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TÜRKİYE CUMHURİYET MERKEZ BANKASI

# RESEARCH NOTES IN ECONOMICS

## Update: Normalized CES Production Function for Turkey

Selen Andıç

**Özet:** Bu çalışma, normalize edilmiş sabit ikame esneklikli üretim fonksiyonunu Türkiye'nin revize edilmiş milli gelir verileriyle tekrar tahmin etmeyi amaçlamaktadır. Bunu yaparken, üretim fonksiyonu ve iki adet birinci derece koşullarından oluşan arz yönlü bir denklem sistemi kullanılmıştır. Sonuçlar, ikame esnekliğinin 0,6 ile 1'in hala anlamlı bir şekilde altında olduğunu göstermektedir. İşgücü verimliliği yavaşlayarak artmakta, sermaye verimliliği ise azalmaktadır. Uzun vadede işgücü verimliliğinin sermaye verimliliğine kıyasla baskın olduğu bulunmakta, bu durum ekonomi genelindeki verimlilik büyümesinin pozitif olmasında kendini göstermektedir. Revize edilmiş veri, eski veriye kıyasla, Türkiye'de potansiyel büyümenin arttığını ortaya koymaktadır. Bu artışın sebebi, güçlü sermaye birikimi ve verimlilikteki iyileşme olarak gözükmektedir.

**Abstract:** This study re-estimates the normalized CES production function for Turkey with the revised GDP data. It employs a supply-side system of equations, which incorporates the production function and two first-order conditions. Results indicate that the elasticity of substitution is 0.6 and still significantly below 1. Labor has a positive but slowing productivity growth while capital has a falling productivity. Labor-augmenting technical progress is dominant in the long run as the total factor productivity growth is positive. The revised data implies a higher potential growth for Turkey compared to the old data. Strong capital accumulation and improvement in the productivity seem to be the reasons for the change in potential growth.

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## 1. Introduction

The production function is a mathematical expression that describes the relationship between inputs and output of an economy. It is also a tool to assess the potential output from the supply side.

The most popular production function is Cobb-Douglas since it is easy to work with. However, it is appropriate to use only if the labor and capital have constant shares in national income. With the unit elasticity of substitution between capital and labor, Cobb-Douglas is a special case of constant elasticity of substitution (CES) production function. Finally, it should be kept in mind that the productivity of labor and capital cannot be differentiated in Cobb-Douglas because everything related to productivity goes to the "Solow residual."

It is possible that labor and capital have different levels of productivity. Moreover, the factor productivities can change over time at different speeds. If the technical progress becomes more labor or capital-augmenting, then the factor shares will not be constant. Hence, Cobb-Douglas will not be an appropriate production function for that economy. Trends in the factor shares call for the CES production function, which lets bias in the technical change.

The share of labor and capital in Turkish national income is presented in Graph 1. Labor's share of income has a downward trend as opposed to capital's during 1991-2016 period.[1] Hence, factor income shares and their ratio are non-stationary. These observations imply that there are productivity changes both in labor and capital but their dynamics are different. Therefore, differing technical progress in factors necessitates CES production function for Turkey.

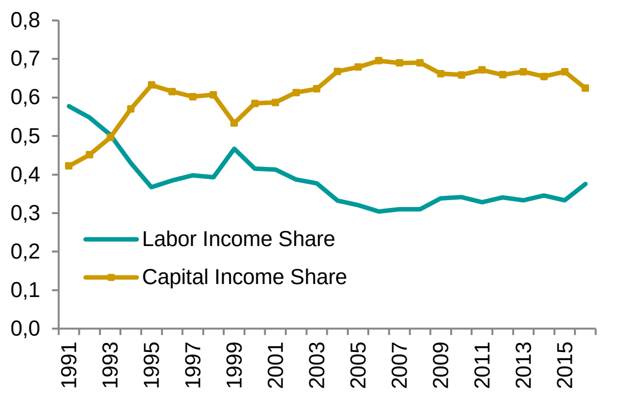
Andıç (2016a) has estimated a normalized CES production function for Turkey a la Klump et al. (2007). All the technical details of the model can be found in those papers. This study only aims to re-estimate the model with the new Turkish GDP series and find out the revised parameter estimates. Thereby, through the lens of this production function, it intends to compare old and new potential GDP and the sources of potential growth. The information on the potential level of output is deemed to be crucial as it enables policy-makers to build

[1] Graph 1 indicates that the share of labor is around 0.4, while that of capital, including profit, is around 0.6 in on average in Turkey since early the 2000s. This is consistent with the findings for Turkey in Senhadji (2000), İsmihan and Metin-Özcan (2006) and Abu-Qarn and Abu-Bader(2007). On the Other hand, the share of labor is typically taken as 0.65 for developed countries, for example see Musso and Westermann (2005) and Husabo (2013).

macroeconomic policies with the objective of converging to the frontier economies or providing an equitable income distribution.

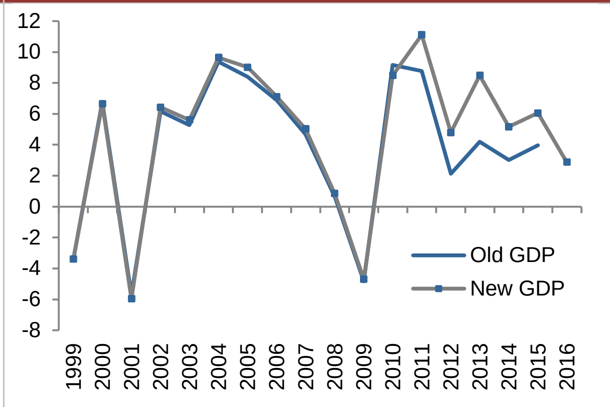
The remainder of this paper is organized as follows: Section 2 sheds light on the revised data. Section 3 briefly discusses the normalized CES production function. Section 4 recalls the model estimated. Section 5 shows the updated parameter estimates. Section 6 and 7 argue the new potential growth and its sources while comparing them with the old ones, respectively. Finally, Section 8 concludes.

Graph 1: Labor and capital income shares in national income in Turkey, 2009 based GDP




Source: SSI, AMECO, TurkStat, Author's calculations.

Graph 2: Growth in real GDP, annual % change




Source: Turkstat.

## 2. Revision in data

In December 2016, TurkStat has switched from ESA95 to ESA2010 methodology in the implementation of System of National Accounts in Turkey. After this revision, not only the level of GDP has increased in real and nominal terms, but also the growth rates have scaled up after 2009. While the old GDP series measured the average growth rate as 5.2 percent from 2010 to 2015, the new series has shown that it was indeed 7.4 percent. The main reason for this rise is the up-shift in the construction investments, which resulted from the improvement in data sources. According to the TurkStat, new data sources rest more on official recordings and are more representative of the whole economy.[2]

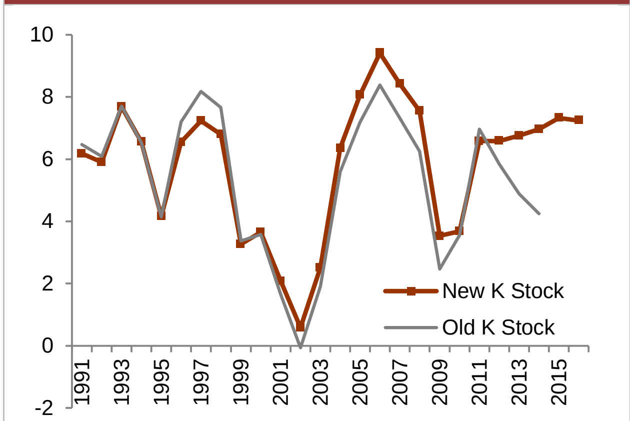
Graph 2 and 3 depicts the old and new growth rates of real GDP, $Y_t$, and capital stock, $K_t$.[3] New growth rates are higher after 2009. Growth in capital stock soared upon the rise in construction investments. Growth in machinery and equipment stock is almost unchanged.

