentz & Mortensen (forthcoming) find that the welfare loss of the decentralized economy in Lentz & Mortensen (2008) is equivalent to 21% tax on social planner consumption path.

Table 8: Decomposition of Labor Across Activities in Counterfactuals

	$l_f$	$l_{R\&D}$	$l_i$	$l_e$	$l_{f_m}$	<i>Total</i>
Benchmark	0.387	0.066	0.544	0.000	0.002	1
$\tau = 1.12$	0.378	0.064	0.556	0.000	0.002	1
$\zeta = 1$	0.387	0.066	0.545	0.000	0.002	1
$\sigma = 2.5$	0.363	0.123	0.511	0.000	0.002	1
$\sigma = 3.1$	0.401	0.032	0.565	0.000	0.002	1

Notes: This table shows decomposition of how total labor is allocated across different activities in equilibriums with different parameter specifications. Benchmark results are produced using the parameters in Table 2. Each row presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. As seen in the last column each rows adds up to 1. Each cell in a row shows the share of aggregate labor in the economy is allocated to the activity represented by the column. In all of the computed equilibriums aggregate labor supply is the same, and labor markets clear.

## 5.2 Shutting Down the Spillover Effect

When we shut down the spillover effect of imported inputs, average innovation effort  $\lambda$  and aggregate innovation rate of the economy declines by 3.3% and 4%, respectively. These declines come from mainly the importer firms whose aggregate innovation rates decline by 20%.

Share of growing firms slightly declines from 3.5% to 3.3% while fraction of multi-product firms stayed with multiple products between two periods increases from 31.6% to 33%. This mainly because of the creative destruction rate which comes down to 38% from 39%.

Eliminating the spillover effect does not change the price levels drastically. The equilibrium number of importer firms declines by less then 1% while aggregate price level decreases by less then 1%. Again, this finding results from the inherent inefficiency in the decentralized economy. Since the wage rate does not change much, reallocation of labor among different types of activities like R&D, final good production, intermediate goods production is at similar levels. Since aggregate entry is higher, labor allocated to entry goes up by 4%. R&D labor also shuffles across importer and non-importer firms.

### 5.3 Robustness to Changing Elasticity Parameter

As explained in the previous sections, we have taken the demand elasticity from the literature, and set it equal to 2.8. To understand the role of the role of this elasticity we both reduce  $\sigma$  to 2.5 and increase to 3.1.

When the demand elasticity is lower, aggregate price, and average profits are higher by 58%, and 13.5%, respectively. Less elastic demand increases returns to per product created by a firm. What happens in innovative activity and the labor market is also very interesting. Average innovative effort  $\lambda$  increases by 51% while aggregate innovations increase by 15% and average number of products decline by 16%. In the labor market, aggregate labor allocated to final and intermediate goods production declines. While, on average, final and intermediate goods production firms are smaller where average number of employees per final and intermediate goods producer firm is lower by 35%. Total labor allocated to R&D is higher by 85% and the average increase in R&D labor of incumbent firms is by 81%. Incumbent firms increase their innovation effort mainly because now the final goods are less substitutable to consumers, and average profitability of a new good is higher. Since returns to a new good are higher more firms prefer to be an importer which is also evidenced by 13.5% higher fix import costs in Panel B of Table 8 . When we increase the elasticity to 3.1 the mechanism works in the reverse direction<sup>20</sup>.

## 6 Further Discussions

### 6.1 Multi-Product Firms

Similar to ours, a key implication of their model of Bernard, Redding and Schott (2010) is that a firm's product range increases in its productivity. In their setup, a firm incurs fixed production cost for each product while in our specification innovation effort is endogenously determined, and is not attached to any specific product.

One limitation of our framework is not allowing heterogeneity across products of a firm. Hence, the quantitative analysis cannot target some moments related to asymmetries across

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<sup>20</sup>When reduce elasticity excessively, let's say 1.1, every firm imports. Inversely, when we increase the elasticity excessively, let's say 6, importing never becomes better than non-importing for any firm. Both cases are not very interesting for targeted moments of the data. In numerical calculation terms for both cases we cannot calculate a  $\hat{\varphi}$ .

products within a firm, and abstracting from the findings of Bernard, Redding, Schott (2010, 2011). On the other hand, by making this simplifying assumption we are able to develop a general equilibrium model that allows the study of innovation behavior in multi-product firms.

