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Optimal Resolution Procedures and Dividend Policy for Global-Systemically-Important-Banks*

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Abstract

Following the global financial crisis of 2007-2009, bank regulators have adopted special resolution procedures for global systemically important banks. They now have the power to seize these banks when their capital falls below some threshold, and to sell them back to new investors after having restructured them. This paper characterizes the optimal intervention thresholds and studies the interactions with the dividend and equity issuance policies of global systemically important banks. The main findings of our analysis for the optimal regulatory policy are: First, when the restructuring costs are high, it is optimal for regulators to impose dividend payout restrictions for undercapitalized banks. Second, in states of the economy, where capital supply becomes more scarce, these results are aggravated. In particular, regulators intervene relatively earlier, set stricter dividend payout restrictions and require relatively higher initial capital of the banks. Finally, capital supply constraints have an important impact on the financing decisions of shareholders. Banks recapitalize less frequently when the cost of raising equity is high or when the external capital supply is plentiful.

Key words: capital supply uncertainty, banking regulation, optimal dividend policy, resolution procedures, global systemically important bank.

JEL classifications: $G21$, $G32$, $G33$, $G35$.

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1 Introduction

The 2007-2009 financial crisis has brought into the attention of regulatory authorities the massive risks generated by the so-called Global Systemically Important Banks (G-SIBs).[1] During the crisis, costly government interventions such as bail-outs of large, complex banks or other financial institutions[2] and the disruptive bankruptcies of Lehman Brothers or other systemically important financial institutions have been followed by a turmoil in the world's financial markets. As a direct reaction to the financial crisis, governments have agreed on the necessity of reforms of the existing regulations. In particular, the development of special resolution mechanisms to deal with the distress or even failure of Systemically Important Financial Institutions (SIFIs) (such as G-SIBs) has been in the focus of the joint effort to design, promote and implement reforms of the financial regulations.[3] The Dodd-Frank Wall Street Reform and Consumer Protection Act, which was established in the US in July 2010, authorizes the Federal Deposit Insurance Corporation (FDIC) to step-in and resolve a situation of severe financial distress of a systemically important financial firm.[4] This mechanism should assure a fast stabilization of the financial firm in order to protect the taxpayers' rights and the financial system by dispensing from the necessity of a bail-out and ensuring the continuation of the systemically important financial intermediary. The key feature is that the FDIC obtains the full power over the management of the distressed financial institution. In particular, it has the right to seize, restructure and subsequently sell a G-SIB that is in a financially distressed situation. In addition, the Basel Committee on Banking Supervision introduced Basel III decisions in September 2010 which focus on increasing the loss absorbing capacity of G-SIBs by proposing higher capital requirements and by introducing capital surcharges for these institutions.[5] As part of these new regulations, the Financial Stability Board (FSB) introduced new international standards for effective resolution procedures [9] in November 2011, which also emphasize the more intensive and effective supervision of all G-SIBs. Finally, European Union (EU) is currently adopting new reforms on the structure of EU banking sector which aim at eradicating

[1] The term Global Systemically Important Bank refers to a financial institution whose distress or a close-down has substantial adverse effects on an economy due to its complexity, size, national and global interconnectedness, and central role as financial intermediary in an economy. The Basel Committee on Banking Supervision developed an indicator-based measurement approach to determine which banks are global systemically important. For each individual bank, the method calculates the weighted average of the indicator values representing five categories of systemic importance, which are: size, cross-jurisdictional activity, interconnectedness, substitutability, and complexity. The Financial Stability Board annually announces the list of G-SIBs with regard to this approach.

[2] AIG, Freddie Mac and Fannie Mae are popular examples for the bail-outs during the crisis.

[3] Since the main focus of this study is on Global Systemically Important Banks, we refer in what follows only to G-SIBs even though the new regulation apply to systemically important financial institutions in general.

[4] See *https://www.sec.gov/about/laws/wallstreetreform-cpa.pdf* for detail.

[5] See *http://www.bis.org/bcbs/basel3.htm* for detail.

too-big, too-complex, too-interconnected-to-fail properties embodied by G-SIBs.

One of the main problems that the regulators face in the implementation of these new policy rules is, however, the actual management of the resolution procedures. That is, when it is optimal, from a social welfare point of view, that the regulator steps-in, decides to restructure and subsequently sells a G-SIB. Moreover, from a regulatory point of view, it is also crucial to prevent G-SIBs from reaching a highly critical situation at all. In order to do so, it might be optimal to restrict the dividend policy of these banks. Furthermore, it is unclear whether the new regulatory policy might actually have adverse effects on shareholders' financing decisions. In particular, shareholders of the bank might anticipate the potential restructuring interventions of the regulator and therefore issue new equity earlier than it would be optimal.

This paper presents a dynamic model that contributes to our understanding about optimal regulatory intervention policies and their interaction with the optimal financing policies of G-SIBs. Importantly, the model incorporates also the impact of capital supply constraints. One main reason why solvency problems of a bank might be exacerbated and potentially even lead to insolvency, is the capital supply uncertainty. During market downturns and credit crises, outside investors are not always standing by to finance firms. The 2007-2009 financial crisis is a recent illustration of a situation of severe capital supply constraints. Therefore, we directly incorporate these constraints in our model by assuming that investors that are willing to inject fresh equity only arrive at an uncertain (i.e. stochastic) rate. Decreasing the intensity of investor arrival allows us to directly study the impact of capital supply constraints on the equity issuance decisions of firms and on the optimal resolution and dividend payout restriction policies of the regulator.

The main features of the model are as follows. The cash flows of the bank follow an arithmetic Brownian motion and the bank has a fixed size.[6] The regulator continuously audits the bank's capital. Depending on the level of the capital, the regulator can decide on the optimal single-point-of-entry for a global systemically important bank. The regulator endogenously defines a capital threshold under which the bank is restructured. In particular, every time the bank's capital falls below a certain nonnegative level, the shareholders are expropriated, the bank is restructured and privatized by the regulator.[7] Another important feature of the model is that we allow the regulator to restrict the dividend payout policy of the bank depending on the capital. The intuition is that the interests of the regulator and shareholders potentially run counter to each other since the shareholders maximize the market value of equity instead of the social welfare that the regulator aims to maximize.

[6]For the sake of simplicity, we take the size of the bank to be given in our model. Moreover, we assume that the bank is systemically important and leave the question of when a bank is to be considered as systemically important for future research.

[7]Such a policy has been successfully implemented in Scandinavian countries, in particular in Sweden, during the banking crises in early 1990′s. Regulators did nationalize almost one third of banks, restructured and sold them to the private sector with a profit.

The initial capital of the bank is also endogenously chosen by the regulator.[8] Finally, in addition to the capital supply uncertainty, we introduce a fixed cost of new equity issuance. Due to this model feature, the management of the bank chooses endogenously the time of the new equity issuance by aiming to maximize the value of the equity.[9]

The main findings of our analysis for the optimal regulatory policy are: First, when the restructuring costs are high, it is optimal for regulators to impose dividend payout restrictions for undercapitalized banks. Second, initial capital that should be invested by the shareholders of the bank is positively related with the restructuring costs. More specifically, when the restructuring costs are high, regulators require banks to invest higher initial capital to prevent possible costly restructuring. Third, in states of the economy where capital supply becomes more scarce these results are aggravated. For instance, regulators intervene relatively earlier, set stricter dividend payout restrictions and require relatively higher initial capital of the banks. We further find that capital supply constraints have an important impact on the financing decisions of shareholders. Banks recapitalize less frequently when the cost of raising equity is high or when external capital supply is high. We believe that, overall, our findings make an important step towards solving the problem of the implementation of the new resolution procedures for G-SIBs and improve our understanding about their potential adverse effects on shareholders' financing decisions.

The remainder of the paper is organized as follows. Section 2 gives a brief review of the literature. We describe our model in Section 3. In Section 4 we provide the characterization of the value function and the restructuring, equity issuance, and payout policies. Section 5 contains the numerical analysis and section 6 finally concludes. The proofs and Figures are gathered in Appendix A and B, respectively.

2 Related Literature

Our paper builds on recent studies in the corporate finance literature that analyze the implications of financing constraints on corporate policies by using continuous time models. We extend these papers by including essential properties that are specific to banks and characterize the optimal regulatory intervention policies and their interaction with the optimal financing policies of G-SIBs under financial frictions that are mainly related to capital supply uncertainty.

The first strand of the related literature comprises the banking regulation studies that rely on the 'valuation approach'. This strand dates back to Merton [18] who derives deposit insurance costs by utilizing the famous Black-Scholes framework. Merton defines an isomorphism between deposit insurance and put options on firm equity and utilizes explicit

[8] The regulator chooses initial capital, the restructuring and dividend thresholds of the bank with the objective of social welfare maximization.

[9] Even though it might be an interesting avenue for future research, we do not consider any moral hazard problem between managers and equity holders in this work.

formulas from the Black-Scholes model. He extends the framework by incorporating auditing costs and random auditing times in a subsequent paper [19]. Marcus [17] emphasizes the impact of franchise value and bankruptcy costs to the banks' policy.[10] He concludes that as the franchise value of the banks decreases (increases), banks become more risk-loving (risk-averse). Fries et al. [11] investigate optimal bank closure rules and their implications on deposit insurance pricing in a setting where the regulator continuously audits. They show that the regulator optimally balances the monitoring and bankruptcy costs. An optimal closure rule has the feature that, given lower monitoring costs and the independence of bankruptcy costs and profitability, the regulator postpones the closure until the bank's asset value is low enough to decrease the bankruptcy costs. Milne and Whalley [22] is the closest paper to our study in the banking regulation literature. They build a model that examines the effects of capital regulation and audit frequency on the incentives of commercial banks. The model is an extension of the continuous-time capital structure trade-off model of Milne and Robertson [21] by means of incorporating Poisson distributed audits of the regulator, which result in either liquidation or restructuring of the bank, and which are subject to a fixed cost.[11] Their analysis shows that the fear of liquidation provides an incentive for banks to hold an extra capital buffer with respect to the regulatory threshold. Our paper differs from Milne and Whalley [22] in the following ways: First, we introduce capital supply uncertainty instead of assuming perfect elasticity of the capital supply. Second, we assume continuous audit of the regulator and the direct restructuring of the bank in case of a financial distress. Milne [20] utilizes the incentive mechanism in Milne and Whalley [22] for examining the banks' portfolio choice. He shows that, given the ex-post penalty of capital requirements violation, the main impact of capital regulation is reflected as ex-ante incentives of the bank to avoid these capital requirement breaches. Thus, in contrast to the literature, he proposes strengthening regulatory ex-post penalties. Bhattacharya et al. [3] analyze optimal closure rules for banks in a regulatory structure consisting of audit frequency, capital replenishment and closure rules depending on the risk level of banks. They demonstrate that the excessive risk taking behavior of solvent banks can be deterred by an optimal combination of capital adequacy, closure and auditing rules. Decamps, Rochet, and Roger [7] propose a dynamic model to grasp the interaction of the three pillars of Basel II: capital adequacy requirement, supervisory review, and market discipline. They find that the capital adequacy requirement should be used as a vehicle to oblige the closure before the bank becomes insolvent. Moreover, when the cash flows of the bank are not visible without high monitoring costs, the bank should be required to raise subordinated debt with a cash flow contingent payoff, which yields a cash flow threshold

[10] Marcus denotes it as 'charter value', which represents the present value of the expected future earnings that is lost in case of liquidation.

[11] The paper characterizes the optimal corporate policies in an environment, in which the firm is inelastic to raise debt or equity and is subject to liquidation when the cash flows drop to a certain level. Therefore, optimal firm behavior shows a risk aversion, which is negatively correlated with the liquid assets in hand.

for auditing. However, their results are contingent on non-volatile market prices and the independence of regulators from political pressure. Our contribution to the existing literature in banking regulation is to introduce capital supply uncertainty and to investigate the impact of this friction to global systemically important banks' capital management policies under regulation.

The second strand of the related literature includes the recent corporate finance papers on financing frictions. In these models, cash hoarding is precautionary due to either external financing costs or capital supply uncertainty.[12] Décamps et al. [6] develop a dynamic model of cash management with two financial constraints: internal agency frictions and external financing costs. The model is solved in closed-form and the optimal payout and equity issuance policies are fully characterized. Implications of financial frictions on the issuance and dividend policies, corporate cash value, and the stock price dynamics are presented. In a contemporaneous study, Bolton et al. [4] extend Décamps et al. [6] by incorporating flexible firm size which allows them to investigate investment in a dynamic manner. They enhance the existing results by demonstrating that the investment depends on the ratio of marginal q to the marginal cost of financing. An extension of these two papers is the study of Bolton et al. [5], which allows for time varying investment and stochastic financing opportunities.[13] The key observations are: First, during market downturns or weak financing conditions the firm has a precautionary motive for holding cash, reduces investment, and postpones pay-outs. However, during favorable market conditions the firm may rationally time the market and issue equity even when it is not necessary. The models in previous studies have a common feature that firms always follow a double barrier policy for issuance and payout. On the contrary, by introducing capital supply uncertainty and lumpy investment, Hugonnier et al. [12] show that optimal financing and payout policies of firms may differ from standard double barrier (S,s) policies. Another appealing result in their study is that smooth-pasting conditions in preceding papers are necessary but not sufficient. Our model is built on the setup in Hugonnier et al. [12] without growth option. We adapt the existing setup in Hugonnier et al. [12] to a continuous time framework for a G-SIB and analyze capital supply effects to the optimal dividend, equity issuance, and restructuring policies of these institutions.

3 Model

We model a bank with capital (equity) (C_t) that collects deposits (D) from the public, transforms them into risky assets (A), and optimizes its cash buffer (M_t) by retaining earnings or raising new equity from outside investors. For the sake of simplicity, the deposit

[12]The precautionary motive for holding cash was introduced by the grandfather of modern macroeconomics, Keynes [14]. In the recent literature, e.g., Kim et al. [16], Almeida et al. [1] and Bates et al. [2] emphasize this motive as well.

[13] In particular, there exist two states for capital at any point in time: cheap and expensive.

volume is taken as a constant and the interest rate paid to the depositors is assumed to be zero. Asset value (A) represents the loans and is taken as a constant as well. Hence, the balance sheet of the bank (book value) is given as

A	D
M_t	C_t

We have a continuous time model with no agency frictions: the manager acts in the best interest of shareholders. Shareholders and the manager of the bank are risk neutral and discount the future payments at a constant rate r. Uncertainty in the model is described by $(\Omega, \mathbb{F}, \mathbb{P})$, a filtered probability space satisfying the usual assumptions.[14] Risky Assets (A) of the bank generate random cash flows that follow an arithmetic Brownian motion

$$dY_t = \mu dt + \sigma dB_t, \tag{3.1}$$

where μ and σ are constant (mean and volatility of the cash flows), and B_t is a standard Brownian motion.[15] Therefore, the bank may have operating losses, which are financed through cash reserves. The bank uses additional financing through raising new equity.[16] The core financial friction in the model is the capital supply uncertainty. The bank has to search for outside investors in order to raise new equity. There is no search cost but the outside investors appear at the jump times of a Poisson process N_t with intensity λ. Thus, the expected outside financing lag is $\frac{1}{\lambda}$ years. Raising outside financing has a fixed cost, F. In particular, to obtain $f \geq 0$, the bank has to raise $f + F$ from outside financiers. Hence, when outside investors arrive, the bank will raise outside equity only if it is profitable to do so. Shareholders have limited liability and the cash reserves must remain nonnegative. Dividend payments are chosen subject to these restrictions. Cumulative dividend payments are expressed by L_t, which is an adapted, nondecreasing, càdlàg process with $L_0 = 0$. The returns on cash reserves are assumed to be zero. Thus, there is an opportunity cost of holding cash, which makes the liquidity management crucial. In light of these assumptions, the dynamics of the bank's cash reserves satisfy

$$dM_t = \mu dt + \sigma dB_t - dL_t + f_t dN_t, \tag{3.2}$$

where f_t denotes the outside fund process, which is nonnegative and predictable. Finally,

[14]See Karatzas and Shreve [15] for details.

