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Interest Rate Fluctuations and Equilibrium in the Housing Market

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Interest Rate Fluctuations and Equilibrium in the Housing Market*

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Abstract

I study the general equilibrium of the housing market in an economy populated by overlapping generations of households. A contribution of the present paper is to solve for the housing market equilibrium in the presence of aggregate (interest rate) uncertainty with a realistic mortgage contract. In addition, households also face idiosyncratic uncertainty resulting from stochastic changes over the lifecycle in tastes (or need) for housing. In this environment, profit-maximizing banks offer fixed-rate mortgage (FRM) contracts to homebuyers. As seems plausible, each housing market transaction is subject to a fixed cost, which gives rise to S-s policy rules for housing transactions: existing homeowners change the size of their houses only if there is a sufficiently large change in the state of the economy (i.e., in interest rates, in their taste for housing, etc.). A plausibly calibrated version of the model is consistent with three empirically documented features of the housing market: (i) highly volatile housing prices and transaction volume, (ii) a strong positive correlation between transaction volume and housing prices, and (iii) a significant negative relationship between interest rates and housing prices, which can rationalize a large part of the recent boom in housing prices in the U.S. and around the world.

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1 Introduction

Until recently, mainly due to technical difficulties, the dynamic macroeconomics literature generally abstained from a detailed modeling of housing and mortgage markets in their frameworks. As is well known, the rigidities in the housing market, together with the complicated structure of realistic mortgage contracts, make the computation of those models cumbersome. This paper contributes to the dynamic macroeconomics literature by providing and solving a dynamic macro model with such features.

In particular, I study the effects of interest rate fluctuations on the equilibrium of the housing market in an economy populated by overlapping generations of households. I combine three realistic features of the housing market in the particular model studied here: first, I allow for stochastic fluctuations in mortgage interest rates; second, I model long-term mortgage contracts explicitly; and third, I model housing as an illiquid asset that is also consumed.

I show that the model is quantitatively successful in matching several important features of the housing market that have proved hard to match in the earlier literature. The first one of these features is the high volatility of housing prices compared to most of the other macroeconomic variables such as GDP.[1] This feature was especially pronounced in large cities: for example, in places like Boston, New York, San Francisco, and Los Angeles, the volatility of real housing prices is more than the national average (see Table 1, Figures 1 and 2.) In addition to housing prices, another highly followed statistic of the housing market is the transaction volume, which I define as the number of existing homes sold in a given time period. Similar to housing prices, transaction volume is highly volatile, as can be seen in Table 1 and Figure 3. Furthermore, the movements in transaction volume appear to be closely associated with the movements in housing prices: the correlation between the two series is 0.71 in the national US data (Figure 4), and as shown by Genesove and Mayer (2001) the comovement is even more pronounced in local housing markets.

Clearly, this paper is not the first to observe that (mortgage) interest rates are likely to have an important impact on the housing market. Nevertheless, the difficulty of solving a model with interest rate uncertainty and realistic mortgage contracts (as well as some other key features of housing market transactions, such as fixed costs, which I consider here) has previously prevented a quantitative assessment of the channels explored here.

Using a plausibly calibrated version of the model, I find that housing prices are quite sensitive to interest rates: a 1-percentage-point decline in interest rates causes a 5 percent increase in real median housing prices. An interesting implication of the model is that current interest rates affect not only current housing prices but also the next period's housing prices. Higher interest rates in the current period induce young agents to save more, which increases the total wealth in the next period. Since wealthier agents will demand more housing, prices will go up in the next period.

[1] See Rios-Rull, J. and Sanchez-Marcos, V.(2008).

Quantitatively, if current interest rates increase 1 percent, housing prices will increase around 4 percent in the next period.

In the model, transaction volume also varies significantly with interest rates. With lower interest rates in the current period, homeowners are more willing to sell their houses, since the mortgage contract they got in the previous period is relatively more costly to hold. With a similar intuition, if homebuyers purchase their home with low interest rates in the current period, they will be less likely to sell their houses in the next period, because they will already be holding a good contract. The model predicts that, on average, a 1 percent decline in the current interest rate causes a 3 percent increase in transaction volume in the current period and a 5 percent decrease in transaction volume in the next period. The combination of the two effects further implies that transaction volume will decline when interest rates rise.

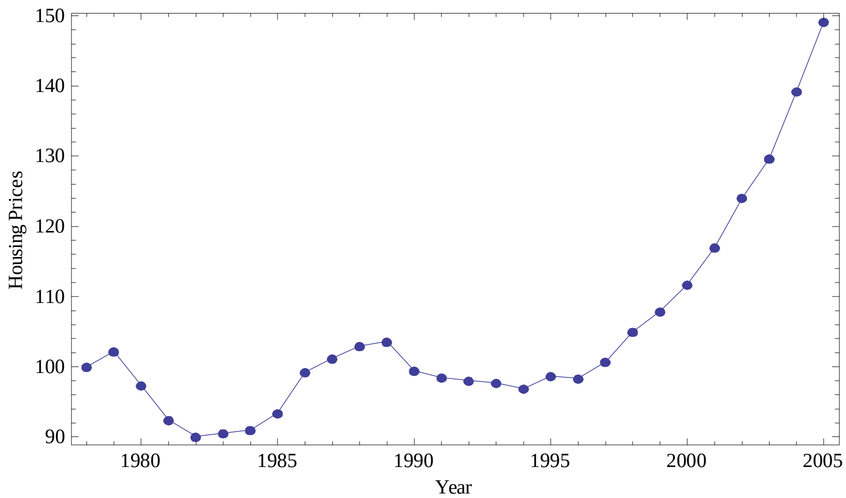
Consistent with the US data, transaction volume and housing prices implied by the model comove with a large positive correlation (around 0.8) as interest rates affect both variables similarly. Moreover, although I abstract from the sources of dispersion other than interest rates, the dispersions of housing prices and transaction volume—measured by the coefficient of variation—are very close to the dispersions observed in the US data.