Bernard, Redding and Schott (2010) highlight the importance of shocks that are idiosyncratic to firm-product pairs in explaining the product switching behavior observed for the US firms. Our model cannot capture the relationship between number of products produced by a firm and the share of each product in firm output. (Table 10 of BRS(2010)) They find out that the intensive margin of output per product dominant in determining firm size. In our setup we emphasize innovation and the role of imported intermediate varieties in the innovation effort of firms.

In their specification, taste shocks idiosyncratic to firm-product pairs are determinant in output share of each product for firms. In fact a concordance between the vector of these shocks at firm level in BRS (2010) setup and the knowledge capital in KK (2004) setup can be built. If we treat groupings in the knowledge capital  $n$  in the KK (2004) framework as distinct products, then each of these groups can be treated as the firm-product taste shocks of the BRS (2010).

An extension of product heterogeneity within a firm would make the model closer to the data at the expense of making the model less tractable. Still, achieving a better comprehension of the interrelation between firm innovation behavior and heterogeneity of products within a firm would be useful, and it is a part of future research agenda.

## **6.2 Firm and Industry Dynamics and Imported Intermediates**

We augment the previous studies (Klette and Kortum (2004), Luttmer (2011) and Şeker (2012)) by linking innovation and firm dynamics and allowing imported inputs to impact both firm productivity and innovative effort. Luttmer (2011) pays special attention to the thick right tail behavior of firm size distribution for US firms and he also discusses the shortcomings of Gibrat's law in capturing the firm age structure in the data and his improvement on this issue by relaxing the Gibrat's conjecture.

## 7 Conclusion

In this study we develop a general equilibrium model of multi-product firms to explain the relationship between importing, innovation, and firm growth. Following the structural model of Klette and Kortum (2004) and their extension in Şeker (2012), we present a stochastic dynamic model of firm and industry evolution. In the model we introduce heterogeneity in firms' efficiency levels as in Melitz (2003). The novel feature of our model is that unlike many of the recent trade models that follow Melitz (2003), our model has a dynamic feature. Firms invest in R&D which results in introduction of new varieties to the economy. Each period these products face a probability of being destructed. The birth and death process of products yield the stochastic growth process of firms. Incorporating the importing decision in this setup allows us to relate trade with innovation and growth.

Firm's efficiency is the main driver of its evolution. Only the most efficient firms can participate in import markets as they can compensate the sunk costs of trade. These firms are also more innovative. With the learning they obtain through knowledge spillover from the use of foreign intermediates, their innovation rates increase even further relative to non-importing firms. With the additional benefits of importing on their revenues and innovation rates, these firms grow faster and exit less often.

We test the model's ability to explain product distribution, dynamics of firm evolution, and the relationship between importing and innovation through a calibration exercise. We fit the model to Indian panel of firms for 1989-1997 time period. The model explains the targeted moments in the data relatively well. It also produces reasonable estimates of some un-targeted moments such as the variation in output with respect to extensive margin, between share of annual growth, and product distribution of multi-product firms.

We also present a comparison of autarky and trade equilibrium. This exercise shows that as in Melitz (2003), in trade resources are reallocated to more efficient firms and this leads to an increase in average size and number of products produced by firms. Moreover, average product creation rate increases which leads to faster growth. Higher innovation rates of importing firms cause a reduction in the entry rate. Our comparative statics show that although opening up an economy to trade boosts domestic innovation, further reductions in trade costs, although increases welfare, does not necessarily result in the same outcome. Finally, one other novelty

of the study for understanding the impact of trade on innovation rates which is the knowledge spillover from imported intermediates onto innovation costs seems to undertake 20% of the innovation rates of importer firms.