[2] Details of the mentioned revision can be found at http://www.tuik.gov.tr/duyurular/duyuru_3244.pdf.
[3] The machinery-equipment investment and construction investment series in the revised GDP start from 2009. To compute the capital stock dating back to 1987, we have used 1998 and 1987 based GDP data. While

Labor, $L_t$, is another important ingredient in the production function. Andıç (2016a) measures it as employment taken from the 2005 based Household Labor Force Survey. In early 2014, TurkStat has made amendments to this survey to align it more with the EU standards.[4] This study rests on the new employment data, which is slightly lower than the one used in Andıç (2016a) (Graph 4).

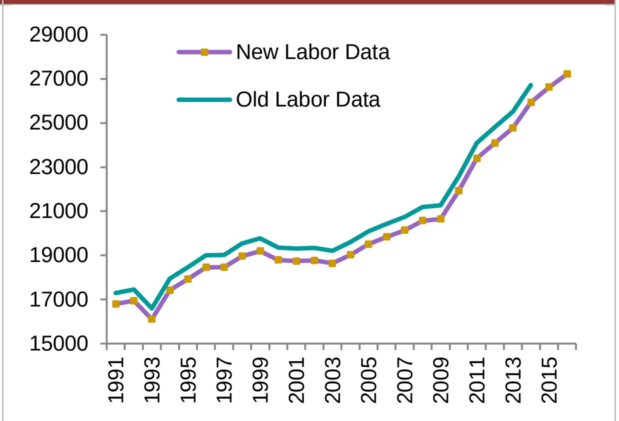
The data are annual and cover the 1991-2016 period. The fact that Turkey experienced crises in 1994, 1999, 2001 and 2009 increases the volatility in the data. Therefore, as in Andıç (2016a), to smooth out fluctuations, five-year centered moving averages of the series are used in the estimations.

Graph 3: Growth in capital stock, annual % change




Source: TurkStat, Author's calculations.

Graph 4: Level of employment, 1000 person




Source: TurkStat, Author's calculations.

## 3. What is a normalized CES production function?

The constant elasticity of production function is indeed derived from the definition of elasticity, which is the percentage change in capital-labor ratio divided by the percentage change in the marginal rate of technical substitution. The latter equals the marginal product of labor over the marginal product of capital. When this definition is transformed into a

interpolating the values of subcomponents of investments from 1987 to 2008, we have taken into account the shares of machinery-equipment and construction investments in GDP, and the year-on-year changes in these investments. The methodology used in the construction of capital stock draws from Demiroğlu (2012). The depreciation rate ($\delta$) of construction investment (machinery-equipment) is taken as 2% (16%) per year. Capital is assumed to accumulate according to $K_{t+1} = (1-\delta)K_t + I_{t+1}$. The growth in the total capital stock is a weighted average of the growth in machinery-equipment and construction capital stocks; i.e., $\frac{\dot{K}_{total}}{K_{total}} = w_{mac.equ.}\frac{\dot{K}_{mac.equ.}}{K_{mac.equ.}} + w_{cons.}\frac{\dot{K}_{cons.}}{K_{cons.}}$. The weights are determined by the level of capital stocks, user costs of capital and prices of investments relative to the price of output. In fact, it is computed that $w_{mac.equ.} \approx 0.4$ and $w_{cons.} \approx 0.6$. The initial levels of capital stocks are determined judgmentally and in such a way that the ratio of capital stock to GDP at the beginning of the sample will be in accordance with the ratio computed for the rest of the sample.

[4] Details can be found at http://www.tuik.gov.tr/MicroVeri/Hia_2014/turkce/downloads/aciklamalar.pdf.

second order partial differential equation in per capita capital in productivity units, then the general CES production function is obtained:

$$Y_t = C\left[\pi\left(e^{g_K(t)}K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi)\left(e^{g_L(t)}L_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}} \quad (1)$$

$\sigma$ is the elasticity of substitution. If $\sigma > 1$ ($\sigma < 1$), $L$ and $K$ are substitutes (complements) (Acemoğlu, 2002). $e^{g_K(t)}$ and $e^{g_L(t)}$ capture the capital and labor-augmenting technical progress, or the productivity level of capital and labor, respectively. $C$ and $\pi$ are unknown parameters that arise a result of the transformation mentioned above. So, how can we identify those parameters? They are identified by normalization; i.e., by selection of baseline points, which involves two steps. First, we know that elasticity is determined at a particular output level with particular levels of labor, capital, technology and returns to inputs. That is, it is determined at a particular point in time. Using this notion, after some algebra we can write;

$$Y_t = Y_0\left[\pi_0\left(e^{g_K(t)}\frac{K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(e^{g_L(t)}\frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}} \quad (2)$$

where $\pi_0 = r_0K_0/(r_0K_0 + w_0L_0)$, that is the capital's share in total earnings of capital and labor, $r_0$ is the real rental price of capital and $w_0$ is the real wage rate $t = t_0$. Equation (2) is more intuitive than equation (1) as all the parameters are economically meaningful. However, what should we use for $Y_0$, $K_0$, and $L_0$? Also, the same question applies for $t_0$, which will appear in the technical progress functions shown in the next section. In the second step, we assume some values for these variables. Klump et al. (2004) argue that an appropriate value for these should be detected from the data and include as much information as possible. Therefore, we use sample averages. Specifically, $K_0 = \bar{K}$, $L_0 = \bar{L}$, and $Y_0 = A\bar{Y}$ are calculated as geometric averages, while $t_0 = \bar{t}$ is measured as a simple average. Note that though we will show $\pi_0$ as $\bar{\pi}$, it will be an estimated parameter of the model as it is associated with the markup parameter.[5] $A$ is a scaling parameter that comes out due to the non-normality of the CES production function and expected to be close to 1.[6] Hence, the normalized CES production function is[7]:

[5] Klump et al. (2004) argue that if we had perfect competition assumption, we would not have markup parameter in the model. Hence, we could directly calculate $\bar{\pi}$ from the data as a simple average.
[6] Because of the non-normality of the CES function, sample averages of the inputs, whether geometric or simple, need not exactly coincide with the sample average of output. The possible emergence of this fact is captured by the scaling parameter $A$.
[7] It should be noted that though the CES production function is more flexible than the Cobb-Douglas, for instance as $\sigma$ is not constrained to be 1, it still relies on the assumption of constant returns to scale. Moreover, in the context of this paper, as the CES production function does not involve other factors of production such as human capital, ideas, land, etc. it is still an incomplete picture of the economy.

$$Y_t = A\bar{Y}\left[\bar{\pi}\left(e^{g_K(t)}\frac{K_t}{\bar{K}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\left(e^{g_L(t)}\frac{L_t}{\bar{L},}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}} \quad (3)$$