[15]The rationale behind the choice of arithmetic Brownian motion for the cash flow process is that the arithmetic Brownian motion fits well to the setup with fixed asset size and it captures the potential operating losses.

[16] As shown in the pioneering work of Jeanblanc-Picqué and Shiryaev [13], any form of debt (straight, contingent convertible (coco), subordinated, etc.) is sub-optimal in the absence of tax benefits or public subsidy, and some asymmetric information and corporate governance problems (moral hazard, asset substitution, cash flow diversion, etc.).

the book value of the equity is given by the balance sheet equation as

$$C_t = M_t + A - D.$$[17]

The bank is global systemically important: its closure would entail huge cost for the society. Hence, the regulator adopts a special resolution procedure: every time that the bank's capital falls below a certain threshold $\underline{c} \geq 0$, the shareholders are expropriated, the bank is restructured and privatized by the regulator again. The restructuring threshold $\underline{c}$ is chosen by the regulator so as to maximize social welfare at date 0. Let $\mathcal{A}$ be the set of dividend strategies such that $\mathbb{E}_c\left[\int_0^\tau e^{-rt}dL_t\right] < \infty$ for all $c \geq \underline{c}$ and $\tau := \inf\{t \geq 0 | C_t = \underline{c}\}$. The bank maximizes the expected present value of future payments to the incumbent shareholders until the first restructuring time τ, net of claim of the new (outside) investors on future cash flows by choosing the bank's payout ($L \in \mathcal{A}$) and financing (f) policies:

$$V_s(c) = \max_{(L\in\mathcal{A},f)} \mathbb{E}\left[\int_0^\tau e^{-rt}(dL_t - (f_t + \mathbf{1}_{\{f_t>0\}}F)dN_t)|C_0 = c\right], \tag{3.3}$$

where V_s is the value function (market value of equity) of the bank and $\mathbf{1}$ is the indicator function. There is a trade-off for the shareholders in the choice of the optimal dividend policy: When the bank distributes higher amount of dividends, the shareholders wealth will be higher but the risk of restructuring will increase. However, if the bank keeps a higher level of cash in the bank, the shareholders will get few dividends and will be subject to the cost of holding cash. Therefore, as shown in the extant literature for firms, there exists a target m^* at which the marginal cost and benefit of holding cash are equalized, and it becomes optimal for the shareholders to start paying dividends. This target cash level corresponds to a capital threshold $c^* = m^* + A - D$.
In addition, at any time t such that $C_t < c^*$ and the outside investors arrive, the bank raises outside funds to bring the capital to c^* if it is profitable to do so. Considering the fixed cost (F), the net gain for the bank from raising outside equity is

$$(V_s(c^*) - V_s(c)) - (c^* - c) - F.$$

Assuming concavity of the value function, $V_s'(c^*) = 1$ and $V_s'(c) \geq 1,\ \forall c \leq c^* \Leftrightarrow V_s(c) - c$ is increasing in c.[18] Then, taking into account the fixed cost of issuance (F), the bank raises outside funds only if the net gain is positive: $V_s(c^*) - c^* - F > V_s(c) - c$.
The left hand side of the above inequality is constant, whereas the right hand side is increasing in c. Therefore, $\exists\ c_1 < c^*$ at which

$$V_s(c^*) - c^* - F = V_s(c_1) - c_1,$$

[17]The basic setup in our model yields $dC_t = dM_t$, which means that one can interpret the regulatory policies in terms of either capital or reserve requirements.

[18]Concavity is proved in Appendix 2.A by adapting the methodology presented in Hugonnier et al. [12] to our setup.

i.e., marginal cost and benefit of raising outside equity are equal. Therefore, the bank raises outside equity only if $c \le c_1$.
On the other hand, the interests of the regulator and shareholders conflict in our framework since the shareholders maximize the market value of equity (V_s) instead of social welfare as the regulator does. Hence, we allow the regulator to prohibit dividends until the bank's capital reaches another threshold $\overline{c} \ge c^*$. This threshold is chosen by the regulator via social welfare maximization:

$$W = \max_{c_0, \underline{c}, \overline{c}} \mathbb{E}\left[\int_0^\tau e^{-rt}(dL_t - (f_t + 1_{\{f_t>0\}}F)dN_t) + e^{-r\tau}[-\xi + \underline{c} + W]\right] - c_0. \tag{3.4}$$

The interpretation of the above maximization problem is as follows: The regulator initially gives a licence to the bank to operate. However, shareholders should invest c_0 to get this licence and initialize the bank, which gives them a certain value. The last term in the expectation represents the expected discounted welfare gain from the next restructuring. In particular, when the bank's capital drops to $\underline{c}$ at time τ, the regulator expropriates shareholders and restructures the bank by paying the fixed cost (ξ). Moreover, it takes the bank's capital ($\underline{c}$) at that moment and the continuation welfare of the bank (W) since the bank is restarted. The below figure summarizes the new issuance policies of the shareholders and the regulatory policies in different regions:

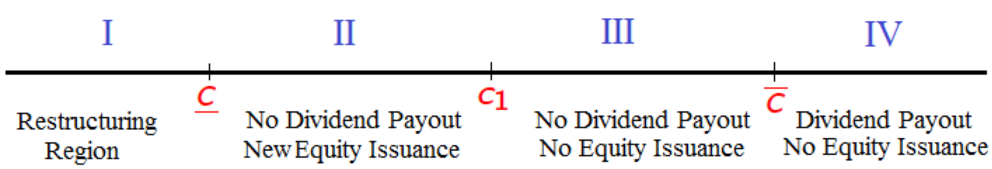

Figure 1: Policies in Different Regions.

In region I, the bank is restructured and privatized by the regulator. Region II is the financial distress region, in which the bank raises new equity as soon as the new investors arrive but it is not allowed to distribute dividends. In region III, raising new equity is not profitable for the bank anymore and the dividend payout is still forbidden by the regulator. Finally, the bank distributes excess cash as dividend to the shareholders in region IV.

4 Characterization of the Solution

In this section, we characterize the solution of the model. We start with the particular case, $F = \infty$, i.e., the option to raise outside equity is never exercised. Then, we'll move to the general case with an outside financing option and investigate the optimal dividend, equity issuance and restructuring policies for a global systemically important bank.

4.1 Particular Case: $F = \infty$

We start with a case where the voluntary recapitalization by shareholders is infinitely costly. Under this circumstance, restructuring and dividend thresholds $(\underline{c}, \bar{c})$ are policy variables chosen by the regulator. To better evaluate the effects of regulatory policies, we first investigate the optimal dividend threshold for the shareholders in the case where regulators do not restrict dividends. In this case, the manager decides on the optimal dividend policy by maximizing the expected present value of all dividend payouts until restructuring and the value function of the shareholders, $V_s(c \mid \underline{c}) := V_s(c)$ satisfies the following ordinary differential equation (ODE):

$$\begin{aligned} rV_s(c) &= \mu V_s'(c) + \frac{\sigma^2}{2} V_s''(c) && (4.1) \\ s.t. \quad & V_s(\underline{c}) = 0, && (4.2) \\ & V_s'(c^*) = 1, && (4.3) \\ & V_s''(c^*) = 0. && (4.4) \end{aligned}$$

The left hand side of equation (4.1) represents the expected return demanded by shareholders. The first and second terms on the right hand side represent the change in the bank's value via expected cash flows and the cash flow volatility, respectively. Condition (4.2) says that the bank's value is zero at the time of restructuring. Condition (4.3) is the smooth-pasting condition which implies that the marginal value of one dollar inside or outside the bank are equal at the optimal dividend threshold. Condition (4.4) is the super contact condition proved by Dumas [8] to be necessary and sufficient for optimality. Boundary conditions imply that the value of the bank at the optimal dividend threshold equals the first best value, i.e., $V_s(c^*) = \frac{\mu}{r}$. Given $\underline{c}$, one can solve the above second order homogenous ODE explicitly and the Proposition 4.1 summarizes the results.

Proposition 4.1 *The value of the bank and the optimal dividend threshold for the shareholders are given by*

$$V_s(c) = \begin{cases} \frac{-\eta_2^2 e^{\eta_1(c-c^*)} + \eta_1^2 e^{\eta_2(c-c^*)}}{\eta_1\eta_2(\eta_1-\eta_2)}, & \underline{c} \leq c \leq c^* \\ V_s(c^*) + c - c^*, & c \geq c^* \end{cases} \tag{4.5}$$

$$c^* = \underline{c} + \frac{ln(\frac{\eta_2^2}{\eta_1^2})}{\eta_1 - \eta_2}, \tag{4.6}$$

where $\eta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}$ *and* $V_s(c^*) = \mu/r$. [19]

[19] Note that $\eta_1 > 0 > \eta_2$ and $|\eta_1| < |\eta_2|$.

Proposition 4.1 shows that the optimal dividend threshold c^* is equal to the restructuring threshold chosen by the regulator plus a constant term that depends on the average profitability (μ), cash flow volatility (σ), and the discount rate (r).
We now proceed to the regulator's problem. The restructuring and dividend thresholds of the bank ($\underline{c}, \overline{c}$) and the initial capital (c_0) are assumed to be policy variables chosen by the regulator so as to maximize social welfare (W). W is itself the solution of the following equation

$$W = \max_{c_0, \underline{c}, \overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + W]. \tag{4.7}$$

As mentioned above, shareholders invest c_0 to get the license of the bank, which gives them a value $V(c_0; \underline{c}, \overline{c})$. Hence, $V(c_0; \underline{c}, \overline{c}) - c_0$ is the revenue raised by the public authorities when granting the license to shareholders. The last term represents the expected discounted welfare gain from the next restructuring where the function $G(c_0; \underline{c}, \overline{c}) := \mathbb{E}[e^{-r\tau}]$ is the stochastic discount factor.[20] This procedure is repeated to guarantee that the global systemically important bank operates forever.
We start by solving $V(c_0; \underline{c}, \overline{c}) := V(c_0)$ and $G(c_0; \underline{c}, \overline{c}) := G(c_0)$. In the region $(\underline{c}, \overline{c})$, functions $V(c_0)$ and $G(c_0)$ satisfy the following ODE's:

$$\begin{aligned} rV(c_0) &= \mu V'(c_0) + \frac{\sigma^2}{2} V''(c_0) \\ s.t. \quad V(\underline{c}) &= 0, \\ V'(\overline{c}) &= 1. \end{aligned} \tag{4.8}$$

$$\begin{aligned} rG(c_0) &= \mu G'(c_0) + \frac{\sigma^2}{2} G''(c_0) \\ s.t. \quad G(\underline{c}) &= 1, \\ G'(\overline{c}) &= 0. \end{aligned} \tag{4.9}$$

With the same intuition as in the shareholders' problem, the initial condition in (4.8) arises from the fact that the regulator stops the bank when the capital drops to $\underline{c}$. Moreover, the marginal value of capital will be one for the regulator at the level $\overline{c}$, which implies the boundary condition. On the other hand, the initial condition in (4.9) is due to the fact that the bank will be directly restructured if it starts with the lower threshold level. In addition, when the bank's capital is at the level $\overline{c}$, small positive changes in the cash reserves will be suddenly distributed as dividends, which returns the capital back to $\overline{c}$. Therefore, the regulator will be indifferent, which implies the boundary condition. The closed form solutions of (4.8) and (4.9) are given in Proposition 4.2.

[20] When r varies, this stochastic discount factor corresponds to the Laplace transform of the stopping time.

Proposition 4.2 *The closed form solutions for the functions $V(c_0; \underline{c}, \overline{c})$ and $G(c_0; \underline{c}, \overline{c})$ are given by*

$$V(c_0; \underline{c}, \overline{c}) = \frac{e^{\eta_2(c_0-\underline{c})} - e^{\eta_1(c_0-\underline{c})}}{\eta_2 e^{\eta_2(\overline{c}-\underline{c})} - \eta_1 e^{\eta_1(\overline{c}-\underline{c})}}, \tag{4.10}$$

$$G(c_0; \underline{c}, \overline{c}) = \frac{-\eta_2 e^{\eta_1(c_0-\overline{c})} + \eta_1 e^{\eta_2(c_0-\overline{c})}}{\eta_1 e^{-\eta_2(\overline{c}-\underline{c})} - \eta_2 e^{-\eta_1(\overline{c}-\underline{c})}} \tag{4.11}$$

where $\eta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}$.

In light of the Proposition 4.2, we therefore seek the fixed point

$$W_M = \max_{c_0, \underline{c}, \overline{c}} \mathcal{H}(c_0, \underline{c}, \overline{c}, W_M), \tag{4.12}$$

where

$$\begin{aligned} \mathcal{H}(c_0, \underline{c}, \overline{c}, W_M) \equiv\ & \frac{e^{\eta_2(c_0-\underline{c})} - e^{\eta_1(c_0-\underline{c})}}{\eta_2 e^{\eta_2(\overline{c}-\underline{c})} - \eta_1 e^{\eta_1(\overline{c}-\underline{c})}} - c_0 \\ & + \frac{-\eta_2 e^{\eta_1(c_0-\overline{c})} + \eta_1 e^{\eta_2(c_0-\overline{c})}}{\eta_1 e^{-\eta_2(\overline{c}-\underline{c})} - \eta_2 e^{-\eta_1(\overline{c}-\underline{c})}} [-\xi + \underline{c} + W_M]. \end{aligned} \tag{4.13}$$

Proposition 4.3 *Let $\mathbb{T}(W) = \max_{c_0, \underline{c}, \overline{c}} \mathcal{H}(c_0, \underline{c}, \overline{c}, W)$. Then, $\mathbb{T}$ is a contraction mapping and (4.12) has the unique fixed point W_M.*

Next, we solve the above fixed point problem iteratively and find the values for the initial capital, the restructuring and dividend thresholds. The numerical calculations show that when the voluntary recapitalization is impossible, optimal regulatory policies are:

1. The restructuring threshold is $\underline{c} = A - D$.
2. The regulator chooses a (weakly) higher dividend threshold than the shareholders: $\overline{c} \geq c^*$.
3. When the restructuring cost ξ is higher than a critical value ξ^* this inequality is strict.

Our first observation shows that the regulator always waits for the restructuring until the bank's capital falls below A-D or equivalently until the cash reserves of the bank drop to zero. Possible explanations for this result are: First, the regulator continuously audits the bank's capital and the intervention of the regulator is immediate in our model. In addition, bank's assets and the capital are fully observable and we do not allow for negative jumps in the cash flow process. Therefore, it may be reasonable for the regulator to wait until the last point in time due to the possibility that the bank can recover itself from the financial distress region. The second observation is that when the restructuring cost is lower than a critical value ξ^*, the optimal dividend thresholds of the regulator and shareholders coincide. Hence, the regulator does not put any restriction on dividend payouts. However, when the restructuring cost is high, i.e., $\xi > \xi^*$, the regulator prevents shareholders from distributing dividends until $\bar{c}$ is reached. Figure 2 illustrates the optimal thresholds.

[Insert Figure 2 Here]

Figure 2 shows that the initial capital (c_0) is positively related with the restructuring cost. In particular, when the restructuring cost is high, the regulator starts the bank with higher initial capital with an incentive to postpone the next costly restructuring event. This incentive is not so strong for small restructuring cost levels, which implies lower c_0 values. In addition, the regulator starts to install dividend payout restrictions to prevent the bank from financial distress region and the costly restructuring for high restructuring cost levels. Figure 3 presents the value function of the bank from the perspectives of the shareholders and the regulator for high restructuring cost levels, i.e., $\xi > \xi^*$.