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## A Solving Firm's Static Problem

In a symmetric equilibrium, the static profit maximization problem of a final good producer can be formalized as follows

$$\begin{aligned}
 & \text{Max } PC^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} - w \int_0^1 q_d(j) dj - w\tau \int_0^N q_f(j) dj - wl \\
 & \text{st} \\
 y &= \varphi l^\alpha \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}.
 \end{aligned}$$

The first order conditions of this maximization problem with respect to  $q_d(j)$ ,  $q_f(j)$  for  $\forall j$  and  $l$  in respective order are given as follows:

$$w = \tag{20}$$

$$PC^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} y^{\frac{\sigma-1}{\sigma}-1} \varphi l^\alpha \frac{(1-\alpha)\gamma}{\gamma-1} \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1} \frac{\gamma-1}{\gamma} q_d(j)^{\frac{\gamma-1}{\gamma}-1} \tag{21}$$

$$w\tau =$$

$$PC^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} y^{\frac{\sigma-1}{\sigma}-1} \varphi l^\alpha \frac{(1-\alpha)\gamma}{\gamma-1} \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1} \frac{\gamma-1}{\gamma} q_f(j)^{\frac{\gamma-1}{\gamma}-1}$$

$$w = C^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} y^{\frac{\sigma-1}{\sigma}-1} \alpha \varphi l^{\alpha-1} \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}. \tag{22}$$

From equation 20, we get  $q_d(j) = q_d$  for all  $j$ , and from equation 21 we get  $q_f(j) = q_f$  for  $\forall j$ . Then, taking ratios of equations 20 and 21, for all firms that import intermediate products we get

$$\begin{aligned} \frac{1}{\tau} &= \left( \frac{q_d}{q_f} \right)^{-\frac{1}{\gamma}} \\ q_f &= \tau^{-\gamma} q_d \text{ for } \forall j. \end{aligned} \tag{23}$$

Total output of a final good is found as

$$\begin{aligned} y(\varphi, I) &= \varphi l^\alpha \left[ q_d^{\frac{\gamma-1}{\gamma}} + IN (\tau^{-\gamma} q_d)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \\ &= \varphi l^\alpha \left[ (1 + IN \tau^{1-\gamma}) q_d^{\frac{\gamma-1}{\gamma}} \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \\ &= \varphi l^\alpha (1 + IN \tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha}. \end{aligned} \tag{24}$$

This equation could further be simplified. Taking the ratios of equations 20 and 22 and using the results that  $q_d$  and  $q_f$  are constant, we get

$$\frac{l^\alpha \frac{(1-\alpha)\gamma}{\gamma-1} \left[ q_d^{\frac{\gamma-1}{\gamma}} + IN q_f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1} \frac{\gamma-1}{\gamma} q_d^{\frac{\gamma-1}{\gamma}-1}}{\alpha \varphi l^{\alpha-1} \left[ q_d^{\frac{\gamma-1}{\gamma}} + IN q_f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}} = 1.$$

Simplifying this ratio and using the finding  $q_f = \tau^{-\gamma} q_d$ , we get

$$\begin{aligned}
\frac{l(1-\alpha)q_d^{-\frac{1}{\gamma}}}{\alpha \left[ q_d^{\frac{\gamma-1}{\gamma}} + INq_f^{\frac{\gamma-1}{\gamma}} \right]} &= 1 \\
\frac{l(1-\alpha)q_d^{-\frac{1}{\gamma}}}{\alpha \left[ (1+IN\tau^{1-\gamma})q_d^{\frac{\gamma-1}{\gamma}} \right]} &= 1 \\
(1+IN\tau^{1-\gamma})q_d^{\frac{\gamma-1}{\gamma}} q_d^{\frac{1}{\gamma}} &= l \left( \frac{1-\alpha}{\alpha} \right) \\
(1+IN\tau^{1-\gamma})q_d \left( \frac{\alpha}{1-\alpha} \right) &= l. \tag{25}
\end{aligned}$$

