

On the Uncertainty-Investment Relationship: An Overview with an Application to the Power Plant Investments in Turkish Electricity Sector

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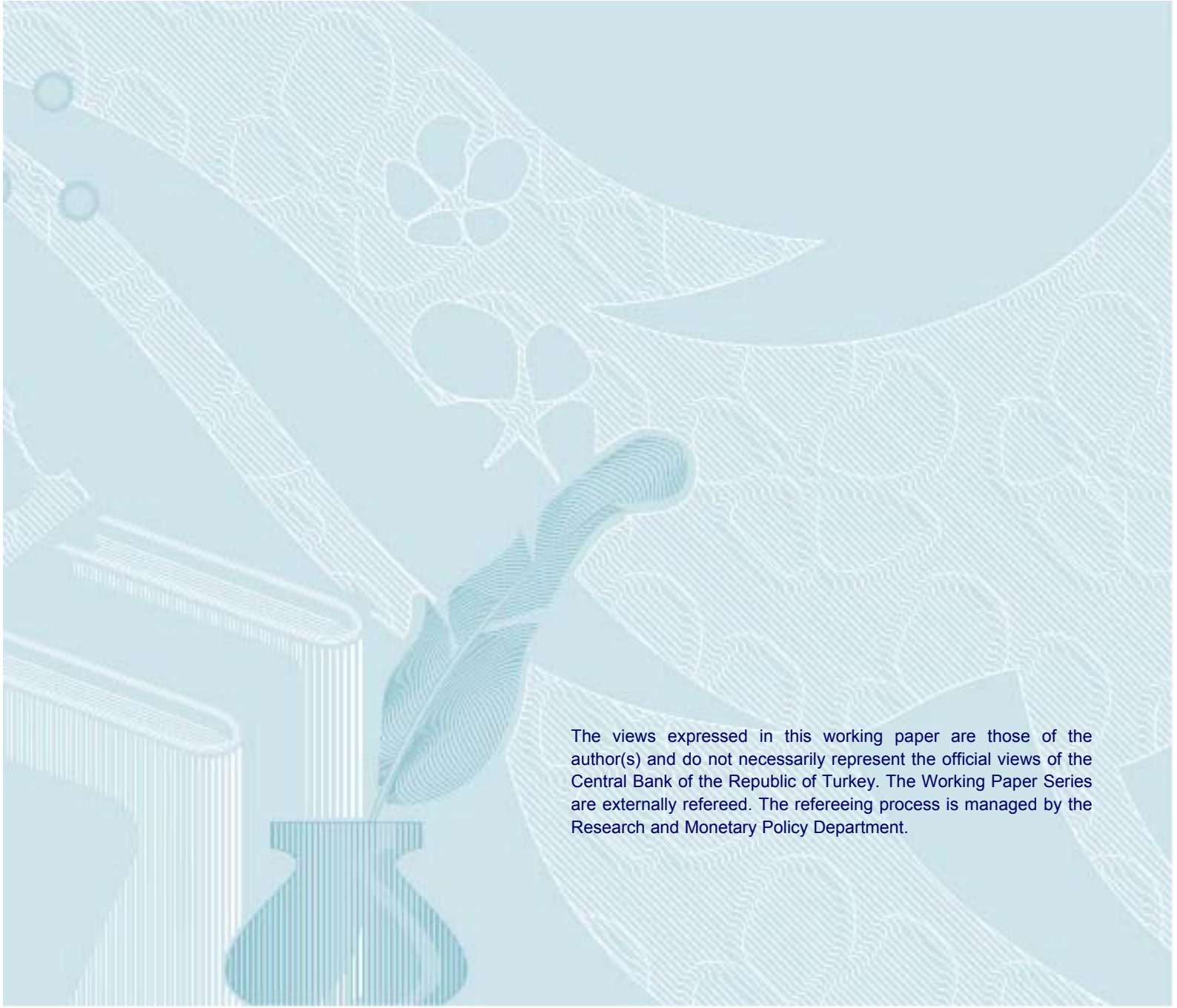
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On the Uncertainty-Investment Relationship: An Overview with an Application to the Power Plant Investments in Turkish Electricity Sector¹

Erdal Yılmaz²

Abstract

The effect of uncertainty on investment is widely considered to have a negative sign in the real option literature. Contrary to prediction of conventional real option theory, there are studies pioneered by Sarkar (2000) and Gryglewicz et al. (2008) with the argument that this negative relationship is not always correct. Such result is exceptional, since they show that uncertainty may accelerate irreversible investment without building on the convexity of the marginal product of capital in the real option framework. Major contribution of this paper, by applying the Gryglewicz et al. (2008) approach, is to show numerically that the uncertainty-investment relationship in Turkish electricity plant investment is non-monotonic and U-shaped. We also numerically compare those two studies and investigate whether certain conditions in Sarkar (2000) are associated with the parameter support by Gryglewicz et al. (2008) or not. Finally, we numerically demonstrate partial effect of the interest rate changes on optimal investment trigger based on Gryglewicz et al. (2008) framework.

Keywords: Investment, real option, uncertainty

JEL classification: D92, E22, G31.

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1. Introduction

Impact of uncertainty on investment is one of the important issues for policymakers and academics for a long time. Research that emphasizes the roles played by macroeconomic and policy uncertainty in curtailing economic activity has exploded after the 2008 global crisis in focusing on the old questions such as *How important is uncertainty in driving economic activity?* or *How do fluctuations in uncertainty affect economic activity or behavior of firms?* Starting with the seminal contribution of Bloom (2009); many studies; such as Bloom et al. 2012; Stock and Watson, 2012; Bachman et al., 2013; Baker et al., 2013; Bekaert et al., 2013; have focused on the aggregate effects of time-varying uncertainty.³ In general, this research agenda relates to at least two strands of literature: (i) research on the impact of economic uncertainty on investment; (ii) research on policy uncertainty.

In this paper, we study firms' specific investment decisions under uncertainty. Before proceeding, it is worth to discuss the ways of evaluating investment projects and taking optimal investment decisions. The neo-classical theory of investment emphasizes the importance of the simple net present value (NPV) rule. According to this rule, a firm should invest in a project as long as the NPV is positive and it should discard projects with negative NPVs. However, this classical view neglects many characteristics associated with investment decisions, such as irreversibility, uncertainty, and timing of investment. In that regard, for most investments, the usefulness of the NPV rule is severely limited and the view that accepts all projects with positive NPVs is quite generally wrong. For example,

³ See Bloom, 2014 for a very recent and detailed survey on fluctuations in uncertainty; and see Kose and Terrones, 2012; Bloom et al., 2013 for short, policy-oriented writings.