## 4. The system estimation

Assuming imperfect competition, the profit maximization yields equation (4) and (5). Additionally, equation (3) can be rearranged as equation (6). Hence, the three-equation supply-side system is arrived:

$$ln\left(\frac{W_tL_t}{P_tY_t}\right) = ln\left(\frac{1-\bar{\pi}}{1+\mu}\right) + \frac{(1-\sigma)}{\sigma}\left[ln\left(\frac{Y_t/\bar{Y}}{L_t/\bar{L}}\right) - ln(A) - g_L(t)\right] \quad (4)$$

$$ln\left(\frac{R_tK_t}{P_tY_t}\right) = ln\left(\frac{\bar{\pi}}{1+\mu}\right) + \frac{(1-\sigma)}{\sigma}\left[ln\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - ln(A) - g_K(t)\right] \quad (5)$$

$$ln\left(\frac{Y_t}{L_t}\right) = ln\left(\frac{A\bar{Y}}{\bar{L}}\right) + g_L(t) - \frac{\sigma}{(1-\sigma)}ln\left[\bar{\pi}e^{\frac{(1-\sigma)}{\sigma}(g_L(t)-g_K(t))}\left(\frac{K_t/\bar{K}}{L_t/\bar{L}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\right] \quad (6)$$

$W_tL_{t/}/P_tY_t$ is the labor share of income. $R_tK_{t/}/P_tY_t$ shows the share of non-profit capital, since due to imperfect competition $P_tY_t = W_tL_t + R_tK_t + Profit_t$. $\mu$ is the markup rate that arises due to the assumption of imperfect competition in the goods market. Technical progress functions of inputs, i.e., $g_L(t)$ and $g_K(t)$ are modeled as follows:

$$g_i(t) = \begin{cases} \frac{\gamma_i\bar{t}}{\lambda_i}\left[\left(\frac{t}{\bar{t}}\right)^{\lambda_i} - 1\right] & for\ \lambda_i \neq 0 \\ \gamma_i\bar{t}\,ln\left(\frac{t}{\bar{t}}\right) & for\ \lambda_i = 0 \end{cases} \quad (7)$$

where $i = L$ and $K$. These functions give flexibility to model productivity of labor and capital (i.e., $e^{g_L(t)}$ and $e^{g_K(t)}$) differently -thanks to parameters $\lambda$ and $\gamma$-, and in a time-varying fashion. Then, the total factor productivity, ($TFP_t$), must only be a function of $g_L(t)$ and $g_K(t)$. However, we are unable to calculate the overall productivity in the economy unless we do a second order Taylor series expansion to equation (6) for $\sigma$ around $\sigma = 1$. This is called Kmenta approximation, and it gives $TFP_t$:[8]

[8] The Kmenta approximation (Kmenta, 1967) yields:

$$\ln\left(\frac{Y_t}{L_t}\right) \approx \bar{\pi}g_K(t) + (1-\bar{\pi})g_L(t) - \left(\frac{1-\sigma}{\sigma}\right)\frac{\bar{\pi}(1-\bar{\pi})}{2}\left(g_L(t) - g_K(t)\right)^2 + \ln\left(\frac{A\bar{Y}}{\bar{L}}\right)$$
$$+ \bar{\pi}ln\left(\frac{K_t/\bar{K}}{L_t/\bar{L}}\right) - \left(\frac{1-\sigma}{\sigma}\right)\frac{\bar{\pi}(1-\bar{\pi})}{2}\left(ln\left(\frac{K_t/\bar{K}}{L_t/\bar{L}}\right)\right)^2.$$

As in Klump et al. (2004) the term $\left(\frac{1-\sigma}{\sigma}\right)\bar{\pi}(1-\bar{\pi})\left(g_L(t) - g_K(t)\right)\ln\left(\frac{K_t/\bar{K}}{L_t/\bar{L}}\right)$ is dropped from the approximation for convenience. Yet the results on the TFP stay broadly the same if we do not drop it, too. It should be addressed that the closer the elasticity of substitution to unity, the more precise the approximation is. As shown in Section

$$TFP_t = e^{\bar{\pi} g_K(t) + (1-\bar{\pi}) g_L(t) - \frac{(1-\sigma)}{\sigma} \frac{\bar{\pi}(1-\bar{\pi})}{2} (g_L(t) - g_K(t))^2}. \quad (8)$$

Since the factor and overall productivities are modeled as exponential functions, obtaining productivity growths is simple calculus: take the natural logarithm, and then take the derivative with respect to time.[9]

Table A2 in Appendix 2 presents explanations of the abbreviations and parameters used in equations (4) through (8). $\bar{\pi}, \mu, \sigma, A, \gamma_i$ and $\lambda_i$, where $i = L$ and $K$, are the estimated parameters of the model. As the equations (4), (5) and (6) are linked through their coefficients, they make up a non-linear seemingly unrelated equations system and the system is estimated by the iterated feasible GLS.

## 5. Updated parameter estimates

The parameters estimated with the revised data are shown in column (2) of Table 1. The results of Andıç (2016a), which uses old GDP and labor data, are also given in column (3) of the same table. All the parameters are significant at 1 percent level. According to the new data, the elasticity of substitution is estimated around 0.6 in Turkey. It is found to be significantly below 1, and lower than the previous estimate. As before, this finding suggests that labor and capital are complements.

The markup, $\mu$, is unchanged as it is still found to be around 36 percent. Roughly, this means that a good or service that costs 1 TL is sold for 1.36 TL in Turkey. $\bar{\pi}$ is estimated as 0.49, which is slightly higher than the old estimate. The rise in capital stock with the new GDP data seems to significantly increase the capital's share in total earnings of labor and capital.

$\gamma_L$ and $\lambda_L$ are estimated as 0.023 and 0.7, respectively. $\hat{\gamma}_L$ is significantly different than the one found in Andıç (2016a). Consistent with the previous findings, the new parameters show that productivity of labor increases at a decreasing rate (Graph 5 and 7). The productivity level of labor is higher than the one found in Andıç (2016a) (Graph 6). It is intuitive given that $Y_t$ is revised upwards while $L_t$ is revised downwards in the new data. Also, productivity growth in labor is now calculated as 2.5 percent on average, whereas it was measured around 2 percent before.

5, the updated elasticity is estimated at 0.6 and found to be significantly below the previous estimate of 0.8. Therefore, the Kmenta approximation is less precise with the revised GDP data.

[9] Resulting functions can be found at Appendix 1.

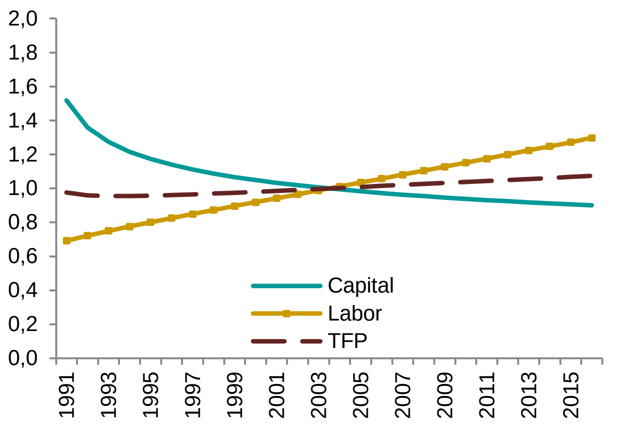
| Table 1: Estimation results of equations (4)-(8) | | |
|---|---|---|
| | **This Study** | **Andıç (2016a)** |
| $\bar{\pi}$ | 0.493***<br>(0.004) | 0.445***<br>(0.003) |
| $\mu$ | 0.363***<br>(0.026) | 0.374***<br>(0.029) |
| $\sigma$ | 0.639***<br>(0.043) | 0.799***<br>(0.026) |
| $A$ | 0.989***<br>(0.008) | 0.998***<br>(0.012) |
| $\gamma_L$ | 0.023***<br>(0.001) | 0.015***<br>(0.001) |
| $\lambda_L$ | 0.713***<br>(0.074) | |
| $\gamma_K$ | -0.012***<br>(0.001) | -0.011***<br>(0.002) |
| $\lambda_K$ | | -0.490***<br>(0.087) |
| TFP growth | 0.005 | 0.002 |
| **p-values of restrictions** | | |
| $\sigma = 1$ | 0.0000 | 0.0000 |
| $\bar{\pi} = 0.445$ | 0.0000 | |
| $\mu = 0.374$ | 0.7675 | |
| $\sigma = 0.799$ | 0.0002 | |
| $\gamma_L = 0.015$ | 0.0008 | |
| $\gamma_K = -0.011$ | 0.0950 | |

