CENTRAL BANK OF THE REPUBLIC OF TURKEY

WORKING PAPER NO: 10/04

Optimal Monetary Policy under Sectoral Heterogeneity in Inflation Persistence

March 2010

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Optimal Monetary Policy under Sectoral Heterogeneity in Inflation Persistence *

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Abstract

This paper analyzes the relevance of sectoral inflation persistence differentials for optimal monetary policy using a two-sector sticky price model, which generalizes the standard models by introducing backward looking price setting into both sectors. The results show that even if the sectors have the same degree of inflation persistence, optimal inflation targeting policy attaches different weights to these unless they have exactly the same price setting mechanism. In particular, different combinations of price change frequency and backward looking price setting parameters can produce the same inflation persistence but have different implications for the optimal inflation targeting policy. However, the optimal inflation targeting rule attaches a higher weight to the inflation of the sector with a flatter Philips curve whether it is more persistent or not.

Keywords: Relative prices, Optimal monetary policy, Inflation persistence
JEL Codes: E31, E32, E52

*I thank Refet Gürkaynak for invaluable advice and encouragement, Giancarlo Corsetti, Pelin İlbaş, Tommaso Monacelli, and especially Kosuke Aoki, Selim Ali Elekdağ, Ali Hakan Kara, Eric Swanson and Andrea Tambalotti for helpful comments on an earlier version of this paper, entitled "The (Ir)relevance of Inflation Persistence for Inflation Targeting Policy Design", and Michel Juillard for the help with the use of Dynare. I also thank the continuous feedback provided by Harun Alp as well as participants in seminars at Bilkent University and Central Bank of the Republic of Turkey. Contact information: Research and Monetary Policy Department, Head Office, İstiklal Cad. 10 Ulus, 06100, Ankara, Turkey. E-mail: sevim.alp@tcmb.gov.tr. Usual disclaimer applies.

1. Introduction

Several recent empirical studies provide evidence that inflation persistence varies across sectors of the economy.[1] This evidence has important implications for monetary policy. Understanding sectoral responses to monetary policy shocks can be helpful in explaining the mechanism through which monetary policy affects the real economy.[2] Moreover, heterogeneity across sectors determines the way monetary policy should be designed since the hands of the policy makers are tied with a single policy instrument. Under an inflation targeting regime, the question is how different degrees of inflation persistence across sectors affects the optimal measure of inflation that the central bank should target.

One possible source of the difference in the estimated degree of inflation persistence is the difference in the degree of backward looking price setting across sectors. As this source of persistence results from the structure of the price setting mechanism itself, it is called endogenous, or intrinsic, inflation persistence, which is measured by the coefficient of backward looking component of the hybrid New Keynesian Phillips curve (NKPC). Leith and Malley (2005) and Massidda (2005) show that the degree of endogenous persistence is different across sectors and this differential arises from the difference not only in the frequency of price change, which is the case popularly analyzed in the literature, but also in the fraction of backward looking price setters across sectors.[3]

Consistent with these empirical findings, this paper analyzes the implications of endogenous inflation persistence differential across sectors for: (i) the design of the optimal measure of inflation to target and (ii) the social welfare. To this end, I introduce backward looking price setting as in Gali and Gertler (1999) in both sectors into an otherwise standard two sector sticky price model. This is important because I show that an equal degree of endogenous inflation persistence can be produced with different combinations of price change frequency and fraction of backward looking price setters. Thus, it is important to investigate whether a *higher* degree of endogenous persistence in one sector implies a weight *higher than* its expenditure share in the optimal inflation targeting rule when this inflation persistence being due to different structural features of the model.

First, I find that targeting the inflation of the sector that has higher endogenous persistence is not a parameter robust policy and for some parameter combinations it is optimal to target the inflation of the less persistent sector. This is particularly interesting because having estimated a two sector sticky price model with only one sector having backward looking price setters, Benigno and Lopez-Salido (2006) concludes that central bank should attach a higher weight to the inflation of the sector that *has endogenous*

[1] See, among many others, Aucramanne and Collin (2005), Altissimo et al. (2007), Bilke (2004), Lünnemann and Mathä (2004).

[2] See, for example, Carvalho (2006), Barsky et al. (2007), Nakamura and Steinsson (2008), Golosov and Lucas (2007) and Bouakez et al. (2008).

[3] See Huang and Liu (2005), Carvalho (2006), Erceg and Levin (2006), Bils and Klenow (2004), Carlstorm et al. (2006), Sheedy (2007), Wolman (2008), Bodenstein et al. (2008) and Bouakez et al. (2009)

persistence.[4] This finding is later interpreted by Levin and Moessner (2004) as the central bank should target the inflation of the sector that has *a higher degree of endogenous persistence*. Contrary to this, this paper shows that a monotonic relationship between endogenous inflation persistence and the weight in the optimal inflation targeting policy does not exist.

Second, I analyze the implications of the equal degree of endogenous inflation persistence across sectors. This analysis is done in order to understand whether homogeneity in degree of endogenous persistence can be a summary statistic and help the central bank avoid considering the underlying sectoral heterogeneities. If so, equal persistence across sectoral inflations would imply the optimality of the CPI targeting.[5] Results, however, show that CPI targeting is optimal only if the sectors have exactly the same price setting mechanisms. That is, inflation persistence is a reduced form parameter and for policy analysis its micro foundations are of crucial importance.

Another key finding that results from the analysis of this most general model in terms of price setting dynamics is that while endogenous inflation persistence is not monotonically related to optimal policy weights, the slope of the NKPC is. That is, the policy maker should attach a higher weight to the inflation of the sector that implies a higher short run trade off between sectoral inflation and sectoral output regardless of whether its inflation has endogenous persistence that is higher than that of the other sector or not. An analogue of this result exists in models with price stickiness and no backward looking price setting as in Aoki (2001) and Benigno (2004), where the slope of the NKPC is a function of price change frequency and therefore their analysis done by changing that frequency is equivalent to changing the slope of the NKPC. Here it is shown that, if anything, it is the slope of the NKPC that can serve as a guide for optimal policy weights irrespective of the sources that determine the slope, i.e. changes in the slope of the NKPC should be reflected in the optimal policy even when the price change frequency remain constant.

To compare the performance of inflation targeting rule relative to the optimal policy, I derive a central bank loss function as a second order approximation to social welfare. Three findings arise from the welfare analysis. First, optimal inflation targeting policy coincides with the optimal policy only if the sectors have exactly the same price setting mechanism. For other parameter combinations, optimal inflation targeting rule implies a welfare loss almost twice as high as that of optimal policy and exhibits significantly

[4] Benigno and Lopez-Salido (2006) analyzes the relevance of heterogeneity in endogenous persistence across countries in a monetary union rather than across sectors in a country. Note that, the analysis of the optimal monetary policy under a currency union with heterogeneous countries and in a single country with heterogeneous sectors is analogous (Woodford, 2003). The only difference is that what is called terms of trade in the two country model corresponds to a relative price in the two sector model.

[5] In an earlier version Benigno and Lopez-Salido (2002) propose an approximate nominal rigidity measure for the sector with persistent inflation and suggest that when this measure implies the same degree of nominal rigidity across sectors, optimal inflation targeting policy is targeting the CPI inflation. For feasible calibrations, Kösem-Alp (2009) shows that targeting CPI inflation on the basis of equivalence of this measure across sectors implies significant welfare loss.

different impulse responses.

Second, keeping the degree of heterogeneity constant, an increase in the level of endogenous inflation persistence in the economy as a whole implies a higher or a lower additional welfare loss depending on its source. As summarized by opposite changes in the NKPC slope, the two sources of higher endogenous persistence have opposite welfare implications. If the source of higher persistence is an increase in the fraction of backward looking price setters and therefore implies the flattening of the sectoral NKPCs, optimal inflation targeting policy produces a higher additional loss. However, additional welfare loss decreases when the source of higher persistence is an increase in the frequency of price change, which implies steeper sectoral NKPCs.

Third, as the magnitude of heterogeneity in endogenous persistence across sectors increases, the additional welfare loss implied by the optimal inflation targeting policy instead of the optimal policy increases. That is, optimal inflation targeting policy approximates the optimal policy relatively better when the sectors have the same degree of endogenous inflation persistence.

The rest of the paper is structured as follows. In Section 2, I bring the estimated measure of inflation persistence and endogenous inflation persistence together. Section 3 presents the model and the utility based welfare function that policymakers seek to maximize. The emphasis will be on how the existence of backward looking price-setters affects this welfare function. Section 4 shows the optimal inflation targeting rule under homogenous and heterogeneous degrees of inflation persistence across sectors. Section 5 displays the welfare implications of adopting optimal inflation targeting policy instead of the optimal policy and impulse response analysis. Section 6 concludes.

2. Measuring the Inflation Persistence

Aggregate or disaggregated inflation persistence is usually computed under univariate models by summing the autoregressive coefficients or looking at the largest root of the auto-regressive (AR) lag polynomial.[6] However, under this reduced form analysis the sources behind the inflation persistence are not clear. Angeloni et al. (2004) address this issue and distinguish the possible sources of inflation persistence by employing a hybrid NKPC of the following form:

$$\pi_t = \kappa_1(Y_t - Y_t^n) + \kappa_2\pi_{t-1} + \kappa_3 E_t\pi_{t+1} + u_t$$

where π_t is inflation and $(Y_t - Y_t^n)$ is output gap. In this context, κ_2 is called endogenous or intrinsic persistence and measures the dependence on past inflation due to the price

[6] Note that, when the auto-regressive process is of first order, both approaches produce the same degree of inflation persistence which is the serial correlation of inflation.

setting mechanism. Since the focus of this paper is the heterogeneity in inflation persistence arising from the existence of different price setting mechanisms across sectors, the measure of inflation persistence to be employed is κ_2.[7]

Walsh (2003) shows that in a model with endogenous persistence, under optimal commitment policy, it is possible to write a reduced form AR(1) process for inflation as:[8]

$$\pi_t = \rho_\pi \pi_{t-1} + e_t$$

where ρ_π is serial correlation in inflation. Levin and Moessner (2005) shows that the serial correlation in inflation increases as endogenous persistence increases. Following Levin and Moessner (2005), I compute the serial correlation in sectoral inflations as a reduced form measure of persistence in sectoral inflations both under the optimal commitment policy and the simple Taylor rule of the form:[9]

$$i_t = 1.5\pi_t + 0.5(Y_t - Y_t^n) + e_t$$

Panels of Figure 1 display the difference in serial correlations in inflation persistence across sectors as a function of the difference in degree of endogenous persistence. The figure shows that the sector with a higher degree of endogenous persistence also has a higher degree of reduced form inflation persistence and the difference in serial correlation of sectoral inflations increases as the difference in endogenous persistence increases. Moreover, this finding is robust not only to the choice of the monetary policy rule but also to alternative calibrations of the sectoral price setting mechanisms, which will be explained in detail in the following section. Therefore, within the set up of this paper, the word endogenous is redundant and I refer to the sector with a higher κ_2 as more persistent.

3. The Model

The model studied in this paper is a standard stochastic general equilibrium representative household model with two monopolistically competitive sectors. The sectors are assumed to be of the same expenditure share.[10] Both sectors are characterized by sluggish price adjustment and a fraction of producers in each sector are unsophisticated price setters, who adjust their prices to according to a rule of thumb. In this paper, I generalize the standard two sector models in the literature by introducing backward indexing

[7] In order to differentiate the intrinsic inflation persistence from extrinsic persistence, the sources of inflation persistence exogenous to the price setting, I calibrate the sectors with equally persistent supply shocks.

[8] Clarida et al. (1999) shows that an AR(1) representation of this form can also be obtained in a model with endogenous persistence, under optimal discretionary policy.

[9] The serial correlation in sectoral inflations under each policy are obtained numerically by solving the model under the corresponding policy.

[10] Alternative calibrations of the share in consumption or equivalently the expenditure share is also possible and do not change the results.

producers into both sectors.