Now combining the result from equation 22, the finding that  $p(j) = \left(\frac{y}{C}\right)^{-\frac{1}{\sigma}} P$ , and the result in equation 25, we get an equation that gives the relationship between price and wage rate

$$\begin{aligned}
w &= \frac{\sigma-1}{\sigma} p \alpha \varphi l^{\alpha-1} \left[ \int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}} dj + I \int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \\
w &= \frac{\sigma-1}{\sigma} p \alpha \varphi l^{\alpha-1} (1+IN\tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha} \\
p &= \frac{\frac{\sigma-1}{\sigma} p \alpha \varphi l^{\alpha-1} (1+IN\tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha}}{w} \\
&= \frac{\frac{\sigma-1}{\sigma} p \alpha \varphi \left( (1+IN\tau^{1-\gamma})q_d \left( \frac{\alpha}{1-\alpha} \right) \right)^{\alpha-1} (1+IN\tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha}}{w} \\
&= \frac{\frac{\sigma-1}{\sigma} \alpha^\alpha (1-\alpha)^{1-\alpha} \varphi (1+IN\tau^{1-\gamma})^{\alpha-1} (1+IN\tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}}}{w} \\
p &= \frac{\frac{\sigma-1}{\sigma} \alpha^\alpha (1-\alpha)^{1-\alpha} \varphi (1+IN\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}}{w}. \tag{26}
\end{aligned}$$

The result obtained from equation 26 shows that the final good price for a good that only uses domestic intermediate goods is  $p^h(\varphi) = \frac{\sigma}{\sigma-1} \frac{w}{\alpha^\alpha (1-\alpha)^{1-\alpha} \varphi}$ . If imported intermediate goods are used in production, then the price is  $p^m(\varphi) = p^h(\varphi) / (1+IN\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}$ .

Next we derive the equilibrium values of  $q_d$  and  $l$ . From the solution of the profit maximization problem of the final good producer, replacing the equilibrium value of labor  $l$  from

equation 25 into equation 24 we get

$$\begin{aligned}
y(\varphi, I) &= \varphi \left( (1 + IN\tau^{1-\gamma})q_d \left( \frac{\alpha}{1-\alpha} \right) \right)^\alpha (1 + IN\tau^{1-\gamma})^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha} \\
y(\varphi, I) &= \varphi (1 + IN\tau^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \left( \frac{\alpha}{1-\alpha} \right)^\alpha q_d.
\end{aligned} \tag{27}$$

Recall that from composite good producer's maximization problem we had

$$y(\varphi) = p(\varphi)^{-\sigma} \frac{E}{P^{1-\sigma}}. \tag{28}$$

Combining equations 27 and 28, we get

$$\begin{aligned}
p(\varphi)^{-\sigma} \frac{E}{P^{1-\sigma}} &= \varphi (1 + IN\tau^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \left( \frac{\alpha}{1-\alpha} \right)^\alpha q_d \\
q_d &= \frac{p(\varphi)^{-\sigma} \frac{E}{P^{1-\sigma}}}{\varphi (1 + IN\tau^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \left( \frac{\alpha}{1-\alpha} \right)^\alpha}.
\end{aligned}$$

We can get rid of the price in this equation by plugging in the value of  $p(\varphi)$  from equation 26

$$\begin{aligned}
q_d &= \frac{E}{P^{1-\sigma}} \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \left[ \varphi (1 + IN\tau^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \right]^{-1} \left[ \frac{\sigma}{\sigma-1} \frac{w}{\alpha^\alpha (1-\alpha)^{1-\alpha} \varphi (1 + IN\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}} \right]^{-\sigma} \\
&= \frac{E}{P^{1-\sigma}} \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \varphi^{\sigma-1} (1 + IN\tau^{1-\gamma})^{\frac{(1-\alpha)\sigma-\gamma+\alpha}{\gamma-1}} \left( \frac{\sigma-1}{\sigma} \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{w} \right)^\sigma.
\end{aligned}$$

Finally labor value can be found by incorporating  $q_d$  into equation 25

$$l = \frac{E}{P^{1-\sigma}} \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \varphi^{\sigma-1} (1 + IN\tau^{1-\gamma})^{\frac{(1-\alpha)(\sigma-1)}{\gamma-1}} \left( \frac{\sigma-1}{\sigma} \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{w} \right)^\sigma.$$