The model predicts that the welfare effects of housing price movements differ across agents. In the case of a housing price increase, an old agent who downsizes his house will be better off, since his return from the housing transaction will be higher (due to the difference in the size of the houses.) Old agents who move to bigger houses and young agents will be worse off.

In the earlier general equilibrium models of housing market, such as Heathcote and Davis (2005) and Iacoviello (2005), the volatility of housing prices was much lower compared to the data. Recently, Iacoviello and Neri (2010) obtain reasonable housing price volatility compared to the data but most of the volatility comes from the exogenous preference shifts toward housing. Although it has been hard to generate high housing price response to interest rates in general equilibrium models, it has been empirically shown by Himmelberg, Mayer and Sinai (2005) that housing prices respond strongly to interest rate fluctuations. One of the goals of this paper is to fill the gap between the empirical estimates and the general equilibrium models.

The comovement of housing prices and transaction volume, which contradicts the efficient market prediction of Lucas (1978), has attracted significant attention in the literature. Stein (1995) and Ortalo-Magne and Rady (2004) explain the comovement by using credit constraints and large down payment requirements. Wheaton (1990) and Berkovec and Goodman (1996) use search models to explain the comovement. Using a data set for the Boston condominium market, Genesove and Mayer (2001) argue that loss aversion can explain price-volume correlation. This paper contributes to the literature by showing that mortgage contracts offer another mechanism that can generate the comovement of the two series. Besides, the model of this paper successfully matches the volatility of the transaction volume, while the studies mentioned are silent about it.

Figure 1: Real, Median Housing Prices, US, in 2004 Dollars. Source: National Association of Realtors.



The paper is organized as follows. The next section documents some observations on the housing market. Section 3 describes the model. In Section 4, I outline the algorithm that I use to solve the model. I do the calibration in Section 5. The main results of the paper are reported in Section 6. In Section 7, I look at the effect of housing prices movements on the welfare of consumers. In Section 8, I conclude.

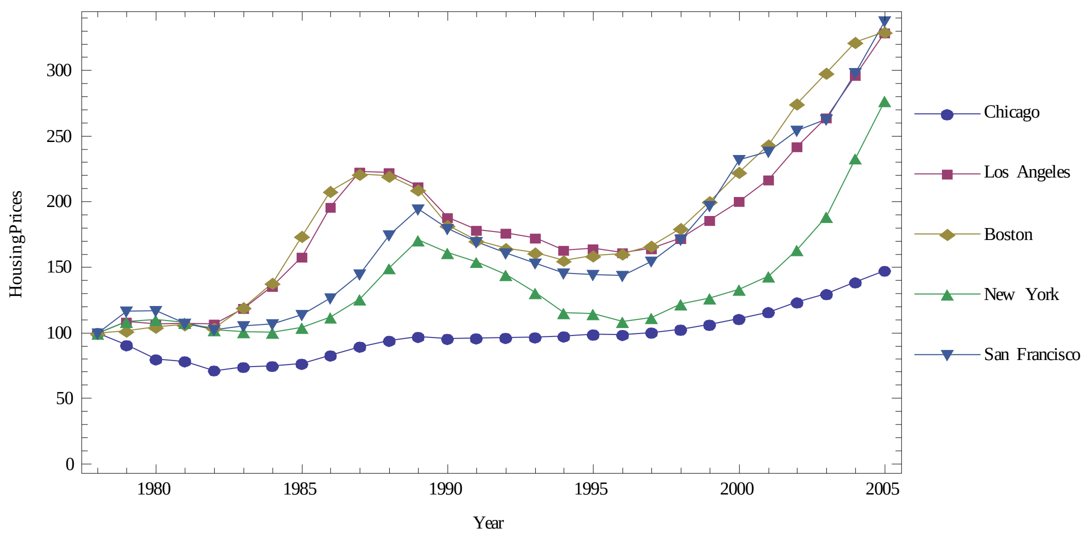
2 Some Features of the Housing Market

The Federal Housing Finance Agency (FHFA), National Association of Realtors (NAR), S&P/Case-Shiller and Census Bureau are the major sources of housing price data in the US. McCarthy and Peach (2006) show that the Census constant quality index consistently predicts less housing price appreciation than the other two. The NAR index and the FHFA index have very close housing price predictions. But the bottom line is that all of them predict that housing prices have been rising exceptionally from 1994 to 2005. Here I present the housing price index from the NAR, after adjusting for inflation.[2] Figure 1 shows that the real median price of a house in the US has increased more than 50 percent in the last 20 years. Housing prices have been increasing more than the national average for most big cities. The median price in New York, San Francisco, Boston and Los Angeles almost doubled from 1995 to 2005 (Figure 2).[3]

[2] I use CPI-less shelter as the deflator.

[3] In the model I assume that housing supply is fixed. This assumption makes the model findings more comparable to the data for large cities.

Figure 2: Real, Median, City Level Housing Prices, in 2004 Dollars. Source: National Association of Realtors.