3.1. Utility of a Representative Household

Each household consumes all of the differentiated goods in both sectors, and produces a single good. The objective of household j is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(\xi_t^d C_t^j) - v(\xi_{i,t}^s y_{i,t}^j) \right] \tag{1}$$

where $u(\cdot)$ represents the utility of consumption and $v(\cdot)$ represents the disutility of production. I make the usual assumptions that $u(\cdot)$ is increasing and concave, and that $v(\cdot)$ is increasing and convex. The constant $\beta \in (0,1)$ is the discount factor and the argument C_t^j, which represents a CES index of representative household purchases of the differentiated goods of both sectors, is defined as

$$C_t^j = \frac{1}{2} \left(C_{1,t}^j \right)^{1/2} \left(C_{2,t}^j \right)^{1/2} \tag{2}$$

where $C_{i,t}^j$ itself is a CES aggregate of sectoral goods.

$$C_{i,t}^j = \left[\int_0^1 c_{i,t}^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

Here $i \in \{1,2\}$ indexes sectors. The elasticity of substitution between any two differentiated goods in each sector, θ, is assumed to be greater than unity and uniform across sectors. The argument $y_{i,t}^j$ is the output of the good that representative household j in sector i produces. Following Aoki (2001), I assume that the preference shock ξ_t^d is identical across all households. I also assume that $\xi_{i,t}^s = \xi_{1,t}^s$ for all households producing one of the differentiated goods of the first sector and $\xi_{i,t}^s = \xi_{2,t}^s$ for all households producing one of the differentiated goods of the second sector, where ξ_t^d and $\xi_{i,t}^s$ are stationary random shocks. These assumptions imply that all of the producers in each sector face the same supply shocks and that there is no sector specific demand shock in this economy.

3.2. The Consumption Decision

The model assumes complete financial markets with no obstacles to borrowing against future income, so that each household faces a single intertemporal budget constraint. Therefore households can insure one another against idiosyncratic income risk. These assumptions imply that, if all households have identical initial wealth, they will choose identical consumption plans. The optimal allocation for a given level of nominal spending across all of the differentiated goods of both sectors at time t leads to the Dixit-Stiglitz demand relations as functions of relative prices. For the following, the index j is suppressed, since the consumption decision is identical across all households. The total expenditure required to obtain a given level of consumption index C_t is given by P_tC_t, where P_t is defined as

$$P_t = (P_{1,t})^{1/2}(P_{2,t})^{1/2} \tag{4}$$

Here $P_{i,t}$ is the price index of the sector i defined below. I assume that the share of each sectors' composite good comprises half of the total consumption. Demand for the sectoral composite differentiated goods of sector i are the usual Dixit-Stiglitz demand relations as functions of relative prices, which are given by

$$C_{i,t} = \frac{1}{2}\left(\frac{P_{i,t}}{P_t}\right)^{-1} C_t \tag{5}$$

where $P_{i,t}$ is the Dixit-Stiglitz price index defined as

$$P_{i,t} = \left[\int_0^1 p_{i,t}(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{6}$$

where $p_{i,t}(z)$ is the price of differentiated good in sector i indexed as z at time t. Demand for each differentiated good z, $c_{i,t}(z)$, is given by

$$c_{i,t}(z) = \frac{1}{2}\left(\frac{P_{i,t}}{P_t}\right)^{-1}\left(\frac{p_{i,t}(z)}{P_{i,t}}\right)^{-\theta} C_t \tag{7}$$

The optimal consumption plan of the household must satisfy

$$\frac{\xi_t^d u'(\xi_t^d C_t)}{P_t} = \Lambda_t \tag{8}$$

where Λt is marginal utility of nominal income, which follows the rule of motion

$$\Lambda_t(1 + \mathrm{R_t}) = \beta\Lambda_{t+1} \tag{9}$$

with R_t the risk-free nominal interest rate at time t.

3.3. The Production Decision

It is assumed, as is standard in this literature, that prices in both sticky-price sectors are changed at exogenous random intervals in the fashion of Calvo (1983). The producers in each sector can change their prices with a constant probability $1 - \alpha_i$. Following Gali and Gertler (1999), I assume that only a fraction $1-\psi_i$ of the households who can change their prices behave optimally when making their pricing decisions. The remaining producers, a fraction ψ_i, are backward looking as they instead use a simple backward-looking rule-of-thumb when setting their prices.

Given the complete markets and symmetric initial steady state assumptions, all forward-looking producers that are able to adjust their price at date t, will choose the same price,

$P^f_{i,t}$. I assume that all backward-looking producers who change their price at date t also set the same price, $P^b_{i,t}$.

The forward looking producer who is able to choose his price in period t chooses $P^f_{i,t}$ to maximize the discounted future profits

$$E_t \left\{ \sum_{k=0}^{\infty} \left\{ (\alpha_i \beta)^k [\Lambda_{t+k} P_{i,t} y_{i,t+k} - v(\xi^s_{t+k} y_{i,t+k} \right\} \right\} \tag{10}$$

First term is the expected revenue in utility terms. Since the cost of production is in terms of utility, the revenue is multiplied by the marginal utility of income. Maximizing the objective function with respect to $P^f_{i,t}$ gives the following first order condition:

$$E_t \left\{ \sum_{k=0}^{\infty} \left\{ (\alpha_i \beta)^k \Omega_{i,t+k} (P^f_{i,t} - \frac{\theta}{\theta - 1} S_{i,t+k} \right\} \right\} = 0 \tag{11}$$

where

$$\Omega_{i,t+k} \equiv \frac{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})}{\xi^d_t u'(\xi^d_t C_t)} c_{i,t+k}(z) \tag{12}$$

and

$$S_{i,t+k} = \frac{\xi^s_{i,t+k} v'(\xi^s_{i,t+k} y_{i,t+k})}{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})} P_{i,t+k} \tag{13}$$

is interpreted as the nominal marginal cost of sector i. Since the household is both worker and the owner of the firm in sector i, the cost of production is the disutility resulting from working.

As in Gali and Gertler (1999), I assume that the backward-looking firms set their prices according to the following rule:

$$P^b_{i,t} = P^*_{i,t-1} \pi_{i,t-1} \tag{14}$$

where $\pi_{i,t-1}$=$P_{i,t-1}/P_{i,t-2}$ and $P^*_{i,t-1}$is an index of prices set at *t-1*,given by

$$P^*_{i,t-1} = (P^f_{i,t-1})^{1-\psi_i} (P^b_{i,t-1})^{\psi_i} \tag{15}$$

According to equation (14) the backward looking firms adjust their prices to equal the geometric mean of the prices that they saw chosen in the previous period, $P^*_{i,t-1}$, adjusted for the sectoral inflation rate they observed in the previous period, $\pi_{i,t-1}$. That is, these firms use the inflation observed in the previous period a proxy for that of the current period. This way of price setting, while not optimal, keeps their relative prices same across periods when inflation is constant, for example at steady state.

The aggregate price level will then evolve according to

$$P_{i,t} = \left(\alpha_i P^{1-\theta}_{i,t-1} + (1 - \alpha_i)(1 - \psi_i)(P^f_{i,t})^{1-\theta} + (1 - \alpha_i)\psi_i (P^b_{i,t})^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{16}$$

Each period, a fraction α_i of the producers keeps charging the price of the previous period. The remaining $1-\alpha_i$ of the firms change their prices but only $1-\psi_i$ of them choose the optimal price and the remaining producers set their prices according to the rule of thumb.

Unsophisticated price setters are introduced into both sectors because standard New Keynesian models with purely forward looking price setting mechanisms fail to explain the hump shaped responses of sectoral inflations to demand and supply shocks. Introducing this type of backward looking behavior helps to alleviate this problem.[11] Moreover, it allows studying the relative importance of price stickiness versus backward lookingness in assigning the optimal weights for optimal inflation targeting policy.

3.4. Log-linearization of the Model

In this paper, the equations of the model, which is a general form of the model used by Benigno and Lopez-Salido (2006), are a quite complicated system of stochastic non-linear difference equations. I log-linearize the model around its steady state with zero inflation and study the dynamics of this approximate model.

The log-linearized Euler conditions (9) and (10) imply the following IS curve [12]

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(r_t - E_t\hat{\pi}_{t+1} - \hat{\xi}_t^d + E_t\hat{\xi}_{t+1}^d) \tag{17}$$

The NKPC of the sector$i$ is given by[13]

$$\hat{\pi}_{i,t} = \kappa_{i1}(\hat{Y}_{i,t} - \hat{Y}_{i,t}^n) + \kappa_{i2}\hat{\pi}_{i,t-1} + \kappa_{i3}E_t\hat{\pi}_{i,t+1} + \kappa_{i4}\hat{x}_{i,t} \tag{18}$$

where $x_{i,t} = P_{i,t}/P_t$ denotes the relative price of each sector and

$$\kappa_{i1} = \frac{(1-\alpha_i\beta)(\omega^{-1}+\sigma^{-1})(1-\alpha_i)(1-\psi_i)}{(1+\frac{\theta}{\omega})(\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta))}$$

$$\kappa_{i2} = \frac{\psi_i}{\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta)}$$

$$\kappa_{i3} = \frac{\alpha_i\beta}{\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta)}$$

$$\kappa_{i4} = -\frac{(1-\alpha_i\beta)(1+\omega^{-1})(1-\alpha_i)(1-\psi_i)}{(1+\frac{\theta}{\omega})(\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta))}$$

$$\hat{Y}_{i,t}^n \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}_{i,t}^s - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}_t^d$$

[11] Rumler (2004) and Jondeau and Le Bihan (2004) show that the degree of endogenous inflation persistence differs across the countries in the euro area. Therefore, the analysis done in this paper also has an international appeal.

[12] Variables with hats denote the log deviations from the steady state.

[13] See Appendix A for the derivation.

The parameter ω is the elasticity with respect to output of the disutility of supplying production, and σ is the elasticity the intertemporal elasticity of substitution.[14] $\hat{Y}_{i,t}^n$ represents a change in the natural rate of output in sector i, which is the level of supply that keeps the real marginal cost in sector i constant at the flexible price level.

As both sectors have backward looking price setting producers, the NKPC of each sector exhibit inflation persistence. The measure of inflation persistence, the coefficient of the lagged inflations in the NKPC of the sectors, is a function of two parameters of price setting: probability of not changing the price, α, and fraction of backward looking price setters, ψ. Thus, it is possible to produce the same degree of inflation persistence with different combinations of these parameters. Moreover, $\lim_{\alpha_i \to 1} \kappa_{i2} = \psi_i/(1+\psi_i\beta) < 1$, $\lim_{\psi_i \to 1} \kappa_{i2} = 1/(1+\alpha_i\beta) > 0$, $\lim_{\alpha_i \to 0} \kappa_{i2} = 1$ and $\lim_{\psi_i \to 0} \kappa_{i1} = 0$. Therefore, the degree of persistence in a sector is higher the higher the frequency of price change and the higher the fraction of backward indexing producers, as higher frequency of price change and higher fraction of backward indexing producers imply a higher total fraction of backward looking price setters given by $(1-\alpha)\psi$. Note that when κ_{12} is zero, the first sector does not display inflation persistence and the model reduces to that of Benigno and Lopez-Salido (2006).

The existence of backward looking price setters has implications also for the slope of the sectoral NKPCs. $\lim_{\alpha_i \to 1} \kappa_{i1} = 0$, $\lim_{\psi_i \to 1} \kappa_{i1} = 0$, $\lim_{\alpha_i \to 0} \kappa_{i1} = \psi_i/(1-\psi_i) > 0$ and $\lim_{\psi_i \to 0} \kappa_{i1} = (1-\alpha_i\beta)(1-\alpha_i)/\alpha_1 > 0$. Thus, as the frequency of price change decreases, α_i increases, and the fraction of backward indexing producers increases, the NKPC becomes flatter and sectoral inflation becomes less elastic to the changes in the output gap. In other words, sectoral inflation fails to change inline with the efficient fluctuations in the output gap.