[Insert Figure 3 Here]

The regulator predicts a relatively lower market value for the bank and hence sets a relatively higher dividend payout threshold.

4.2 General Solution

In the general case, shareholders have the option to raise outside equity when the outside investors are present and when it is profitable to do so, i.e., $c < c_1$.[21] The outside financing threshold (c_1) is optimally chosen by the shareholders via value maximization. By contrast, restructuring and dividend thresholds of the bank ($\underline{c}, \bar{c}$) and the initial capital (c_0) are assumed to be policy variables chosen by the regulator. In line with the previous case, we start by solving the shareholders' problem in the absence of dividend payout restrictions. For $c \in (\underline{c}, c^*)$, the shareholders' value function $V_s(c \mid \underline{c}) := V_s(c)$ satisfies the following ODE:

$$\begin{aligned} rV_s(c) &= \mu V_s'(c) + \frac{\sigma^2}{2} V_s''(c) + \lambda \max\left[V_s(c^*) - (c^* - c) - F - V_s(c), 0\right] \\ s.t. & \qquad (4.14) \\ V_s(\underline{c}) &= 0, \qquad (4.15) \\ V_s'(c^*) &= 1, \qquad (4.16) \\ V_s''(c^*) &= 0, \qquad (4.17) \\ V_s(c_1) &= V_s(c^*) - (c^* - c_1) - F, \qquad (4.18) \end{aligned}$$

[21] Note that the outside investors appear with a Poisson arrival rate λ.

where the term in brackets on the right hand side of (4.14) represents the expected change in the bank's value obtained by raising new equity. In particular, the last term is the product of the probability that the outside financiers arrive and the surplus from raising capital to the target level. The last boundary condition is incorporated into the general problem to reflect the fact that when the bank's capital is at the level c_1, the total surplus from raising new equity is zero, thus the shareholders are indifferent between raising new equity or not. Note that the upper bound for the fixed cost of issuance is $F^* = V_s(c^*) - c^*$. If $F > F^*$, it is never optimal to issue new equity for the bank. Therefore, we concentrate in the following analysis on those cases where $F < F^*$. Under the circumstances, the above problem has different solutions in 3 regions. More specifically, when the bank is in the financial distress region (i.e., $c \in (\underline{c}, c_1)$), the bank raises outside funds as soon as the outside financiers appear. On the other hand, when the bank is in the safe region (i.e., $c \in (c_1, c^*)$), the shareholders never exercise the issuance option even if the investors arrive since the surplus from raising outside equity is negative. Therefore, the last term in (4.14) vanishes. Finally, when the firm has excess cash (i.e., $c \in [c^*, \infty)$), the bank distributes this amount as a dividend. Proposition 4.4 presents the closed form solutions for the value function in different regions and the unique outside financing and dividend thresholds chosen by the shareholders.

Proposition 4.4 *The shareholders' value function with an outside financing option is a piecewise C^2 function, which is given by*

$$V_s(c) = \begin{cases} \frac{\mu\lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}\left[\frac{\mu}{r} + c - c^* - F\right] + z_1 e^{\theta_1(c-c_1)} + z_2 e^{\theta_2(c-c_1)}, & \underline{c} < c \le c_1 \\ \frac{-\eta_2^2 e^{\eta_1(c-c^*)} + \eta_1^2 e^{\eta_2(c-c^*)}}{\eta_1\eta_2(\eta_1-\eta_2)}, & c_1 \le c < c^* \\ V_s(c^*) + c - c^*, & c \ge c^* \end{cases} \tag{4.19}$$

where η_1, η_2 are defined as above, z_1, z_2 are constants[22]*, $V_s(c^*) = \mu/r$, and*

$$\theta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2(r+\lambda)}}{\sigma^2}.$$

In addition, there exist unique constants y_1^* and y_2^* such that the dividend and outside financing thresholds of the shareholders are given by

$$\begin{aligned} c_1 &= \underline{c} + y_2^*, \\ c^* &= \underline{c} + y_1^* + y_2^*. \end{aligned}$$

[22]These coefficients are found uniquely by using the boundary conditions and derived in Appendix 2.A.

Proposition 4.4 shows that the solution of the shareholders' problem depends only on the restructuring threshold. In addition, optimal thresholds for raising outside equity and dividend payouts are linear functions of this threshold.

As a next step, we deal with the regulator's welfare maximization problem to find the optimal thresholds. As in the particular case, we assume that the restructuring and dividend thresholds of the bank $(\underline{c}, \overline{c})$ and the initial capital (c_0) are policy variables chosen by the regulator so as to maximize welfare at each restructuring date:

$$W = \max_{c_0, \underline{c}, \overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + W].$$

However, the outside financing threshold (c_1) is chosen by shareholders optimally and incorporated into welfare maximization. We start by solving $G(c_0; \underline{c}, \overline{c}) := G(c_0)$[23] which satisfies in the region $(\underline{c}, \overline{c})$ the following ODE:

$$\begin{aligned} rG(c_0) &= \mu G'(c_0) + \frac{\sigma^2}{2} G''(c_0) + \lambda \, \max\left[G(\overline{c}) - G(c_0), 0\right] \quad (4.20) \\ s.t. \quad G(\underline{c}) &= 1, \\ G'(\overline{c}) &= 0. \end{aligned}$$

The last term in (4.20) reflects the impact of raising new equity on the stochastic discount factor. We will also incorporate the continuity and smooth pasting properties of the function G at c_1 to obtain a closed-form solution. We solve the problem in two regions:

$$G(c_0) = \begin{cases} G_1(c_0), & \underline{c} \leq c_0 \leq c_1, \\ G_2(c_0), & c_1 \leq c_0 \leq \overline{c}. \end{cases}$$

Proposition 4.5 *The closed form formula for the function $G(c_0)$ is given as follows:*

$$G(c_0) = \begin{cases} \frac{\lambda k}{(r+\lambda)} + \left(\frac{k(q-\theta_2 p)}{\theta_1 - \theta_2}\right) e^{-\theta_1(\underline{c} + y_2^* - c_0)} + \left(\frac{k(q-\theta_1 p)}{\theta_2 - \theta_1}\right) e^{-\theta_2(\underline{c} + y_2^* - c_0)}, & \underline{c} \leq c_0 \leq c_1 \\ \\ \frac{\eta_2 k e^{\eta_1(c_0 - \overline{c})} - \eta_1 k e^{\eta_2(c_0 - \overline{c})}}{\eta_2 - \eta_1}, & c_1 \leq c_0 \leq \overline{c} \end{cases}$$

$$\begin{aligned} where \quad k &= \frac{1}{\frac{\lambda}{(r+\lambda)} + \left(\frac{q-\theta_2 p}{\theta_1 - \theta_2}\right) e^{-\theta_1 y_2^*} + \left(\frac{q-\theta_1 p}{\theta_2 - \theta_1}\right) e^{-\theta_2 y_2^*}}, \\ p &= \frac{\eta_2 e^{\eta_1(\underline{c} - \overline{c} + y_2^*)} - \eta_1 e^{\eta_2(\underline{c} - \overline{c} + y_2^*)}}{\eta_2 - \eta_1} - \frac{\lambda}{(r+\lambda)}, \\ q &= \frac{\eta_2 \eta_1 e^{\eta_1(\underline{c} - \overline{c} + y_2^*)} - \eta_1 \eta_2 e^{\eta_2(\underline{c} - \overline{c} + y_2^*)}}{\eta_2 - \eta_1}. \end{aligned}$$

[23] For notational simplicity, we supress $(\underline{c}, \overline{c})$ when we refer to the function G.

Secondly, we solve for $V(c_0; \underline{c}, \overline{c}) := V(c_0)$ which satisfies the following ODE in the region $(\underline{c}, \overline{c})$:

$$\begin{aligned} rV(c) &= \mu V'(c) + \frac{\sigma^2}{2} V''(c) + \lambda \max[V(\overline{c}) - (\overline{c} - c) - F - V(c), 0] \\ s.t. \quad V(\underline{c}) &= 0, \\ V'(\overline{c}) &= 1. \end{aligned}$$

One should note that the super contact condition is not satisfied at $\overline{c}$. This is because $\overline{c}$ is chosen by the regulator, not by the shareholders. We solve the problem in two regions as above:

$$V(c_0) = \begin{cases} V_1(c_0), & \underline{c} \le c_0 \le c_1, \\ V_2(c_0), & c_1 \le c_0 \le \overline{c}. \end{cases}$$

Proposition 4.6 *The closed form formula for the function $V(c_0)$ is given as follows:*

$$V(c_0) = \begin{cases} e_1 c_0 + e_2 + f_1 e^{\theta_1 c_0} + f_2 e^{\theta_2 c_0}, & \underline{c} \le c_0 \le c_1 \\ \frac{(1-\eta_2 n) e^{\eta_1 (c_0 - \overline{c})} - (1-\eta_1 n) e^{\eta_2 (c_0 - \overline{c})}}{(\eta_1 - \eta_2)}, & c_1 \le c_0 \le \overline{c} \end{cases}$$

where

$$\begin{aligned} n &= \frac{e^{-\eta_2(\overline{c} - \underline{c} - y_2^*)} - e^{-\eta_1(\overline{c} - \underline{c} - y_2^*)} + (\eta_2 - \eta_1)(\overline{c} - \underline{c} - y_2^* + F)}{\eta_1 e^{-\eta_2(\overline{c} - \underline{c} - y_2^*)} - \eta_2 e^{-\eta_1(\overline{c} - \underline{c} - y_2^*)} + (\eta_2 - \eta_1)}, \\ e_1 &= \frac{\lambda}{r + \lambda}, \\ e_2 &= \frac{\mu \lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}(n - \overline{c} - F), \\ f_1 &= \frac{(\eta_1 - \theta_2)(1 - \eta_2 n) e^{\eta_1(\underline{c} + y_2^* - \overline{c})} - (\eta_2 - \theta_2)(1 - \eta_1 n) e^{\eta_2(\underline{c} + y_2^* - \overline{c})}}{(\eta_1 - \eta_2)(\theta_1 - \theta_2) e^{\theta_1(\underline{c} + y_2^*)}} \\ &\quad - \frac{[e_1(1 - \theta_2(\underline{c} + y_2^*)) - \theta_2 e_2]}{(\theta_1 - \theta_2) e^{\theta_1(\underline{c} + y_2^*)}}, \\ f_2 &= \frac{(\eta_1 - \theta_1)(1 - \eta_2 n) e^{\eta_1(\underline{c} + y_2^* - \overline{c})} - (\eta_2 - \theta_1)(1 - \eta_1 n) e^{\eta_2(\underline{c} + y_2^* - \overline{c})}}{(\eta_1 - \eta_2)(\theta_2 - \theta_1) e^{\theta_2(\underline{c} + y_2^*)}} \\ &\quad - \frac{[e_1(1 - \theta_1(\underline{c} + y_2^*)) - \theta_1 e_2]}{(\theta_2 - \theta_1) e^{\theta_2(\underline{c} + y_2^*)}}. \end{aligned}$$

As a next step, we deal with the regulator's welfare maximization problem to find the optimal thresholds $(\underline{c}, \overline{c})$ and the initial capital (c_0).[24] The problem is discussed in two

[24]The solution of the welfare maximization exists since the welfare function is continuous and bounded. This is ensured by the properties of the functions V and G.

cases since the functions V and G are defined piecewise:

$$W = \begin{cases} \max_{\underline{c},\bar{c},c_0} V_1(c_0;\underline{c},\bar{c}) - c_0 + G_1(c_0;\underline{c},\bar{c})[-\xi+\underline{c}+W], & \underline{c} \le c_0 \le c_1 \\ \max_{\underline{c},\bar{c},c_0} V_2(c_0;\underline{c},\bar{c}) - c_0 + G_2(c_0;\underline{c},\bar{c})[-\xi+\underline{c}+W], & c_1 \le c_0 \le \bar{c} \end{cases}$$

Intuitively, it is not reasonable for the regulator to start the bank in the financial distress region. Therefore, we are interested in the solution in region 2, i.e., $c_0 \in [c_1, c^*]$. The welfare function in this region is denoted by W_2. Hence, the regulator maximizes

$$\begin{aligned} W_2 \quad &= \quad \max_{\underline{c},\bar{c},c_0} \frac{(1-\eta_2 n)e^{\eta_1(c_0-\bar{c})} - (1-\eta_1 n)e^{\eta_2(c_0-\bar{c})}}{(\eta_1-\eta_2)} - c_0 \\ &\quad\quad + \frac{\eta_2 k e^{\eta_1(c_0-\bar{c})} - \eta_1 k e^{\eta_2(c_0-\bar{c})}}{\eta_2-\eta_1}[-\xi+\underline{c}+W_2], \\ s.t. \quad &\quad\quad \bar{c} \ge c_0 > \underline{c} \ge 0. \end{aligned}$$

We therefore investigate the fixed point problem:

$$W_{2M} = \max_{c_0,\underline{c},\bar{c}} \mathcal{H}(c_0,\underline{c},\bar{c},W_{2M}), \tag{4.21}$$

where

$$\begin{aligned} \mathcal{H}(c_0,\underline{c},\bar{c},W_{2M}) \quad &\equiv \quad \frac{(1-\eta_2 n)e^{\eta_1(c_0-\bar{c})} - (1-\eta_1 n)e^{\eta_2(c_0-\bar{c})}}{(\eta_1-\eta_2)} - c_0 \\ &\quad\quad + \frac{\eta_2 k e^{\eta_1(c_0-\bar{c})} - \eta_1 k e^{\eta_2(c_0-\bar{c})}}{\eta_2-\eta_1}[-\xi+\underline{c}+W_{2M}]. \end{aligned} \tag{4.22}$$

Proposition 4.7 *Let $\mathbb{T}(W_2) = \max_{c_0,\underline{c},\bar{c}} \mathcal{H}(c_0,\underline{c},\bar{c},W_2)$. Then, $\mathbb{T}$ is a contraction mapping and* (4.21) *has the unique fixed point W_{2M}.*

As in the particular case, we solve the fixed point problem iteratively and find the numerical solutions of the initial capital, the restructuring and dividend thresholds. When the voluntary recapitalization is possible, the optimal regulatory policies are found as follows:

1. The restructuring threshold is $\underline{c} = A - D$.
2. The regulator chooses a (weakly) higher dividend threshold than the shareholders: $\bar{c} \ge c^*$.
3. When the restructuring cost ξ is higher than a critical value $\xi^{**} > \xi^*$ this inequality is strict.

Similar to the particular case, where raising outside equity is impossible, the regulator always prefers to restructure the bank when the cash reserves fall below zero, or equivalently when the bank's capital drops to A-D. As the bank has a voluntary recapitalization option upon the arrival of the outside investors, the bank may prevent itself from the financial distress region by raising outside equity. This option and the complete transparency of the bank's capital might provide an incentive to the regulator to wait until the last moment. The main difference from the particular case is that the critical restructuring cost level is relatively higher in the general case. In particular, the regulator intervenes relatively later. This result can be explained with the intuition that when the bank has the opportunity to issue new equity, its ability to prevent itself from financial distress is relatively stronger. Therefore, the intervention threshold of the regulator is relatively higher.
Figure 4 illustrates one example for the initial capital and the optimal dividend thresholds.

[Insert Figure 4 Here]

Comparison of Figures 2 and 4 shows that the divergence of the shareholders' and the regulator's dividend thresholds is relatively smaller when we introduce an outside financing option. Hence, the regulations are less stringent in the general case due to the relatively higher probability of the bank's self-prevention from the costly restructuring by raising new equity. Another observation is that the regulator starts the bank with relatively lower initial capital in the general case, which also shows that the opportunity of raising new equity relaxes the regulations.
Finally, Figure 5 presents the value function of the bank for the shareholders and the regulator in an environment with a voluntary recapitalization option of the bank.