*Notes:* Standard errors in parenthesis. */**/*** show the 10, 5 and 1 percent significance level, respectively. TFP growth is average. Sample covers 1991-2016 period in this study, while it spans 1991-2014 in Andıç (2016a).

$\hat{\gamma}_K$ is -0.012. However, there is weak evidence that it differs significantly from the previous estimate. Different than Andıç (2016a), this study implies that technical progress in capital is modeled best when its curvature parameter, $\lambda_K$, is 0. Yet these results still mean that the productivity of capital decreases at a decreasing rate (Graph 7 and 8). The average growth in capital productivity is -1.8 percent. A negative productivity growth is not surprising given that output to capital has a decreasing trend.[10] These parameters imply that the overall productivity of the Turkish economy grows at around 0.5 percent in the last 20 years. Hence, labor-augmenting technical progress is dominant in the long-run.[11]

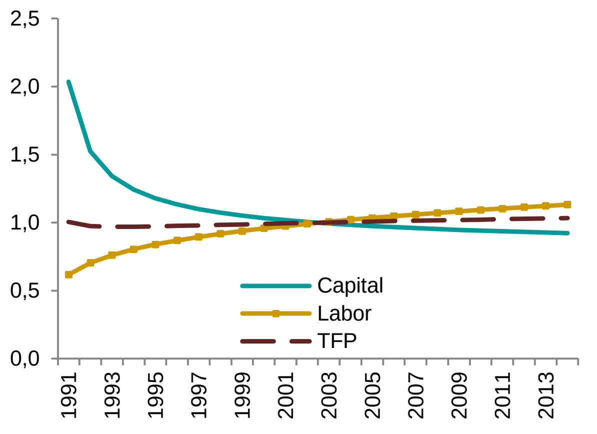
[10] The related graph can be found in Appendix 2.
[11] Robustness checks of the results are shown in Appendix 3.

Graph 5: Capital, labor and overall productivity in levels, 2009 based GDP



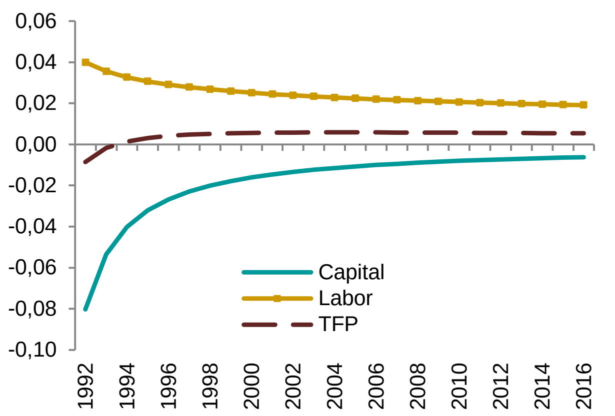

Source: Author's calculations.

Graph 6: Capital, labor and overall productivity in levels, 1998 based GDP



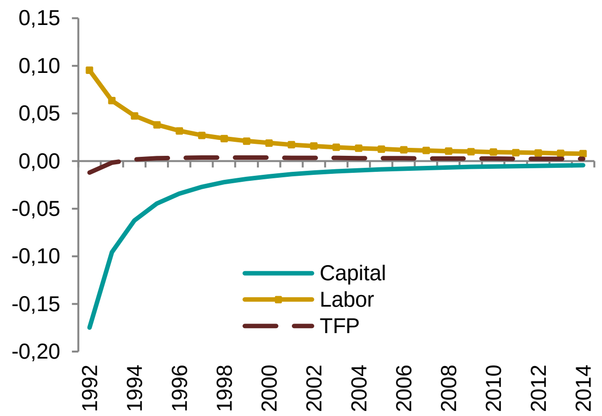

Source: Author's calculations.

Graph 7: Capital, labor and overall productivity growth, 2009 based GDP




Source: Author's calculations.

Graph 8: Capital, labor and overall productivity growth, 1998 based GDP




Source: Author's calculations.

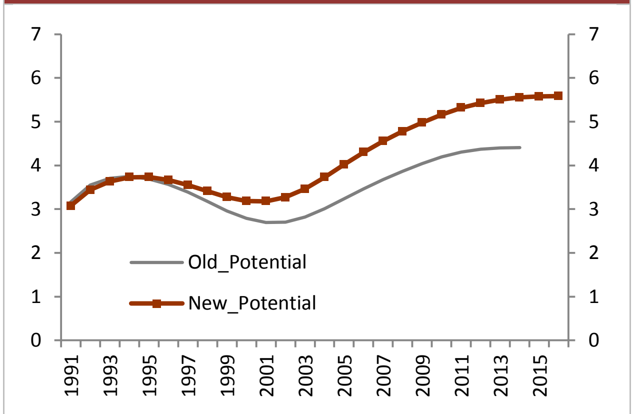
## 6. Potential growth

The potential level of GDP is defined in different ways. Okun (1962) describes it as the level of output consistent with stable inflation. This definition mostly matters for the central banks whose primary objective is to achieve price stability. On the other hand, macro textbooks define the potential GDP as the maximum level of output that can be produced with the existing resources. Following a textbook-type of approach gives a smoother potential growth. However, it does not guarantee price stability. The potential output obtained using production functions falls into the second category, and the CES production function is no exception.

To measure the potential output via the CES production function, first, $L_t$ and $K_t$ are HP-filtered taking the smoothing parameter as 100. Then, potential $Y_t$ is obtained using the trends in labor and capital, estimated parameters and equation (6). Graph 9 presents the year-on-year growth in this series. In the early 1990s, Turkey had a potential growth around 3.5 percent. However, through the end of the decade, it lost momentum and reached a trough with the 2001 crisis. New data imply a faster recovery compared to the old one. Specifically, revised estimates show that it took three years for the potential growth to return to its pre-crisis levels.

The next eight years are marked by a continuous increase in the potential growth of the Turkish economy. Interestingly, as opposed to the previous crisis, 2009 crisis does not seem to have lowered the potential growth. A reason for this finding may be related with the developments in the labor market, which seems to have been more depressed by the stream of crises in 1994, 1999 and 2001 compared to 2009. More recently, new data imply that the growth in the maximum output is approximately one percentage point higher than what the old data suggest.[12]

Graph 9: Potential Growth, annual % change




Source: Author's calculations.