## B Balanced Growth in the Economy

We assume that efficiency levels grow at rate  $g$  (i.e.  $\dot{\varphi}/\varphi = g$ ). Since there is no population growth,  $L$  is constant. Then from equation 4 we get  $\dot{y}(\varphi, I)/y(\varphi, I) = g$ . Equation 1 gives

$$\begin{aligned}\dot{C} &= \frac{\sigma}{\sigma-1} \left( \int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}-1} \left( \int_{j \in J} \frac{\sigma-1}{\sigma} y_t(j)^{\frac{\sigma-1}{\sigma}-1} y(j) \frac{\dot{y}(j)}{y(j)} dj \right) \\ \dot{C} &= \frac{\sigma}{\sigma-1} \frac{\sigma-1}{\sigma} Cg \\ \frac{\dot{C}}{C} &= g.\end{aligned}$$

Since we normalize aggregate expenditure to a constant in the model  $E = P_t C_t$  implies that  $P$  and  $p(j)$  for  $\forall j$  decreases at rate  $g$ .

$$\begin{aligned}P_t &= \left( \int_{j \in J} p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \\ \dot{P} &= \frac{1}{1-\sigma} \left( \int_{j \in J} p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}-1} \left( \int (1-\sigma) p(j)^{-\sigma} \frac{\dot{p}(j)}{p(j)} p(j) dj \right) \\ \frac{\dot{P}}{P} &= -g = \frac{\dot{p}(j)}{p(j)} \text{ for } \forall j.\end{aligned}$$

From the final good producer's optimization problem we found that

$$p(\varphi) = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}.$$

Since  $p(\varphi)$  decreases at rate  $g$  and  $\varphi$  grows at rate  $g$ , wage is constant. Then profit per product  $\pi(\varphi) = \left( \frac{p(\varphi)}{P} \right)^{1-\sigma} \frac{E}{\sigma}$  is constant. This allows us to get the stationary solution in the Bellman equation. Note that although wage is constant, real wage which is  $w/P_t$  and real output  $\frac{r(\varphi)}{P}$  grow at rate  $g$ .

## C Steady State Size Distribution of Firms

Klette and Kortum (2004) show that in steady state mass of firms of any type converges to

$$M_n(\varphi) = \frac{\eta\phi(\varphi)}{\mu n} \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1}. \quad (29)$$

Using this equation, we can derive total mass of firms of type  $\varphi$  as

$$\begin{aligned} M(\varphi) &= \sum_{n=1}^{\infty} M_n(\varphi) = \frac{\eta\phi(\varphi)}{\mu} \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1} \\ &= \frac{\eta\phi(\varphi)}{\lambda(\varphi)} \ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right). \end{aligned} \quad (30)$$

Here, convergence to a stationary size distribution requires  $\mu > \lambda(\varphi)$  for all  $\varphi$ . Taking the ratio of equations 29 and 30 gives the steady state size distribution for the number of products

$$\frac{M_n(\varphi)}{M(\varphi)} = \frac{\frac{1}{n} \left( \frac{\lambda(\varphi)}{\mu} \right)^n}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)}.$$

This is probability distribution for the logarithmic distribution with parameter value of  $\lambda(\varphi) / \mu$ .

Finally total mass of products produced by  $\varphi$ -type firms is found as

$$\Lambda(\varphi) = \sum_{n=1}^{\infty} n M_n(\varphi) = \sum_{n=1}^{\infty} \frac{\eta\phi(\varphi)}{\mu} \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1} = \frac{\eta\phi(\varphi)}{\mu - \lambda(\varphi)}. \quad (31)$$

## D Deriving Entry Type Distribution in Steady State

From equation 30 we get

$$\begin{aligned} \eta\phi(\varphi) &= \frac{M(\varphi) \lambda(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} \\ &= \frac{M \lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)}. \end{aligned}$$