Transaction volume (number of existing units sold) has been increasing steadily during the last three decades other than the sharp decline around the 1980s. After that decline, transaction volume has more than doubled in the last two decades (Figure 3).[4] The data include existing detached single-family homes and townhomes but does not include condos and co-ops.

An interesting and also puzzling feature of the housing market is the high positive correlation between transaction volume and housing prices (the correlation coefficient is around 0.72 for the national data). In Figure 4 I plot the HP-filtered housing prices and transaction volume series divided by their respective standard deviations. The figure shows the strong comovement of the two series.

Housing prices and transaction volume have been highly volatile. Genesove and Mayer (2001) report that in a real estate cycle, from peak to trough, it is not unusual to see a 50 percent decline in transaction volume, especially in local markets. Rios-Rull and Sanchez-Marcos (2008) show that housing prices and transaction volume are much more volatile than GDP in both Canada and the US. In Table 1 I report the dispersion of some housing market variables measured by the coefficient of variation. Housing prices in big cities are more dispersed than national prices and transaction volume is more dispersed than housing prices.

[4] I control for the increase in the number of households in the US. The data for the number of households can be obtained at http://www.census.gov/population/www/socdemo/hh-fam.html#ht (Table HH-4)

Figure 3: Transaction Volume (Number of Existing Houses Sold), US (Yearly). Source: Bloomberg.

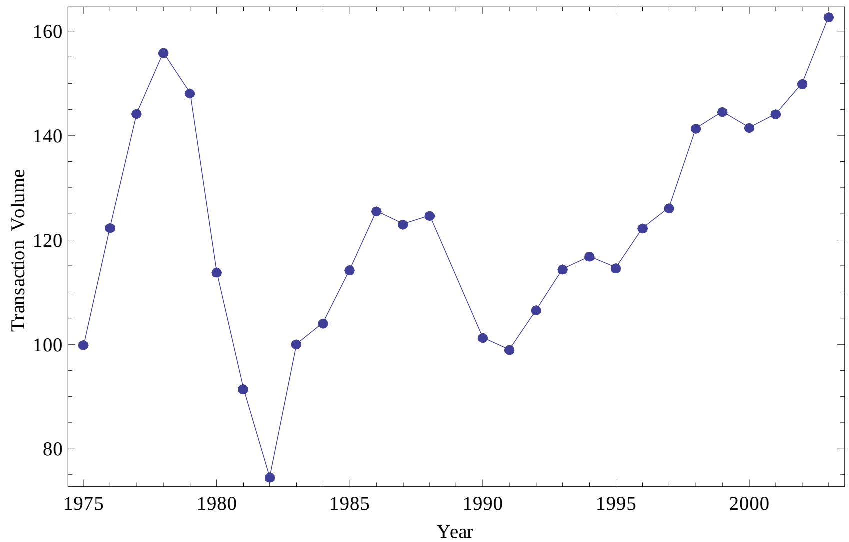


Figure 4: Comovement of Housing Prices and Transaction Volume (HP filtered and standardized). Correlation is 0.72.

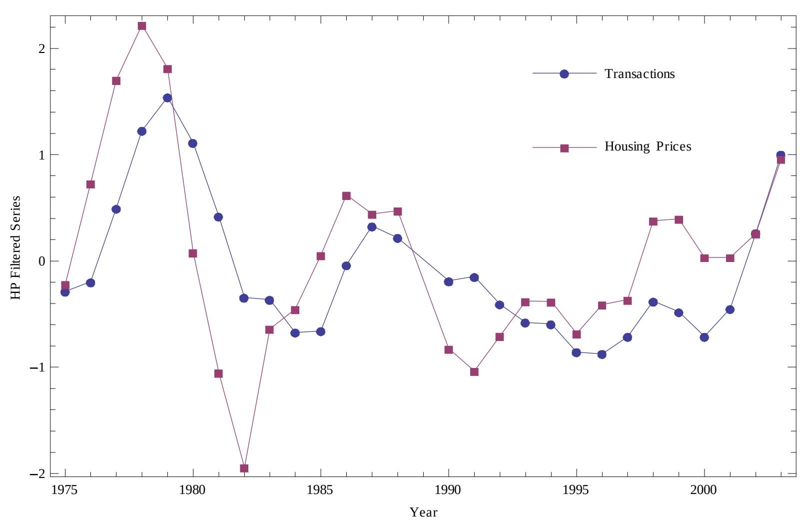


Table 1: Housing Market Statistics

	Data
CV(housing price, national)	0.08
CV(housing price, big cities)	0.18
CV(transaction volume, national)	0.15

Note.– CV is the coefficient of variation calculated as the standard deviation divided by the mean. All series are detrended.

3 The Model

The model is a two-period overlapping generations model.[5] At the beginning of each period, the economy is hit by an exogenous interest rate shock. A young agent enters the economy with a limited amount of resources, which makes him borrow in the mortgage market to finance his housing purchase. After he buys his house, he receives his remaining income, saves in the bond market and consumes the non-housing consumption good. At the end of the period he makes the first mortgage payment. Old agents receive idiosyncratic housing taste shocks, which can be interpreted as changes in family size or changes in the need for housing, causing optimal housing to be different from the one obtained when young. Since there are fixed transaction costs, some of the agents will choose to sell their existing houses and some will choose to stay. If an old agent sells his house, he gets the return from selling the house and pays the remaining mortgage debt from the previous house. Then he borrows in the mortgage market to finance his new house. If he does not sell his house, he makes the second mortgage payment and uses the remaining income to purchase non-housing consumption. At the end of their lives, all agents sell their houses and leave the economy. There are risk-neutral profit-maximizing banks that offer mortgage contracts. In equilibrium, banks are indifferent between lending in the bond market and the mortgage market. For simplicity, I assume that aggregate house supply is fixed and there is no rental market.