Ingersoll and Ross (1992) show that even for the simplest projects with deterministic cash flows, interest-rate uncertainty has a significant effect on investment. Accordingly, a firm can postpone investment to obtain more information about future. As Ross (1995) states “*when evaluating investments, optionality is ubiquitous and unavoidable.*”

The possibility of delaying/abandoning an irreversible investment project can lead to better investment decisions without being obliged to. In this regard, an investment opportunity is viewed as an American option to invest and exercise optimally. This is the main theme of the “real options” approach, the body of literature about the effects of uncertainty on investment, which roots back to Bernanke, 1983; Brennan and Schwartz, 1985; McDonald and Siegel, 1986; Majd and Pindyck, 1987; Pindyck, 1988; and Dixit and Pindyck, 1994. By analogy with financial options (see Black and Scholes, 1973; Merton, 1973), an opportunity to invest is a call option and to invest is to exercise the option. As Trigeorgis (1993) states, such real options may occur naturally, via deferring, contracting, shutting down or abandoning; or they may be planned at some extra cost; via expanding capacity, building growth options, defaulting when investment is staged.

A general prediction of the real options literature is that a higher level of uncertainty has a negative effect on investment (Mauer and Ott, 1995; Leahy and Whited, 1996; Dixit and Pindyck, 1994). However, in earlier research, there are some studies that contrast to the main theme of the real option approach. For example, Hartman (1972) and Abel (1983) show that increased uncertainty in the future price of output leads competitive firms to hasten investment in a setting which is based on convexity of marginal profits in price and convex costs of capital adjustments. Bar-Ilan and Strange (1996) and Caballero (1991) observe that depending on a specific parameter, higher uncertainty may increase or decrease investment. These papers depart from conventional result of the real options

literature, since their models create convexities in line with Hartman (1972) and Abel (1983).

Another line of research, pioneered by Sarkar (2000) and Gryglewicz et al. (2008), shows that the investment-uncertainty relationship is not necessarily negative and monotonic. The results of these papers are exceptional in the real option approach in the sense that uncertainty may hasten irreversible investment without building on convexity of the marginal product of capital. In fact, many recent papers in this literature are influenced by these studies.⁴ The main contribution of Gryglewicz et al. (2008) is that they show that the uncertainty-investment relationship is positive when project life is finite, level of uncertainty is low and the risk adjusted rate of return on the project is positively related to uncertainty. In other words, uncertainty has a U-shaped influence on the value of the investment threshold. This is the non-monotonicity result of their work. Sarkar (2000) analyzes effect of the various parameters on uncertainty-investment relationship and concludes that the uncertainty–investment relationship is more likely to be positive for the following situations: (i) the current level of uncertainty σ is low, (ii) λ is high, (iii) ρ is high, (iv) r is high, (v) μ is low, and (vi) T is short.

In this paper, we numerically investigate whether certain conditions in Sarkar (2000) are associated with parameters support by Gryglewicz et al. (2008) or not. We construct a set of parameters in line with the model assumptions of Gryglewicz et al. (2008) and the arguments of Sarkar (2000). Then, we numerically demonstrate that the parameters proposed by Sarkar (2000) support non-monotonicity result related to the investment and uncertainty in Gryglewicz et al. (2008).

⁴⁴ See, for example, Lund (2005), Wong (2007), Lucas and Welling (2014) and the references therein.

We change the level of interest rate without altering the characteristics of investment project in order to understand, partially, the possible effects of interest rate on the optimal investment trigger strategy in Gryglewicz et al. (2008) model. Changing the level of interest rate affects the risk adjusted expect return of the project and, thus, has impact on opportunity cost of investing project or option value. This helps us to see the impact of expansionary and contractionary monetary policy on the optimal trigger. We numerically demonstrate that in the case of the relatively lower interest rate; the positive relationship between uncertainty–investment is more profound and the optimal trigger investment level increases as expected.

Major contribution of our paper is that this is one of the first papers on uncertainty-investment relationship in Turkish electricity plant investment. Madlener and Stoverink (2012) focus on the economic feasibility of constructing a 560 MW coal-fired power plant in Turkey. We use some of their parameters and construct rest of the parameters needed for model and the project life of electricity sector in Turkey is short. So, the availability of data and short project life provide us a good opportunity to analyze uncertainty-investment relationship in Turkish electricity plant investment based on the theoretical findings of Gryglewicz et al. (2008) for an emerging market. We numerically show that uncertainty-investment relationship in Turkish electricity plant investment based on Gryglewicz et al. (2008) framework is non-monotonic and U-shaped.

The rest of the paper is organized as follows. In section 2, we focus on the contingent claim analysis, Gryglewicz et al. (2008) model and their results. Section 3 provides an economic analysis of the non-monotonicity result and we present results of some numerical simulations. In Section 4, we analyze power plant investment in the

Turkish electricity sector based on the model of Gryglewicz et al. (2008). Section 5 concludes the paper.

2. Contingent Claims Analysis, Model and Optimal Investment Decision

In this section, we revisit Gryglewicz et al. (2008) model and their solution. They use contingent claims valuation technique in real options theory, when they analyze optimal investment policy. In order to understand clearly the reason why they implement this technique in their study, it is worth to begin with discussion on the techniques used in real options theory.

There are basically two techniques in the real options theory in order to calculate the value of waiting to invest (investment opportunity); dynamic programming (DP) and contingent claims analysis (CCA) (Dixit and Pindyck, 1994). Although these two techniques are strongly associated with each other, and lead to yield identical outcome in many applications, two techniques differ from each other due to the fact that they have different assumptions about financial markets, and discount rates that firms use to value future cash flows according to Dixit and Pindyck (1994).

DP is an older approach, which dates to Richard E. Bellman and his associates at the Rand Corporation in the 1950s, and used extensively in economics and management science (see Bellman, 1957). Essentially, DP is an optimization approach that transforms a complex problem into a sequence of simpler problems. In practice, dynamic programming typically involves adopting an exogenous constant discount rate. On the other hand, the discount rate is determined endogenously as an implication of the overall equilibrium in capital markets in the CCA method as compared to DP and hence CCA suggests a better dealt with the discount factor. To summarize why Gryglewicz et al. (2008) prefer to use

CCA of real investment opportunities, the assumption of uncertainty affecting the discount rate and convenience yield appears to be the most plausible one.

On the other hand, one of the core assumptions in the CCM is that existing assets with a price that is perfectly correlated with Q (stochastic revenue per unit time t) so that uncertainty over future values of Q can be replicated by existing assets must span the stochastic variations in Q . With this assumption, CCM allows to make analysis the equilibrium impact of the systematic risk on the discount rate, and, on the value of investment option and, the investment policy by using the intertemporal CAPM of Merton (1973) as pointed out by Gryglewicz et al. (2008). Using the spanning technique let P be the price of the asset that is perfectly correlated with Q . Let ρ_{PM} be the correlation of P with market portfolio M , then, $\rho_{PM} = \rho_{QM}$. Since P is perfectly correlated with Q , P is assumed to evolve the same way:

$$dQ_t = \mu Q_t dt + \sigma Q_t dz_t, \quad (1)$$

$$dP_t = \pi P_t dt + \sigma P_t dz_t, \quad (2)$$

where μ is the drift parameter or the expected rate of change in Q , σ is the volatility (uncertainty) of revenue faced by firm, and dz_t is the increment of a standard Brownian motion process which is log-normally distributed. π is risk-adjusted rate of return on this asset. It is crucial to note that $\mu, \sigma > 0$. By the Capital Asset Pricing Model (CAPM), π also reflects the asset's systematic risk and it is given by:

$$\pi = r + \lambda \rho_{PM} \sigma, \quad (3)$$

where $\lambda = (r_M - r)/\sigma_M$ is the aggregate market price of risk. r_M is the expected return on the market which can also be considered as return of the whole market portfolio that provides availability of diversification. r is risk-free interest rate and assumed to be exogenous. The risk premium is determined by the covariance between r_M and r . It is assumed that $\pi > \mu$ in order to guarantee that a firm invest in the project. Convenience yield of the investment opportunity is described as the difference between π , risk-adjusted expected return of the project, or risk-adjusted discount rate and μ , the expected rate of return of project or the expected rate of change in Q. The difference is shown by δ or put it differently, which is an opportunity cost of delaying investing in the project and keeping the option to invest alive. And therefore, δ satisfies:

$$\delta = \pi - \mu = r + \lambda \rho_{PM} \sigma - \mu. \quad (4)$$

In case of $\delta = 0$, that is $\pi = \mu$, then this implies that there would be no opportunity cost to keeping the option alive, and the firm never invest in this project. Therefore, it is worth to analyze the case where $\delta > 0$, this case implies that the investment occurs; otherwise it is never optimal to exercise the option. From Equation (4), it is clear that change in volatility faced by firm leads to change in π . In order to satisfy Equation (4), an adjustment needs either in μ or in δ or both. A certain rule in this respect could be Pindyck (2004), i.e., a volatility shock has a significant effect on the convenience yield (δ) and only a small effect on the price. Consistent with this evidence, the related literature on the investment–uncertainty relationship commonly assume that μ is fixed and δ changes with σ (Sarkar, 2000, 2003). Therefore, adjustment comes from δ and δ becomes an endogenous parameters affected by volatility, σ and function of σ in Gryglewicz et al. (2008). This

assumption is important for both our numerical simulation and their model set up that we will focus on later.

After brief introduction on technique and main assumptions, it is worth to focus on how Gryglewicz et al. (2008) analyze the option to invest and get the optimal trigger investment level. Their model is based on value of the project, denoted by $V(Q)$. The project value is a function of the stochastic revenues and evolves over time and depends on the current realization of Q . The project value, $V(Q)$, can be obtained by the expected present value of the revenue stream discounted by the risk-adjusted discount rate as it is standard in the literature. If the project has a finite life of T years, which is important assumption for their contribution, then the project value at the time of the investment as formulated by Gryglewicz et al. (2008) is

$$V(Q) = E \left[\int_0^T e^{-\pi t} Q_t d_t \middle| Q_0 = Q \right] = \int_0^T e^{-(\pi-\mu)t} Q d_t = Q \frac{1 - e^{-(r+\lambda\rho_{PM}\sigma-\mu)T}}{r + \lambda\rho_{PM}\sigma - \mu} \quad (5)$$

Value of project depends on Q , revenue of project, convenience yield, $\delta = r + \lambda\rho_{PM}\sigma - \mu$, and project life, T . Prior to installation of project, the firm holds an option to invest. The option is held until the stochastic revenue flow reaches a sufficiently high level at which it is optimal to exercise the option and invest. The option value, $F(Q)$, can be found by constructing a risk-free portfolio, determining its expected rate of return, and equating that return to the risk free rate of interest rate, r . Value of option, $F(Q)$, follows second order differential equation of the form specified in Dixit and Pindyck (1994) as below:

$$\frac{1}{2} \sigma^2 Q^2 F''(Q) + (r - \delta) Q F'(Q) - r F(Q) = 0. \quad (6)$$

$F(Q)$ also satisfies the following boundary conditions:

$$F(0) = 0. \quad (7)$$

$$F(Q^*) = V(Q^*) - I. \quad (8)$$

$$F'(Q^*) = V'(Q^*). \quad (9)$$

Again $V(Q^*)$ represents value of the project at which it is optimal to invest.⁵ Equation (7) states that when $Q = 0$, the value of the option to invest has no value. Equation (8) is the value-matching condition that is upon investing; the firm receives a net payoff $V(Q^*) - I$. Rewriting Equation (8) as $I = V(Q^*) - F(Q^*)$, which implies that when the firm invests in the project, it gets the value $V(Q^*)$, but gives up the opportunity to invest $F(Q^*)$. The critical value Q^* is obtained when this net gain, $V(Q^*) - F(Q^*)$, is equal to the direct cost of I (investment). Equation (9) is the smooth-pasting condition. That is, if $F(Q^*)$ were not continuous and smooth at the critical value Q^* , it is better for firm to wait Δt to observe next step of Q . To solve for the critical value Q^* , we must solve Equation (6) subject to the boundary conditions Equation (7), (8) and (9). McDonald and Siegel (1986) suggest that the solution satisfies the condition (7) and thus, must take the form⁶:

$$F(Q) = A Q^\beta. \quad (10)$$

Equation (10) indicates that β and Q plays important role for value of option. After solving (10), we end up following quadratic equation:

⁵ Cox and Ross (1976) prove that the same solution is obtained by implementing dynamic programming technique under the assumption that all agents are risk-neutral.

⁶ $F(Q) = A_1 Q^{\beta_1} + A_2 Q^{\beta_2}$. Since boundary condition (7) is $F(0) = 0$ which implies that $A_2 = 0$.

$$\frac{1}{2}\sigma^2\beta(\beta-1)+(r-\delta)\beta-r=0. \quad (11)$$

We are looking for the positive root ($\beta > 1$) of quadratic Equation (11). Then we obtain β as follows in terms parameter,

$$\beta = \frac{1}{2} - \frac{(r - (r + \lambda\rho\sigma - \mu))}{\sigma^2} + \sqrt{\left(\left[\frac{(r - (r + \lambda\rho\sigma - \mu))}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}\right)} > 1. \quad (12)$$

or

$$\beta = \frac{1}{2} - \frac{(\mu - \lambda\rho\sigma)}{\sigma^2} + \sqrt{\left(\left[\frac{(\mu - \lambda\rho\sigma)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}\right)}$$

And more simplifying of Equation (12) can be also written as follows:

$$\beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\left[\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}\right)}. \quad (13)$$

Therefore, β depends on value of the parameters; δ , σ and, r . Since r is a constant, β is a function of convenience yield (δ) and volatility (σ), $\beta(\sigma, \delta(\sigma))$. Furthermore, we plug β in order to solve for Q^* in equation 13 which yields the investment trigger:

$$Q^* = \frac{\beta}{(\beta-1)} \frac{r + \lambda\rho\sigma - \mu}{1 - e^{-(r+\lambda\rho\sigma-\mu)T}} I, \quad \text{or} \quad Q^* = \frac{\beta}{(\beta-1)} \frac{\delta}{1 - e^{-(\delta)T}} I. \quad (14)$$

From Equation (14), we can conclude that the investment trigger value depends on β , δ , I , and T , $Q^*(\delta(\sigma), \beta(\sigma, \delta(\sigma)))$. Gryglewicz et al. (2008) reach important result of real option theory from Equation (14) as follows: Q^* is higher than the level of revenue flow that would make investment decision under the net present value (NPV) rule.