3.5. The Loss Function

The central bank is concerned with maximizing the welfare of the households. Following Rotemberg and Woodford (1998, 1999) and Woodford (2003, ch. 6), the welfare measure is the expected utility of the households given by

$$W = E\left\{\sum_{t=0}^{\infty} \beta^t U_t\right\} \tag{19}$$

where

$$U_t = 2u(\xi_t^d Y_t/2) - \int_0^1 \upsilon(\xi_{1,t}^s y_{1,t}(z))dz - \int_0^1 \upsilon(\xi_{2,t}^s y_{2,t}(z))dz \tag{20}$$

Following Aoki (2001), I assume that only half the households produce a good that both households consume. Therefore, in equilibrium each household consumes the half of the

[14] Here $\sigma^{-1} = u''\xi^d C/u'$ and $\omega^{-1} = \upsilon''\xi_i^s Y_i/\upsilon'$ for $i = 1, 2$, evaluated at steady state. Following Aoki (2001), ω is assumed to be uniform across sectors.

goods it produces and half of the goods that households in the other sector produces. As in Aoki (2001) and Benigno and Lopez-Salido (2006), I assume that this steady state involves a lump-sum tax, which is set such that the steady state levels of output in both sectors are efficient.

A second order Taylor series approximation of equation (20) around the zero inflation steady state is[15]

$$U_t = -\frac{1}{2}u'\bar{Y}L_t \tag{21}$$

$$L_t = \lambda_1 \hat{\pi}_{1,t}^2 + \lambda_2 \hat{\pi}_{2,t}^2 + \lambda_3(\hat{Y}_t - \hat{Y}_t^n)^2 + \lambda_4((\hat{Y}_{1,t} - \hat{Y}_{2,t}) - \kappa(\hat{Y}_{1,t}^n - \hat{Y}_{2,t}^n))^2 + \lambda_5(\Delta\hat{\pi}_{1,t})^2 + \lambda_6(\Delta\hat{\pi}_{2,t})^2$$

where L_t is the loss function and the coefficients are given by

$$\lambda_1 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\alpha_1}{(1-\alpha_1)(1-\alpha_1\beta)}$$

$$\lambda_2 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\alpha_2}{(1-\alpha_2)(1-\alpha_2\beta)}$$

$$\lambda_3 = \sigma^{-1} + \omega^{-1}$$

$$\lambda_4 = \frac{1}{4}(1 + \omega^{-1})$$

$$\lambda_5 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\psi_1}{(1-\alpha_1)(1-\psi_1)(1-\alpha_1\beta)}$$

$$\lambda_6 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\psi_2}{(1-\alpha_2)(1-\psi_2)(1-\alpha_2\beta)}$$

The introduction of backward looking price setters makes the deviation of the current inflation from inflation of the previous period a concern of optimal policy, since the relative price of the backward looking price setters are distorted as much as this deviation. Moreover, as ψ increases the weight of the deviation of this period's inflation from that of the previous period, λ_5 or λ_6, increases. Therefore, for a constant level of price change frequency, as the fraction of backward indexing producers increases, the weights attributed to changes in inflation increases. Notice that, when $\psi_1 = 0$, the loss function simplifies to that of Benigno and Lopez-Salido (2006), where central bank takes into account inflations of both sectors and change in the inflation of the second sector only. It will be shown that allowing for backward looking price setting in both sectors implies additional insights.

Note also that when $\psi_1 = \psi_2 = 0$ the loss function obtained is that of Benigno (2004). Since there exists no backward indexing producers in the economy, deviation in inflation is not a concern of the central bank in itself. Moreover, since the only parameter governing the price setting dynamics in a sector is α, once it is equal across sectors, the weights of the sectoral inflations is equal in the loss function. This clearly implies attaching equal

[15] See Appendix B for the derivation.

weights to sectoral inflations in the optimal inflation targeting rule.

3.6. Optimal Inflation Targeting

As in Benigno (2004) and Benigno and Lopez-Salido (2006), the model is closed by introducing a strict inflation targeting rule, which has the following form[16]

$$\zeta\pi_{1,t} + (1-\zeta)\pi_{2,t} = 0 \tag{22}$$

where ζ is the weight that is attributed to the inflation of the first sector. The weight is chosen to maximize the welfare criterion (19) subject to the structural equations of the model ((17) and (18)). Once sectoral asymmetries are introduced, under the inflation targeting regime, the concern of the central bank becomes which inflation to target. Therefore, under the strict inflation targeting rule, the central bank defines the optimal basket, which is determined by optimally choosing the weights that should be attached to each sector.

In order to stabilize the optimal inflation measure, central bank sets the policy rate so that one percent inflation in the first sector is accompanied with $1\backslash\zeta - 1$ percent deflation in the second sector. Therefore, attaching a higher weight to the inflation of a sector implies that central bank allows for a higher rate of inflation or deflation in the other sector.

3.7. Model Calibration

The calibrations follow those of Benigno and Lopez-Salido (2006). The discount rate β is calibrated as 0.99. I set the parameter θ equal to 6, which corresponds to a steady-state mark-up of 1.2. The elasticity of substitution in consumption, σ, is 6 and inverse of elasticity of the disutility of producing the differentiated goods, ω, is 2, as in Steinsson (2003). This parameter is estimated to be 2.13 in Rotemberg and Woodford (1997) and calibrated in Benigno and Lopez-Salido (2006) as 1.67. The average duration of price, which is given by $1/(1-\alpha)$, is calibrated alternatively as 3, 4, 5 and 6 quarters. The fraction of rule of thumb price setters are assumed to be 0.01, 0.3, 0.5 and 0.8 as the fraction of backward indexing producers are estimated to be less than 0.8 for each sector in Massidda (2005) and Leith and Malley (2005). The sectors are assumed to be equal in expenditure share. The asymmetric supply shocks and the symmetric demand shock follow an AR(1) process of the kind:

$$\mathrm{X}_t = \rho \mathrm{X}_{t-1} + \varepsilon_t$$

where X_t is the vector of shock processes, X_t=($\hat{\xi}^s_{1,t}$, $\hat{\xi}^s_{2,t}$, $\hat{\xi}^d_t$), ρ is 0.9 and ε_t is the vector of independently identified disturbances. The shocks $\hat{\xi}^s_{1,t}$, $\hat{\xi}^s_{2,t}$ and $\hat{\xi}^d_t$ have standard deviations

[16] See Mankiw and Reis (2003) for a more general targeting rule.

of unity.

Benigno and Lopez-Salido (2006) calibrate the model so that the persistent sector has not only a lower frequency of price change than the other sector, but also has backward looking price setters. It is this assumption that implies a monotonic relation between the optimal weights and inflation persistence, as both lower frequency of price change and a higher degree of backward lookingness imply a flatter NKPC. Here I relax that assumption to consider all possible calibrations and allow a higher degree of inflation persistence to be resulting from different sources.

4. Inflation Persistence and Optimal Inflation Targeting

In this section I analyze the relevance of the inflation persistence for the optimal inflation targeting policy for the two cases: sectors have same degree of inflation persistence and one of the sectors is more persistent than the other sector.

4.1. Homogenous Degrees of Inflation Persistence across Sectors

I first calibrate the sectors with a uniform degree of inflation persistence and analyze the relevance of the same reduced form dynamics for the optimal inflation targeting policy. Therefore, the concern of this section is that whether the central bank can ignore the underlying heterogeneities across sectors when they imply same degree of inflation persistence and target the CPI inflation. Since inflation persistence is determined by the two parameters of the price setting mechanism, it is possible to produce same degree of inflation persistence with different combinations of probability of not changing the price, α, and fraction of backward looking price setters, ψ.

Table 1(a) displays the optimal weight attached to the inflation of the first sector. First column is the duration of prices in the second sector, which is given by $1/(1\text{-}\alpha)$, and second column is that of the first sector. For each calibration of the price setting parameters of the second sector and the frequency of the price setting in the first sector, the fraction of backward indexing producers in the first sector is computed consistent with calibrations of other parameters so that the degree of inflation persistence of the first sector is equal to that of the second sector. The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.[17] Note that, CPI targeting is optimal only if the sectors are characterized by exactly the same price setting mechanism. That is, optimal weight is equal to the expenditure share of each sector, 0.5, when they have the same frequency of price change and fraction of backward looking price setters. When the sectors have heterogenous price setting mechanisms, the optimal weight takes values between 0.20 and 0.79. This implies that even if both sectors have the same degree of inflation persistence,

[17] Negative values or values higher than 0.8 are excluded from the analysis.

the optimal inflation targeting rule is not CPI targeting.

Since the degree of inflation persistence is the same across sectors, a higher duration of price in one sector is accompanied with a higher fraction of backward looking price setters than the other sector. Therefore, as the duration of price increases in the first sector, the weight attached to its inflation increases.

4.2. Heterogenous Degrees of Inflation Persistence across Sectors

The previous section analyzed whether the CPI inflation targeting is the optimal inflation targeting rule under uniform calibration of the inflation persistence. It is shown that even if the sectors have uniform degree of inflation persistence, the differences in the price setting mechanism should be considered when designing the optimal inflation targeting measure. In this section the relevance of higher degree of inflation persistence in one sector for optimal monetary policy is analyzed.

First note that persistence in one sector can be made higher than that of the other sector by assuming a higher total fraction of the backward looking price setters in that sector, namely $(1-\alpha)\psi$. Clearly, this can happen in many ways for different combinations of α and ψ.

Benigno and Lopez-Salido (2006) show that for a constant fraction of backward indexing producers, the weight attached to the inflation of the sector increases with the increase in the duration of price. Similarly, keeping the duration of price constant, the weight of the inflation of the sector increases as the fraction of backward price setters increases. In this paper, the calibrations span all possible cases changing both parameters to find out which effect dominates.

For different feasible calibrations of the parameters of the price setting mechanisms of both sectors, the second sector is calibrated to be more persistent than the first sector and the weights in the optimal inflation targeting rule are computed. The first panel of Table 2 displays the optimal weights attached to the inflation of the first sector when the second sector is more persistent than the first sector by 0.1 points. Since the weight in the table is that of the less persistent sector, one would expect that it takes values less than 0.5. However, results show that for some calibrations the optimal weight is higher than the expenditure share of 0.5 and implies that central bank should pay higher attention to the inflation of the less persistent sector. The weight attached to the inflation of the first sector decreases as the fraction of backward looking price setters in the second sector increases.

The robustness of the result is checked by calibrating the inflation of the second sector to be 0.3 points higher than that of the first sector. The optimal weights are reported in the first panel of Table 3. Results show that for some of the parameter calibrations, although the first sector is significantly less persistent than the second sector, a higher weight is attributed to its inflation in the optimal inflation targeting rule.

The difference between the first panels of Table 2 and Table 3 is that the weights in Table 2 are smaller than the corresponding weights in Table 3. That is, for 3 quarters of duration of prices in both sectors and 0.3 of fraction of backward looking price setters in the second sector, the weight attached to the inflation of the first sector in Table 2 is 0.456, whereas the corresponding weight in Table 3 is 0.398. Therefore, as the gap between the degree of persistence of the sectoral inflations increases, the weight attached to the less persistent sector decreases.

Having shown that the optimal inflation targeting rule is not designed considering the degree of inflation persistence, the next section explores whether the reason behind this fact is the implications related to the slope of the NKPC.

4.3. Slope of the NKPC and Optimal Inflation Targeting

As mentioned before, models without inflation persistence suggest that the central bank should attribute a higher weight to the inflation of the sector with a higher duration of price. Although it is not emphasized by Aoki (2001) or Benigno (2004), this is equivalent to the suggestion that stabilization of the inflation of the sector that has a flatter NKPC should be more of a concern for monetary policy. In the model presented in this paper, the slope of the NKPC is not only a function of the frequency of price change but also the fraction of backward looking price setters. Therefore, this section addresses the question whether optimal inflation targeting policy is designed according to the relative slope of the sectoral NKPCs in a generalized model. The reason for the policy maker to consider the relative flatness is that a flatter NKPC implies a higher short run trade off between inflation and output and an equal level of inflation implies a wider output gap for the sector with a flatter NKPC.