[Insert Figure 5 Here]

5 Numerical Analysis

In this section, we investigate how the optimal thresholds change with respect to the model parameters: expected profitability of the bank (μ), volatility of the cash flows (σ), cost of holding cash (r), cost of raising outside equity (F), and the arrival rate of the outside investors (λ). As Figures 2 and 4 illustrate, the regulator intervenes when the restructuring cost is higher than a certain threshold. Therefore, we provide figures for both low and high restructuring cost regimes. We also present comparative statics for the critical restructuring cost for the regulator.
We start our analysis with the particular case where there is no outside financing option. Figures 6 and 7 illustrate the sensitivity of the optimal dividend thresholds and the initial capital to the changes in the profitability of the bank, volatility of the cash flows, and the cost of holding cash.

[Insert Figure 6 Here]

When the restructuring cost is low, the regulator does not put any restrictions on the dividend payouts of the bank. Thus, the optimal dividend thresholds of the shareholders and the regulator coincide as shown in Figure 6. The most salient observation is that the optimal dividend thresholds have an inverse U-shaped, left-skewed relationship with the profitability of the bank's operations. Obviously, when the bank is highly profitable, early dividend payments can be expected since the risk of financial distress is low. However, when the bank is scarcely profitable, the surprising result of early dividend payments could be explained with the intuition that the potential losses from the restructuring is low.[25] In addition, the optimal dividend thresholds increase with the volatility of cash flows since the probability of financial distress is high for banks with more volatile cash flows, which provides incentives to these institutions to retain earnings and hold more capital for precautionary reasons. Furthermore, the cost of holding cash is inversely related to the optimal thresholds. Hence, when the internal cash holdings are very costly, the bank distributes them as a dividend as soon as possible. The regulator starts the bank with an initial capital that is closer but smaller than the optimal dividend threshold. Surprisingly, the difference is small when the bank is more profitable and has less risky cash flows.[26] Intuitively, the regulator starts the bank with higher capital when the cost of holding cash is low.

Figure 7 provides the comparative statics for the high restructuring cost regime.

[Insert Figure 7 Here]

The main observation is that the optimal dividend thresholds of the shareholders and the regulator diverge. In this case, the regulator forces the bank to retain earnings until the capital reaches a higher threshold, which aims at preventing the bank from financial distress. The difference between optimal dividend thresholds is high when the bank is less profitable or the cash flows of the bank are more risky. Hence, the bank is subject to more stringent regulations in these cases. Finally, the comparison of Figures 6 and 7 shows that the regulator starts the bank with relatively higher capital in the high restructuring cost regime to prevent the bank from costly restructuring.

Secondly, we move to the general case with the bank's voluntary recapitalization option and investigate the comparative statics for the initial capital, optimal dividend and the outside financing thresholds. Figures 8 and 9 illustrate the low and high restructuring cost regimes, respectively.

[Insert Figure 8 Here]

[25] See Rochet and Villeneuve [23] for more details.

[26] Note that the critical restructuring cost changes with the parameter values as given in Figures 10 and 11. We always take the low restructuring cost as half the amount of the evaluated critical restructuring cost. This counter-intuitive result may change with different specifications of the restructuring cost values.

[Insert Figure 9 Here]

In both regimes, the sensitivity of the optimal dividend thresholds with respect to the profitability, cash flow volatility and the cost of holding cash parameters are quite similar to the particular case. However, the optimal dividend thresholds are lower than the particular case and the divergence of the shareholders' and the regulator's dividend thresholds is not as big as in the particular case. These observations can be explained with the higher ability of the bank to prevent itself from financial distress region due to the option to raise outside equity. The novel implications arise with respect to the fixed cost of raising outside equity and the arrival rate of the outside financiers, which reflect the level of credit market frictions. The optimal level of capital is positively related with the fixed cost of raising outside equity. Hence, when the outside financing is very costly, the bank retains earnings and postpones dividend payouts to utilize the internal capital, which is relatively cheaper. The bank distributes dividends earlier when the outside investors appear more frequently, which can be explained with an easy access to the external market. Results for the outside financing threshold are threefold. First, when the bank is highly profitable, has less volatile cash flows, or the cost of holding cash is higher, the manager defers raising new equity since the risk of financial distress is low. Second, the outside financing threshold is negatively related with the fixed cost of raising new equity. In particular, the bank prefers to exercise the outside financing option rarely and lumpy when it is very costly. Third, the bank waits longer before raising new equity when the outside investors appear more frequently, i.e., the external capital supply is high, due to lower credit market frictions.

Finally, we consider how the restructuring decisions of the regulator change with respect to the model parameters. Figures 10 and 11 illustrate the cases with or without voluntary recapitalization option, respectively.

[Insert Figure 10 Here]

[Insert Figure 11 Here]

When there is no outside financing option, the critical restructuring cost for the regulator increases with the profitability of the bank and decreases with the volatility of the cash flows and the cost of holding cash. The intuition is that when the bank has higher and less volatile cash flows, the regulator is relatively less likely to intervene since the risk of financial distress is low. In addition, when the cost of holding cash is higher, the intervention region of the regulator becomes larger. These results are still valid when we introduce an outside financing option. However, the critical restructuring cost is relatively higher when the bank has the option to raise new equity since the bank's ability to recover itself from financial distress is stronger with the outside financing opportunity. Finally, when the cost of raising new equity is high or the outside investors appear rarely, the critical restructuring cost is relatively lower since the capital supply frictions have a negative impact on the bank's situation, which forces the regulator to intervene earlier.

6 Conclusion

This paper investigates optimal resolution procedures and dividend policy for global systemically important banks. For this purpose, we build a dynamic model that considers the trade-off for regulators when to optimally step-in and restructure a bank and how to optimally restrict the dividend payout policies of G-SIBs. Moreover, the model incorporates the interaction of regulatory intervention policies with the equity issuance decisions of a bank. Importantly, the model features also supply side credit market frictions. This allows us to analyze capital supply effects on the optimal dividend, equity issuance, and restructuring policies of G-SIBs. The core financial friction in our model is that the bank has to search for outside investors in order to raise new equity and the outside investors only arrive at an uncertain (i.e., a stochastic) rate.

Given this modeling framework, the main suggestions that we derive for an optimal regulatory policy are twofold. First, the regulator intervenes by setting a capital threshold under which the bank is restructured. In particular, every time the bank's capital drops to a certain nonnegative threshold, the shareholders are expropriated and the regulator restructures the bank with the aim of a subsequent re-privatization. Second, the regulator imposes dividend payout restrictions to G-SIBs, hinging on restructuring costs. Our analysis shows that the regulator always restructures the bank when the cash reserves fall below zero. In addition, when the restructuring costs are high, the regulator prohibits dividend payouts to prevent a situation of, from a social welfare point of view, costly restructuring. Another crucial result is that the bank postpones new equity issuance when the cost of raising equity is high or when the capital supply is plentiful. Finally, when the bank is relatively constrained with regard to external capital supply, the regulator intervenes earlier, imposes relatively higher dividend thresholds to the bank, and initializes the bank with higher capital.

Our simple stylized model is a first step to solve the implementation problem of the regulators of the resolution procedures and to examine their potential adverse effects on shareholders' behavior. Further extensions of our model with regard to other regulatory tools such as contingent capital contracts or the introduction of moral hazard problem are interesting avenues for future research.

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Appendix A

Proof of Proposition 4.1

Let $\mathcal{A}$ be the set of dividend strategies such that $\mathbb{E}_c\left[\int_0^\tau e^{-rt}dL_t\right] < \infty$ for all $c \geq \underline{c}$ where τ is the first time that the bank's capital falls to $\underline{c}$ and $\mathbb{E}_c[.]$ denotes an expectation conditional on the initial capital $C_0 = c$.

The first step of the proof is to define the dynamic programming equation (DPE) for the shareholders' problem, which is given by using standard stochastic optimal control results (see Fleming and Soner [10] for detail) as

$$\min\{rV_s(c) - \mu V_s'(c) - \frac{\sigma^2}{2}V_s''(c), V_s'(c) - 1\} = 0 \qquad \forall c > \underline{c}, \tag{6.1}$$

where $V_s(\underline{c}) = 0$.

The second step is to construct a solution $(\hat{V}_s)$ to the system (4.1 - 4.4), which solves the dynamic programming equation given by (6.1). Since we have a second order homogenous ordinary differential equation (ODE), we conjecture the following solution form for $c \in (\underline{c}, c^*)$:

$$\hat{V}_s(c) = \alpha_1 e^{\eta_1 c} + \alpha_2 e^{\eta_2 c},$$

where $\alpha_i,\ i = 1, 2$ are constants and $\eta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}$ are the roots of the characteristic equation

$$\frac{\sigma^2}{2}\eta^2 + \mu\eta - r = 0.$$

Then, by using boundary conditions (4.3) and (4.4), we have

$$\begin{aligned} \alpha_1\eta_1 e^{\eta_1 c^*} + \alpha_2\eta_2 e^{\eta_2 c^*} &= 1, \\ \alpha_1\eta_1^2 e^{\eta_1 c^*} + \alpha_2\eta_2^2 e^{\eta_2 c^*} &= 0. \end{aligned}$$

Solving the above equations yields

$$\begin{aligned} \alpha_1 &= \frac{-\eta_2}{\eta_1(\eta_1 - \eta_2)}e^{-\eta_1 c^*}, \\ \alpha_2 &= \frac{\eta_1}{\eta_2(\eta_1 - \eta_2)}e^{-\eta_2 c^*}. \end{aligned}$$

Finally, plugging α_1 and α_2 into the initial condition (4.2) provides the free boundary as

$$c^* = \underline{c} + \frac{ln(\frac{\eta_2^2}{\eta_1^2})}{\eta_1 - \eta_2}.$$

Moreover, we conjecture that the function satisfies $\hat{V}_s(c) = \hat{V}_s(c^*) + c - c^*$, for $c \geq c^*$. Now, we have to show that the constructed solution solves the DPE, i.e.,

1. $\hat{V}_s'(c) \geq 1, \quad \forall c \in (\underline{c}, c^*)$ and

2. $r\hat{V}_s(c) - \mu\hat{V}_s'(c) - \frac{\sigma^2}{2}\hat{V}_s''(c) \geq 0, \quad \forall c \geq c^*$.

One can easily see that the conjectured function $\hat{V}_s(c)$ is increasing (since $\hat{V}_s'(c) \geq 0$) and concave (since $\hat{V}_s''(c) \leq 0$) in the region $(\underline{c}, c^*)$ with $\hat{V}_s'(c^*) = 1$. Therefore, $\hat{V}_s'(c) \geq 1$, $\forall c \in (\underline{c}, c^*]$. Now, we verify the second condition: $\forall c \geq c^*$,

$$r\hat{V}_s(c) - \mu\hat{V}_s'(c) - \frac{\sigma^2}{2}\hat{V}_s''(c) = r[\hat{V}_s(c^*) + c - c^*] - \mu = r(c - c^*) \geq 0$$

since $\hat{V}_s(c^*) = \mu/r$. Finally, we present the verification step.

Verification Theorem.

Let $\hat{V}_s$ be the constructed function and V_s be the value function. Then,

$$\hat{V}_s(c) = V_s(c) = \mathbb{E}_c\left[\int_0^\tau e^{-rt} dL_t^*\right],$$

where

$$L_t^* = \sup_{\{0\leq s\leq t\}} \{(c + \mu s + \sigma B_s - c^*)^+\}.$$

Proof.

(⇒:) Let $L \in \mathcal{A}$ be any admissible dividend strategy. Then, by Ito's formula

$$\begin{aligned} d\left[e^{-rt}\hat{V}_s(C_t)\right] &= e^{-rt}\left[-r\hat{V}_s(C_t) + \mu\hat{V}_s'(C_t) + \frac{\sigma^2}{2}\hat{V}_s''(C_t)\right] dt \\ &\quad + e^{-rt}\hat{V}_s'(C_t)\sigma dB_t - e^{-rt}\hat{V}_s'(C_t) dL_t. \end{aligned}$$

Integrating both sides from 0 to $T \wedge \tau$ yields

$$\begin{aligned} e^{-r(T\wedge\tau)}\hat{V}_s(C_{T\wedge\tau}) &= \hat{V}_s(c) + \int_0^{T\wedge\tau} e^{-rt}\left[\underbrace{-r\hat{V}_s(C_t) + \mu\hat{V}_s'(C_t) + \frac{\sigma^2}{2}\hat{V}_s''(C_t)}_{\leq 0}\right] dt \\ &\quad + \underbrace{\int_0^{T\wedge\tau} e^{-rt}\hat{V}_s'(C_t)\sigma dB_t}_{martingale} - \underbrace{\int_0^{T\wedge\tau} e^{-rt}\hat{V}_s'(C_t) dL_t}_{\geq 0}, \end{aligned} \qquad (*)$$

where the second term is non-positive and the last term is non-negative due to the DPE. Then, taking the expectation of both sides and plugging 1 instead of $\hat{V}_s'$ into the last term provide the following inequality:

$$\hat{V}_s(c) \quad \geq \quad \mathbb{E}_c\left[e^{-r(T\wedge\tau)}\hat{V}_s(C_{T\wedge\tau})\right] + \mathbb{E}_c\left[\int_0^{T\wedge\tau} e^{-rt} dL_t\right].$$

Finally, letting $T \uparrow \infty$ and using Fatou's lemma we obtain

$$\begin{aligned}
\hat{V}_s(c) &\geq \mathbb{E}_c\left[e^{-r\tau}\underbrace{\hat{V}_s(C_\tau)}_{=0}\right] + \lim_{T\to\infty}\mathbb{E}_c\left[\int_0^{T\wedge\tau} e^{-rt}dL_t\right] \\
&\geq \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}dL_t\right] = V_s(c)
\end{aligned}$$

(⇐:) In the second part of the proof we will show that all above inequalities turn into equalities when we use L^*. More specifically, the second term in (*) vanishes for the dividend strategy L^*, which keeps the bank's capital in the region $(\underline{c}, c^*]$ where the expression in brackets is zero due to the DPE. In addition, L^* is only activated when $C_t = c^*$, so $\hat{V}_s'(C_t) = 1$ for L^*. Then, we end up with

$$\hat{V}_s(c) = \mathbb{E}_c\left[e^{-r(T\wedge\tau)}\hat{V}_s(C^*_{T\wedge\tau})\right] + \mathbb{E}_c\left[\int_0^{T\wedge\tau} e^{-rt}dL^*_t\right].$$

When $T \to \infty$,

$$\lim_{T\to\infty}\mathbb{E}_c\left[e^{-r(T\wedge\tau)}\hat{V}_s(C^*_{T\wedge\tau})\right] = \mathbb{E}_c\left[e^{-r(\tau)}\hat{V}_s(\underline{c})\right] = 0.$$

Finally, since the function L_t is positive, non-decreasing and bounded from below, letting $T \to \infty$ and using the dominated convergence theorem provide

$$\hat{V}_s(c) = \lim_{T\to\infty}\mathbb{E}_c\left[\int_0^{T\wedge\tau} e^{-rt}dL^*_t\right] = \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}dL^*_t\right] = V_s(c).$$

■

Proof of Proposition 4.2

We start with the function $V(c_0)$, which satisfies a second order homogenous ODE having following solution form:

$$V(c_0) = \delta_1 e^{\eta_1 c_0} + \delta_2 e^{\eta_2 c_0},$$

where $\eta_{1,2}$ are defined as above. Then, by using initial and boundary conditions in (4.8) we have

$$\begin{aligned}
\delta_1 &= \frac{e^{(\eta_2-\eta_1)\underline{c}}}{\eta_1 e^{\eta_1\bar{c}+(\eta_2-\eta_1)\underline{c}} - \eta_2 e^{\eta_2\bar{c}}}, \\
\delta_2 &= \frac{1}{\eta_2 e^{\eta_2\bar{c}} - \eta_1 e^{\eta_1\bar{c}+(\eta_2-\eta_1)\underline{c}}}.
\end{aligned}$$