## 7. Sources of potential growth

Why does the new data imply a higher potential growth? Analyzing contributions of labor, capital and overall productivity sheds light on this question. However, this requires some simple algebra: taking derivative of the Kmenta approximated output equation with respect to time. Then, one gets the following equation:

$$\frac{\dot{Y}_t}{Y_t} \approx \frac{T\dot{F}P_t}{TFP_t} + \underbrace{\frac{\dot{K}_t}{K_t}\left[\bar{\pi} + \left(\frac{\sigma-1}{\sigma}\right)\bar{\pi}(1-\bar{\pi})\left(ln\left(\frac{K_t}{\bar{K}}\right) - ln\left(\frac{L_t}{\bar{L}}\right)\right)\right]}_{part\ 1} + \underbrace{\frac{\dot{L}_t}{L_t}\left[(1-\bar{\pi}) + \left(\frac{\sigma-1}{\sigma}\right)\bar{\pi}(1-\bar{\pi})\left(ln\left(\frac{L_t}{\bar{L}}\right) - ln\left(\frac{K_t}{\bar{K}}\right)\right)\right]}_{part\ 2}. \quad (9)$$

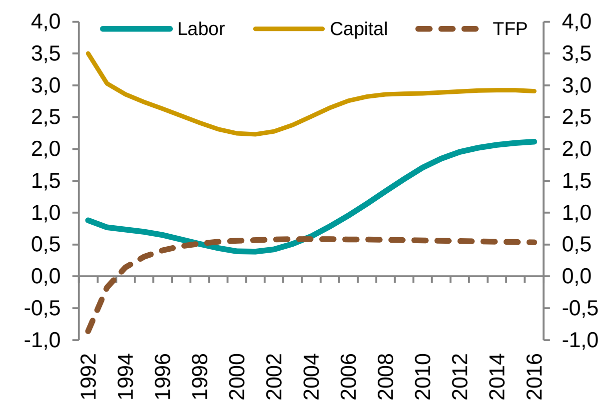
where $\dot{x}$ shows the change in variable $x$ with respect to time. We compute the potential growth in labor and capital by using the trends of $L_t$ and $K_t$, which are obtained by the HP

[12] There is a sample-end bias in filtering. Hence, the more recent potential growth estimates are less reliable.

filter. $\frac{T\dot{F}P_t}{TFP_t}$ is the growth in and contribution of overall productivity to potential growth. It is the series obtained under the Kmenta approximation and is shown in Graph 7. As $\frac{T\dot{F}P_t}{TFP_t}$ is non-volatile, we do not smooth out the level of productivity. $part\ 1\ (2)$ is the contribution of capital (labor).

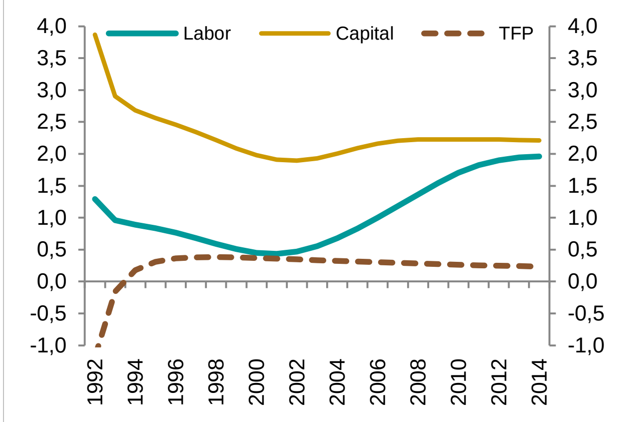
It should be noted that there is some labor (capital) left in the capital's (labor's) contribution. In other words, $part\ 1\ (2)$ does not show how much "pure" capital (labor) adds to potential growth. However, equation (9) is expressed in this way to comply with the notation prevalent in the growth literature.[13] Hence, this is the equation that underlies Graph 10 and 11.

Graph 10: Contribution of inputs to potential growth, percentage points, 2009 based GDP




Source: Author's calculations.

Graph 11: Contribution of inputs to potential growth, percentage points, 1998 based GDP




Source: Author's calculations.

The revised data show that the largest contribution to potential growth comes from capital, which is recently around 3 percentage points (Graph 10). Labor ranks second as it currently adds 2 percentage points. Finally, overall productivity in the economy contributes about 0.5 percentage points to potential growth. Ranking of the factors was same with the old data. Capital and labor contributed 2.2 and 2 percentage points in the last 5 years, respectively (Graph 11). However, the contribution of productivity remained weak with 0.2 percentage points. These numbers lead us to the answer of the question asked at the beginning of this section. Potential growth increased with the new data due to higher contribution from capital and productivity.

With the rise in construction investments after the revision in national accounts, capital stock has scaled up. It now grows stronger and adds more to potential growth. The

[13] See Appendix 4 for a discussion of "pure" contributions in CES specification.

improvement in the overall productivity is due to the rise in the productivity of labor with the new data, which outweighs fall in that of capital in the long-run.

How does the total factor productivity growth of 0.5 percent compare to other countries? Klump et al. (2007) show that total factor productivity growth in the US and Euro area is 1.4 and 1.1 percent, respectively[14]. Andıç (2016b) argues that the TFP growth in South Korea, the second miracle of Asia, is around 2 percentage points and it contributes to growth more than labor. Furthermore, these findings hold even when South Korea used to be a developing country in the 1990s. So, these simple comparisons suggest that productivity growth is low in Turkey. Economic growth is driven by factors and contribution of productivity falls further behind. In other words, Turkish economy operates with the old engines of growth.[15]

## 8. Conclusion

In December 2016, TurkStat has made a methodological change in the measurement of national accounts, which resulted in significant revisions in GDP and capital stock in Turkey. Indeed, the revisions are so substantial that they call for updates to studies resting on the old GDP data. Accordingly, this paper re-estimates the normalized CES production function for Turkey, which was first presented in Andıç (2016a).

The findings show that the elasticity of substitution has decreased to 0.6 from 0.8. It is significantly below unity. This is suggestive of the fact that CES is a more appropriate production function than the Cobb-Douglas for Turkey. Results indicate that labor's productivity increases at a decreasing rate and its average productivity growth is higher with the revised data. On the other hand, capital's productivity decreases at a decreasing rate and growth in its productivity is still negative. Consistent with the previous findings, labor-augmenting technical progress surpasses capital-augmenting one, and the total factor productivity growth is positive.

This study sheds light on the potential growth of Turkey, as well. The results obtained with the revised data show that potential growth has increased in the last two decades. Yet the old GDP data were signaling a milder improvement in the same period. Growth

[14] Klump et al. (2007) use non-residential capital stock.

[15] The contribution of capital to (actual) growth is the highest even in the developed countries. Yet different than the developing countries, they tend to have contribution of productivity more than or close to the contribution of labor. According to the Total Economy Database (TED) of Conference Board, some countries whose productivity contributes more than or almost equal to labor are United States, Denmark, Finland, Germany, Greece, Iceland, Malta, Norway, Sweden, Austria, France, Ireland, United Kingdom, South Korea and Taiwan. The calculations rest on the TCB adjusted data of the TED for the period between 1990 and 2007.

accounting reveals that the new GDP data imply a higher potential growth upon the higher contributions of capital and overall productivity in the economy.

It should be kept in mind that the potential level of output obtained from a production function is the maximum level of output that can be achieved with the existing inputs. Hence, there is no guarantee for the price -or indeed any macro-variable- stability when an economy grows at the rate suggested by these functions. Last but not least, despite the improvement in the productivity with the new data, TFP growth seems to be low in Turkey. As put by Paul Krugman “productivity isn't everything, but, in the long run, it is almost everything.” At the aggregate level, an improving capability of producing more from available resources will mean a higher income for a country.