Panels of Figure 2 display the optimal weights attached to the inflation of the first sector obtained in the previous sections as a function of the relative slope of the first sector when the differential in inflation persistence across sectors is 0, 0.1 and 0.3 points and fraction of backward looking producers is 0.5 in the second sector.[18] The first panel shows that for a given duration of prices in the second sector as the relative slope increases, the weight attached to the inflation of the first sector decreases. Moreover, when the NKPC of the first sector is flatter than that of the second sector, the optimal weight attached to the inflation of it is higher than 0.5. Similarly, when the NKPC of the first sector is steeper than that of the second sector, a lower weight is attached to its inflation.

The second and the third panels of Figure 2 presents that this result is robust across different calibrations of inflation persistence differential across sectors. Therefore, the weight attached to the inflation of the first sector is a function of the slope of the NKPC but not that of the inflation persistence per se. Even though the second sector is calibrated

[18] Relative slope is obtained by dividing the slope of the NKPC of the first sector to that of the second sector.

to be more persistent than the first sector, unless it has a flatter NKPC, stabilization of its inflation becomes less of concern for the central bank.[19] Therefore, the suggestion of Goodfriend and King (1997) and King and Wolman (1999), and the findings of Aoki (2001) and Benigno (2004) that central bank should pay higher attention to the inflation of the sector that implies higher real distortions carry over to a generalized framework.

5. Welfare Analysis

Once sectoral differences are not considered and the economy is modeled as a single sector, Rotemberg and Woodford (1998) show that inflation targeting policy coincides with optimal policy. Inflation targeting policy still approximates the optimal policy in a two sector environment as in Aoki (2001), Benigno (2004) and in a multi-sector environment as in Eusepi et al. (2009). However, when inflation persistence is introduced the optimal inflation targeting policy fails to approximate the optimal policy in single sector models.[20] Benigno and Lopez-Salido (2006) shows that optimal inflation targeting policy is outperformed by output gap targeting policy for some parameter calibrations. In the following subsections, I report the welfare cost of adopting inflation targeting policy instead of the optimal policy for the two cases: first, when the sectors have same degree of inflation persistence, and second when one of the sectors is more persistent than the other sector. Combining the results, I analyze the relevance of the *level* of inflation persistence in the economy and the *magnitude of the difference* in inflation persistence across sectors for welfare.

5.1. Welfare Analysis under Homogenous Degrees of Inflation Persistence across Sectors

For each calibration of the optimal inflation targeting rule when sectors have same degree of inflation persistence, Table 1(b) shows the welfare cost of inflation targeting policy instead of optimal policy. The welfare measure is the additional loss as a percentage of optimal loss which is given by the expression:

$$D_{IT} = \frac{E(W_{InflationTargeting}) - E(W_{Optimal})}{E(W_{Optimal})} \times 100 \tag{23}$$

where $W_{InflationTargeting}$ and $W_{Optimal}$ are the welfare loss in terms of steady state consumption under optimal inflation targeting rule and the optimal policy respectively. When both sectors have the same price setting mechanism, i.e. same duration of price, the optimal inflation targeting rule coincides with the optimal policy as is the case in the models

[19] For some calibrations that are not reported here, the optimal weight is not equal across sectors when they have NKPCs that have the same slope, which can be produced by different price setting mechanisms. Because of that the slope of the NKPC is not a sufficient summary statistic for the optimal inflation targeting policy design here, unlike in the case of models without inflation persistence.

[20] See Smets and Wouters (2003), Steinsson (2003) and Levin and Williams (2003) for the implications of inflation persistence in single sector models.

without persistence.

As far as heterogenous price setting mechanisms producing the same level of persistence are concerned, two findings emerge. First, when the fraction of backward indexing price setters in the second sector is 0.01 optimal inflation targeting policy approximates the optimal policy well. Therefore, inflation targeting is nearly optimal when sectoral inflations do not display significant inflation persistence. Second, for given durations of price across sectors, the welfare cost increases as the fraction of backward indexing producers in the second sector increases. Optimal inflation targeting policy implies an additional welfare loss as high as 7.4% when the fraction of backward indexation is 0.8 and duration of prices is 6 quarters in the second sector given that frequency of price change is 3 quarters in the first sector.

5.2. Welfare Analysis under Heterogenous Degrees of Inflation Persistence across Sectors

In this section, the welfare cost of optimal inflation targeting rule is considered relative to the optimal rule according to the welfare measure (23) for a wide set of feasible calibrations. Persistence of the first sector is calibrated to be 0.1 and 0.3 points less than that of the second sector, keeping this difference constant. Therefore, as persistence in the second sector increases that of first sector also increases. Thus, level of inflation persistence in the economy as a whole increases.

The second panel of the Table 2 displays the extra welfare loss resulting from optimal inflation targeting as a percentage of the optimal loss when the degree of inflation persistence in the second sector is 0.1 points higher than that of the first sector. Optimal inflation targeting rule approximates the optimal policy best when the duration of price and the fraction of backward looking price setters in the second sector are lowest. The percentage welfare loss increases as the average duration of prices and the fraction of backward looking price setters increases. That is, keeping the fraction of backward indexing producers in the second sector at 0.3, when duration of prices in the first sector is 3 quarters, the percentage loss increases from 2.27 to 3.59, 4.85 and 5.97 as the duration of prices in the second sector increases from 3 quarters to 4, 5 and 6 quarters. Similarly, keeping the average duration of prices at 3 quarters in both sectors, as the fraction of backward looking price setters in the second sector increases from 0.3 to 0.5 and 0.8, the percentage welfare loss increases from 2.27 to 4.99 and 20.39.

Note that, higher duration of prices and higher fraction of backward looking price setters have different implications for the level of the inflation persistence. Higher duration implies lower degree of inflation persistence, whereas higher fraction of backward looking price setters implies a higher degree of inflation persistence for both sectors. Therefore, it is not possible to draw conclusions regarding the relevance of the *level* of inflation persistence in the economy for the welfare loss resulting from optimal inflation targeting. Instead of the level of inflation persistence in the economy, its source is relevant for welfare.

Second panel of the Table 3 presents the welfare loss when second sector is 0.3 points more persistent than the first sector. Similar to the results in Table 2(b), optimal inflation targeting policy fails to approximate the optimal policy and the welfare loss does not increase as the level of persistence increases. The welfare loss increases as the fraction of backward indexing producers in the second sector increases and implies almost as high as twice the welfare loss under optimal rule when average duration of price change is 6 quarters in both sectors and $\psi_2 = 0.8$.

Another interesting finding is that, for a given calibration of frequency of price change in both sectors and fraction of backward indexing price setters in the second sector, as the difference in the degree of inflation persistence across sectors increases the welfare loss also increases. To illustrate, for $D_2 = 4$ and $\psi_2 = 0.8$ the degree of inflation persistence in the second sector is 0.52. Keeping $D_1 = 3$ and calibrating the first sector so that the persistence is equal to 0.52, 0.42 and 0.22 implies additional welfare losses of 2.82, 25.36 and 58.58 percent, respectively. Therefore, it appears that as the degree of inflation persistence differential across sectors increases, optimal inflation targeting rule implies higher welfare loss when compared to the optimal policy.

5.3. Impulse Responses to a Negative Supply Shock

To gain insights of these results, Figures 3 to 5 display the responses of the sectoral inflations and output gaps, which is the deviation of the actual output from its natural rate, to a negative supply shock to the second sector. Following a negative supply shock, inflation of the second sector increases while that of the first sector decreases. A common feature of all three figures is that optimal inflation targeting policy approximates the impulse responses of the sectoral inflations under optimal policy well. However, optimal inflation targeting policy imply impulse responses of sectoral output gaps that are different than those under optimal policy.

The implications of the magnitude of the differential in sectoral inflation persistence can be analyzed by comparing the responses in the three columns of the Figures 3 to 5. The first column of Figure 3 displays the impulse responses when both sectors have same the degree of inflation persistence and the second and third columns display the responses when the second sector is 0.1 points and 0.3 points more persistent than the first sector, respectively. In the first column, responses under optimal inflation targeting rule approximates that of the optimal policy well. However, as the difference between the persistence of sectoral inflations increases, the responses under optimal inflation targeting rule diverges from those of the optimal policy. Therefore, the welfare loss relative to the optimal loss increases as the degree of inflation persistence differential increases.

Similar to Figure 3, Figures 4 and 5 show that as the sectors diverge in terms of degree of inflation persistence, optimal inflation targeting policy fails to approximate the optimal policy and implies significant differences in terms of responses of sectoral inflations and output gaps.

As far as the implications of higher level of inflation persistence are concerned, the analysis can be done by comparing impulse response of each variable across figures, for a given degree of differential in inflation persistence across sectors. Inflation persistence in the second sector, which determines the level of persistence in overall economy, is 0.43, 0.4 and 0.52 in Figure 3, Figure 4 and Figure 5, respectively. The source of higher persistence in Figure 3 than that in Figure 4 is higher frequency of price change. Whereas, Figure 5 displays higher persistence than Figure 4 as it is generated by a higher fraction of backward looking price setters.

The impulse responses in all three columns of the Figure 3 provide slightly better approximations to the optimal responses than the ones in the less persistent economy, namely Figure 4. That is, although the level of persistence is lower in Figure 4, the gap between the optimal policy and the inflation targeting policy in terms of impulse responses is higher. Therefore, when the source of higher persistence in the economy is a higher frequency of price change, higher level of persistence implies a lower additional cost of adopting optimal inflation targeting policy.

However, in Figure 5 the difference between the impulse responses of each variable under the optimal policy and optimal inflation targeting policy is significantly higher than the one observed in Figure 4. Therefore, if the higher persistence is resulting from a higher fraction of backward looking price setters, as inflation persistence increases, the deviation of the inflation targeting policy from the optimal policy increases.

6. Conclusions

In this paper, I extend the standard two sector sticky price model in order to introduce inflation persistence into two sectors and analyze the relevance of the inflation persistence differential for the optimal inflation targeting policy design. I show that even if the sectoral inflations have the same degree of inflation persistence, optimal inflation targeting policy attaches different weights to them unless they are characterized by equal frequency of price change and fraction of backward looking price setters.

The main contribution of this paper is the finding that the optimal inflation targeting rule does not always attach a higher weight to the inflation of the more persistent sector. Rather, the weight attached to a sector is higher when the sector has a flatter NKPC than the other sector. That is, monetary authority should stabilize the inflation of the sector that implies higher real distortions.

The welfare implications of the optimal inflation targeting policy are analyzed using a loss function that is produced as a second order approximation to social welfare. The results show that in contrast to the models without inflation persistence, in this model, optimal inflation targeting policy fails to approximate the optimal policy. Moreover, keeping the degree of heterogeneity across sectors constant, I differentiate between the sources of higher inflation persistence in the overall economy and show that if the persistence is resulting from a higher fraction of backward looking price setters, the optimal inflation targeting rule implies higher additional welfare loss. However, if the source of higher inflation persistence is lower duration of prices, the extra welfare loss implied by optimal inflation targeting decreases as inflation persistence increases.

As far as the relevance of the magnitude of the heterogeneity in inflation persistence across sectors is concerned, as the difference between the degree of sectoral inflation persistence increases, the welfare loss implied by the optimal inflation targeting policy increases. Therefore, optimal inflation targeting policy may not be an appropriate policy when the inflation persistence differential across sectors is high.

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Appendix

This section presents the derivations of the New Keynesian Phillips curve and the central bank loss function.