Mortgage Market

Risk-neutral profit-maximizing banks offer fixed-rate 30-year mortgage contracts. I have two conditions to find the mortgage rates and the corresponding payments. The first condition is the *present value condition*, which means that the present value of the payments should be equal to the loan amount.

$$1 = \frac{D_t}{1 + d_t} + \frac{D_t}{(1 + d_t)^2} \tag{1}$$

An agent should pay D_t if he borrows 1 dollar in the mortgage market when the mortgage rate is

[5] In this section I present the two-period version of the model for clarity. In Section 5 I extend it to 30 periods to have a more realistic calibration.

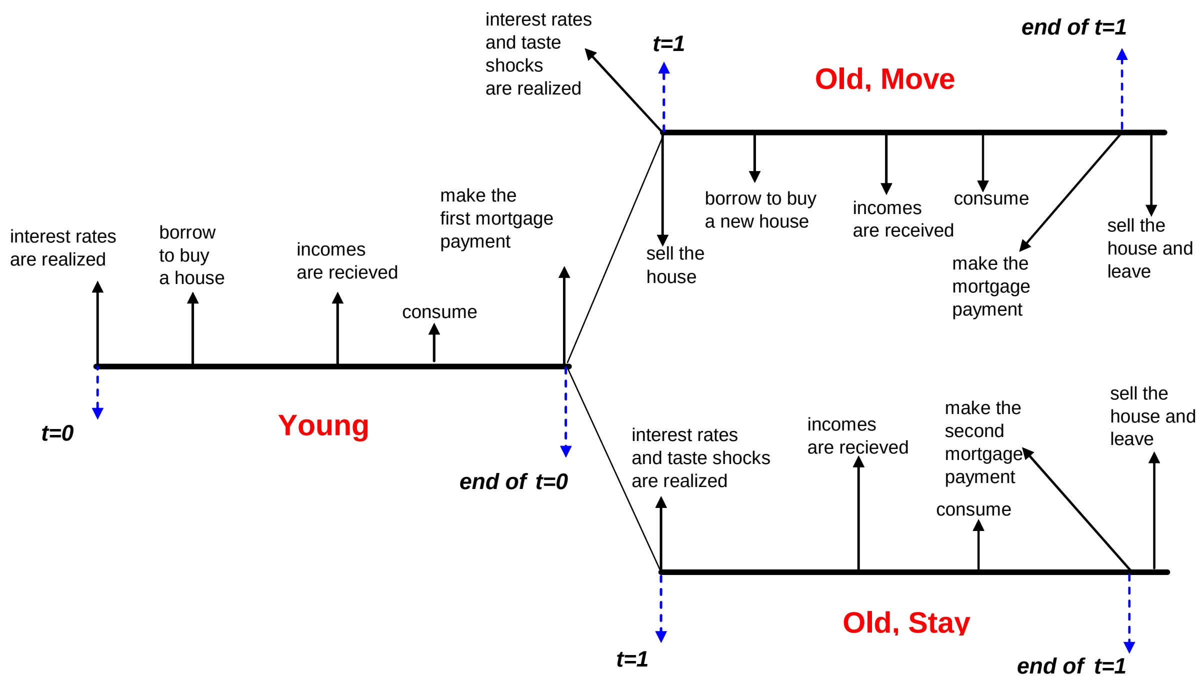


Figure 5: Timing of Events

d_t. The next condition is the *no-arbitrage condition*. In equilibrium, banks should be indifferent between lending in the bond market and lending in the mortgage market.

$$1 = \frac{D_t}{1+r_t} + (1-\pi_t)\frac{D_t}{(1+r_t)E[(1+r_{t+1})]} + \pi_t\frac{1-D_t+d_t}{1+r_t} \tag{2}$$

The lender will receive the first mortgage payment D_t at the end of the first period, and he will discount this payment with the current interest rate, $1+r_t$.[6] With probability $1-\pi_t$ the agent will not move and he will make his second mortgage payment, which has a present value of $\frac{D_t}{(1+r_t)E[(1+r_{t+1})]}$. With π_t probability the agent will move and prepay his loan by the beginning of the second period. The discounted value of this payment is $\frac{1-D_t+d_t}{1+r_t}$. In the solution of the model, I should also find D_t, d_t and π_t as functions of state variables. The old agents who buy a house get a 15-year mortgage contract, and the interest on the mortgage is the same as the interest rate on the bond.