Investment occurs as soon as the risk adjusted project value exceeds the investment cost, $Q_{npv}^* = \frac{r + \lambda\rho\sigma - \mu}{1 - e^{-(r+\lambda\rho\sigma-\mu)T}} I$ under NPV case. This value is always lower than Q^* in Equation (14), since $\beta > 1$. They conclude that “*there are states where the expected payoff of investment is positive and the firm chooses to wait and not to invest. The option to invest captures this value of waiting*”. The difference between NPV and real option trigger value is driven by β , as it is the critical parameter to determine value of option.

Equation (4) and (13) and the condition related to them ($\delta > 0$, ($\beta > 1$)) provide us opportunity to check whether the parameter chosen by Gryglewicz et al. (2008) satisfy these conditions or not. Equation (14) helps us to analyze impact of interest rate on investment trigger value. Furthermore, we implement the same procedure for the example that we study on power plant investments in the Turkish electricity sector.

3. Economic Analysis of the non-Monotonicity Result

This section is related to economic analysis of non-monotonicity result and examining the consistency of the parameters and presenting results of our simulations. We begin with presenting an economic interpretation of the non-monotonic effect of uncertainty on investment in Gryglewicz et al. (2008).⁷ Rewriting the optimal investment trigger obtained in the previous section as follows:

$$Q^* = \frac{\beta}{(\beta - 1)} \frac{r + \lambda\rho\sigma - \mu}{1 - e^{-(r+\lambda\rho\sigma-\mu)T}} I.$$

⁷ In Gryglewicz et al. (2008), the proposition related to non-monotonic result states that if project life is finite and ($\lambda\rho > 0$), effect of uncertainty on investment trigger is non-monotonic: it decreases in σ for low level of σ and then increases. The length of the σ interval where the negative effect occurs, is negatively related to the project life.

At this point, it is a good starting point to trace all the variables that are influenced by uncertainty and consider the trigger value as a function of two parameters: $Q^*(\delta(\sigma), \beta(\sigma, \delta))$. Then the derivative of the investment trigger with respect to σ can have three effects in the following way:

$$\frac{d}{d\sigma} Q^*(\delta(\sigma), \beta(\sigma, \delta)) = \underbrace{\frac{\partial Q^*}{\partial \delta} \frac{\partial \delta}{\partial \sigma}}_{\text{Discounting Effect}} + \underbrace{\frac{\partial Q^*}{\partial \beta} \frac{\partial \beta}{\partial \sigma}}_{\text{Volatility Effect}} + \underbrace{\frac{\partial Q^*}{\partial \beta} \frac{\partial \beta}{\partial \delta} \frac{\partial \delta}{\partial \sigma}}_{\text{Convenience yield Effect}} \quad (15)$$

The three effects have a clear explanation and each has an unambiguous sign (for the case of $\lambda\rho > 0$). The first effect is associated with discount rate via the risk premium component. This channel works the same way as case of NPV. Increase in uncertainty raises the discount rate, which results in reducing the NPVs of the investment and thus raises the investment threshold. This means that it is less attractive or profitable to invest in this project, which ends up an increase of the trigger value. Therefore, it is concluded that the discounting effect is always positive.

Since the derivative of the trigger with respect to β has two effects due to fact β is a function of σ and δ from Equation (13). The first effect is called by Gryglewicz et al. (2008) as volatility effect and second one is called convenience yield effect. These two effects capture the impact of uncertainty on the value of the option to wait. Since β is directly important determinant of option value (equation 10), and δ is the key determinant of β , Equation (13), and thus δ has indirect effect on option value. Thus, these two effects combined as the option effect. The volatility effect is characterized by the second term in

⁸ Please see the result of total derivative á la Gryglewicz et al. (2008) in Appendix.

Equation (15). Increased uncertainty raises the upside potential payoff from the option, leaving the downside payoff unchanged at zero (since the option will not be exercised at low payoff values). This is the well-known positive impact of uncertainty on the option value with respect to Gryglewicz et al. (2008) and Dixit and Pindyck (1994). An increase option value means that the firm has more incentive to wait. This increases the opportunity cost of investing and consequently the investment trigger increases. Hence, the effect is clearly positive.

The last term in derivatives is related to the effect of uncertainty on the option value via the convenience yield. Higher uncertainty increases risk premium of expected rate of return and thus also convenience yield (from Equation 4). Increase in convenience yield creates less incentive to hold option and this leads to decrease option value. So, it is more attractive to invest earlier, which reduce investment trigger threshold. Therefore, last impact of uncertainty on trigger investment is negative. All in all, from above discussion on impact of uncertainty on investment, one can conclude that the convenience yield effect is negative, whereas the discounting and volatility effects are positive. It is also clear that, if the convenience effect dominates the two other effects, one can observe the positive relationship between uncertainty and investment. In other words, negative sign of derivative of Equation (15) is driven by domination of convenience yield effect over discounting and volatility effect. Last but not least, if total derivative sign is negative (positive), uncertainty has positive (negative) effect on investment.

In order to analyze the effects of uncertainty of the investment trigger, Gryglewicz et al. (2008) use the following the set of parameters: $\mu=0.08$, $r=0.1$, $\rho=0.7$, $\lambda=0.4$, $I=10$. The source of parameters that are used in their paper is from Sarkar (2000). Sarkar(2000)

chooses these values for the following reason: $\rho=0.7$ reflects a projects imperfectly (but positively) correlated with market, he states that this number assigns for the correlation is a description of the majority of the projects; and the market price of risk value ($\lambda=0.4$) is the approximate historical average (see Bodie et al., 1996, p.185). Risk-free interest rate, r , is chosen following Dixit and Pindyck (1994) and μ is chosen such that it guarantees the condition that $\delta > 0$. Before numerically show non-monotonic result, it is crucial to check whether ($\delta > 0$, $\beta > 1$, $\lambda\rho > 0$, and $\mu > 0$). We can calculate δ and β as follows:

$$\delta = r + \lambda\rho_{PM}\sigma - \mu \quad \text{and} \quad \beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}\right]}.$$