The most important element of the CES production function is the flexible modeling of the technical progress in factors. Yet we think it is still an open question why they do not show any cyclicality with crises or shifts with creative destructions. So, in our view the most valuable contributions will come from future works which will introduce these ideas to technical progress functions. Also, the supply-side system estimation by sectors will be an extension to this paper.

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## Appendix 1

The productivity growth in inputs is:

$$\frac{dg_i(t)}{dt} = \begin{cases} \gamma_i \left(\frac{t}{\bar{t}}\right)^{\lambda_i - 1} & for\ \lambda_i \neq 0 \\ \gamma_i \frac{\bar{t}}{t} & for\ \lambda_i = 0 \end{cases} \tag{10}$$

where $i = K, L$. The growth in total factor productivity is:

$$\frac{T\dot{F}P_t}{TFP_t} = \bar{\pi}\dot{g_K}(t) + (1-\bar{\pi})\dot{g_L}(t) - \frac{(1-\sigma)}{\sigma}\,\bar{\pi}(1-\bar{\pi})(g_L(t) - g_K(t))(\dot{g_L}(t) - \dot{g_K}(t)) \tag{11}$$

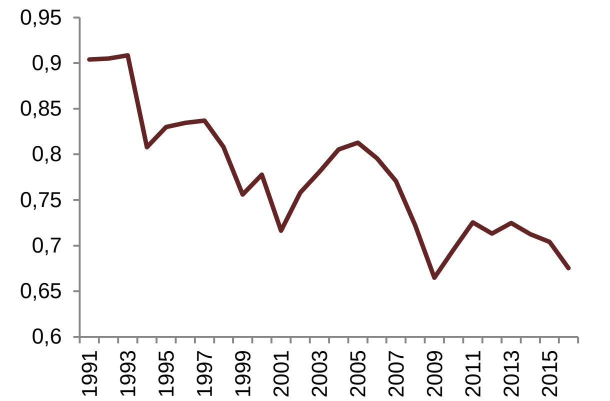
where $\dot{x}$ shows the change in variable $x$ with respect to time.

## Appendix 2

Table A2: Definitions of abbreviations and parameters in equations (1)-(5)

| Data | | | Parameters | | |
|---|---|---|---|---|---|
| | **Explanation** | **Data Source** | | **Meaning** | **Explanation** |
| $W_t$ | nominal wage | Social Security Institution, Ministry of Finance | $\bar{\pi}$ | capital's share in total earnings of capital and labor | expected to be roughly equal to $\frac{R_t K_t}{R_t K_t + W_t L_t}$ on average. |
| $L_t$ | employment | TurkStat | $\mu$ | markup rate. | expected to be roughly equal to $\frac{P_t Y_t}{R_t K_t + W_t L_t} - 1$ on average. |
| $P_t Y_t$ | nominal GDP | TurkStat | $\sigma$ | elasticity of substitution | $[0, \infty)$. If $\sigma > 1$ $(\sigma < 1)$, $L$ and $K$ are substitutes (complements). |
| $Y_t$ | real GDP | TurkStat | $A$ | scaling parameter | expected to be roughly equal to 1. |
| $R_t$ | nominal return on capital | Demiroğlu (2012) | $\gamma_L$ | Productivity growth of labor at $t = \bar{t}$ | |
| $K_t$ | capital services index | author's calculations based on Demiroğlu (2012) | $\gamma_K$ | Productivity growth of capital at $t = \bar{t}$ | |
| $\bar{Y}$ | geometric mean of $Y_t$ | | $\lambda_L$ | curvature parameter for labor | If $\lambda$=1 (=0) [<0], the technical progress has linear (log-linear) [hyperbolic] dynamics. |
| $\bar{L}$ | geometric mean of $L_t$ | | $\lambda_K$ | curvature parameter for capital | If $\lambda$=1 (=0) [<0], the technical progress has linear (log-linear) [hyperbolic] dynamics. |
| $\bar{K}$ | geometric mean of $K_t$ | | | | |
| $t_0$ | simple mean of $t$ | | | | |
| $t$ | time | | | | |

Graph A2.1: Output to capital ratio, real, 2009 based GDP




Source: Author's calculations.

## Appendix 3

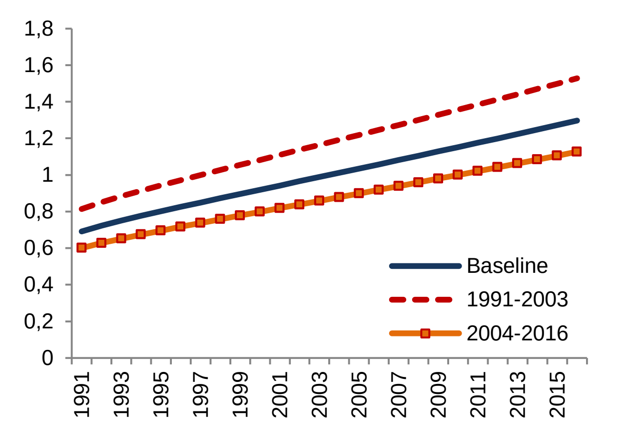
To provide evidence on the robustness of the results two exercises are conducted. In the first one, the estimations are done using the full sample but assuming different baseline values for $Y_0$, $K_0$, $L_0$, and $t_0$. In the second one, both the sample size and the baseline values are changed as recursive estimations are performed.

Table A3.1 shows the results of the first robustness check. The first column is obtained if the baseline values are assumed to be equal to the median of $Y_t, K_t, L_t$ and $t$ in the full sample. The second (third) column is acquired if $\bar{Y}, \bar{K}, \bar{L}$, and $\bar{t}$ are averaged out using 1991-2003 (2004-2016) period. Intuitively, the approach in second (third) in column imposes the assumption that the first (last) 13 years of the sample gives the “long-run” values of the economy, which sounds less convincing compared to selecting a benchmark point using the full sample information. The first column replicates the results in Table 1. Therefore, as long as we use full sample data, the results are insensitive to the choice of baseline points. In second and third columns, the most noticeable changes in the parameter estimates are seen in $\gamma_L$ and $\gamma_K$. Other parameter estimates stay broadly the same. $\hat{\gamma}_L$ and $\hat{\gamma}_K$ differs from the results shown in Table 1 due to the change in $\bar{t}$. The changes in these parameters yield differences in the productivity levels, yet they do not induce a change in the productivity growth of inputs (Graphs A3.1-A3.4) However, as the TFP growth depends on both the level of and growth in productivity of inputs, it exhibits minor shifts (Graphs A3.5, A3.6).