Plugging $\delta_i,\ i = 1, 2$, into the conjectured solution form provides

$$\begin{aligned} V(c_0) &= \frac{e^{(\eta_2-\eta_1)\underline{c}+\eta_1 c_0} - e^{\eta_2 c_0}}{\eta_1 e^{\eta_1 \overline{c}+(\eta_2-\eta_1)\underline{c}} - \eta_2 e^{\eta_2 \overline{c}}} \\ &= \frac{e^{\eta_2(c_0-\underline{c})} - e^{\eta_1(c_0-\underline{c})}}{\eta_2 e^{\eta_2(\overline{c}-\underline{c})} - \eta_1 e^{\eta_1(\overline{c}-\underline{c})}}, \end{aligned}$$

where the last equality holds by multiplying and dividing the right hand side of the previous equality by $e^{-\eta_2 \underline{c}}$.
Secondly, the function $G(c_0)$ has the same solution form:

$$G(c_0) = \varsigma_1 e^{\eta_1 c_0} + \varsigma_2 e^{\eta_2 c_0}.$$

Similarly, using the initial and boundary conditions in (4.9) yields

$$\begin{aligned} \varsigma_1 &= \frac{\eta_1 e^{-\eta_1 \underline{c}}}{\eta_1 e^{(\eta_2-\eta_1)\underline{c}} - \eta_2 e^{(\eta_2-\eta_1)\overline{c}}}, \\ \varsigma_2 &= \frac{-\eta_2 e^{(\eta_2-\eta_1)\overline{c}-\eta_1 \underline{c}}}{\eta_1 e^{(\eta_2-\eta_1)\underline{c}} - \eta_2 e^{(\eta_2-\eta_1)\overline{c}}}. \end{aligned}$$

Hence, the function $G(.)$ becomes

$$\begin{aligned} G(c_0) &= \frac{-\eta_2 e^{(\eta_2-\eta_1)\overline{c}+\eta_1(c_0-\underline{c})} - \eta_1 e^{\eta_2 c_0-\eta_1 \underline{c}}}{\eta_1 e^{(\eta_2-\eta_1)\underline{c}} - \eta_2 e^{(\eta_2-\eta_1)\overline{c}}} \\ &= \frac{-\eta_2 e^{\eta_1(c_0-\overline{c})} + \eta_1 e^{\eta_2(c_0-\overline{c})}}{\eta_1 e^{-\eta_2(\overline{c}-\underline{c})} - \eta_2 e^{-\eta_1(\overline{c}-\underline{c})}}, \end{aligned}$$

where the last equality is satisfied by multiplying and dividing the right hand side of the previous equality by $e^{\eta_2 \overline{c}-\eta_1 \underline{c}}$. ■

Proof of Proposition 4.3

Consider the complete metric space $(\mathbb{R}, d)$ where

$$d(W_1, W_2) = |W_1 - W_2|, \ \ \forall W_1, W_2 \in \mathbb{R}.$$

We will show that the real valued function

$$\mathbb{T}(W) = \max_{c_0,\underline{c},\overline{c}} \mathcal{H}(c_0, \underline{c}, \overline{c}, W) = \max_{c_0,\underline{c},\overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + W]$$

is a contraction mapping in $(\mathbb{R}, d)$, i.e. $\exists k < 1$ such that for all $W_1, W_2 \in \mathbb{R}$ we have

$$|\mathbb{T}(W_2) - \mathbb{T}(W_1)| \leq k|W_2 - W_1|.$$

We start by noting that the function $G(c_0; \underline{c}, \overline{c}) = \mathbb{E}[e^{-r\tau}]$ is the stochastic discount factor with a range $[0, 1]$. Let $W_1 < W_2$. The function $\mathbb{T}(W)$ is increasing in W since

$$\frac{\partial \mathbb{T}(W)}{\partial W} = \max_{c_0, \underline{c}, \overline{c}} G(c_0; \underline{c}, \overline{c}) \geq 0.$$

Then, we have $\mathbb{T}(W_1) \leq \mathbb{T}(W_2)$. In addition,

$$\begin{aligned}
\mathbb{T}(W_2) &= \max_{c_0, \underline{c}, \overline{c}} \mathcal{H}(c_0, \underline{c}, \overline{c}, W_2) \\
&= \max_{c_0, \underline{c}, \overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + W_2] \\
&= \max_{c_0, \underline{c}, \overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + (W_2 - W_1) + W_1] \\
&\leq \max_{c_0, \underline{c}, \overline{c}} V(c_0; \underline{c}, \overline{c}) - c_0 + G(c_0; \underline{c}, \overline{c})[-\xi + \underline{c} + W_1] \\
&\quad + \max_{c_0, \underline{c}, \overline{c}} (W_2 - W_1) G(c_0; \underline{c}, \overline{c}) \\
&= \mathbb{T}(W_1) + (W_2 - W_1) \max_{c_0, \underline{c}, \overline{c}} G(c_0; \underline{c}, \overline{c}),
\end{aligned}$$

where the inequality follows from a basic math inequality:

$$\max_{c_0, \underline{c}, \overline{c}} \{f_1(c_0, \underline{c}, \overline{c}) + f_2(c_0, \underline{c}, \overline{c})\} \leq \max_{c_0, \underline{c}, \overline{c}} \{f_1(c_0, \underline{c}, \overline{c})\} + \max_{c_0, \underline{c}, \overline{c}} \{f_2(c_0, \underline{c}, \overline{c})\},$$

for all real valued functions f and g. Then, using the increasing property of $\mathbb{T}(W)$ yields

$$0 \leq \mathbb{T}(W_2) - \mathbb{T}(W_1) \leq (W_2 - W_1) \max_{c_0, \underline{c}, \overline{c}} G(c_0; \underline{c}, \overline{c}).$$

Intuitively, the regulator never starts the bank at zero cash level. Therefore, $G(c_0; \underline{c}, \overline{c}) < 1$ for all $c_0, \underline{c}, \overline{c}$. Hence, defining $k := \max_{c_0, \underline{c}, \overline{c}} G(c_0; \underline{c}, \overline{c}) < 1$ yields

$$\mathbb{T}(W_2) - \mathbb{T}(W_1) \leq k(W_2 - W_1),$$

or equivalently,

$$|\mathbb{T}(W_2) - \mathbb{T}(W_1)| \leq k|W_2 - W_1|,$$

since the both sides of the previous inequality are positive. One can easily show the above inequality for $W_2 < W_1$ by using exactly the same steps. Therefore, $\mathbb{T}$ is a contraction mapping and has a unique fixed point in $\mathbb{R}$ by the Banach Fixed Point Theorem. ■

Proof of Proposition 4.4

In this proof, we adapt the methodology given in Hugonnier et al. [12] to our setup. Let $\mathcal{A}$ be the set of dividend and financing strategies such that

$$\mathbb{E}_c\left[\int_0^\tau e^{-rt}(dL_t + f_t dN_t)\right] < \infty, \qquad \forall c \geq \underline{c}$$

where τ is the first time that the bank's capital falls to $\underline{c}$ and $\mathbb{E}_c[.]$ denotes an expectation conditional on the initial capital $C_{0-} = c$. We define the following operators:

$$\mathcal{L}\varphi(c) := r\varphi(c) - \mu\varphi'(c) - \frac{\sigma^2}{2}\varphi''(c),$$
$$\mathcal{F}\varphi(c) := \max_{f\geq 0}\ \lambda\{\varphi(c+f) - \varphi(c) - f - \mathbf{1}_{\{f\geq 0\}}F\}.$$

which will be used throughout the proof.

STEP 1

First, we define the dynamic programming equation (DPE) in the general case with outside financing option by using the singular stochastic control theory (see Fleming and Soner [10] for detail):

$$\min\{\mathcal{L}V_s(c) - \mathcal{F}V_s(c), V_s'(c) - 1\} = 0, \qquad \forall c > \underline{c} \tag{6.2}$$

with $V_s(\underline{c}) = 0$.

STEP 2

Second, we construct a solution $(\hat{V}_s)$ to the system (4.14 - 4.18). Our conjecture is that the optimal dividend and financing policies are of threshold forms. Let $d \geq \underline{c}$ be a fixed target capital level for the bank and $\hat{V}_s(c) := \hat{V}_s(c; d)$ denote the value of a bank that follows the barrier strategy d. We will construct the solution for any target level d, then we will show that there exists a unique target capital c^* satisfying $\hat{V}_s(\underline{c}; c^*) = 0$. For the notational simplicity, we supress d when we refer to the conjectured function $\hat{V}_s$.

$\hat{V}_s(c)$ is a piecewise-defined function as follows:

$$\hat{V}_s(c) = \begin{cases} \hat{V}_{s1}(c), & \underline{c} \leq c \leq c_1 \\ \hat{V}_{s2}(c), & c_1 \leq c \leq d \\ \hat{V}_{s2}(c) + c - d, & c \geq d \end{cases}$$

where the functions $\hat{V}_{s1}(c)$ and $\hat{V}_{s2}(c)$ represent the constructed solutions in the financial distress region and the safe region, respectively. In the second region, $\hat{V}_{s2}(c)$ solves

$$\begin{aligned} r\hat{V}_{s2}(c) &= \mu\hat{V}_{s2}'(c) + \frac{\sigma^2}{2}\hat{V}_{s2}''(c) \\ s.t.& \\ \hat{V}_{s2}(c_1) &= \hat{V}_{s2}(d) - (d - c_1) - F, \\ \hat{V}_{s2}'(d) &= 1, \\ \hat{V}_{s2}''(d) &= 0. \end{aligned} \tag{6.3}$$

The closed form solution for $\hat{V}_{s2}$ is the same as the value function in the particular case:

$$\hat{V}_{s2}(c) = \frac{-\eta_2^2 e^{\eta_1(c-d)} + \eta_1^2 e^{\eta_2(c-d)}}{\eta_1\eta_2(\eta_1 - \eta_2)}.$$

On the other hand, $\hat{V}_{s1}(c)$ solves

$$\begin{aligned}
(r+\lambda)\hat{V}_{s1}(c) &= \mu\hat{V}_{s1}'(c) + \frac{\sigma^2}{2}\hat{V}_{s1}''(c) + \lambda\left(\hat{V}_{s2}(d) - (d-c) - F\right) \qquad (6.4)\\
s.t. \quad \hat{V}_{s1}(\underline{c}) &= 0,\\
\hat{V}_{s1}(c_1) &= \hat{V}_{s2}(c_1) = \hat{V}_{s2}(d) - (d-c_1) - F,\\
\hat{V}_{s1}'(c_1) &= \hat{V}_{s2}'(c_1).
\end{aligned}$$

The general solution form for $\hat{V}_{s1}(c)$ is given as

$$\hat{V}_{s1}(c) = \underbrace{\alpha c + \beta}_{\hat{V}_{s1p}(c)} + \underbrace{\gamma_1 e^{\theta_1 c} + \gamma_2 e^{\theta_2 c}}_{\hat{V}_{s1h}(c)},$$

where $\hat{V}_{s1p}(c)$ is the particular solution of (6.4), $\hat{V}_{s1h}(c)$ is the solution of the homogenous part of it, and $\theta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2(r+\lambda)}}{\sigma^2}$[27] are the roots of the characteristic equation

$$\frac{\sigma^2}{2}\theta^2 + \mu\theta - (r+\lambda) = 0.$$

Plugging $\hat{V}_{s1p}(c)$ into (6.4) and equating the constants and the coefficients of c provide

$$\begin{aligned}
\alpha &= \frac{\lambda}{r+\lambda},\\
\beta &= \frac{\mu\lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}\left[\frac{\mu}{r} - d - F\right].
\end{aligned}$$

We will use the initial and boundary conditions to find $\gamma_1, \gamma_2, c_1, d$:

$$\begin{aligned}
\hat{V}_{s1}(c_1) &\equiv \alpha c_1 + \beta + \gamma_1 e^{\theta_1 c_1} + \gamma_2 e^{\theta_2 c_1} = \frac{\mu}{r} - d - F + c_1\\
&= \frac{\eta_1^2 e^{\eta_2(c_1-d)} - \eta_2^2 e^{\eta_1(c_1-d)}}{\eta_1\eta_2(\eta_1-\eta_2)} \equiv V_2(c_1),
\end{aligned}$$

where the first and second equalities follow from the definition of c_1 and the continuity of $\hat{V}_s$, respectively. Define $y_1 := d - c_1$ and

$$\psi(y_1) := \frac{\eta_1^2 e^{-\eta_2 y_1} - \eta_2^2 e^{-\eta_1 y_1}}{\eta_1\eta_2(\eta_1-\eta_2)} + y_1 - \frac{\mu}{r} + F,$$

[27] $\theta_1 > 0 > \theta_2$ and $|\theta_2| > |\theta_1|$.

where $\psi(0) = F > 0$.[28] Moreover,

$$\begin{aligned}
\psi'(y_1) &= \frac{\eta_1\eta_2^2 e^{-\eta_1 y_1} - \eta_2\eta_1^2 e^{-\eta_2 y_1}}{\eta_1\eta_2(\eta_1-\eta_2)} + 1 \;\Rightarrow\; \psi'(0) = 0 \; and \; \psi'(y_1) < 0, \; \forall y_1 > 0. \\
\psi''(y_1) &= \frac{\eta_1\eta_2}{\eta_1-\eta_2}[e^{-\eta_2 y_1} - e^{-\eta_1 y_1}] \\
\Rightarrow & \quad \psi''(0) = 0, \;\; \psi''(y_1) < 0, \; \forall y_1 \;\; and \;\; \lim_{y_1\to\infty} \psi(y_1) = -\infty.
\end{aligned}$$

Thus,

$$\exists! \; y_1^* > 0 \;\; s.t. \;\; \psi(y_1^*) = 0.$$

Therefore, c_1 is monotone increasing in d, i.e., $d = c_1 + y_1^*$. In addition, the smooth pasting condition at c_1 provides

$$\begin{aligned}
\hat{V}_{s1}'(c_1) &= \alpha + \theta_1\gamma_1 e^{\theta_1 c_1} + \theta_2\gamma_2 e^{\theta_2 c_1} \\
&= \frac{\eta_1 e^{\eta_2(c_1-d)} - \eta_2 e^{\eta_1(c_1-d)}}{\eta_1-\eta_2} = \hat{V}_{s2}'(c_1).
\end{aligned}$$

Define $z_1 := \gamma_1 e^{\theta_1 c_1}$ and $z_2 := \gamma_2 e^{\theta_2 c_1}$. Then, with the help of boundary conditions, we obtain a system of linear equations of (z_1, z_2) as follows:[29]

$$\begin{aligned}
z_1 + z_2 + \frac{\mu\lambda}{(r+\lambda)^2} - \frac{r}{r+\lambda}\left(\frac{\mu}{r} - F - y_1^*\right) &= 0, \\
\theta_1 z_1 + \theta_2 z_2 - \frac{\eta_1 e^{-\eta_2 y_1^*} - \eta_2 e^{-\eta_1 y_1^*}}{\eta_1-\eta_2} + \frac{\lambda}{r+\lambda} &= 0,
\end{aligned}$$

where z_1 and z_2 can be solved uniquely from the above equations as

$$\begin{aligned}
z_1 &= \frac{b - \theta_2 a}{\theta_1 - \theta_2}, \\
z_2 &= \frac{b - \theta_1 a}{\theta_2 - \theta_1},
\end{aligned}$$

where

$$\begin{aligned}
a &= \frac{r}{r+\lambda}[\frac{\mu}{r} - F - y_1^*] - \frac{\mu\lambda}{(r+\lambda)^2}, \\
b &= \frac{\eta_1 e^{-\eta_2 y_1^*} - \eta_2 e^{-\eta_1 y_1^*}}{\eta_1-\eta_2} - \frac{\lambda}{r+\lambda}.
\end{aligned}$$

[28] We use $\eta_1\eta_2 = -\frac{2r}{\sigma^2}$ and $\eta_1 + \eta_2 = -\frac{2\mu}{\sigma^2}$.