A. Derivation of the New Keynesian Phillips Curve

Derivation done in this section mostly follows that of Steinsson (2003). Equation (11) can be written in terms of stationary variables as follows:

$$E_t \sum_{k=0}^{\infty} (\alpha_i \beta)^k \Omega'_{i,t+k} x^f_{i,t} = E_t \sum_{k=0}^{\infty} (\alpha_i \beta)^k \Omega'_{i,t+k} \frac{\theta}{\theta - 1} s_{i,t+k} \tag{A.1}$$

where $x^f_{i,t} = P^f_{i,t}/P_t$, $x_{i,t} = P_{i,t}/P_t$, $s_{i,t+k} = S_{i,t+k}/P_{t+k}$, $\pi_{i,t+s} = P_{t+s}/P_{t+s-1}$ and $\pi_{i,t+s} = P_{i,t+s}/P_{i,t+s-1}$

$$\Omega'_{i,t+k} = \frac{\Omega_{i,t+k}}{P_{t+k}} = \frac{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})}{\xi^d_t u'(\xi^d_t C_t)} x^{-1}_{i,t} (x^f_{i,t} \prod_{s=1}^{k} \pi^{-1}_{i,t+s})^{-\theta} C_{t+k} \tag{A.2}$$

Log-linearization of the equation (A.1) is

$$\sum_{k=0}^{\infty} (\alpha_i \beta)^k \hat{x}^f_{i,t} + \hat{x}_{i,t} - E_t \sum_{s=0}^{k} \pi_{t+1+s} = \sum_{k=0}^{\infty} (\alpha_i \beta)^k E_t \hat{s}_{t+k} \tag{A.3}$$

This can be written as

$$(\hat{x}^f_{i,t} + \hat{x}_{i,t}) \sum_{k=0}^{\infty} (\alpha_i \beta)^k = \sum_{k=0}^{\infty} (\alpha_i \beta)^k E_t (\hat{s}_{t+k} + \sum_{s=0}^{k} \pi_{t+1+s}) \tag{A.4}$$

This equation can be recursively written as follows

$$\hat{x}^f_{i,t} + \hat{x}_{i,t} = (\alpha_i \beta)(\hat{x}^f_{i,t} + \hat{x}_{i,t}) + (1 - \alpha_i \beta) \left\{ \hat{s}_t + \frac{1}{1 - \alpha_i \beta} \pi_{t+1} \right\} \tag{A.5}$$

$\hat{s}_{t+k}$ is obtained by loglinearizing the equation (13) in the text

$$\hat{s}_{t+k} = \hat{\xi}^s_{i,t+k} (1 + \frac{1}{\omega}) + \frac{1}{\omega} \hat{y}(z)_{i,t+k} - \hat{\xi}^d_{t+k} (1 - \frac{1}{\sigma}) + \frac{1}{\sigma} \hat{Y}_{t+k} \tag{A.6}$$

where loglinearizing the demand condition (7)

$$\hat{y}(z)_{i,t+k} = -\hat{x}_{i,t+k} - \theta \hat{x}^f_t + \theta \sum_{s=0}^{k-1} \pi_{i,t+1+s} + \hat{Y}_{t+k} \tag{A.7}$$

plugging this back to (A.7)

$$\begin{aligned} \hat{s}_{t+k} &= (\frac{1}{\omega} + \frac{1}{\sigma}) \hat{Y}_{t+k} + \hat{\xi}^s_{i,t+k} (1 + \frac{1}{\omega}) - \frac{1}{\omega} (\hat{x}_{i,t+k} - \theta \hat{x}^f_t + \theta \sum_{s=0}^{k} \pi_{t+1+s}) - \hat{\xi}^d_{t+k} (1 - \frac{1}{\sigma}) \quad \text{(A.8)} \\ &= (\frac{1}{\omega} + \frac{1}{\sigma}) (\hat{Y}_{t+k} - \hat{Y}^n_{i,t+k}) - \frac{1}{\omega} (\hat{x}_{i,t+k} - \theta \hat{x}^f_t + \theta \sum_{s=0}^{k} \pi_{t+1+s}) \end{aligned}$$

where

$$\hat{Y}^n_{i,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^s_{i,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^d_t$$

Next, loglinearization of price indices (14)-(16) and the demand condition (5) produce the following identities:

$$\hat{x}^b_{i,t} = \hat{x}^*_{t-1} - \pi_{i,t} + \pi_{i,t-1} \quad (A.9)$$

$$\hat{x}^*_{t-1} = (1-\psi_i)\hat{x}^f_t + \psi_i \hat{x}^b_t \quad (A.10)$$

$$\pi_t = \frac{\alpha}{1-\alpha}((1-\psi_i)\hat{x}^f_t + \psi\hat{x}^b_t)) \quad (A.11)$$

$$\hat{Y}_{i,t} = -\hat{x}_{i,t} + \hat{Y}_t \quad (A.12)$$

Combining the equations (A.9) - (A.12) produce the New Keynesian Phillips curve (18) in the text.

B. Derivation of the Loss Function

In this section I derive the utility based loss function for the model. The period t welfare of the economy is the average utility over the continuum of households is given by the expression:

$$W_t = 2U_t(\xi^d_t Y_t/2) - \int_0^1 v(\xi^s_{1,t} y_{1,t}(z))dz - \int_0^1 v(\xi^s_{2,t} y_{2,t}(z))dz \quad (B.1)$$

First, a second order Taylor series approximation is taken for the first term of the equation

$$\begin{aligned} U_t(\xi^d_t Y_t/2) = \; & U_{Y_1}(Y_{1,t}-\bar{Y_1}) + U_{Y_2}(Y_{2,t}-\bar{Y_2}) + \frac{1}{2}U_{Y_1Y_1}(Y_{1,t}-\bar{Y_1})^2 \\ & + \frac{1}{2}U_{Y_2Y_2}(Y_{2,t}-\bar{Y_2})^2 + U_{Y_1Y_2}(Y_{1,t}-\bar{Y_1})(Y_{2,t}-\bar{Y_2}) \\ & + U_{\xi^d Y_1}(\xi^d_t - \bar{\xi^d})(Y_{1,t}-\bar{Y_1}) + U_{\xi^d Y_2}(\xi^d_t - \bar{\xi^d})(Y_{2,t}-\bar{Y_2}) + t.i.p + O(3) \end{aligned} \quad (B.2)$$

where $U_{Y_i} \equiv \partial U/\partial Y_i$ and $U_{Y_iY_i} \equiv \partial^2 U/\partial {Y_i}^2$. $\bar{Y_1} and \bar{Y_2}$ are the optimal equilibrium levels of output of sectors and $\bar{\xi^d}$ is the steady state value of the identical demand shock. Using the fact that $Y_i = \bar{Y_{i,t}}(1+\hat{Y_{i,t}}+\frac{1}{2}\hat{Y_{i,t}}^2) + O(3)$, (B.2) can be written as:

$$\begin{aligned} U_t(\xi^d_t Y_t/2) = \; & U_{Y_1}\bar{Y_1}\hat{Y}_{1,t} + U_{Y_2}\bar{Y_2}\hat{Y}_{2,t} + \frac{1}{2}(U_{Y_1}\bar{Y_1} + U_{Y_1Y_1}\bar{Y_1}^2)\hat{Y}^2_{1,t} \\ & + \frac{1}{2}(U_{Y_2}\bar{Y_2} + U_{Y_2Y_2}\bar{Y_2}^2)\hat{Y}^2_{2,t} + U_{Y_1Y_2}\bar{Y_1}\bar{Y_2}\hat{Y}_{1,t}\hat{Y}_{2,t} \\ & + U_{\xi^d Y_1}\bar{\xi^d}\bar{Y_1}\hat{\xi}^d_t\hat{Y}_{1,t} + U_{\xi^d Y_2}\bar{\xi^d}\bar{Y_2}\hat{\xi^d}_t\hat{Y}_{2,t} + t.i.p + O(3) \end{aligned} \quad (B.3)$$

where $\hat{Y}_{i,t} \equiv \log(Y_{i,t}/\bar{Y})$. A second order approximation to the second and the third terms of (B.1) and similar manipulations give the following expression

$$\begin{aligned} v(\xi^s_{i,t} y_{i,t}(z)) = \; & v_{y_i}\bar{Y_i}\hat{y}_{i,t}(z) + \frac{1}{2}(v_{y_i}\bar{Y_i} + v_{y_iy_i}\bar{Y_i}^2)\hat{y}_{i,t}(z)^2 \\ & + v_{\xi^s_i y_i}\bar{\xi^s_i}\bar{Y_i}\hat{y}_{i,t}(z)\hat{\xi}^s_{i,t} + t.i.p + O(3) \end{aligned} \quad (B.4)$$

where $v_{y_i} \equiv \partial v/\partial y_i$, $v_{y_iy_i} \equiv \partial^2 v/\partial {y_i}^2$ and $\hat{y}_t(z) \equiv \log(y_t(z)/\bar{Y})$. $\bar{\xi^s_i}$ is the steady state value of sector specific supply shock. Integrating (B.4) over z

$$\int_0^1 \upsilon(\xi^s_{i,t} y_{i,t}(z))dz = \upsilon_{y_i}\bar{Y}_i E_z[\hat{y}_{i,t}(z)] + \frac{1}{2}(\upsilon_{y_i}\bar{Y}_i + \upsilon_{y_i y_i}\bar{Y}_i^2) var_z[\hat{y}_{i,t}(z)] + \frac{1}{2}(\upsilon_{y_i}\bar{Y}_i + \upsilon_{y_i y_i}\bar{Y}_i^2) E_z[\hat{y}_{i,t}(z)]^2 + \upsilon_{\xi^s y_i}\bar{\xi}^s \bar{Y}_i \hat{\xi}^s_{i,t} E_z[\hat{y}_{i,t}(z)] + t.i.p + O(3) \tag{B.5}$$

Here $E_z[\cdot]$ and $var_z[\cdot]$ represent the population average and variance of the outputs of the producers, respectively. A Taylor series expansion of the Dixit-Stiglitz aggregator (3) is given by

$$\hat{Y}_{i,t} = E_z[\hat{y}_{i,t}(z)] + \frac{1}{2}\frac{\theta-1}{\theta} var_z[\hat{y}_{i,t}(z)] + O(3) \tag{B.6}$$

Solving for $E_z[\cdot]$ and plugging it back to (B.5) yields

$$\int_0^1 \upsilon(\xi^s_{i,t} y_{i,t}(z))dz = \upsilon_{y_i}\bar{Y}_i \hat{Y}_{i,t} + \frac{1}{2}(\upsilon_{y_i}\bar{Y}_i + \upsilon_{y_i y_i}\bar{Y}_i^2)\hat{Y}^2_{i,t} + \frac{1}{2}(\theta^{-1}\upsilon_{y_i}\bar{Y}_i + \upsilon_{y_i y_i}\bar{Y}_i^2) var_z[\hat{y}_{i,t}(z)] + \upsilon_{\xi^s y_i}\bar{\xi}^s \bar{Y}_i \hat{\xi}^s_{i,t}\hat{Y}^2_{i,t} + t.i.p + O(3) \tag{B.7}$$

Substituting (B.5) and (B.7) into welfare equation (B.1) gives

$$\begin{aligned} W_t = {} & \frac{1}{2}(2U_{Y_1Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1)\bar{Y}_1^2\hat{Y}^2_{1,t} + \frac{1}{2}(2U_{Y_2Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2)\bar{Y}_2^2\hat{Y}^2_{2,t} \\ & + (2U_{\xi^d Y_1}\bar{\xi}^d\bar{Y}_1\hat{\xi}^d_t - \upsilon_{\xi^s_1 y_1}\bar{\xi}^s_1\bar{Y}_1\hat{\xi}^s_{1,t})\hat{Y}_{1,t} \\ & + (2U_{\xi^d Y_2}\bar{\xi}^d\bar{Y}_2\hat{\xi}^d_t - \upsilon_{\xi^s_2 y_2}\bar{\xi}^s_2\bar{Y}_2\hat{\xi}^s_{2,t})\hat{Y}_{2,t} \\ & + 2U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{2}(2\theta^{-1}U_{Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1^2) var_z[\hat{y}_{1,t}(z)] \\ & - \frac{1}{2}(2\theta^{-1}U_{Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2^2) var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3). \end{aligned} \tag{B.8}$$