Old Agent's Problem

An old agent decides whether or not to move and, if he moves, how big a house to purchase depending on the taste shock θ_h he received and the state of the market. The maximization problem

[6] Since the agents in the model can transact only after 15 years, the corresponding discount factor will be "15 to the power of *yearly interest rate*."

is,

$$V_{old}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = Max\left\{V_{move}, V_{stay}\right\}. \tag{3}$$

The aggregate state of the economy is summarized by $P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t$. The price ($P_{t-1}$), the quantity ($H_{t-1}$), and the interest rate (r_{t-1}) at $t-1$ are all effective for the old agents' decisions today. I use $t-1$, because it is the time when they first bought their houses. The aggregate amount of housing held by the oldest generation which is leaving the economy is H_{t-2}.[7] The current period interest rate is denoted as r_t. The quantity of housing of an old agent is h_{t-1}, and the savings that that he/she brings from the previous period is s_{t-1}.[8] The value of moving can be formulated as,

$$V_{move}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = \underset{h_{t,o,m}}{Max}\left\{U(c_{t,o,m}, h_{t,o,m}) + \beta E[U(c_{t,o,l}, 0)]\right\}. \tag{4}$$

In equation 4, $c_{t,o,m}$ is the consumption and $h_{t,o,m}$ is the housing choice of an old moving agent. I assume that when agents leave the economy, they sell their houses and use the returns from the transaction to purchase non-housing consumption good, $c_{t,o,l}$ (no housing for the leavers).[9] The value of the houses to the agents when they leave the economy is then $\beta E[U(c_{t,o,l}, 0)]$.

The utility function is quadratic in both consumption and housing. By the quadratic utility function, I am able to find most of the decision rules and all of the value functions analytically which simplifies the computation significantly.

$$U(c_{t,o,m}, h_{t,o,m}) = -\alpha_c/2(\theta_c - c_{t,o,m})^2 - \alpha_h/2(\theta_h - h_{t,o,m})^2 \tag{5}$$

The parameters of the utility function of an old agent are $\alpha_c, \theta_c, \alpha_h, \theta_h$. The taste shock that an old agent receives at the beginning of the period is θ_h and it is distributed uniformly with a lower bound $\theta_{\min}$ and a higher bound $\theta_{\max}$. I assumed a taste shock in this form because for convenience.

$$\begin{aligned} c_{t,o,m} &= w_{old} + s_{t-1}(1+r_{t-1}) - \lambda P_t h_{t,o,m} - \frac{(1-\lambda)P_t h_{t,o,m}(1+r_t)}{1+r_t} + \\ &\quad P_t h_{t-1} - (1-\lambda)P_{t-1}h_{t-1}(1-D_{t-1}+d_{t-1}) - \Delta \end{aligned} \tag{6}$$

The consumption of an old, moving agent is formulated in equation 6. If an old agent chooses to move, first he will sell his old house, then pay off the remaining mortgage debt. The seller will receive

[7] In this two-period set-up I do not need the total housing of the leaver, H_{t-2}, as a state variable, since it can be calculated by using aggregate supply and H_{t-1}. But when I make the model thirty periods in the calibration, I will need to know the quantity of the housing held by the generation that leaves the economy.

[8] In the equilibrium, the housing held by an old agent at the beginning of the period, h_{t-1}, and the aggregate amount of housing held by the old agents at the beginning of the period, H_{t-1}, will be equal to each other.

[9] Allowing agents to sell their houses at the end of their lives gives an additional wealth motive to homebuyers.

$P_t h_{t-1}$ as the price. The remaining debt from the old mortgage is $(1-\lambda)P_{t-1}h_{t-1}(1-D_{t-1}+d_{t-1})$.[10] Once he is done with the old house, he purchases a new house by borrowing in the mortgage market. His mortgage payment at the end of the period is $(1-\lambda)P_t h_{t,o,m}(1+r_t)$. I discount this payment with $1+r_t$ to measure the real cost of mortgage payment to the agent. The buying process has a fixed transaction cost of Δ and agents should make a down payment equivalent to $\lambda P_t h_{t,o,m}$, where λ is the down payment ratio. In equation 6, w_{old} is the old agent's income and $s_{t-1}(1+r_{t-1})$ is the return from savings made when young. The consumption of an agent who leaves the economy, $c_{t,o,l}$, is equal to his return from selling his house.

$$c_{t,o,l} = P_{t+1}h_{t,o,m} \tag{7}$$

The optimal housing decision of an old agent is,

$$\begin{aligned} h_{t,o,m} &= \frac{\alpha_c P_t(-\Delta + h_{t-1}[P_t + (-1+\lambda)P_{t-1}(1-D_{t-1}+d_{t-1})] + s_{t-1}r_{t-1} - \theta_c)}{\alpha_h + \alpha_c(P_t^2 + \beta E[P_{t+1}^2])} \\ &\quad + \frac{\alpha_h\theta_h + \beta\alpha_c\theta_c E[P_{t+1}]}{\alpha_h + \alpha_c(P_t^2 + \beta E[P_{t+1}^2])}. \end{aligned} \tag{8}$$

Although equation 8 seems complicated, there are intuitive comparative static results. For example, if the transaction cost Δ increases, $h_{t,o,m}$ will decrease. This is intuitive, since higher transaction costs will leave the moving agents with less wealth, and with less wealthy agents cannot afford larger houses. With a similar intuition, higher mortgage debt from the previous period decreases the optimal quantity of the mover's housing. If expected prices $E[P_{t+1}]$ increase, movers' optimal housing will increase, since the return to holding a larger house becomes larger. A higher taste shock θ_h implies a higher optimal quantity of housing. Keeping the expected price constant, if $E[P_{t+1}^2]$ increases, demand for housing decreases. Here, $E[P_{t+1}^2]$ can be thought of as a measure of variance of the price. As variance increases (if the mean stays the same) risk-averse agents will buy less housing.