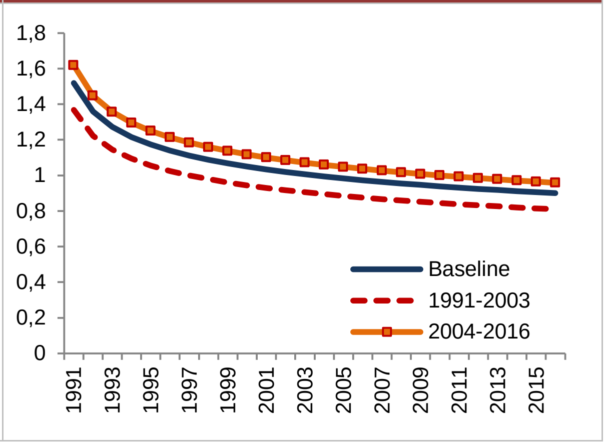
Graphs A3.7-A3.13 show the recursive parameter estimates when the sample size is incrementally extended. In this approach, at each recursive estimation, new baseline points are calculated using definitions stated in Section 3. Due to the small sample size, only 6 recursive estimations are conducted, the first one covers the 1991-2010 period. The estimates of $\bar{\pi}$, $\mu$, and $A$ are fairly constant. $\hat{\sigma}$ seems to be higher at the beginning of the sample. $\hat{\gamma}_L$ and $\hat{\gamma}_K$ seems to converge to their long-run values relatively quickly. The least robust estimate is $\hat{\lambda}_L$, as both the estimated value and uncertainty around it increases through the end of the sample. Though the analysis would be more informative if we could perform it with a larger sample, the overall picture presents some evidence in favor of robustness.

| Table A3.1: Robustness checks of the supply-side system estimation for Turkey | | | |
|---|---|---|---|
| | **Baseline values are the median values in 1991-2016** | **Baseline values are averaged out using 1991-2003** | **Baseline values are averaged out using 2004-2016** |
| $\bar{\pi}$ | 0.495*** (0.004) | 0.488*** (0.004) | 0.488*** (0.004) |
| $\mu$ | 0.363*** (0.026) | 0.363*** (0.026) | 0.363*** (0.026) |
| $\sigma$ | 0.639*** (0.043) | 0.639*** (0.043) | 0.639*** (0.043) |
| $A$ | 0.975*** (0.008) | 1.025*** (0.008) | 0.964*** (0.008) |
| $\gamma_L$ | 0.023*** (0.001) | 0.028*** (0.002) | 0.021*** (0.002) |
| $\lambda_L$ | 0.713*** (0.074) | 0.713*** (0.074) | 0.713*** (0.074) |
| $\gamma_K$ | -0.012*** (0.001) | -0.023*** (0.002) | -0.008*** (0.001) |
| TFP growth | 0.004 | 0.003 | 0.006 |
| | **p-values of restrictions** | | |
| $\sigma = 1$ | 0.00 | 0.00 | 0.00 |
| *Notes:* Standard errors in parenthesis. */**/*** show the 10, 5 and 1 percent significance level, respectively. TFP growth is calculated as an average using equation (8). | | | |