Table 1: Consistency of the model parameters

σ	r	λ	ρ	μ	π	$\delta > 0$	$\beta > 1$
0.0001	0.1	0.4	0.7	0.08	0.100	0.020	1.250
0.02	0.1	0.4	0.7	0.08	0.106	0.026	1.343
0.04	0.1	0.4	0.7	0.08	0.111	0.031	1.446
0.06	0.1	0.4	0.7	0.08	0.117	0.037	1.558
0.08	0.1	0.4	0.7	0.08	0.122	0.042	1.673
0.10	0.1	0.4	0.7	0.08	0.128	0.048	1.788
0.12	0.1	0.4	0.7	0.08	0.134	0.054	1.893
0.14	0.1	0.4	0.7	0.08	0.139	0.059	1.983
0.16	0.1	0.4	0.7	0.08	0.145	0.065	2.054
0.18	0.1	0.4	0.7	0.08	0.150	0.070	2.105
0.20	0.1	0.4	0.7	0.08	0.156	0.076	2.138
0.22	0.1	0.4	0.7	0.08	0.162	0.082	2.156
0.24	0.1	0.4	0.7	0.08	0.167	0.087	2.162
0.26	0.1	0.4	0.7	0.08	0.173	0.093	2.158
0.28	0.1	0.4	0.7	0.08	0.178	0.098	2.147
0.30	0.1	0.4	0.7	0.08	0.184	0.104	2.131
0.32	0.1	0.4	0.7	0.08	0.190	0.110	2.112

We verify that parameters used in Gryglewicz et al. (2008) satisfy their model assumptions (Table 1). Now, we can go one further step to analyze non-monotonic result. In order to do that we calculate investment trigger for different project life ($T=5, T=10, T=20, T=30,$ and $T=1000$ (for infinite case)) using model parameter. We get optimal investment trigger as follow:

$$Q^* = \frac{\beta}{(\beta - 1)} \frac{r + \lambda \rho \sigma - \mu}{1 - e^{-(r + \lambda \rho \sigma - \mu)T}} I.$$

We plug the parameters into optimal investment trigger equation and construct the Table A1 in Appendix and Figure 1.

Figure 1: The impact of uncertainty on investment
($\mu = 0.08, r = 0.1, \rho = 0.7, \lambda = 0.4, I = 10, T = 5$)

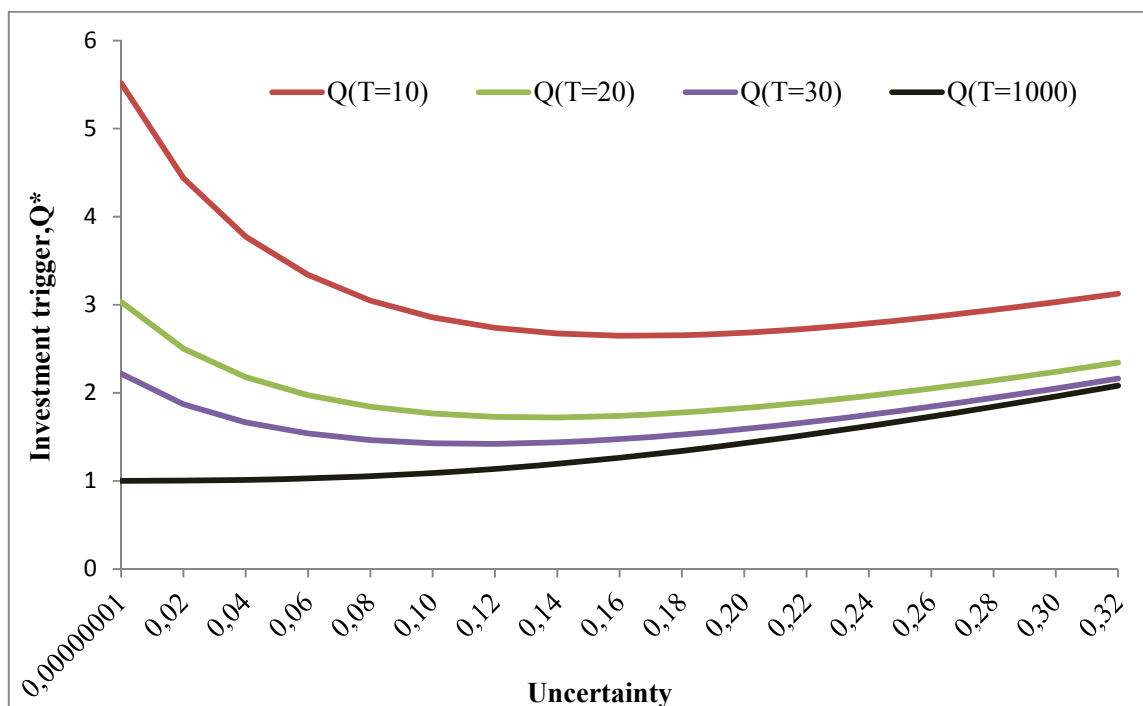


Figure 1 shows that as volatility (σ) increases, optimal investment trigger value (Q^*) decreases for low level of uncertainty. This confirms non-monotonic result of Gryglewicz et al. (2008): “there is a negative relationship between σ and Q^* for low level of uncertainty”. When project life is short, this negativity is more pronounced. For a 30 year project Q^* decreases until σ is around 0.12. The shorter the project life, at the higher the volatility level, investment can accelerate for economically relevant parameter value. When the project life time is 1000 (as infinite case), we can observe negative relation between uncertainty and investment at all level. This numerical example supports the non-monotonicity result.

We make another simulation aiming at compare the Sarkar (2000) arguments with Gryglewicz et al. (2008) theoretical results. According to Sarkar (2000) assumptions, the uncertainty–investment relationship is more likely to be positive when (i) the current level of uncertainty σ is low, (ii) λ is high, (iii) ρ is high, (iv) r is high, (v) μ is low, and (vi) T is short., we choose the following parameters; $\mu = 0.06$, $r = 0.15$, $\rho = 0.85$, $\lambda = 0.7$ taking into account Sarkar (2000) assumption and consistency of parameters ($\delta > 0$, $\beta > 1$, $\lambda\rho > 0$ and $\mu > 0$). We construct Table 2.

Table 2 presents some numerical examples, where parameter values correspond to Sarkar (2000) arguments related to investment-uncertainty relationship. First, we observe that chosen parameters are consistent and satisfy the main assumption of Gryglewicz et al. (2008). Second, there is a negative relation between uncertainty and optimal investment trigger for lower values of σ (up to $\sigma=0,10$). The sign of total derivative of optimal

investment trigger with respect to uncertainty is negative up to point where σ is 20. This implies that investment accelerates up to $\sigma=0,20$, then decelerates. Therefore, we show numerically that Sarkar (2000) argument supports Gryglewicz et al. (2008) theoretical results.