Taking the integrals of both sides and using the fact that  $\int \phi(\varphi) d\varphi = 1$ , we get

$$\begin{aligned}\eta &= \int \eta \phi(\varphi) d\varphi = M \int \frac{\lambda(\varphi) \delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)} d\varphi \\ \phi(\varphi) &= \frac{\frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}}{\int \frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)} d\varphi}.\end{aligned}\tag{32}$$

## E Deriving Aggregate Price Index

We defined  $\Lambda(\varphi)$  as total mass of products produced by  $\varphi$ -type firms. Plugging the value of  $p(\varphi)$  from equation 26 into aggregate price index from equation 2, we get

$$\begin{aligned}P^{1-\sigma} &= \int_0^{\varphi_m^*} p^h(\varphi)^{1-\sigma} \Lambda(\varphi) d\varphi + \int_{\varphi_m^*}^{\infty} p^m(\varphi)^{1-\sigma} \Lambda(\varphi) d\varphi \\ P &= \left( \frac{\sigma}{\sigma-1} \frac{w}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \right) \left( \int_0^{\varphi_m^*} \varphi^{\sigma-1} \Lambda(\varphi) d\varphi + \int_{\varphi_m^*}^{\infty} \left( \varphi(1+N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}} \right)^{\sigma-1} \Lambda(\varphi) d\varphi \right)^{-\frac{1}{\sigma-1}}.\end{aligned}$$

In the simulation exercise, aggregate price index can be computed by replacing  $\Lambda(\varphi)$  by its value in equation 31 and  $\phi(\varphi)$  by its value in equation 32 which yields to

$$\begin{aligned}P^{1-\sigma} &= \int_0^{\infty} p(\varphi)^{1-\sigma} \Lambda(\varphi) d\varphi \\ &= \int_0^{\infty} p(\varphi)^{1-\sigma} \eta \frac{\frac{\lambda(\varphi)\delta(\varphi)}{(\mu-\lambda(\varphi)) \ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}}{\int \frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)} d\varphi} d\varphi.\end{aligned}$$

## F Maximum Attainable Level of Efficiency

Let's define  $\pi^{\max}$  as the maximum attainable level of profit that would satisfy  $\lambda(\varphi) < \mu$  for all  $\varphi$ .

$$\begin{aligned}\frac{\pi^{\max} - \frac{wc_0\mu^{1+c_1}}{(1+N)^{\zeta c_1}}}{r + \mu - \mu} &= \frac{wc_0(1+c_1)\mu^{c_1}}{(1+N)^{\zeta c_1}} \\ \pi^{\max} &= \frac{wc_0\mu^{c_1}}{(1+N)^{\zeta c_1}} (r(1+c_1) + \mu).\end{aligned}$$

The efficiency level that would correspond to this profit level can be found as follows

$$\begin{aligned}
\bar{\pi} &= \pi^{\max} - f_m \\
&= \frac{E}{\sigma} \left[ \frac{\sigma - 1}{\sigma} \frac{P \alpha^\alpha (1 - \alpha)^{1-a} \varphi (1 + N \tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}}{w} \right]^{\sigma-1} - f_m \\
\varphi^{\max} &= \left[ (\bar{\pi} + f_m) \frac{\sigma}{E} \right]^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma - 1} \frac{w}{P \alpha^\alpha (1 - \alpha)^{1-a} (1 + N \tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}}.
\end{aligned}$$

## G Algorithm Steps for the Model Solution

For a given set of model parameters  $\Delta = \{ \Delta_1, \Delta_2 \}$  we simulate a panel of 10,000 firms which are identified by their unique efficiency levels. Using this panel, we compute the simulated moments and seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. The steps to compute the equilibrium are described as follows:

1. The parameter vector is initialized and the vertices of the simplex are determined.
2. For each parameter vector, equilibrium level of aggregate price index  $P$  and aggregate destruction rate  $\mu$  are computed.
3. For each of the 10,000 efficiency level draws, Bellman equation is solved and innovation rates are found.
4. Using these values, moment are computed.
5. The value of the criterion function is checked and using the amoeba routine, the simplex of parameter vectors is updated.
6. The system is iterated until either the value of the criterion function or the parameter vector converges.

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