[29] Note that z_1 and z_2 are independent of d.

Finally, by using the initial condition we have

$$z_1 e^{\theta_1(\underline{c}-c_1)} + z_2 e^{\theta_2(\underline{c}-c_1)} + \frac{\mu\lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}\left(\frac{\mu}{r} - y_1^* - c_1 + \underline{c} - F\right) = 0.$$

Define $y_2 := c_1 - \underline{c}$ and

$$H(y_2) := z_1 e^{-\theta_1 y_2} + z_2 e^{-\theta_2 y_2} + \frac{\mu\lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}\left(\frac{\mu}{r} - y_1^* - y_2 - F\right).$$

Next, we investigate the existence and uniqueness of the root of $H(y_2)$. By the monotone increasing property of $V_{s1}(c)$[30] we have

$$V_{s1}'(c) = \alpha + \gamma_1\theta_1 e^{\theta_1 c} + \gamma_2\theta_2 e^{\theta_2 c} > 0, \ \ \forall\, c \in [\underline{c}, c_1],$$

where $\alpha > 0$, $\theta_1 > 0 > \theta_2$ and $|\theta_1| < |\theta_2|$. Using this property, $\forall\, y_2 \geq 0$ we have

$$\begin{aligned} H'(y_2): &= -\theta_1 z_1 e^{-\theta_1 y_2} - \theta_2 z_2 e^{-\theta_2 y_2} - \frac{\lambda}{r+\lambda} \\ &= -\theta_1\gamma_1 e^{\theta_1 \underline{c}} - \theta_2\gamma_2 e^{\theta_2 \underline{c}} - \alpha = -V_{s1}'(\underline{c}) < 0, \end{aligned}$$

where $lim_{y_2\to\infty} H'(y_2) = -\infty$. Thus, $H(y_2)$ is monotone decreasing. Moreover, since $F < F^* = \frac{\mu}{r} - d$, we have

$$\begin{aligned} H(0) &= \frac{\mu}{r} - y_1^* - F = V_{s1}(c_1) = V_{s2}(c_1) > 0, \\ \lim_{y_2\to\infty} H(y_2) &= -\infty, \end{aligned}$$

which guarantee that

$$\exists!\ y_2^* \ \ s.t.\ \ H(y_2^*) = 0.$$

Therefore, the target capital level (d) is uniquely defined as a function of the restructuring threshold ($\underline{c}$), i.e., $d = \underline{c} + y_1^* + y_2^*$.

STEP 3

The next step is to show that the constructed solution ($\hat{V}_s$) solves the DPE (6.2) i.e.,

1. $\hat{V}_s'(c) \geq 1$, $\forall c \geq \underline{c}$ and
2. $\mathcal{L}\hat{V}_s(c) - \mathcal{F}\hat{V}_s(c) \geq 0$, $\forall c \geq \underline{c}$.

We will first show that the function $\hat{V}_s(c)$ is increasing and concave in the region $[\underline{c}, d]$, which will be useful to prove the above two properties.

[30] We'll prove this property in the next step of the proof.

The function $\hat{V}_{s2}$ is increasing and concave in the region $[c_1, d]$ since

$$\begin{aligned}\hat{V}'_{s2}(c) &= \frac{\eta_1 e^{\eta_2(c-d)} - \eta_2 e^{-\eta_1(c-d)}}{\eta_1 - \eta_2} > 1, \qquad (= 1\ when\ c = d)\\ \hat{V}''_{s2}(c) &= \frac{\eta_1\eta_2 e^{\eta_2(c-d)} - \eta_1\eta_2 e^{-\eta_1(c-d)}}{\eta_1 - \eta_2} < 0 \quad (= 0\ when\ c = d),\end{aligned}$$

by using $\eta_1 > 0 > \eta_2$ and $|\eta_2| > |\eta_1|$.
We need to use the following lemmas to show that $\hat{V}_{s1}$ is increasing and concave in the region $[\underline{c}, c_1]$.

Lemma 2.A.1. Consider a function S which is a solution to

$$-\mathcal{L}S(c) + \varphi(c) = 0 \tag{6.5}$$

for some φ. Then, S does not have negative local minima if $\varphi(c) \geq 0$ and does not have positive local maxima if $\varphi(c) \leq 0$.

Proof. Let $\varphi(c) \geq 0$. At the local minimum, $S'(c) = 0$ and $S''(c) \geq 0$. Then, it folllows from (6.5) that the local minima are non-negative. Similarly, when $\varphi(c) \leq 0$, $S'(c) = 0$ and $S''(c) \leq 0$ at local maximum. Then, (6.5) implies that the local minima are non-positive.

Lemma 2.A.2. Consider a function S which is a solution to (6.5) for some $\varphi(c) \leq 0$. In addition, $S'(\tilde{c}) \leq 0$, $S(\tilde{c}) \geq 0$ and $|S(\tilde{c})| + |S'(\tilde{c})| + |\varphi(\tilde{c})| > 0$ for $\tilde{c} > \underline{c}$. Then, $S(c) > 0$ and $S'(c) < 0$ for all $\underline{c} < c < \tilde{c}$.

Proof. We first prove the decreasing property. Let $S'(c)$ be not always negative for $\underline{c} < c < \tilde{c}$ and let $y < \tilde{c}$ be the largest value at which $S'(c)$ changes sign. Then y is a positive local maximum, which contradicts the fact that S does not have a positive local maxima since $\varphi(c) \leq 0$ as given in Lemma 2.A.1. Therefore, S is decreasing for $c \in (\underline{c}, \tilde{c})$. Secondly, since $S(\tilde{c}) \geq 0$ and S is decreasing, $S(c) > 0$ for all $\underline{c} < c < \tilde{c}$.

Lemma 2.A.3. Consider a function S which is a solution to (6.5) for some φ such that $\varphi'(c) \leq 0$. In addition, $S''(\tilde{c}) \leq 0$, $S'(\tilde{c}) \geq 0$ and $|S'(\tilde{c})| + |S''(\tilde{c})| + |\varphi'(\tilde{c})| > 0$ for $\tilde{c} > \underline{c}$. Then, $S'(c) > 0$ and $S''(c) < 0$ for all $\underline{c} < c < \tilde{c}$.

Proof. Differentiating (6.5) yields $j = S'$ satisfies $-\mathcal{L}j(c) + \varphi'(c) = 0$. Then, we complete the proof by using lemma 2.A.2.
Now, the first and second derivatives of the function $\hat{V}_{s1}$ at c_1 satisfy

$$\hat{V}'_{s1}(c_1) = \hat{V}'_{s2}(c_1) > 1 \quad (by\ smooth\ pasting\ at\ c_1),$$

and

$$\begin{aligned}
\hat{V}_{s1}''(c_1) &= \theta_1^2 z_1 + \theta_2^2 z_2 \\
&= \theta_1^2\left(\frac{b-\theta_2 a}{\theta_1-\theta_2}\right) + \theta_2^2\left(\frac{b-\theta_1 a}{\theta_2-\theta_1}\right) \\
&= b(\theta_2+\theta_1) - a\theta_1\theta_2 \\
&= \frac{-2}{\sigma^2}[\mu b - a(r+\lambda)] \\
&= \frac{-2}{\sigma^2}\left[\mu\left(\underbrace{\frac{\eta_1 e^{-\eta_2 y_1^*} - \eta_2 e^{-\eta_1 y_1^*}}{\eta_1-\eta_2}}_{>1} - 1\right) + r(F+y_1^*)\right] \\
&< 0.
\end{aligned}$$

Define

$$S(c) := \hat{V}_{s1}(c) - \frac{\lambda c}{r+\lambda} - \frac{\mu\lambda}{(r+\lambda)^2} - \frac{\lambda}{r+\lambda}\left[\frac{\mu}{r} - d - F\right],$$

which solves $-\mathcal{L}S(c)+\varphi(c) = 0$ where $\varphi(c) := -\lambda S(c)$. In addition, $S'(c_1) = \hat{V}_{s1}'(c_1) - \frac{\lambda}{r+\lambda} > 0$ and $S''(c_1) = \hat{V}_{s1}''(c_1) < 0$. Therefore, Lemma 2.A.3 yields

$$S'(c) > 0 \textit{ and } S''(c) < 0\ \forall c < c_1,$$

which implies

$$\hat{V}_{s1}'(c) = S'(c) + \frac{\lambda}{r+\lambda} > 0 \ \textit{ and } \ \hat{V}_{s1}''(c) = S''(c) < 0, \ \ \forall c < c_1.$$

Hence, $\hat{V}_{s1}$ is increasing and concave in the region $[\underline{c}, c_1]$.
Having proved that the function $\hat{V}_s$ is increasing and concave for $c \in [\underline{c}, d]$ and using the smooth pasting condition at d provide $\hat{V}_s'(c) \geq 1$, $\forall c \geq \underline{c}$. Moreover, $\forall c \geq \underline{c}$

$$\begin{aligned}
\mathcal{L}\hat{V}_s(c) - \mathcal{F}\hat{V}_s(c) &= \underbrace{(\mathcal{L}\hat{V}_s(c) - \mathcal{F}\hat{V}_s(c))\mathbf{1}_{\{\underline{c}<c<d\}}}_{=0\ (\textit{since } \hat{V}_s \textit{ solves } (4.14))} + (\mathcal{L}\hat{V}_s(c) - \mathcal{F}\hat{V}_s(c))\mathbf{1}_{\{c\geq d\}} \\
&= (r\hat{V}_s(c) - \mu)\mathbf{1}_{\{c\geq d\}} \\
&= (r[\frac{\mu}{r} + c - d] - \mu)\mathbf{1}_{\{c\geq d\}} \\
&= r(c-d) \geq 0.
\end{aligned}$$

Therefore, the constructed function $\hat{V}_s$ solves the dynamic programming equation.

STEP 4

Next, we will show that there exists a unique target level c^* satisfying $\hat{V}_s(\underline{c}; c^*) = 0$. For this purpose, we first need to prove that $\hat{V}_s(c; d)$ is strictly monotone decreasing with respect to the target level d. Let $d_1 < d_2$ and $n(c) := \hat{V}_s(c; d_1) - \hat{V}_s(c; d_2)$. Then, we have

$$-\mathcal{L}n(c) - \lambda(n(c) + d_1 - d_2) = 0,$$

and $n'(d_1) = 1 - \hat{V}_s'(d_1; d_2) < 0$, $n''(d_1) = -\hat{V}_s''(d_1; d_2) \geq 0$. Then, a direct modification of Lemma A.2.3 provides that n is monotone decreasing for all $c < d_1$. Finally,

$$\begin{aligned}
n(d_1) &= \hat{V}_s(d_1; d_1) - \hat{V}_s(d_1; d_2) \\
&= \hat{V}_s(d_1; d_1) - \hat{V}_s(d_2; d_2) + \int_{d_1}^{d_2} \hat{V}_s'(c; d_2) dc \\
&\geq \hat{V}_s(d_1; d_1) - \hat{V}_s(d_2; d_2) + (d_2 - d_1) \\
&= d_2 - d_1 > 0,
\end{aligned}$$

where the inequality follows from the fact that $\hat{V}_s'(c; d_2) > 1$ for $c \in [d_1, d_2]$. Hence, $\hat{V}_s(c; d)$ is monotone decreasing in d. Moreover, $\hat{V}_s(\underline{c}; \underline{c}) = \frac{\mu}{r} > 0$ and $\hat{V}_s(\underline{c}; \infty) < 0$ which imply that $\hat{V}_s(\underline{c}; c^*) = 0$ has a unique solution.

STEP 5

Finally, we proceed with the verification step.

Verification Theorem.

Let $\hat{V}_s$ be the constructed function and V_s be the value function. Then,

$$\hat{V}_s(c) = V_s(c) = \mathbb{E}_c \left[\int_0^\tau e^{-rt} (dL_t^* - (f_t^* + \mathbf{1}_{\{f_t^* > 0\}} F) dN_t) \right],$$

$$\begin{aligned}
\textit{where} \quad L_t^* &= \sup_{\{0 \leq s \leq t\}} \{(h_s - c^*)^+\}, \\
f_t^* &= (c^* - C_t)^+, \\
dC_t &= \mu dt + \sigma dB_t - dL_t^* + f_{t-}^* dN_t, \quad C_{0-} = c, \\
dh_t &= \mu dt + \sigma dB_t + (c^* - h_{t-})^+ dN_t.
\end{aligned}$$

Proof.

(⇒:) Let $(L, f) \in \mathcal{A}$ be any admissible dividend and financing strategies. Define the process

$$X_t = e^{-rt} \hat{V}_s(C_t) + \int_{0+}^t e^{-ru} (dL_u - (f_{u-} + \mathbf{1}_{\{f_{u-} > 0\}} F) dN_u).$$

Applying Ito's formula for semimartingales to X_t yields[31]

$$\begin{aligned} dX_t &= e^{-rt}[-r\hat{V}_s(C_{t-}) + \mu\hat{V}_s'(C_{t-}) + \frac{\sigma^2}{2}\hat{V}_s''(C_{t-})]dt \\ &\quad + e^{-rt}\hat{V}_s'(C_{t-})\sigma dB_t - e^{-rt}\hat{V}_s'(C_{t-})[dL_t - f_{t-}dN_t] \\ &\quad + e^{-rt}\Delta\hat{V}_s(C_{t-}) - e^{-rt}\hat{V}_s'(C_{t-})\Delta C_{t-} + e^{-rt}(dL_t - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)dN_t), \end{aligned}$$

where

$$\begin{aligned} \Delta\hat{V}_s(C_{t-}) &= [\hat{V}_s(C_{t-} + f_{t-}) - \hat{V}_s(C_{t-})]dN_t + \hat{V}_s(C_{t-}) - \hat{V}_s(C_{t-} - \Delta L_t), \\ \Delta C_{t-} &= \Delta L_t + f_{t-}dN_t, \\ \Delta L_t &= L_t - L_{t-}, \ \ (jump\ component\ of\ L_t) \\ L_t^c &= L_t - \Sigma_{s\le t}\Delta L_s. \ \ (continuous\ part\ of\ L_t) \end{aligned}$$

Then, plugging the above definitions into dX_t and compensating the Poisson processes result in

$$dX_t = dQ_t - e^{-rt}dP_t,$$

where

$$dQ_t = e^{-rt}\hat{V}_s'(C_{t-})\sigma dB_t + e^{-rt}[\hat{V}_s(C_{t-} + f_{t-}) - \hat{V}_s(C_{t-}) - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)](dN_t - \lambda dt)$$

is a local martingale (since the first term is a Brownian motion and the second term is a compensated Poisson process) and

$$\begin{aligned} dP_t &= [\mathcal{F}\hat{V}_s(C_{t-}) - \lambda(\hat{V}_s(C_{t-} + f_{t-}) - \hat{V}_s(C_{t-}) - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F))]dt \\ &\quad + (\hat{V}_s'(C_{t-}) - 1)dL_t^c + [\Delta L_t + \hat{V}_s(C_{t-}) - \hat{V}_s(C_{t-} - \Delta L_t)] \end{aligned}$$

is a non-decreasing process since the drift is positive from the definition of $\mathcal{F}$ and $\hat{V}_s'(C_{t-}) \ge 1$, for all $C_t \ge \underline{c}$. Hence, X is a local supermartingale by the Doob-Meyer decomposition. Moreover, the stopped sequence

$$J_t := X_{t\wedge\tau} \ge -\int_{0+}^{\tau} e^{-ru} f_{u-}dN_u$$

is a supermartingale since the admissible dividend strategy f_t is intregrable and constructs