Table 2: The impact of uncertainty on investment
($\mu=0.06, r=0.15, \rho=0.85, \lambda=0.7, I=10$)

Vol	r	λ	ρ	$\mu > 0$	π	$\delta > 0$	$\beta > 1$	T	Q*	TD
0.01	0.15	0.70	0.85	0.06	0.15	0.09	2.50	5	4.14	-43.27
0.02	0.15	0.70	0.85	0.06	0.16	0.10	3.09	5	3.77	-32.06
0.04	0.15	0.70	0.85	0.06	0.17	0.11	3.89	5	3.53	-22.98
0.06	0.15	0.70	0.85	0.06	0.19	0.13	4.81	5	3.40	-15.23
0.08	0.15	0.70	0.85	0.06	0.20	0.14	5.56	5	3.37	-9.50
0.10	0.15	0.70	0.85	0.06	0.21	0.15	5.95	5	3.41	-5.95
0.12	0.15	0.70	0.85	0.06	0.22	0.16	6.04	5	3.49	-3.77
0.14	0.15	0.70	0.85	0.06	0.23	0.17	5.95	5	3.59	-2.32
0.16	0.15	0.70	0.85	0.06	0.25	0.19	5.78	5	3.71	-1.24
0.18	0.15	0.70	0.85	0.06	0.26	0.20	5.57	5	3.83	-0.38
0.20	0.15	0.70	0.85	0.06	0.27	0.21	5.35	5	3.96	0.35
0.22	0.15	0.70	0.85	0.06	0.28	0.22	5.14	5	4.10	0.99
0.24	0.15	0.70	0.85	0.06	0.29	0.23	4.93	5	4.25	1.57
0.26	0.15	0.70	0.85	0.06	0.30	0.24	4.74	5	4.39	2.10
0.28	0.15	0.70	0.85	0.06	0.32	0.26	4.56	5	4.55	2.59
0.30	0.15	0.70	0.85	0.06	0.33	0.27	4.39	5	4.71	3.06
0.32	0.15	0.70	0.85	0.06	0.34	0.28	4.24	5	4.87	3.50

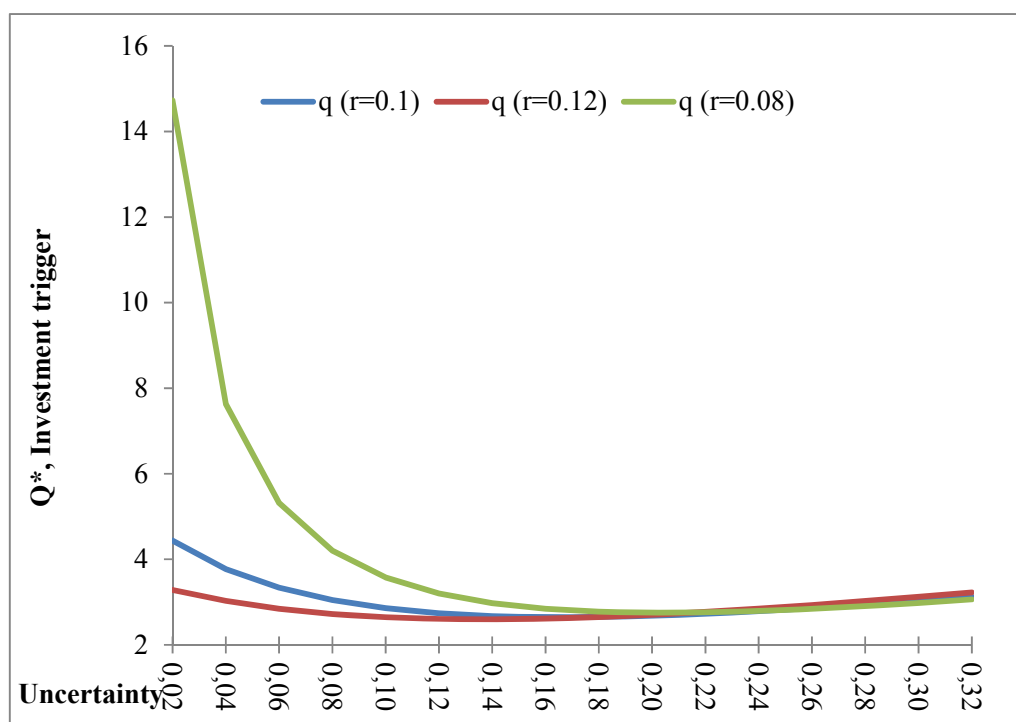
Note: TD value is the total derivatives of trigger investment with respect to volatility. TD<0 implies that uncertainty accelerates investment, TD>0 means that uncertainty decelerates investment

We change the level of interest rate without changing characteristic of investment project in order to understand partially the possible effects of changing interest rate on the uncertainty–investment relationship in Gryglewicz et al. (2008) model. We change the level of interest rate by minus and plus 200 basis points with respect to their model assumption without violating the model basic assumptions (see Table A.2 in Appendix). Changing interest rate affects risk adjusted expect return of the project and thus, has impact on

opportunity cost of investing project or option value. This helps us to see impact of expansionary and contractionary monetary policy on the trigger value of investment.

Figure 2 verifies that changing monetary stance has no impact the non-monotonic result, since interest rate is exogenous. For the case when interest rate is 0.12, investment trigger Q^* decreases until σ is 0.16, then Q^* starts to increase. When interest rate is relatively low, investment trigger Q^* decreases up to point σ is 0.20, and then Q^* starts to increase. Relatively low of interest rate, the positive relationship between uncertainty–investment is more profound. Lastly, lower the interest rate, higher the optimal investment trigger value and this implies low interest rate induces investment. This finding is in line with the literature. We can say that apart from investment characteristics, monetary policy also plays role in optimal investment trigger in Gryglewicz et al. (2008) model.

Figure 2: Impact of Interest rate on uncertainty–investment relationship
 $(\mu = 0.08, \rho = 0.7, \lambda = 0.4, I = 10, T = 5)$



4. Power Plant Investment in Turkey

The study of investment under uncertainty has been revitalized with the developments in the real options approach. The techniques and insights from option pricing are now been used in many areas such as flexible manufacturing, natural resource investments, land development, leasing, large-scale energy projects, research and development efforts, or foreign investments.⁹ In this section our paper, we focus on large-scale energy projects in Turkey to analyze the uncertainty-investment relationship.

One of the main issues in sustainable development is the use of renewable energy sources. In recent years, it has been observed that important steps taken in deregulation process in energy sector in Turkey. Bagdadioglu and Odyatmaz (2009) study Turkish electricity reforms, and point out that Turkish electricity reforms have slowly progressed because of the resistance against privatization. The Electric Market Law of 2001 was arranged in line with the *EU Energy Acquis*, since then reform gained momentum.

Çetin and Oğuz (2007) work on the politics of regulation in Turkish electricity market and emphasize the importance of institutional and political structure of the regulatory reforms. They also observe that the pace of reform aiming to liberalize electricity market is very slow due to the relationship between the government, judiciary and the independent regulator. However, recently, Madlener and Stoverink (2012) state that the Turkish electricity sector has been noticeably restructured. In this regard, the formerly vertically integrated companies have been unbundled in order to open it for private sector. In addition to this, there has been important attempt to privatize in electricity sector since last couple of years in Turkey for enhancing liberalizing in this market.

⁹ See the edited volume by Schwartz and Trigeorgis (2004, Section 6) for a variety of real options applications.

More liberalized energy market may lead to increased investment. It may also lead to a high level of competitiveness and the associated market uncertainty in this area. The real option theory is dealt with risk and uncertainty more properly than traditional approach, discounted cash flow. Therefore, implementation of the real option approach is not only widely used in electricity sector (see, for example, Venetsanos et al., 2002), but also in some other areas, such as R&D investments/programs (see, for example, Davis and Owens, 2003).