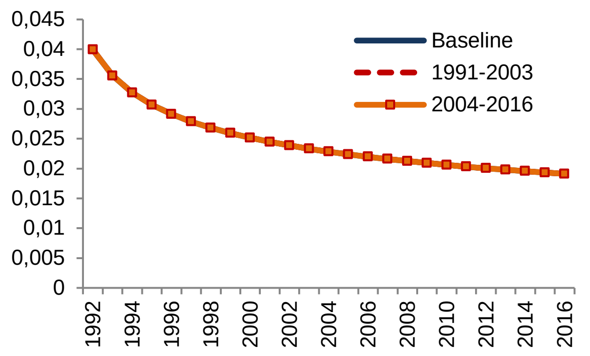
Graph A3.1: L productivity level



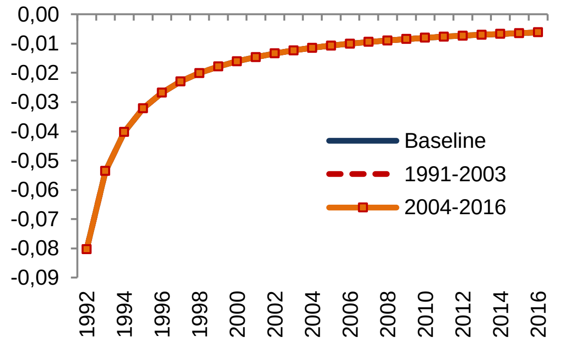

Graph A3.2: K productivity level



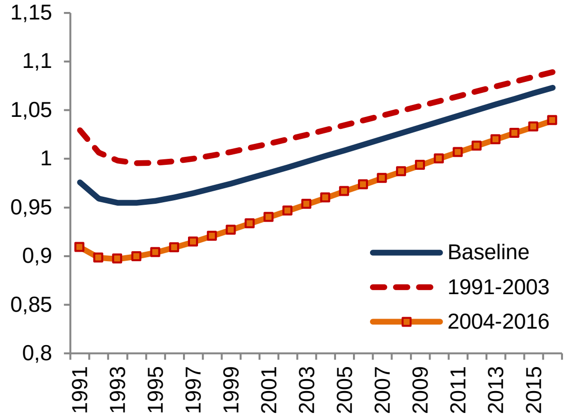
Graph A3.3: L productivity growth



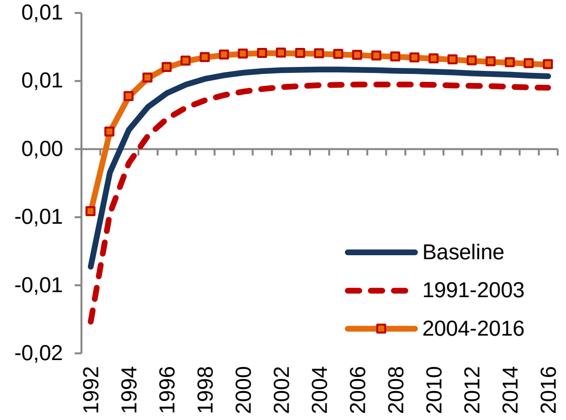

Graph A3.4: K productivity growth



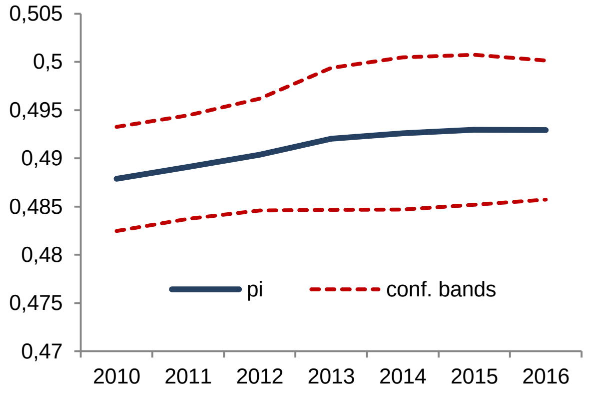

Graph A3.5: TFP level



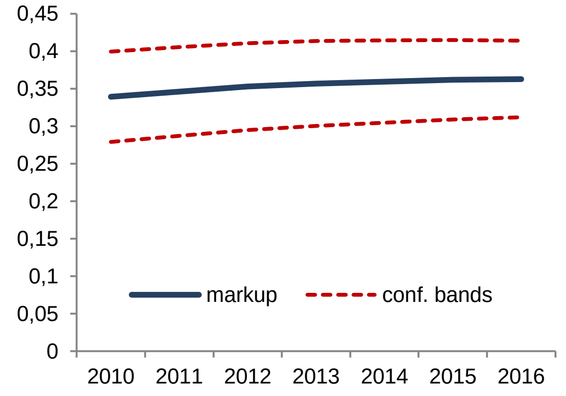

Graph A3.6: TFP growth



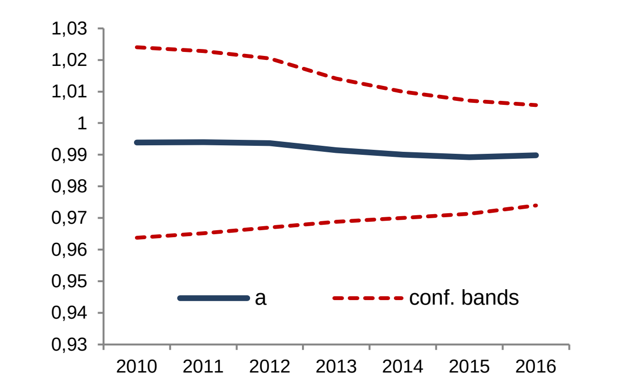

Graph A3.7: $\overline{\pi}$ in recursive estimations



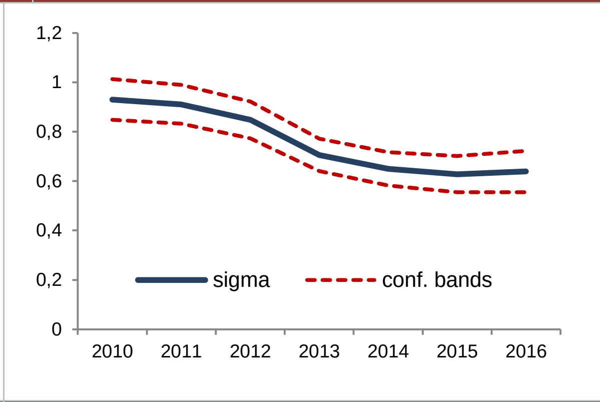

Graph A3.8: $\mu$ in recursive estimations



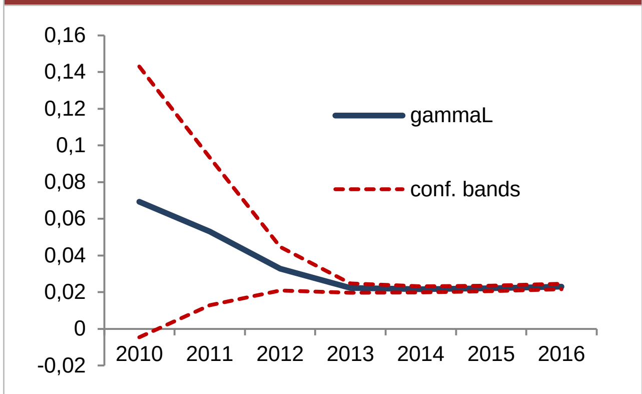
Graph A3.9: $A$ in recursive estimations



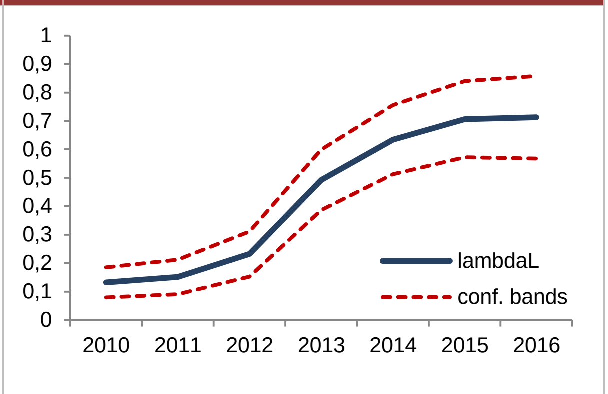

Graph A3.10: $\sigma$ in recursive estimations



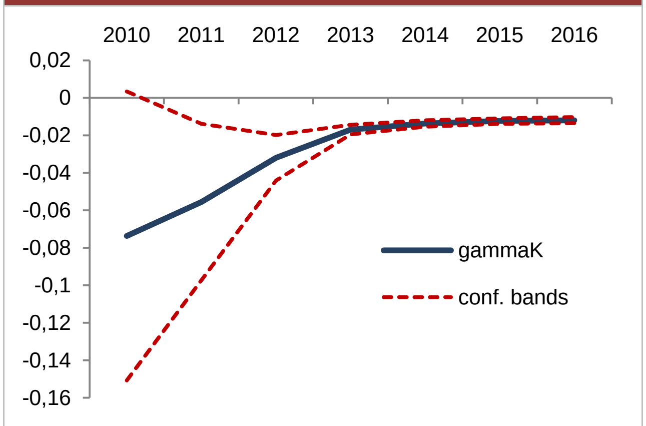

Graph A3.11: $\gamma_L$ in recursive estimations




Graph A3.12: $\lambda_L$ in recursive estimations




Graph A3.13: $\gamma_K$ in recursive estimations



## Appendix 4

A rearrangement of equation (9) can give the “pure” contributions labor and capital to output growth. However, this rearrangement will also include another part showing the contribution of capital intensity, i.e., capital per worker:

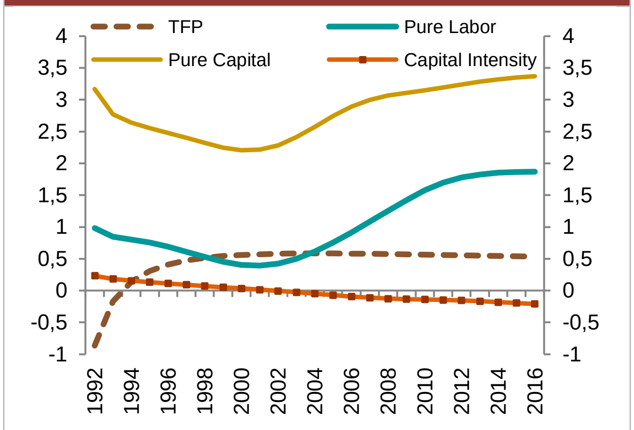
$$\frac{\dot{Y}_t}{Y_t} \approx \frac{T\dot{F}P_t}{TFP_t} + \underbrace{\bar{\pi}\frac{\dot{K}_t}{K_t}}_{pure\ contribution\ of\ capital} + \underbrace{(1-\bar{\pi})\frac{\dot{L}_t}{L_t}}_{pure\ contribution\ of\ labor} + \underbrace{\left(\frac{\sigma-1}{\sigma}\right)\bar{\pi}(1-\bar{\pi})\left(\frac{\dot{K}_t}{K_t}-\frac{\dot{L}_t}{L_t}\right)\left[ln\left(\frac{K_t}{L_t}\right)-ln\left(\frac{\bar{K}}{\bar{L}}\right)\right]}_{contribution\ of\ capital\ intensity}. \quad (12)$$

Equation (12) provides valuable information on the role of capital intensity. If $\sigma < 1$ and growth in capital is greater than growth in labor, i.e., $\left(\frac{\dot{K}_t}{K_t}-\frac{\dot{L}_t}{L_t}\right) > 0$, then a high level of capital per worker will yield a negative contribution from capital intensity to growth.

When the elasticity of substitution is significantly different than one, equation (12) implies that the “Solow residual” of the Cobb-Douglas function will both include the contribution of productivity and capital intensity. That is to say, when $\sigma \neq 1$, but we insist on using Cobb-Douglas, what we call as TFP is not “pure” TFP.

We can decompose potential growth into its components employing equation (12). As in Section 7, we compute potential growth in $L$ and $K$ using HP trends of labor and capital, respectively. $\sigma$ is estimated as 0.6 in Turkey. Capital accumulation is stronger than growth in employment. Capital per worker is currently higher than its historical averages. Hence, the contribution of capital intensity to potential growth is slightly negative (Graph A4.1).

Graph A4.1: Contributions to potential growth with equation (12), pp, 2009 based GDP




Source: Author’s calculations.

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