[31] Note that the function $\hat{V}_s(C_t)$ is C^2 everywhere but at c_1. However, the Lebesque measure of t, for which $C_t = c_1$, is zero. Hence, the value of $\hat{V}_s''(c_1)$ matters little in the follow-up whatever set to be.

a lower bound for J_t. Finally,

$$\begin{aligned}
\hat{V}_s(c) &= \hat{V}_s(C_{0-}) = \hat{V}_s(C_0) - \Delta\hat{V}_s(C_0) = J_0 - \Delta\hat{V}_s(C_0) \geq \mathbb{E}_c[J_\tau] - \Delta\hat{V}_s(C_0) \\
&= \mathbb{E}_c\left[e^{-r\tau}\hat{V}_s(C_\tau) + \int_{0+}^{\tau} e^{-rt}(dL_t - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)dN_t)\right] - \Delta\hat{V}_s(C_0) \\
&= \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}(dL_t - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)dN_t)\right] - \Delta\hat{V}_s(C_0) - \Delta L_0 \\
&\geq \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}(dL_t - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)dN_t)\right],
\end{aligned}$$

where the third equality is due to $\hat{V}_s(C_{0-}) = X_0 = X_{0\wedge\tau} = J_0$, the first inequality follows from the optional sampling theorem for supermartingales, the fifth equality results from $\hat{V}_s(C_\tau) = 0$, and the second inequality comes from the fact that $\hat{V}_s'(c) \geq 1$ for all $c \geq \underline{c}$. Finally, taking the supremum of both sides over all admissible dividend and issuance strategies yields

$$\hat{V}_s(c) \quad \geq \quad \sup_{\{L,f\}\in\mathcal{A}} \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}(dL_t - (f_{t-} + \mathbf{1}_{\{f_{t-}>0\}}F)dN_t)\right] = V_s(c).$$

($\Leftarrow$:) In the second part of the proof, we will show that all above inequalities turn to equalities when we use (L^*, f^*). First, let's prove the admissibility of conjectured policies:

$$\mathbb{E}_c\left[\int_0^{\infty} e^{-rt} f_t^* dN_t\right] \leq \mathbb{E}_c\left[\int_0^{\infty} e^{-rt} c^* dN_t\right] = \frac{\lambda c^*}{r} < \infty,$$

where the inequality results from the definition of f^* and the equality comes by using the mean of the Poisson process (which is λ in our case). In addition, using the cash reserves dynamics we obtain

$$\mathbb{E}_c\left[\int_0^{T} e^{-rt} dL_t^*\right] \quad = \quad c + \mathbb{E}_c\left[\int_0^{T} e^{-rt}\mu dt + \int_0^{T} e^{-rt} f_{t-}^* dN_t\right].$$

Then, letting $T \to \infty$, using the Fatou's lemma and the upper bound for f_t^* yield

$$\begin{aligned}
\lim_{T\to\infty} \mathbb{E}_c\left[\int_0^{T} e^{-rt} dL_t^*\right] &\leq \mathbb{E}_c\left[\int_0^{\infty} e^{-rt} dL_t^*\right] \\
&\leq c + \mathbb{E}_c\left[\int_0^{\infty} e^{-rt}\mu dt + \int_0^{\infty} e^{-rt} f_{t-}^* dN_t\right] \\
&\leq c + \frac{1}{r}(\mu + \lambda c^*),
\end{aligned}$$

which implies that $(L^*, f^*) \in \mathcal{A}$. Now, consider the process

$$X_t = e^{-r(t\wedge\tau)}\hat{V}_s(C_{t\wedge\tau}) + \int_{0+}^{t\wedge\tau} e^{-ru}(dL_u^* - (f_{u-}^* + \mathbf{1}_{\{f_{u-}^*>0\}}F)dN_u).$$

Then, if we apply Ito's formula for semimartingales to X_t with the optimal policies (L^*, f^*), the first term in the dynamics of dP_t (defined in the first part of the proof) vanishes since the optimal issuance policy maximizes $\mathcal{F}$. The second and third terms also disappear since the optimal dividend strategy L^* is only activated when $C_t = c^*$, hence $\hat{V}_s'(C_t) = 1$ for L^*. Therefore, $dX_t = dQ_t$ is a local martingale. Furthermore, for any stopping time τ we have

$$|X_\tau| < |\hat{V}_s(c^*)| + \int_0^\infty e^{-rt}(dL_t^* + f_{t-}^* dN_t),$$

since $\hat{V}_s$ is increasing, the optimal policies are admissible and they keep the bank's capital in the region $(\underline{c}, c^*]$ for all $t \geq 0$. Hence, X_t is uniformly integrable since it is bounded from below and above. Finally,

$$\begin{aligned}
\hat{V}_s(c) &= X_{0-} = X_0 - \Delta X_0 = X_0 + \Delta L_0^* = \mathbb{E}_c[X_\tau] + \Delta L_0^* \\
&= \mathbb{E}_c\left[e^{-r\tau}\hat{V}_s(C_\tau) + \int_{0+}^{\tau} e^{-rt}(dL_t^* - (f_{t-}^* + \mathbf{1}_{\{f_{t-}^*>0\}}F)dN_t)\right] + \Delta L_0^* \\
&= \mathbb{E}_c\left[\int_0^{\tau} e^{-rt}(dL_t^* - (f_{t-}^* + \mathbf{1}_{\{f_{t-}^*>0\}}F)dN_t)\right] = V_s(c),
\end{aligned}$$

where the fourth and fifth equalities follow from the martingale property and the definition of X, respectively. ■

Proof of Proposition 4.5

We first solve for the function $G_2(c_0)$ which satisfies the following ODE:

$$\begin{aligned}
rG_2(c_0) &= \mu G_2'(c_0) + \frac{\sigma^2}{2}G_2''(c_0) \\
s.t. \quad G_2(c_1) &= G_1(c_1), \\
G_2'(c_1) &= G_1'(c_1), \\
G_2'(\overline{c}) &= 0.
\end{aligned}$$

General solution form for the above ODE is

$$G_2(c_0) = a_1 e^{\eta_1 c_0} + a_2 e^{\eta_2 c_0}.$$

Define $G_2(\overline{c}) := k$. This definition and the last boundary condition are used to determine a_1 and a_2 as follows:

$$\begin{aligned}
G_2(\overline{c}) = k \quad &\Rightarrow \quad a_1 e^{\eta_1 \overline{c}} + a_2 e^{\eta_2 \overline{c}} = k, \\
G_2'(\overline{c}) = 0 \quad &\Rightarrow \quad a_1 \eta_1 e^{\eta_1 \overline{c}} + a_2 \eta_2 e^{\eta_2 \overline{c}} = 0.
\end{aligned}$$

By solving the above equations, a_1 and a_2 are found as follows:

$$\begin{aligned} a_1 &= \frac{\eta_2 k}{\eta_2 - \eta_1} e^{-\eta_1 \overline{c}}, \\ a_2 &= \frac{-\eta_1 k}{\eta_2 - \eta_1} e^{-\eta_2 \overline{c}}. \end{aligned}$$

Finally, $G_2(c)$ is given as

$$G_2(c_0) = \frac{\eta_2 k e^{\eta_1 (c_0 - \overline{c})} - \eta_1 k e^{\eta_2 (c_0 - \overline{c})}}{\eta_2 - \eta_1},$$

where k has to be determined. Secondly, we look at the solution of $G_1(c_0)$, which satisfies the following second order non-homogenous ODE:

$$\begin{aligned} (r+\lambda) G_1(c) &= \mu G_1'(c) + \frac{\sigma^2}{2} G_1''(c) + \lambda G_2(\overline{c}) \\ s.t. \quad G_1(\underline{c}) &= 1, \\ G_1(c_1) &= G_2(c_1), \\ G_1'(c_1) &= G_2'(c_1), \end{aligned}$$

which has the following solution form:

$$G_1(c_0) \quad = \quad \underbrace{d_1 c_0 + d_2}_{G_{1p}(c_0)} + \underbrace{b_1 e^{\theta_1 c_0} + b_2 e^{\theta_2 c_0}}_{G_{1h}(c_0)},$$

where $G_{1p}(c_0)$ is the particular solution of the whole ODE and $G_{1h}(c_0)$ is the solution of the homogenous part with $\theta_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2 (r+\lambda)}}{\sigma^2}$.[32] Plugging G_{1p} into the ODE and setting the coefficients of c_0 and the constants yield

$$\begin{aligned} d_1 &= 0, \\ d_2 &= \frac{\lambda k}{r+\lambda}. \end{aligned}$$

Now, we use the initial condition, the continuity and smooth pasting properties at c_1 to find b_1, b_2, k:

$$\begin{aligned} b_1 e^{\theta_1 \underline{c}} + b_2 e^{\theta_2 \underline{c}} + \frac{\lambda k}{r+\lambda} &= 1, \\ b_1 e^{\theta_1 (\underline{c} + y_2^*)} + b_2 e^{\theta_2 (\underline{c} + y_2^*)} &= kp, \\ b_1 \theta_1 e^{\theta_1 (\underline{c} + y_2^*)} + b_2 \theta_2 e^{\theta_2 (\underline{c} + y_2^*)} &= kq, \end{aligned}$$

[32] $\theta_1 > 0 > \theta_2$ and $|\theta_2| > |\theta_1|$.

where

$$p := \frac{\eta_2 e^{\eta_1(\underline{c}-\overline{c}+y_2^*)} - \eta_1 e^{\eta_2(\underline{c}-\overline{c}+y_2^*)}}{\eta_2-\eta_1} - \frac{\lambda}{(r+\lambda)} \text{ and } q := \frac{\eta_2\eta_1 e^{\eta_1(\underline{c}-\overline{c}+y_2^*)} - \eta_1\eta_2 e^{\eta_2(\underline{c}-\overline{c}+y_2^*)}}{\eta_2-\eta_1}.$$

Solving above system of equations yields the followings:

$$\begin{aligned}
k &= \frac{1}{\frac{\lambda}{(r+\lambda)} + \left(\frac{q-\theta_2 p}{\theta_1-\theta_2}\right) e^{-\theta_1 y_2^*} + \left(\frac{q-\theta_1 p}{\theta_2-\theta_1}\right) e^{-\theta_2 y_2^*}}, \\
b_1 &= \left(\frac{k(q-\theta_2 p)}{\theta_1-\theta_2}\right) e^{-\theta_1(\underline{c}+y_2^*)}, \\
b_2 &= \left(\frac{k(q-\theta_1 p)}{\theta_2-\theta_1}\right) e^{-\theta_2(\underline{c}+y_2^*)}.
\end{aligned}$$

■

Proof of Proposition 4.6

We start by solving for the function $V_2(c_0)$, which satisfies the following ODE:

$$\begin{aligned}
rV_2(c_0) &= \mu V_2'(c_0) + \frac{\sigma^2}{2} V_2''(c_0) \\
s.t. \quad V_2(c_1) &= V_2(\overline{c}) - (\overline{c} - c_1) - F, \\
V_2'(\overline{c}) &= 1.
\end{aligned}$$

The general solution form for the above ODE is

$$V_2(c_0) = \zeta_1 e^{\eta_1 c_0} + \zeta_2 e^{\eta_2 c_0}.$$

Define $V_2(\overline{c}) := n$. This definition and the last boundary condition are used to determine ζ_1 and ζ_2 as follows:

$$\begin{aligned}
V_2(\overline{c}) = n \quad &\Rightarrow \quad \zeta_1 e^{\eta_1 \overline{c}} + \zeta_2 e^{\eta_2 \overline{c}} = n, \\
V_2'(\overline{c}) = 1 \quad &\Rightarrow \quad \zeta_1 \eta_1 e^{\eta_1 \overline{c}} + \zeta_2 \eta_2 e^{\eta_2 \overline{c}} = 1.
\end{aligned}$$

By solving the above equations, ζ_1 and ζ_2 are obtained as follows:

$$\begin{aligned}
\zeta_1 &= \frac{1-\eta_2 n}{\eta_1-\eta_2} e^{-\eta_1 \overline{c}}, \\
\zeta_2 &= \frac{1-\eta_1 n}{\eta_2-\eta_1} e^{-\eta_2 \overline{c}}.
\end{aligned}$$

Then, by plugging $\zeta_i,\ i = 1, 2$, into the initial condition, n is found easily as given in the proposition. Secondly, we consider the solution of $V_1(c_0)$, which satisfies a second order non-homogenous ODE having the following solution form:

$$V_1(c_0) = \underbrace{e_1 c_0 + e_2}_{V_{1p}(c_0)} + \underbrace{f_1 e^{\theta_1 c_0} + f_2 e^{\theta_2 c_0}}_{V_{1h}(c_0)},$$

where $V_{1p}(c_0)$ is the particular solution of the whole ODE and $V_{1h}(c_0)$ is the solution of the homogenous part. Plugging V_{1p} into the ODE and equalizing the coefficients of c_0 and the constants yield

$$\begin{aligned} e_1 &= \frac{\lambda}{r+\lambda}, \\ e_2 &= \frac{\mu\lambda}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda}(n - \overline{c} - F). \end{aligned}$$

Remaining steps of the proof are exactly the same as we do above. In particular, we solve for the initial condition, the continuity and smooth pasting equations at c_1 and obtain f_1, f_2, n as given in the proposition. ■

Proof of Proposition 4.7

The proof is straightforward by applying the same steps in the proof of Proposition 4.3 since in the general case with outside financing option the function $G(c_0; \underline{c}, \overline{c})$ is still defined in the range $[0, 1]$.

Appendix B

Figure 2: Optimal Thresholds without Voluntary Recapitalizations.

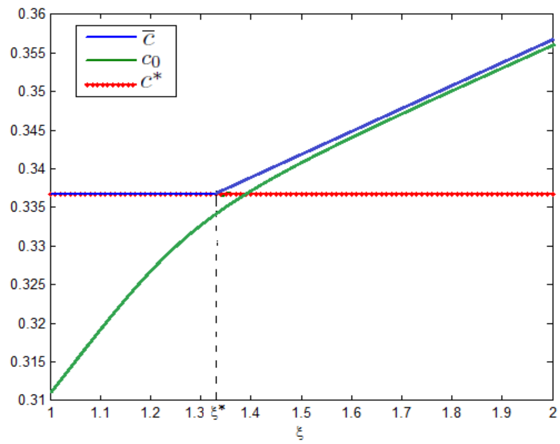

Figure 2 shows how the initial capital (c_0) and the optimal dividend thresholds for the shareholders (c^*) and the regulator ($\overline{c}$) vary with the restructuring cost ξ. Baseline parameters are $\mu = 0.1$, $\sigma = 0.1$, and $r = 6\%$.

Figure 3: Value Function of the Bank without Voluntary Recapitalizations.

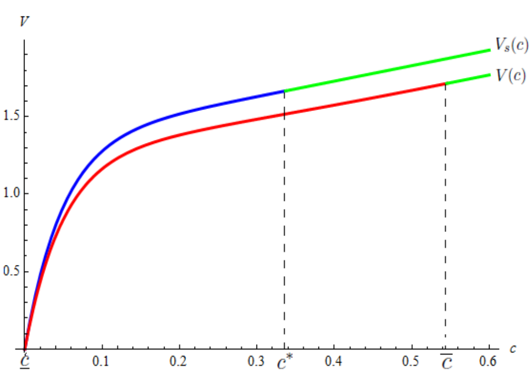

Figure 3 shows the value function of the bank for the cases, in which the dividend threshold is chosen by the shareholders ($V_s(c)$) or by the regulator ($V(c)$). Baseline parameters are $\mu = 0.1$, $\sigma = 0.1$, $r = 6\%$, and $\xi = 2 > \xi^* = 1.31$.

Figure 4: Optimal Thresholds with Voluntary Recapitalizations.