Notice that $2U_{Y_i} = \upsilon_{y_i}$ since the approximation is around efficient steady state and linear terms cancel out.
In order to write (B.8) in terms of natural rates, the following relationships are

$$\begin{aligned} \upsilon_{\xi^s_i y_i} &= \upsilon'(\frac{1}{\omega} + 1) \\ U_{\xi^d Y_i} &= \frac{1}{4}(-\frac{1}{\sigma} + 1)U'\frac{\bar{Y}}{\bar{Y}_i} \end{aligned}$$

where $\sigma^{-1} = -u''\xi^d C/u'$ and $\omega^{-1} = v''\xi^s_i Y_i/v'$. Therefore, the third and the fourth terms in (B.8) can be written as:

$$(2U_{\xi^d Y_1}\bar{\xi}^d\bar{Y}_1\hat{\xi}^d_t - \upsilon_{\xi^s_1 y_1}\bar{\xi}^s_1\bar{Y}_1\hat{\xi}^s_{1,t})\hat{Y}_{1,t} = (\frac{1}{\omega} + \frac{1}{\sigma})2U_{Y_i}\bar{Y}_i\hat{Y}^n_{i,t}\hat{Y}_{i,t} \tag{B.9}$$

where

$$\hat{Y}^n_{i,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^s_{i,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^d_t$$

is the natural rate of output for $i = 1, 2$. Substituting (B.11) back to (B.8) gives:

$$W_t = \frac{1}{2}(2U_{Y_1Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1)\bar{Y}_1^2\hat{Y}_{1,t}^2 + \frac{1}{2}(2U_{Y_2Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2)\bar{Y}_2^2\hat{Y}_{2,t}^2 \quad \text{(B.10)}$$
$$+2(\frac{1}{\omega} + \frac{1}{\sigma})U_{Y_1}\bar{Y}_1\hat{Y}_{1,t}^n\hat{Y}_{1,t} + 2(\frac{1}{\omega} + \frac{1}{\sigma})U_{Y_2}\bar{Y}_2\hat{Y}_{2,t}^n\hat{Y}_{2,t}$$
$$+2U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{2}(2\theta^{-1}U_{Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1^2)var_z[\hat{y}_{1,t}(z)]$$
$$-\frac{1}{2}(2\theta^{-1}U_{Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2^2)var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3).$$

The variance of the sectoral outputs can be written as variance of the prices of the differentiated of products in each sector by utilizing the demand condition (7) as follows:

$$var_z[\hat{y}_{i,t}(z)] = \theta^2 var_z[logp_{i,t}(z)] + O(3) \quad \text{(B.11)}$$

By the definition of variance

$$var_z[logp_{i,t}(z)] = E_z[logp_{i,t}(z)^2] - (E_z[logp_{i,t}(z)])^2$$

Since $E_z[logp_{i,t-1}(z)]$ is a constant in terms of z this can also be written as follows, which gives the price dispersion in each sector:

$$var_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]] = E_z[(logp_{i,t}(z) - E_z[logp_{i,t-1}])^2] \quad \text{(B.12)}$$
$$-(E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]])^2$$

Using the evolution of the aggregate price level of each sector, the square root of second term in (B.12) can be written as:

$$E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]] = \alpha_i E_z[logp_{i,t-1} - E_z[logp_{i,t-1}]] \quad \text{(B.13)}$$
$$+(1-\alpha_i)(1-\psi_i)(logp_{i,t}^f - E_z[logp_{i,t-1}])$$
$$+(1-\alpha_i)\psi_i(logp_{i,t}^b - E_z[logp_{i,t-1}])$$
$$= (1-\alpha_i)(logp_{i,t}^* - E_z[logp_{i,t-1}])$$

The first term in equation (B.12) can be rewritten as:

$$E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]]^2 = \alpha_i E_z[logp_{i,t-1}(z) - E_z[logp_{i,t-1}]]^2 + \quad \text{(B.14)}$$
$$+(1-\alpha_i)(1-\psi_i)(logp_{i,t}^f - E_z[logp_{i,t-1}])^2$$
$$+(1-\alpha_i)\psi_i(logp_{i,t}^b - E_z[logp_{i,t-1}])^2$$

Using

$$p_{i,t}^b = p_{i,t-1}^*\pi_{i,t-1}$$
$$p_{i,t-1}^* = (p_{i,t-1}^f)^{1-\psi_i}(p_{i,t-1}^b)^{\psi_i}$$
$$E_z[logp_{i,t}(z)] = logP_{i,t} + O(2)$$

The second and the third terms in (B.14) can be further expressed as

$$\begin{aligned} logp^b_{i,t} - E_z[logp_{i,t-1}] &= logp^*_{i,t-1} + \pi_{i,t-1} + logP_{i,t-1} + O(2) \\ &= logp^*_{i,t-1} + logP_{i,t-2} + O(2) \end{aligned} \tag{B.15}$$

$$\begin{aligned} logp^f_{i,t} - E_z[logp_{i,t-1}] &= \frac{1}{(1-\psi_i)}(logp^*_{i,t} - \psi_i(logp^*_{i,t-1} + \pi_{i,t-1})) - logP_{i,t-1} + O(2) \\ &= \frac{1}{(1-\psi_i)}(logp^*_{i,t} - logP_{i,t-1} + \psi_i(p^*_{i,t-1} + logP_{i,t-2})) + O(2) \end{aligned} \tag{B.16}$$

Finally by plugging (B.15) and (B.16) back to (B.14)

$$\begin{aligned} var_z[logp_{i,t}(z)] &= \alpha_i var_z[logp_{i,t-1}(z)] + \frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} \\ &+ \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t} + O(3) \end{aligned} \tag{B.17}$$

Solving this equation forward, starting with an initial variance of the prices, $var_z[logp_{i,-1}(z)]$, which is predetermined and independent of policy at time t

$$\begin{aligned} var_z[logp_{i,t}(z)] &= \sum_{s=0}^{t} \alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t}\right) \\ &+ \alpha_i^t var_z[logp_{i,-1}(z)] + O(3) \\ &= \sum_{s=0}^{t} \alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t}\right) + t.i.p. + O(3) \end{aligned} \tag{B.18}$$

The equation (B.10) can be further developed using the following relations

$$\begin{aligned} U_{Y_1}\bar{Y}_1 &= U_{Y_2}\bar{Y}_2 = \frac{1}{4}U'\bar{Y} \\ U_{Y_1Y_1}\bar{Y}_1^2 &= U_{Y_2Y_2}\bar{Y}_2^2 = -\frac{1}{4}U'\bar{Y}[\frac{1}{2\sigma} + \frac{1}{2}] \\ U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2 &= \frac{1}{8}U'\bar{Y}(1 - \frac{1}{\sigma}) \\ \upsilon_{y_1y_1}\bar{Y}_1^2 &= \upsilon_{y_2y_2}\bar{Y}_2^2 = \frac{1}{2\omega}U'\bar{Y} \end{aligned}$$

Substituting these identities back to (B.10) and using the efficient steady state condition $2U_{Y_i} = \upsilon_{y_i}$ yields

$$\begin{aligned} W_t = & -\frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma} + \frac{1}{2} + \frac{1}{\omega})\hat{Y}^2_{1,t} - \frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma} + \frac{1}{2} + \frac{1}{\omega})\hat{Y}^2_{2,t} \\ & + \frac{1}{2}(\frac{1}{\omega} + \frac{1}{\sigma})U'\bar{Y}\hat{Y}^n_{1,t}\hat{Y}_{1,t} + \frac{1}{2}(\frac{1}{\omega} + \frac{1}{\sigma})U'\bar{Y}\hat{Y}^n_{2,t}\hat{Y}_{2,t} \\ & + \frac{1}{4}(-\frac{1}{\sigma} + 1)U'\bar{Y}\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{4}(\frac{1}{\theta} + \frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{1,t}(z)] \\ & - \frac{1}{4}(\frac{1}{\theta} + \frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3). \end{aligned} \tag{B.19}$$

Adding and subtracting $\frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma}+\frac{1}{2}+\frac{1}{\omega})\hat{Y}_{1,t}\hat{Y}_{2,t}$ and arranging gives

$$\begin{aligned} W_t &= -\frac{1}{2}U'\bar{Y}\Phi_t - \frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{1,t}(z)] \\ &\quad -\frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3) \end{aligned} \tag{B.20}$$

where

$$\begin{aligned} \Phi_t &= (\frac{1}{\sigma}+\frac{1}{\omega})(\frac{1}{4}\hat{Y}_{1,t}^2 + \frac{1}{4}\hat{Y}_{2,t}^2 + \frac{1}{2}\hat{Y}_{1,t}\hat{Y}_{2,t}) \\ &\quad -2(\frac{1}{\sigma}+\frac{1}{\omega})(\frac{1}{2}\hat{Y}_{1,t}^n\hat{Y}_{1,t} + \frac{1}{2}\hat{Y}_{2,t}^n\hat{Y}_{2,t}) \\ &\quad +\frac{1}{4}(1+\frac{1}{\omega})(\hat{Y}_{1,t}^2 + \hat{Y}_{2,t}^2 - 2\hat{Y}_{1,t}\hat{Y}_{2,t}) \end{aligned} \tag{B.21}$$

In order to rewrite Φ_t the following identities are used:

$$\begin{aligned} \hat{Y}_t &= \frac{1}{2}\hat{Y}_{1,t} + \frac{1}{2}\hat{Y}_{2,t} \\ \hat{Y}_t^n &= \frac{1}{2}\hat{Y}_{1,t}^n + \frac{1}{2}\hat{Y}_{2,t}^n \end{aligned}$$

Substitution of these together with adding and subtracting $\hat{Y}_t^n$ and (B.19) becomes

$$\begin{aligned} W_t &= -\frac{1}{2}U'\bar{Y}\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2 + \frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}^2-\hat{Y}_{2,t}) - \kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\} \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})(var_z[\hat{y}_{1,t}(z)] + var_z[\hat{y}_{2,t}(z)]) + t.i.p. + O(3) \end{aligned} \tag{B.22}$$

The discounted present value of these terms is obtained as follows

$$\begin{aligned} W &= E_0\sum_{t=0}^{\infty}\beta^t W_t \\ &= -\frac{1}{2}U'\bar{Y}\ E_0\sum_{t=0}^{\infty}\beta^t\left\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2 + \frac{1}{4}(1+\frac{1}{\omega})(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}) - \kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\right\} \\ &\quad - \frac{1}{2}U'\bar{Y}\ E_0\sum_{t=0}^{\infty}\beta^t\left\{(\frac{1}{\theta}+\frac{1}{\omega})(var_z[\hat{y}_{1,t}(z)] + var_z[\hat{y}_{2,t}(z)])\right\} + t.i.p. + O(3) \end{aligned} \tag{B.23}$$

The last term is

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t var_z[\hat{y}_{i,t}(z)] &= \theta^2\sum_{t=0}^{\infty}\beta^t\sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi_{i,t}^2 + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi_{i,t}^2\right) + t.i.p. + O(3) \\ &= \frac{\theta^2}{1-\alpha_i\beta}\sum_{t=0}^{\infty}\beta^t\left(\frac{\alpha_i}{1-\alpha_i}\pi_{i,t}^2 + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi_{i,t}^2\right) \end{aligned} \tag{B.24}$$

Substituting this back:

$$\begin{aligned} W_t &= -\frac{1}{2}U'\bar{Y}\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}^2-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\} \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}(\frac{\alpha_1}{1-\alpha_1}\pi_{1,t}^2+\frac{\psi_1}{(1-\alpha_1)(1-\psi_1)}\Delta\pi_{1,t}^2) \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}(\frac{\alpha_2}{1-\alpha_2}\pi_{2,t}^2+\frac{\psi_2}{(1-\alpha_2)(1-\psi_2)}\Delta\pi_{2,t}^2)+t.i.p.+O(3) \end{aligned} \tag{B.25}$$

Define central bank loss function L_t as $W_t = -\frac{1}{2}U'\bar{Y}L_t$. Then rearranging (B.25) yields

$$\begin{aligned} L_t &= \frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}\frac{\alpha_1}{1-\alpha_1}\pi_{1,t}^2+\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}\frac{\alpha_2}{1-\alpha_2}\pi_{2,t}^2 \\ &\quad +(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^n-\hat{Y}_{2,t}^n))^2 \\ &\quad +\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}\frac{\psi_1}{(1-\alpha_1)(1-\psi_1)}\Delta\pi_{1,t}^2+\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}\frac{\psi_2}{(1-\alpha_2)(1-\psi_2)}\Delta\pi_{2,t}^2 \end{aligned} \tag{B.26}$$