There are studies featuring the real options analysis with applications based on the Turkish electricity supply industry data, such as Madlener et al. (2005), Kumbaroğlu et al. (2008), and Madlener and Stoverink (2012). This section analyzes the impact of uncertainty on investment, based on the theoretical findings of Gryglewicz et al. (2008), in the Turkish electricity and using the parameters of Madlener and Stoverink (2012) for power plant investments.¹⁰ Madlener and Stoverink (2012) study the power plant investment in the Turkish electricity sector. The Turkish case is very relevant for an application of the theoretical views of this study, since Turkey's electricity generation is mainly based on thermal plants. Their share in total electricity production was 75% in 2006 (Bagdadioglu and Odyakmaz, 2009). Particularly, we focus on and examine uncertainty-investment relationship in electricity sector in Turkey.

To do that we implement non-monotonic result of Gryglewicz et al. (2008) by parameters used Madlener and Stoverink (2012). They study on the economic feasibility of a constructing coal-fired power plant investment in Turkish electricity sector, and life of this project is finite. So, the availability of data and short project life provide us a good

¹⁰ Cetin and Oguz (2007) and Bagdadioglu and Odyakmaz (2009) provide detailed background information on reforms in the Turkish electricity reform.

opportunity to test numerically uncertainty-investment relationship in Turkish electricity plant investment based on Gryglewicz et al. (2008).

We take the following parameter in Madlener and Stoverink (2012): the market risk premium of Turkey, $\lambda=0,04$, $\mu =0,14$, $T=5$, cost of project, approximately $I=10$ (million, USD, \$). Systematic risk coefficient ($\beta = \rho_{PM}\sigma_P^2/\sigma_M^2$) is 0.7, volatility of project (σ_P) is 0.25 and we calculate volatility of market portfolio (σ_M) is 0.25 using stock market of Turkey (the BIST100 index). Then, we obtain $\rho_{PM} = 0.77$. In order to calculate risk free interest: we take the average of central bank policy rate for last 10 years, we obtain, $r=0,152$. Turkish case study parameters are dramatically different than Sarkar (2000) and Gryglewicz et al. (2008).

The main source of creating this difference is high interest rate due to high inflation rate in Turkey. The market price of risk is so low because of mainly high interest rate. Lastly, the expected rate of return of project is relatively higher than both and Gryglewicz et al. (2008) and other sector in Turkey. Given these parameters, we construct Table 3 and check whether these parameters satisfy Gryglewicz et al. (2008) model assumption or not. It is clear that the parameters of the Turkish electricity sector satisfy Gryglewicz et al. (2008) assumptions. Second, there is a negative relationship between σ and Q^* up to point where $\sigma =0.26$, then relationship is positive (Figure 3). This evidence supports non-monotonic relationship between σ and Q^* . In other words, uncertainty has a U-shaped influence on the value of investment trigger threshold (Figure 3). It is worth to mention that our case study example based on non-monotonic result of Gryglewicz et al. (2008) present numerically that the uncertainty–investment relationship is more likely to be positive when (i) the current level of uncertainty σ is low, (ii) λ is low, (iii) ρ is high, (iv) r is high, (v) μ is

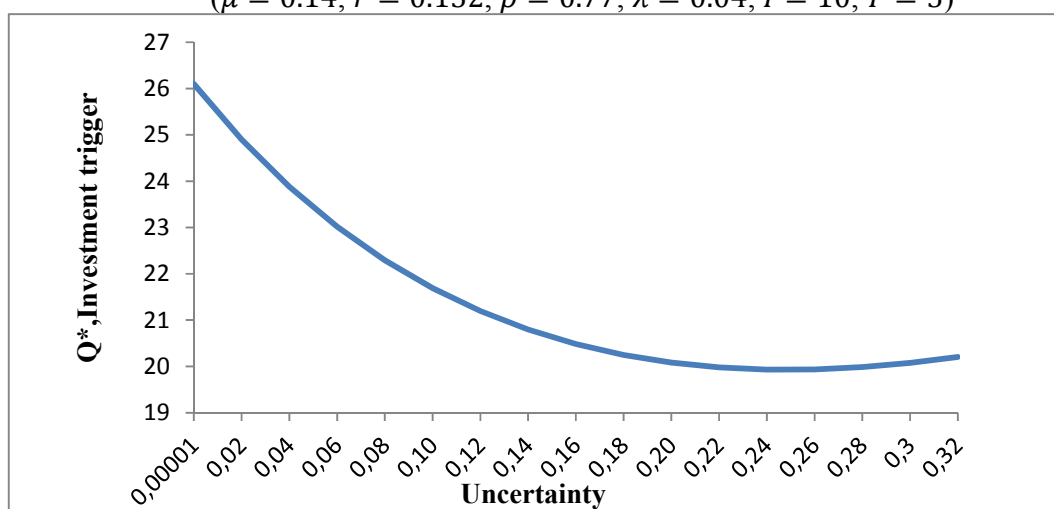
high, and (vi) T is short. Contrary to Sarkar (2000), we show that the uncertainty– investment relationship can be positive, if λ is low, μ is high in our case study.

Table 3: The impact of uncertainty on investment
 $(\mu = 0.14, r = 0.152, \rho = 0.77, \lambda = 0.04, I = 10, T = 5)$

Vol	r	λ	ρ	$\mu > 0$	π	$\delta > 0$	$\beta > 1$	T	Q*	TD
0.01	0.152	0.04	0.77	0.14	0.152	0.012	1.0857	5	26.10	-65.00
0.02	0.152	0.04	0.77	0.14	0.153	0.013	1.0904	5	24.90	-55.32
0.04	0.152	0.04	0.77	0.14	0.153	0.013	1.0948	5	23.88	-46.90
0.06	0.152	0.04	0.77	0.14	0.154	0.014	1.0988	5	23.02	-39.53
0.08	0.152	0.04	0.77	0.14	0.154	0.014	1.1025	5	22.29	-33.05
0.10	0.152	0.04	0.77	0.14	0.155	0.015	1.1059	5	21.69	-27.32
0.12	0.152	0.04	0.77	0.14	0.156	0.016	1.1088	5	21.20	-22.22
0.14	0.152	0.04	0.77	0.14	0.156	0.016	1.1113	5	20.80	-17.68
0.16	0.152	0.04	0.77	0.14	0.157	0.017	1.1134	5	20.49	-13.62
0.18	0.152	0.04	0.77	0.14	0.158	0.018	1.1150	5	20.25	-9.97
0.20	0.152	0.04	0.77	0.14	0.158	0.018	1.1163	5	20.08	-6.70
0.22	0.152	0.04	0.77	0.14	0.159	0.019	1.1172	5	19.98	-3.74
0.24	0.152	0.04	0.77	0.14	0.159	0.019	1.1177	5	19.93	-1.07
0.26	0.152	0.04	0.77	0.14	0.160	0.020	1.1178	5	19.94	1.35
0.28	0.152	0.04	0.77	0.14	0.161	0.021	1.1177	5	19.98	3.55
0.3	0.152	0.04	0.77	0.14	0.161	0.021	1.1173	5	20.08	5.56
0.32	0.152	0.04	0.77	0.14	0.162	0.022	1.117	5	20.21	7.38

Note: TD value is the total derivatives of trigger investment with respect to volatility. TD<0 implies that uncertainty accelerates investment, TD>0 means that uncertainty decelerates investment

Figure 3: Uncertainty investment relationship
 $(\mu = 0.14, r = 0.152, \rho = 0.77, \lambda = 0.04, I = 10, T = 5)$



5. Conclusion

Firms' investment decisions in response to uncertainty are one of the important issues for policy makers and academics for a long time. Although a large body of literature has investigated firms' investment decisions, we mention two basic approaches; net present value and "real options" approach. Net present value approach ignores the impact of uncertainty on investment, whilst the real options approach generally predicts the negative investment-uncertainty relationship. But, in the real options framework, Sarkar (2000) and Gryglewicz et al. (2008) show that uncertainty may accelerate investment under particular conditions.