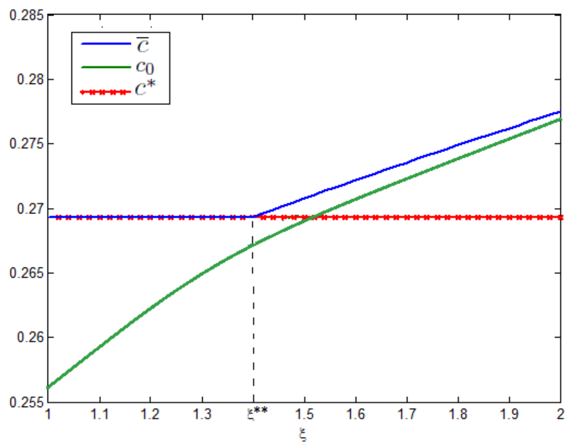

Figure 4 shows how the initial capital (c_0) and the optimal dividend thresholds for the shareholders (c^*) and the regulator ($\overline{c}$) vary with the restructuring cost ξ. Baseline parameters are $\mu = 0.1$, $\sigma = 0.1$, $r = 6\%$, $F = 0.025$, and $\lambda = 6$.

Figure 5: Value Function of the Bank with Voluntary Recapitalizations.

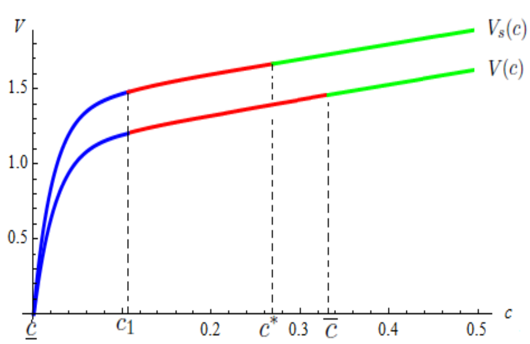

Figure 5 shows the value functions of the bank for the cases in which the dividend threshold is chosen by the shareholders ($V_s(c)$) or by the regulator ($V(c)$). Baseline parameters are $\mu = 0.1$, $\sigma = 0.1$, $r = 6\%$, $F = 0.025$, $\lambda = 6$, and $\xi = 2 > \xi^{**} = 1.41$.

Figure 6: Optimal Thresholds without Voluntary Recapitalizations in the Low Restructuring Cost Regime.

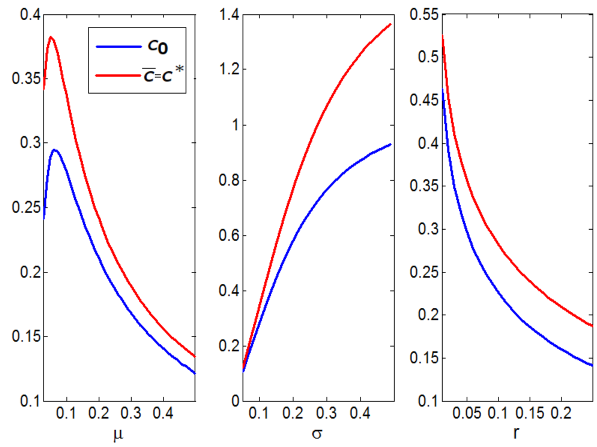

Figure 6 shows how the initial capital (c_0) and the optimal dividend thresholds for the shareholders (c^*) and the regulator ($\bar{c}$) vary with the profitability of the bank (μ), volatility of the cash flows (σ), and cost of holding cash (r) in the case, where raising outside equity is infinitely costly and the restructuring cost is low (i.e. $\xi < \xi^*$).

Figure 7: Optimal Thresholds without Voluntary Recapitalizations in the High Restructuring Cost Regime.

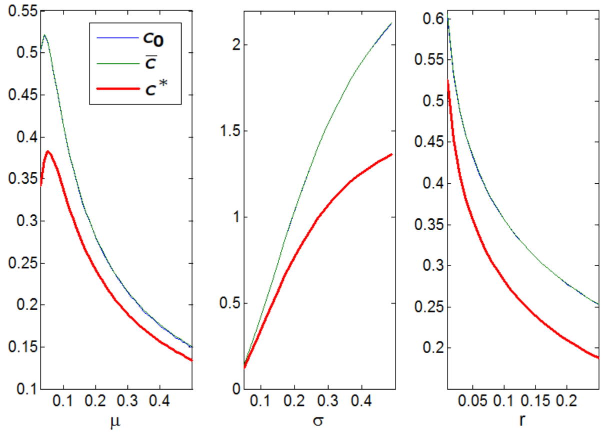

Figure 7 shows how the initial capital (c_0) and the optimal dividend thresholds for the shareholders (c^*) and the regulator ($\bar{c}$) vary with the profitability of the bank (μ), volatility of the cash flows (σ), and cost of holding cash (r) in the case, where raising outside equity is infinitely costly and the restructuring cost is high (i.e. $\xi > \xi^*$).

Figure 8: Optimal Thresholds with Voluntary Recapitalizations in the Low Restructuring Cost Regime.

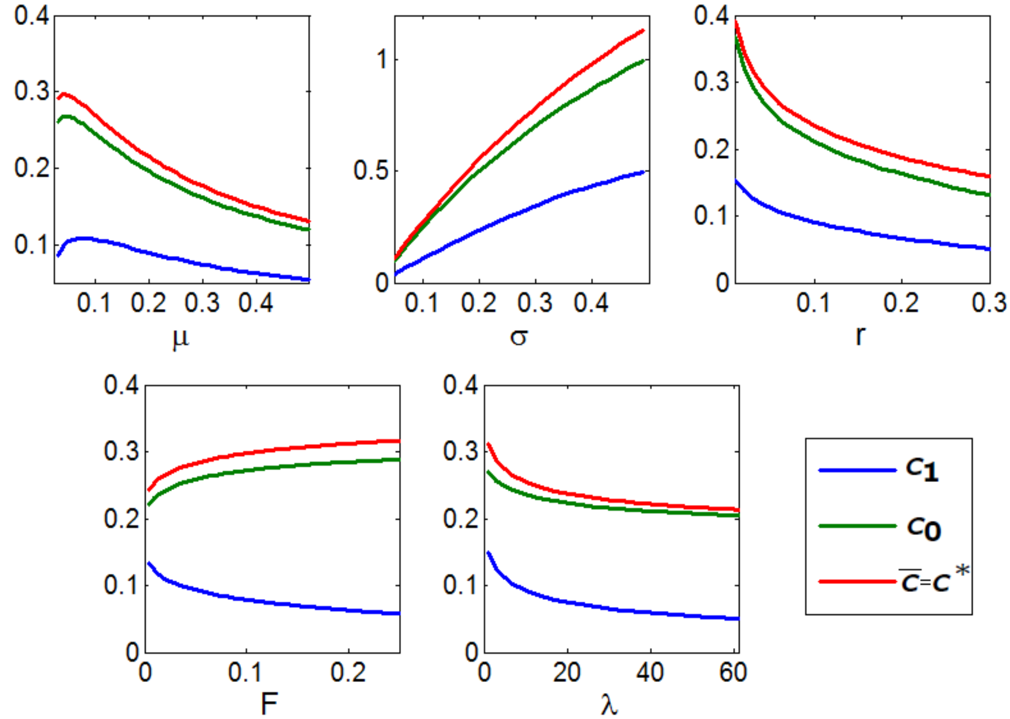

Figure 8 shows how the initial capital (c_0), optimal equity issuance threshold (c_1), and optimal dividend thresholds for the shareholders (c^*) and the regulator ($\bar{c}$) vary with the profitability of the bank (μ), volatility of the cash flows (σ), cost of holding cash (r), cost of raising outside equity (F), and arrival rate of the outside investors (λ) in the case, where the restructuring cost is low (i.e. $\xi < \xi^{**}$). In each plots, the restructuring cost is always taken as half of the critical restructuring cost.

Figure 9: Optimal Thresholds with Voluntary Recapitalizations in the High Restructuring Cost Regime.

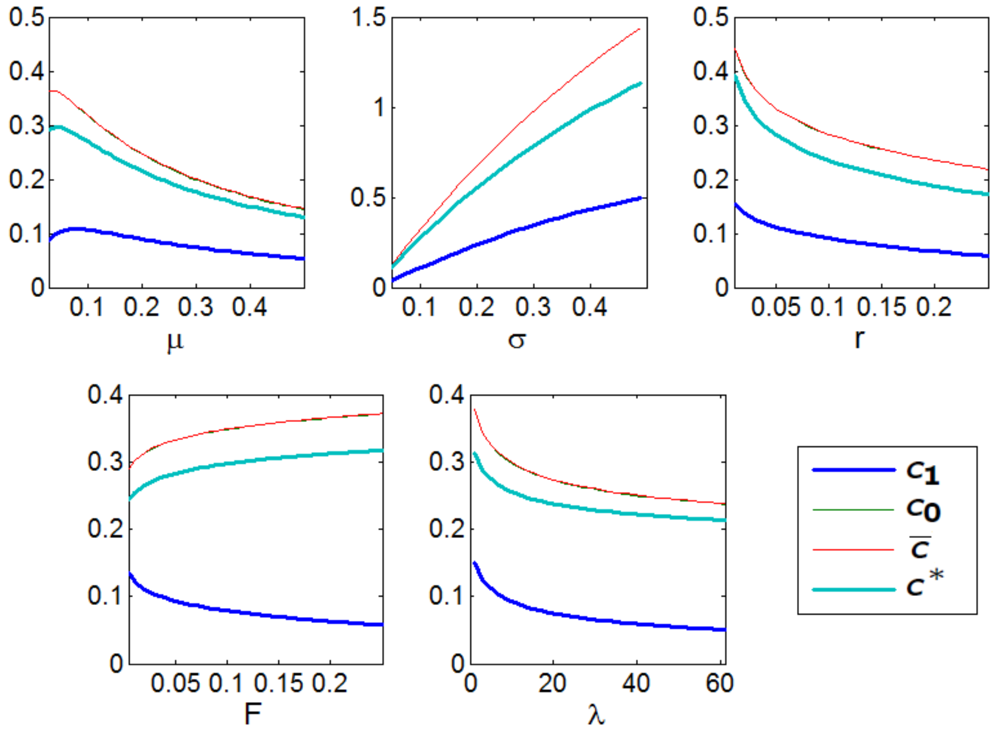

Figure 9 shows how the initial capital (c_0), optimal equity issuance threshold (c_1), and optimal dividend thresholds for the shareholders (c^*) and the regulator ($\bar{c}$) vary with the profitability of the bank (μ), volatility of the cash flows (σ), cost of holding cash (r), cost of raising outside equity (F), and arrival rate of the outside investors (λ) in the case, where the restructuring cost is high (i.e. $\xi > \xi^{**}$). In each plots, the restructuring cost is always taken as two times of the critical restructuring cost.

Figure 10: Critical Restructuring Cost without Voluntary Recapitalizations.

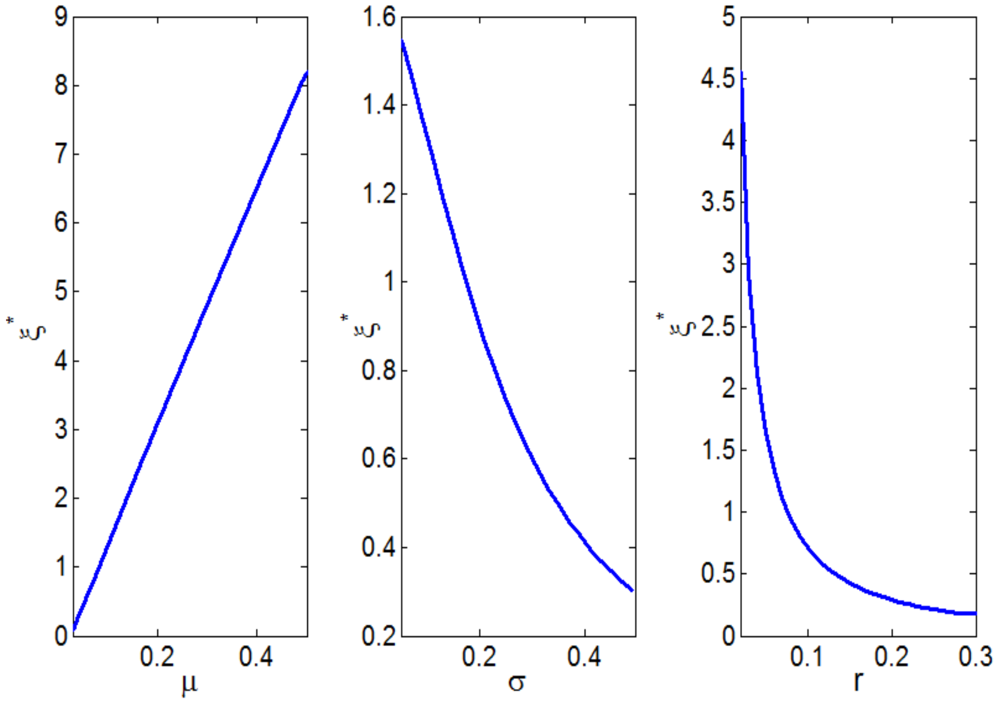

Figure 10 shows the sensitivity of the critical restructuring cost with respect to the expected profitability of the bank (μ), volatility of the cash flows (σ), and cost of holding cash (r) in the case without voluntary recapitalizations.

Figure 11: Critical Restructuring Cost with Voluntary Recapitalizations.

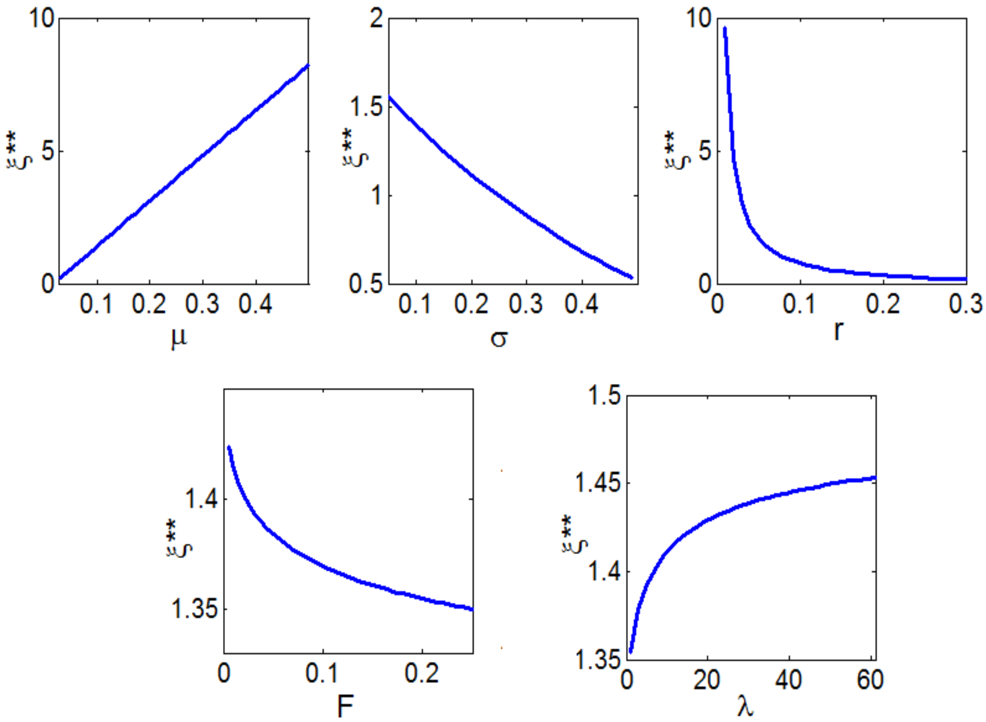

Figure 11 shows the sensitivity of the critical restructuring cost with respect to the expected profitability of the bank (μ), volatility of the cash flows (σ), cost of holding cash (r), cost of raising outside equity (F), and arrival rate of the outside investors (λ) in the case with voluntary recapitalizations.

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