Tables

Table 1: Uniform Degree of Inflation Persistence Across Sectors

(a) Optimal Weights attached to the Inflation of the First Sector

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.01	0.3	0.5	0.8
3	3	0.500	0.500	0.500	
	4	0.629	0.631	0.640	
	5	0.715	0.718	0.732	
	6	0.775	0.779	0.794	
4	3	0.371	0.370	0.364	0.324
	4	0.500	0.500	0.500	
	5	0.596	0.598	0.602	
	6	0.669	0.671	0.678	
5	3	0.285	0.283	0.276	0.231
	4	0.404	0.403	0.399	0.374
	5	0.500	0.500	0.500	0.500
	6	0.578	0.579	0.581	
6	3	0.226	0.223	0.216	0.173
	4	0.331	0.329	0.324	0.291
	5	0.422	0.422	0.419	0.402
	6	0.500	0.500	0.500	0.500

The values are ζ in the optimal inflation targeting rule (22). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

(b) Welfare Cost of Adopting Inflation Targeting Policy

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.01	0.3	0.5	0.8
3	3	0.00	0.00	0.00	
	4	0.10	0.20	0.50	
	5	0.30	0.64	1.57	
	6	0.55	1.17	2.82	
4	3	0.10	0.18	0.38	2.82
	4	0.00	0.00	0.00	
	5	0.07	0.12	0.24	
	6	0.23	0.40	0.78	
5	3	0.30	0.54	1.04	5.41
	4	0.07	0.11	0.21	1.19
	5	0.00	0.00	0.00	0.00
	6	0.05	0.08	0.14	
6	3	0.55	0.95	1.74	7.41
	4	0.23	0.37	0.64	2.88
	5	0.05	0.08	0.13	0.61
	6	0.00	0.00	0.00	0.00

The welfare loss is computed according to (23). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

Table 2: **Higher Degree of Inflation Persistence in the Second Sector (0.1 points)**

(a) Optimal Weights attached to the Inflation of the First Sector

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.3	0.5	0.8
3	3	0.456	0.423	0.283
	4	0.586	0.554	0.415
	5	0.675	0.647	0.520
	6	0.740	0.715	0.602
4	3	0.331	0.301	0.181
	4	0.455	0.422	0.279
	5	0.551	0.517	0.367
	6	0.625	0.593	0.444
5	3	0.251	0.225	0.126
	4	0.361	0.329	0.200
	5	0.453	0.418	0.270
	6	0.530	0.493	0.336
6	3	0.196	0.174	0.093
	4	0.291	0.262	0.150
	5	0.376	0.343	0.206
	6	0.451	0.414	0.260

The values are ζ in the optimal inflation targeting rule (22). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

(b) Welfare Cost of Adopting Inflation Targeting Policy

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.3	0.5	0.8
3	3	2.27	4.99	20.39
	4	1.83	4.04	16.39
	5	1.44	3.20	13.00
	6	1.16	2.54	10.35
4	3	3.59	7.34	25.36
	4	3.10	6.55	23.52
	5	2.66	5.77	21.39
	6	2.31	5.09	19.36
5	3	4.85	9.38	28.84
	4	4.37	8.77	28.26
	5	3.90	8.08	27.00
	6	3.51	7.43	25.56
6	3	5.97	11.10	31.58
	4	5.55	10.69	31.96
	5	5.08	10.11	31.35
	6	4.67	9.52	30.37

The welfare loss is computed according to (23). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

Table 3: Higher Degree of Inflation Persistence in the Second Sector (0.3 points)

(a) Optimal Weights attached to the Inflation of the First Sector

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.3	0.5	0.8
3	3	0.398	0.331	0.097
	4	0.525	0.451	0.147
	5	0.617	0.541	0.201
	6	0.684	0.610	0.256
4	3		0.226	0.055
	4		0.324	0.082
	5		0.405	0.112
	6		0.472	0.144
5	3		0.164	0.036
	4		0.242	0.053
	5		0.309	0.072
	6		0.368	0.092
6	3		0.125	0.026
	4		0.187	0.037
	5		0.241	0.050
	6		0.290	0.064

The values are ζ in the optimal inflation targeting rule (22). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

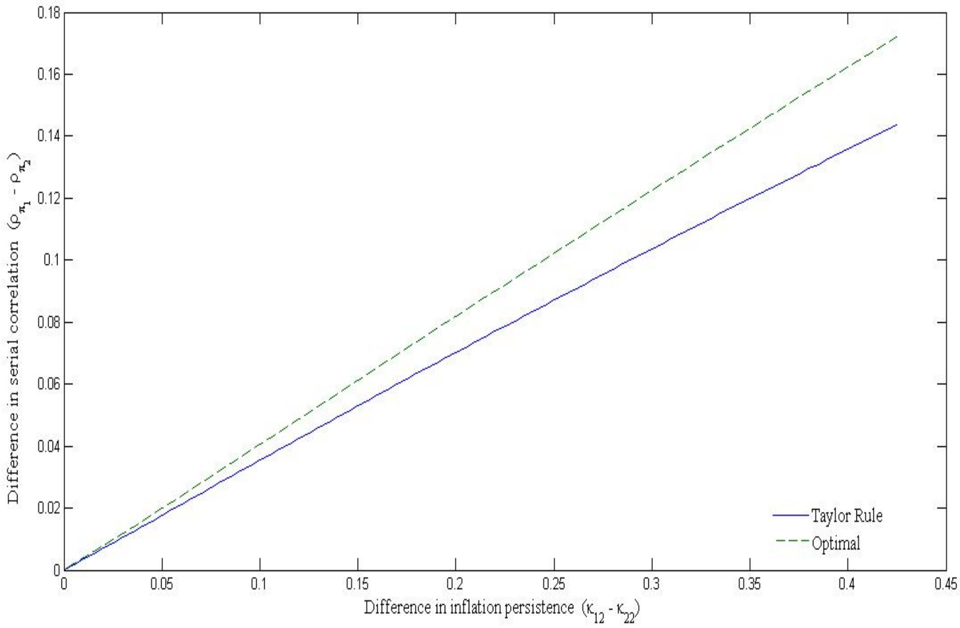
(b) Welfare Cost of Adopting Inflation Targeting Policy

Duration Second Sector	Duration First Sector	Backward looking price setters Second Sector 0.3	0.5	0.8
3	3	13.26	24.74	54.90
	4	14.25	27.44	61.27
	5	14.53	28.58	65.12
	6	14.48	28.90	67.25
4	3		28.89	58.58
	4		32.77	65.46
	5		35.08	69.86
	6		36.38	72.72
5	3		32.45	62.63
	4		37.26	70.06
	5		40.42	74.71
	6		42.49	77.80
6	3		35.38	66.27
	4		41.00	74.43
	5		44.87	79.38
	6		47.54	82.65

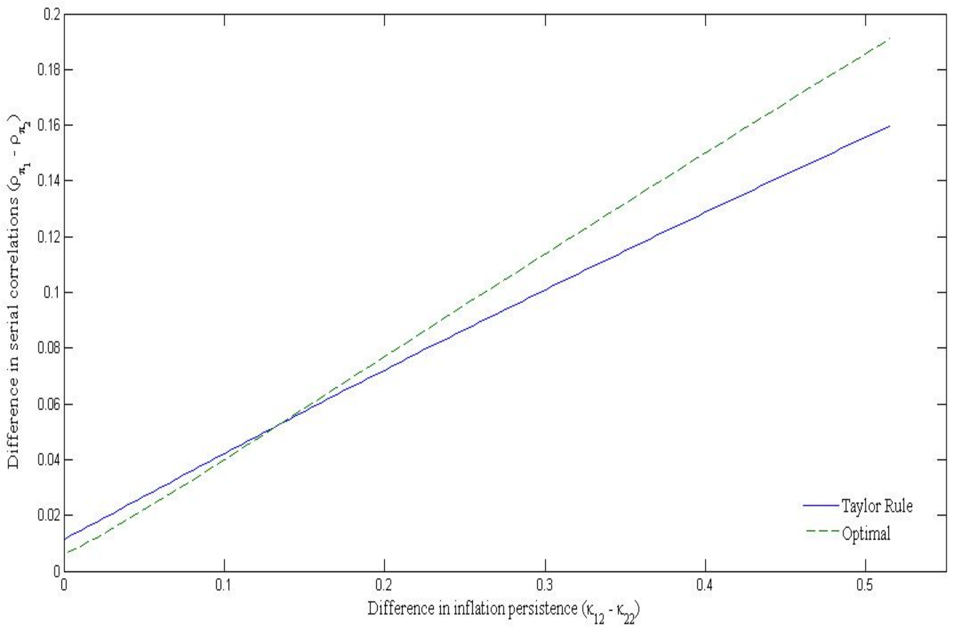
The welfare loss is computed according to (23). The blanks in the table correspond to the cases that a feasible value of fraction of backward looking price setters in the first sector cannot be obtained.

Figures

Figure 1: Difference in Serial Correlation in Sectoral Inflations

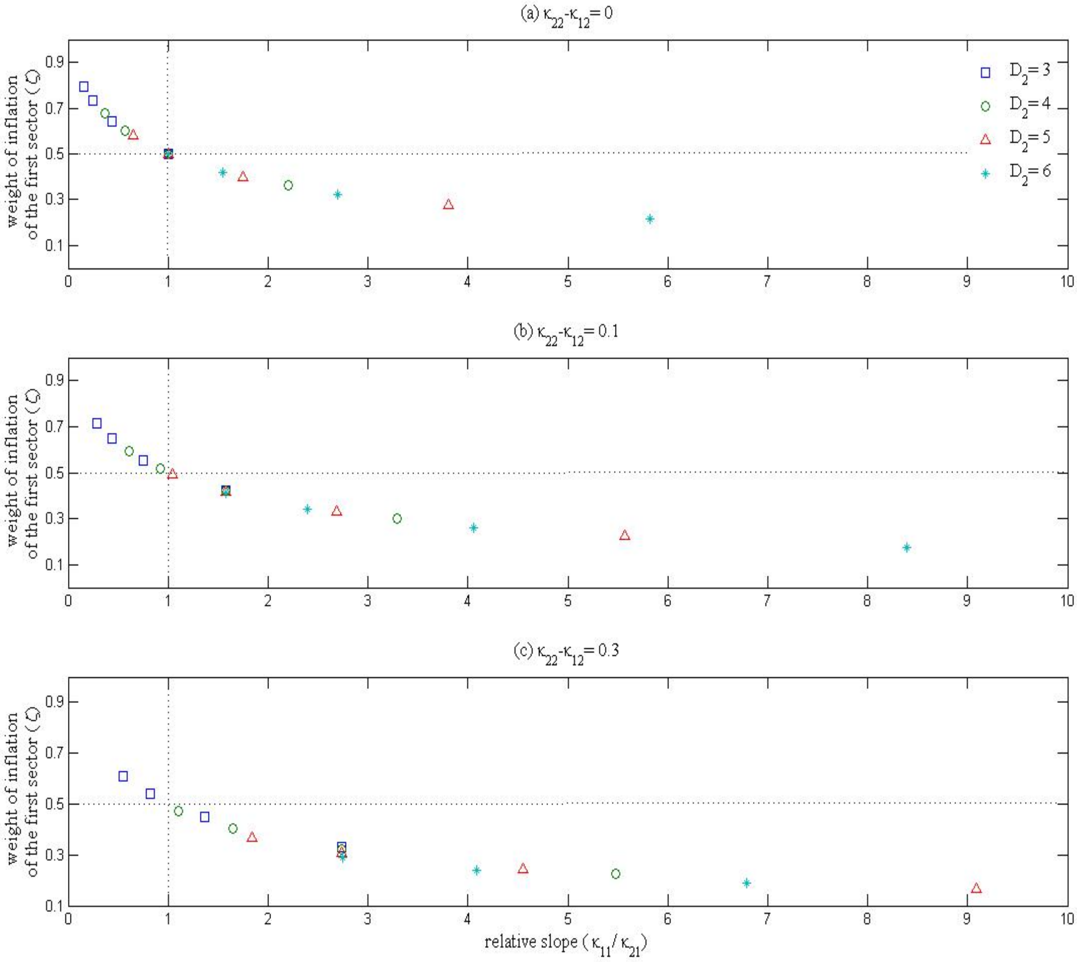


(a)The difference in serial correlation in sectoral inflations, ρ_{π_i}, as a function of the difference in the coefficients of the lagged inflation in the sectoral NKPCs, κ_{i2}. Duration of price is 3 quarters and fraction of backward looking price setters is 0.5 in the second sector and duration of price is 3 quarters in the first sector. The serial correlation in sectoral inflations is obtained numerically by solving the model under each policy.