Because of the existence of transaction costs, not all agents will transact. The value of staying in their old houses is,

$$V_{stay}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = U(c_{t,o,s}, h_{t,o,s}) + \beta E[U(c_{t,o,l}, 0)]. \tag{9}$$

$$c_{t,o,s} = w_{old} + s_{t-1}(1+r_{t-1}) - \frac{(1-\lambda)P_{t-1}h_{t-1}D_{t-1}}{1+r_t} \tag{10}$$

[10] The total payment he made is $(1-\lambda)P_{t-1}h_{t-1}D_{t-1}$, but $(1-\lambda)P_{t-1}h_{t-1}d_{t-1}$ of this payment is goes to the interest.

In equation 9, $c_{t,o,s}$ and $h_{t,o,s}$ are consumption and housing of an old agent if he stays in his old house. Before leaving the economy, old agents sell their houses and get $P_{t+1}h_{t,o,s}$ and use it to buy non-housing consumption good, $c_{t,o,l}$. If the agents stay, they will pay the remaining mortgage on their house, which is $(1-\lambda)P_{t-1}h_{t-1}D_{t-1,}$ and consume the rest.

$$h_{t,o,s} = h_{t-1} \tag{11}$$

Their housing will be the same housing they lived in during the previous 15 periods, h_{t-1}.

Given the state of the economy, there will be θ_{low} and θ_{high} such that the old agents who receive a taste shock θ in between will choose to stay, while the others will move (an S-s rule).[11] Since the old agents with taste parameters θ_{low} and θ_{high} should be indifferent between moving and staying, the value of θ_{low} and θ_{high} can be found by equating the value of staying to the value of moving.[12]

$$\theta_{low}(h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \tag{12}$$

$$\theta_{high}(h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \tag{13}$$

Once the boundaries of the inactivity region are found, it is straightforward to find the transaction volume implied by the model.

Young Agent's Problem

Young agents (no heterogeneity) enter the economy with a limited amount of resources. They borrow in the mortgage market to finance their housing purchases. Then they receive their income, save for the next period, consume and make their first mortgage payments. Young agents solve the following optimization problem,

$$\begin{gathered} V_{young}(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = \\ \max_{s_{t,y}, h_{t,y}} \left\{ \begin{array}{l} U_y(c_{t,y}, h_{t,y}) + \\ +\beta E\left[\int_{\theta_{\min}}^{\theta_{low}} V_{move} f(\theta) d\theta + \int_{\theta_{high}}^{\theta_{\max}} V_{move} f(\theta) d\theta \right] \\ +\beta E\left[\int_{\theta_{low}}^{\theta_{high}} V_{stay} f(\theta) d\theta \right] \end{array} \right\}. \end{gathered} \tag{14}$$

$$c_{t,y} = w_{young} - s_{t,y} - \lambda P_t h_{t,y} - \frac{(1-\lambda)P_t h_{t,y} D_t}{1+r_t} - \Delta \tag{15}$$

[11] The existence and uniqueness of the boundaries hold under very general conditions.
[12] I have the boundaries in closed form, but because of their complexity, I do not present them here.

In the utility function of the young agents, $c_{t,y}$ is consumption, $h_{t,y}$ is housing, and $s_{t,y}$ is saving at time t. Young agents income is w_{young}. They have a quadratic utility function, similar to the old's utility function, the difference being the value of θ_h. I assume,

$$U_y(c_{t,y}, h_{t,y}) = -\alpha_c/2(\theta_c - c_{t,y})^2 - \alpha_h/2(\theta_y - h_{t,y})^2. \tag{16}$$

While making their choices, they take into consideration the fact that it will affect their moving decisions when they are old. Here I know the boundaries in closed form, so I may find the integrals algebraically, which allows me to write the first-order condition of the young agents in closed form.

Market Clearing Condition[13]

There are two market clearing conditions that should be satisfied. The sum of the quantity of housing held by each generation should be equal to the aggregate house supply ($\overline{H_{tot}}$.) The first market clearing condition can be written as,

$$\overline{H_{tot}} = \sum_{i=0}^{1} \overline{H_{t-i}} \tag{17}$$

The total amount of housing owned by age i generation is $\overline{H_{t-i}}$. The market for available houses should also be cleared, since in each period only limited amount of housing will be available for trade. This is equivalent to,

$$\overline{H_{t-2}} + \overline{H_{t-1}} = H_{t,o,s} + H_{t,o,m} + H_{t,y} \tag{18}$$

The left-hand side of the equation is the total available housing for trade. H_{t-2} is the total housing owned by the leavers and H_{t-1} is the housing owned by period t old agents. On the right hand side of equation 18, $H_{t,o,s}$ is the amount of housing held by the old generation who did not move, and $H_{t,o,m}$ is the demand of the movers. The young agents' housing consumption, which I compute by using their optimization problem, is $H_{t,y}$.

$$H_{t,o,s} = \int_{\theta_{low}}^{\theta_{high}} h_{t-1} f(\theta) d\theta \tag{19}$$

$$H_{t,o,m} = \int_{\theta_{\min}}^{\theta_{low}} h_{t,o,m}(\theta) f(\theta) d\theta + \int_{\theta_{high}}^{\theta_{\max}} h_{t,o,m}(\theta) f(\theta) d\theta \tag{20}$$

[13]The market clearing conditions I use here are more general than I need for a two-period setting. But these are necessary when I modify the model while doing the calibration.