In this paper, we compare two important papers in this field. We numerically show that certain conditions in Sarkar (2000) are associated with parameters support Gryglewicz et al. (2008) finding. We also examine the impact of interest rate on relationship between investment and uncertainty based on Gryglewicz et al. (2008) model. We show that lower the interest rate, higher the optimal investment trigger value and this implies low interest rate induces investment. This finding is in line with the literature. We can conclude that apart from investment characteristics, monetary policy also plays role in optimal investment trigger in Gryglewicz et al. (2008) model.

We also show a possible application of the theoretical view of this study, focusing on the power plant investment in an emerging market, Turkey. Our result indicates that the relationship between uncertainty and investment is non-monotonic and U-in the power plant investment in an emerging market, Turkey.

Last but not least, electricity policy in OECD countries over the past decade has been focused on the liberalization of electricity markets (see OECD/IEA, 2003 for a review

of major issues associated with power generation investment in liberalized electricity markets). Turkey began liberalizing its electricity production and retail segments in 2001. In what follows, the Turkish government has shifted the responsibility for financing investment in power generation away from the state-owned monopolies to the private sector. In that regard, we contribute to the uncertainty-investment relationship literature in the context of the Turkish electricity plant investment issues. There are many fruitful areas for future research, such as tax and regulatory issues and the possible effects of such arrangements on investment under uncertainty.

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Appendix

Derivatives of optimal investment trigger:

$$\frac{dQ^*}{d\sigma} = \frac{I \beta_1}{(\beta_1 - 1)^2} \frac{1}{\left(\beta_1 - \frac{1}{2}\right) \sigma^2 + \mu - \lambda \rho \sigma} \frac{1}{1 - e^{-(r + \lambda \rho \sigma - \mu)T}} (M - N \Delta),$$

$$M = (\beta_1 - 1) \left(\beta_1 + \frac{1}{2} \right) \lambda \rho \sigma^2 + (\beta_1 - 1)(r - \mu)\sigma + \beta_1(\mu - \lambda \rho \sigma)\lambda \rho - r \lambda \rho,$$

$$N = (\beta_1 - 1) \left(\beta_1 + \frac{1}{2} \right) \lambda \rho \sigma^2 + (\beta_1 - 1)(\mu - \lambda \rho \sigma)\lambda \rho,$$

$$\Delta = (r + \lambda \rho \sigma - \mu)T [e^{-(r + \lambda \rho \sigma - \mu)T} - 1]^{-1}.$$

Table A.1: The impact of uncertainty on investment ($\mu=0.08, r=0.1, \rho=0.7, \lambda=0.4, I=10$)

Volatility	T=10		T=20		T=30		T=1000	
	Q*	TD	Q*	TD	Q*	TD	Q*	TD
0.00000001	5.517	-69.767	3.033	-17.269	2.216	-7.549	1.000	0.000
0.02	4.439	-41.374	2.502	-9.784	1.870	-4.013	1.003	0.278
0.04	3.774	-26.543	2.179	-5.704	1.664	-1.984	1.012	0.622
0.06	3.339	-17.647	1.973	-3.089	1.538	-0.583	1.028	1.038
0.08	3.049	-11.753	1.843	-1.198	1.464	0.522	1.054	1.525
0.10	2.858	-7.571	1.765	0.279	1.428	1.465	1.089	2.065
0.12	2.739	-4.484	1.728	1.475	1.421	2.287	1.136	2.626
0.14	2.673	-2.168	1.721	2.446	1.438	2.998	1.194	3.172
0.16	2.648	-0.423	1.739	3.228	1.474	3.602	1.263	3.677
0.18	2.653	0.897	1.775	3.855	1.526	4.108	1.341	4.128
0.20	2.682	1.906	1.827	4.361	1.590	4.535	1.428	4.527
0.22	2.728	2.692	1.892	4.778	1.666	4.900	1.522	4.879
0.24	2.788	3.318	1.966	5.129	1.751	5.218	1.623	5.195
0.26	2.860	3.830	2.050	5.434	1.843	5.503	1.729	5.482
0.28	2.941	4.261	2.141	5.707	1.943	5.764	1.842	5.747
0.30	3.030	4.634	2.239	5.956	2.050	6.007	1.959	5.995
0.32	3.126	4.965	2.343	6.190	2.162	6.238	2.081	6.231

Note: TD value is the total derivatives of trigger investment with respect to volatility. TD<0 implies that uncertainty accelerates investment, TD>0 means that uncertainty decelerates investment.

Table A.2: The impact of uncertainty on investment when level of r changes
 ($\mu=0.06, \rho=0.85, \lambda=0.7, I=10, T=10$)

Volatility	Q*($r=0.12$)	TD	Q*($r=0.10$)	TD	Q*($r=0.08$)	TD
0.02	3.286	-15.013	4.439	-41.374	14.729	-711.923
0.04	3.029	-10.837	3.774	-26.543	7.638	-175.731
0.06	2.846	-7.596	3.339	-17.647	5.319	-75.955
0.08	2.721	-4.979	3.049	-11.753	4.203	-40.587
0.1	2.644	-2.834	2.858	-7.571	3.577	-23.833
0.12	2.605	-1.086	2.739	-4.484	3.202	-14.450
0.14	2.598	0.315	2.673	-2.168	2.976	-8.626
0.16	2.616	1.423	2.648	-0.423	2.844	-4.783
0.18	2.653	2.298	2.653	0.897	2.776	-2.150
0.2	2.707	2.994	2.682	1.906	2.753	-0.293
0.22	2.772	3.558	2.728	2.692	2.761	1.053
0.24	2.848	4.026	2.788	3.318	2.793	2.057
0.26	2.933	4.423	2.860	3.830	2.842	2.831
0.28	3.025	4.770	2.941	4.261	2.905	3.447
0.3	3.124	5.080	3.030	4.634	2.979	3.953
0.32	3.228	5.363	3.126	4.965	3.063	4.382

Note: TD value is the total derivatives of trigger investment with respect to volatility. TD<0 implies that uncertainty accelerates investment, TD>0 means that uncertainty decelerates investment

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