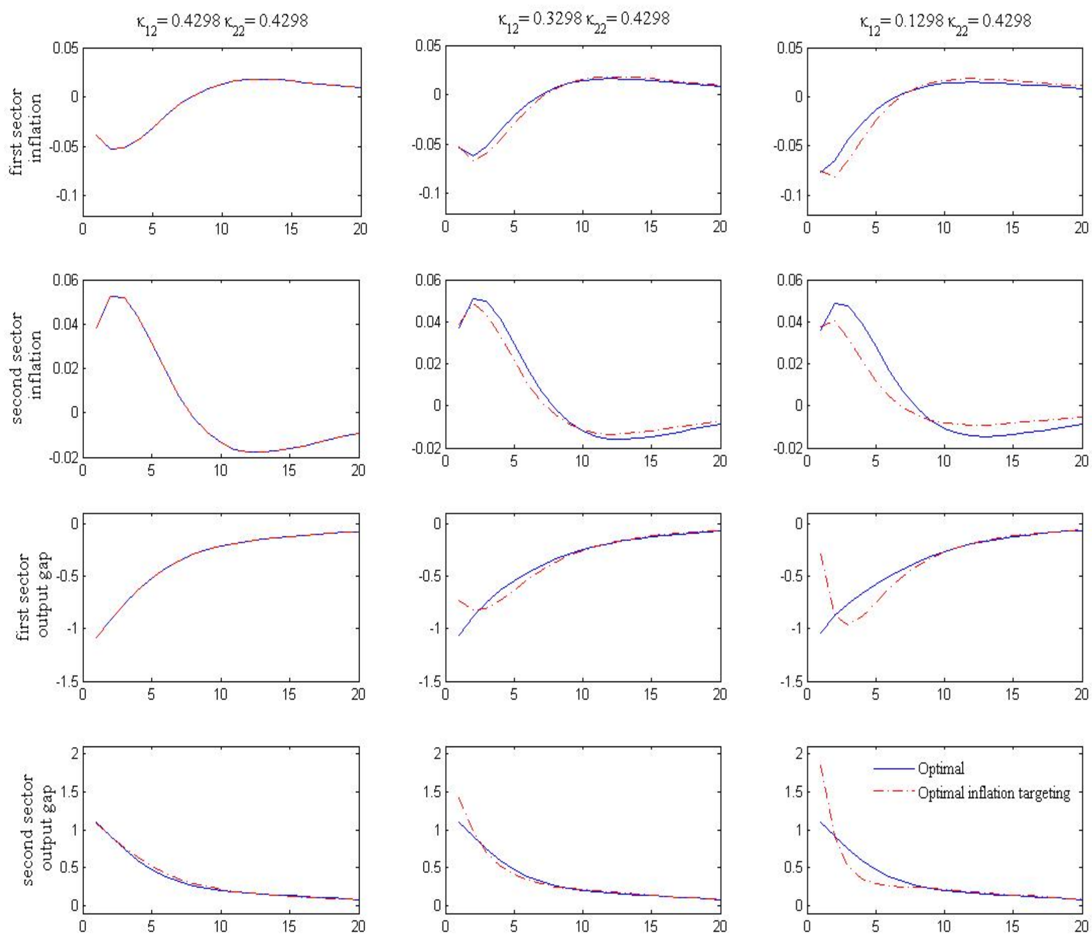
(b)The difference in serial correlation in sectoral inflations, ρ_{π_i}, as a function of the difference in the coefficients of the lagged inflation in the sectoral NKPCs, κ_{i2}. Duration of price is 3 quarters and fraction of backward looking price setters is 0.5 in the second sector and duration of price is 4 quarters in the first sector. The serial correlation in sectoral inflations is obtained numerically by solving the model under each policy.

Figure 2: Optimal Weight and Relative Slope of the NKPC



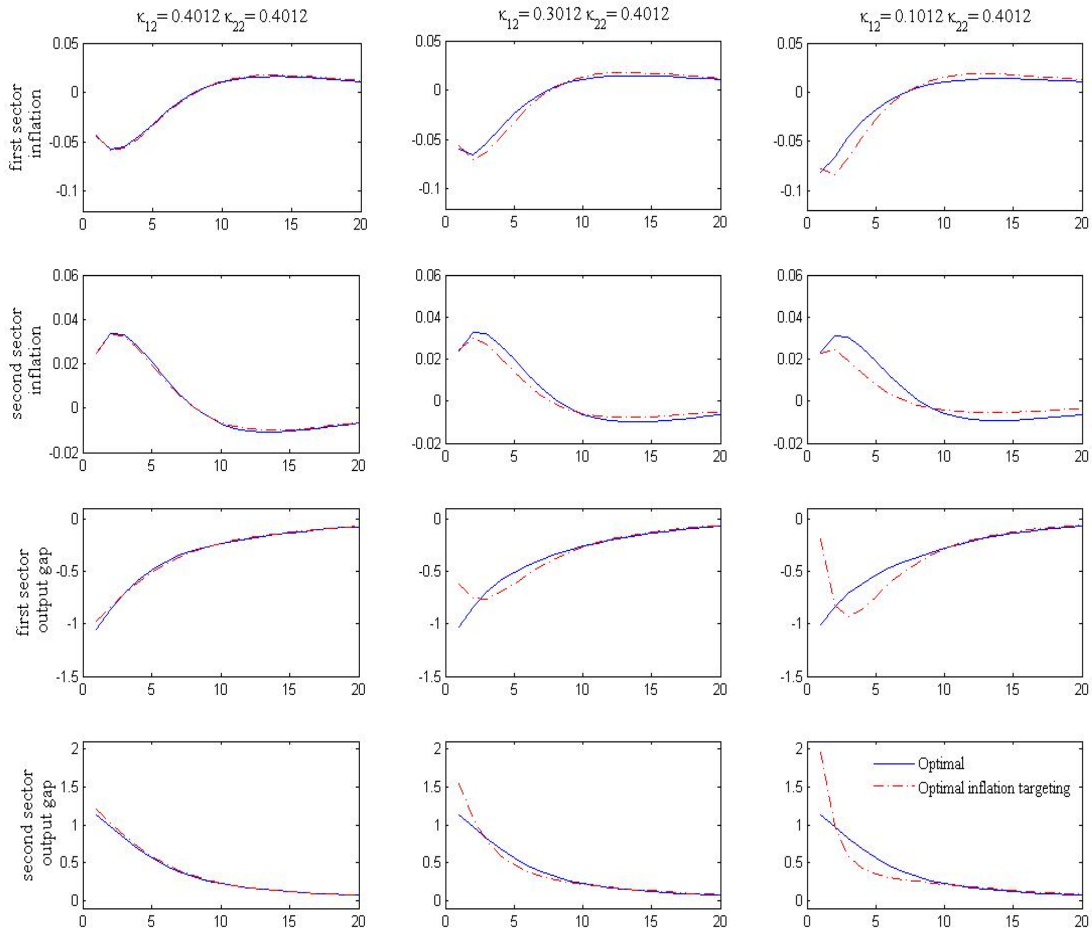
The optimal weight attached to the inflation of the first sector as a function of the relative slope of the sectoral NKPCs for the cases when the differential in degree of inflation persistence is 0, 0.1 and 0.3 points and fraction of backward looking producers is 0.5.

Figure 3: Impulse response functions to a negative supply shock to the second sector



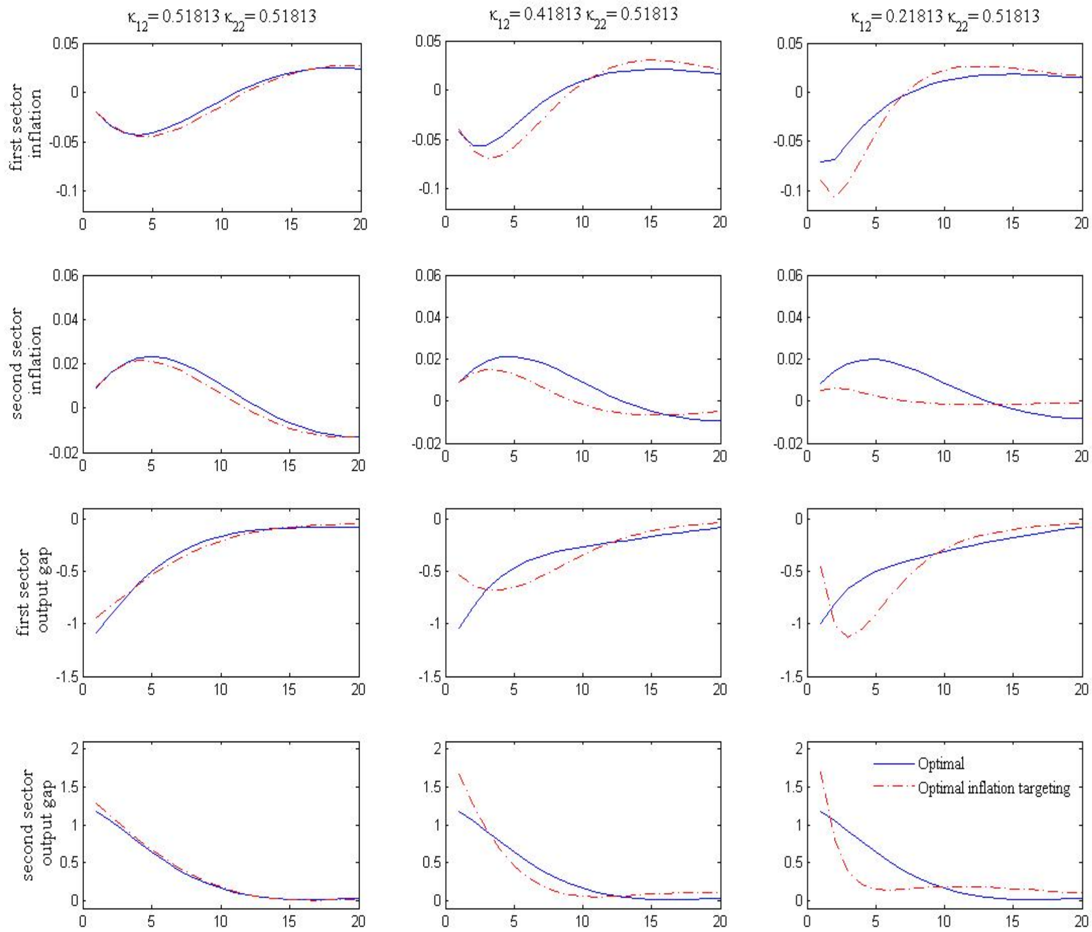
Impulse response functions following a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 3$, $D_1 = 3$ and $\psi_2 = 0.5$. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.5, 0.423 and 0.331 and optimal inflation targeting implies 0, 4.99 and 24.74 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.

Figure 4: Impulse response functions to a negative supply shock to the second sector



Impulse response functions following a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 4$, $D_1 = 3$ and ψ_2= 0.5. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.364, 0.301 and 0.226 and optimal inflation targeting implies 0.38, 7.34 and 28.89 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.

Figure 5: Impulse response functions to a negative supply shock to the second sector



Impulse response functions following a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 4$, $D_1 = 3$ and ψ_2= 0.8. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.324, 0.181 and 0.055 and optimal inflation targeting implies 2.82, 25.36 and 58.58 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.

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