4 Computational Algorithm

I use an algorithm similar to the one introduced by Veraciarto (2002). In his paper he studies the implications of irreversibilities in investments for fluctuations in the business cycles. In his model, due to irreversibilities, S-s type decision rules arise, making the computation more difficult compared to the models without irreversibilities. Instead of using the usual state variables that would be implied by the model, he shows that a limited history of S-s bounds would be enough to have a sufficiently good approximation of the model. The same algorithm has been employed by Veraciarto (2008) where there are S-s type decisions in a different environment. The idea of tracking the limited history instead of the full state space have been utilized by other researchers in the literature as well. For example, Lustig and Chien (2010) track the recent history of the aggregate shocks and show that tracking 4 previous shocks in a yearly model gives precise results. Below I give the details of the algorithm I use in this paper. While the main idea of the algorithm that I use here is the same as in Veraciarto (2002, 2008), the methodologies differ. The particular methodology developed in this paper provides several practical advantages that can be employed in other dynamic macroeconomic models that suffer from the curse of dimensionality.

In my model relevant state variables for the young agents' problem are $P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t$.[14] In the model's formulation, I can state an endogenous variable, say, $h_{t,y}$, as,

$$h_{t,y}= f_h(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t)$$

where f_h is the policy function of the young agents' housing choice. Since P_{t-1}, H_{t-2} and H_{t-1} are also endogenous variables, they can be further written as functions of past realizations of state variables and past interest rate shocks as well.

$$H_{t-1} = f_h(P_{t-2}, H_{t-3}, H_{t-2}, r_{t-2}, r_{t-1})$$

$$H_{t-2} = f_h(P_{t-3}, H_{t-4}, H_{t-3}, r_{t-3}, r_{t-2})$$

$$P_{t-1} = f_p(P_{t-2}, H_{t-3}, H_{t-2}, r_{t-2}, r_{t-1})$$

Inserting the functions of P_{t-1}, H_{t-2} and H_{t-1} into the first equation yields $h_{t,y}$ as,

$$h_{t,y}= f_h(P_{t-3}, H_{t-4}, H_{t-3}, r_{t-3}, r_{t-2}, , r_{t-1}, r_t)$$

Recursive plugging of functions of past endogenous variables into the equation for the current period home bond price function enables me to obtain price as a function of current and past endowment shock realizations.

[14] Since I have the decision variables of the old agents in closed-form, I concentrate on the young agents' problem.

$$h_{t,y} = f(r_0, r_1, ...,, r_{t-1}, r_t)$$

Applying the same logic to the other endogenous variables makes it possible to use interest rate shocks as the sole argument of the functions defining all endogenous variables including prices. Putting it differently, observing the current and past endowment shocks makes it possible to derive current prices and choice variables without a need for any other state variables. Although it is theoretically possible to derive current period endogenous variables as a function of the past history of endowment shocks, it is computationally impossible and inefficient to solve for the whole history. Therefore, I suppose that agents are boundedly rational and they only use the information embedded in the recent history, which can be defined as the current and the recent lags of the exogenous shocks. Although the addition of further lags is always possible, after some history it increases the time and the memory required to come up with a numeric solution without making a significant contribution to the accuracy of the solution.

For the results of this paper I use the current interest rate and the past interest rate as the state variables where interest rates follow a 5-state Markov process. It is possible to increase the histories included in the computation by decreasing the number of states interest rates can take. I have solved the model for 3-state Markov processes with 4 histories and 2-state Markov processes with 8 histories. The main findings of the model do not change. To solve the model, I algebraically find the first order conditions and market clearing conditions for all of the possible states. Then, I use the sum of squared errors of the first order conditions and market clearing conditions across all states to define the objective function. Having obtained the objective function, I use both global and local minimization algorithms to solve for prices and allocations that minimize the objective function. Simulations of the first order condition and the market clearing conditions are presented in Figure 6 in the Appendix.

The algorithm described above has two advantages that can also be used in other macro models. An important advantage of the algorithm is that adding additional endogenous state variables (a new type of asset, for example) would not be a big problem. It is because the new endogenous state variable will also be a function of the history of the aggregate shocks. Consequently, the curse of dimensionality, which is an important problem for many models, will not be a big problem with this algorithm. In addition, the algorithm does not require any functional form guesses for any endogenous variables, which may be time consuming. What is needed are good initial guesses (steady-state values can be a good starting point) for the optimization algorithm to start with. A disadvantage of this algorithm is that increasing the number of values that aggregate shocks can get increases the state space exponentially.

5 Calibration

Although I have most of the decision rules in closed-form, I cannot solve housing prices and the young agent's housing decision analytically. For those, I need to calibrate the model to have a numerical solution comparable to the US data. If they are available, I take the parameter values directly from the US data. For the other ones, I pick some target statistics from the US data and try to match them with the model's predictions by choosing the free parameter values.

In the two-period model described above, there are two generations in the economy and interest rates shocks hit the economy in each period. If I were to use the two-period model for the quantitative exercise and use a 30-year mortgage contract, the period length in the model would correspond to 15 years. Then I would have to calibrate an interest rate process that hits the economy at the beginning of the period and stays the same for 15 years. In that case, whatever the Markov process is, interest rates would be extremely persistent. Another downside of the two-period model is that it is possible to obtain only 15-year aggregate statistics, which would be hard to compare with the data. Hence it is necessary to extend the model without complicating the numerical exercise.

In the extended model that I use for the numerical exercises, I assume that the period length is a year, which enables a realistic calibration of the interest rate process. Each year, a new interest rate shocks hits the economy, a new generation enters the economy and the oldest generation leaves the economy.

In particular, each year a new generation enters the economy and stays in the economy for 30 years. Not to complicate the computational exercise further, I assume that each generation makes consumption, saving and housing choices at the beginning, at the middle (after 15 years), and at the end of their lives (after 30 years). To clarify the model, suppose that we are at time t. At time t, the newborn generation that enters the economy will choose its savings and housing. In addition to the newborn, the generation which entered the economy at t-15 will adjust its saving and will transact to sell and buy housing. The generation that was born at t-30 will sell its houses and leave the economy. The other generations are not allowed to transact. At time t+1, the generations born at t+1, t-14 and t-29 will be transacting. As a result of this simple extension, it is possible to obtain yearly statistics from the extended model where the interest rate process is realistically calibrated from the data.

Normalizations

Housing supply in the model is set to 30 to allow each generation to hold 1 unit of housing on average. I set $\alpha_h = 2$.

Parameters Taken from the Literature

The yearly discount factor, β, is 0.96. The down payment ratio, λ, is 0.2 (Campbell and Cocco (2003).) The fixed transaction cost, Δ, is set to 0.015. This value corresponds to approximately 8 percent of the value of the house purchased, which is in the range of reported values (Smith, Rosen, and Fallis (1988).) I assume that each generation has 1 unit of income ($\frac{w_{young}+w_{old}}{2} = 1$). The income growth of households from young age to middle age (15 years) is calibrated to be around 50 percent (US Census Bureau.)

Statistics used for calibration

I use the following 5 target statistics to calibrate the remaining 5 parameters; θ_y, θ_c, α_c, $\theta_{\min}$, $\theta_{\max}$: (i) in the literature the coefficient of relative risk aversion is widely assumed to be between 1 and 5 [15], (ii) the national average of housing price to income is around 2.6 (Joint Center for Housing Studies of Harvard University, Table A-1, since I have 15 years of income, the adjusted target price to income ratio is 0.22), (iii) approximately, the aggregate housing wealth of people between ages 45-54 is 25 percent higher than the aggregate housing wealth of people between ages 35-44 (Hurst, Luoh, Stafford, and Gale (1998), Table 1.2,) (iv) PSID data show that 5.44 percent of the respondents moved during the last year (Cocco (2005), the corresponding number for 15 years would be 56 percent), (v) Hanushek and Quigley (1980) show that price elasticity of housing is around -0.7.

Interest Rates

I assume the stochastic process in the model is the real interest rate on a 15-year fixed-rate mortgage. This fits into my model, because when old agents borrow in the market, I assume they can get loans with the period interest rate. In reality it will correspond to a 15-year mortgage rate. To find the real rate on a 15-year fixed-rate mortgage, I simply subtract the rate of inflation in that year from the nominal rate.[16] The average of real return on 15-year fixed-rate mortgages during 1975-2005 period is 5 percent. Then I estimate an AR(1) process of these real rates, which yields an autocorrelation coefficient of 0.88. Finally, I use Hussey and Tauchen's (1992) method to approximate the AR(1) process with a 5-state Markov chain. The coefficient of variation of real 15-year fixed-rate mortgage is around 0.5. Because of my computational algorithm,[17] I cannot exactly match both the mean and the coefficient of variation at the same time. In the next sections I report the results for a 6 percent average interest rate with a 0.43 coefficient of variation. I also solve for different combinations of the mean and coefficient of variation and none of the results depend on the specific choice that is made here.

[15] With a quadratic utility function, relative risk aversion is not constant. As consumption changes, relative risk aversion also changes. I report average relative risk aversion that I obtain from the model simulations.

[16] This is equivalent to assuming, current period's inflation is the best predictor of future inflation.

[17] If I increase the coefficient of variation to 0.5, then I have boundary solutions. Even though the algorithm can handle boundary solutions for most of the states, for a small number of states it cannot, which decreases the accuracy of the solution.

Table 2: NORMALIZATIONS

Description	Parameter	Value
A parameter of the utility function	α_h	2.0
Aggregate housing supply	$\overline{H_{tot}}$	30

Table 3: PARAMETERS TAKEN FROM THE DATA AND LITERATURE

Description	Parameter	Value
Discount Factor	β	0.96
Down payment ratio	λ	0.2
Fixed transaction cost	Δ	0.015
Young's income	w_{young}	0.85
Old's income	w_{old}	1.15
Average of real interest rates (1975-2006)	μ	5%
Annual autocorrelation of interest rates (1975-2006)	ρ	0.88
Coefficient of variation of interest rates (1975-2006)	CV	0.5

Table 4: TARGETS

Statistics	Data	Model
The coefficient of relative risk aversion in consumption	1 to 5	1.5
Income to housing price ratio	0.22	0.29
(Old agents' housing wealth)-(Young agents' housing wealth)	25%	24%
Average transaction volume	56%	80%
Price elasticity of housing	~ -0.7	-0.7

Table 5: PARAMETERS CALIBRATED BY USING THE TARGETS

Description	Parameter	Model
A parameter of the young agent's utility function	θ_y	1
Lower bound of the utility function	$\theta_{\min}$	0.1
Upper bound of the utility function	$\theta_{\max}$	2.1
A parameter of the utility function	θ_c	1.4
A parameter of the utility function	α_c	3.0

6 Results

Before going into a more thorough analysis of the model's findings, I will demonstrate the model's response to an interest rate history similar to the history observed in the US. I assume the maximum value of the interest rate is 10 percent and the minimum value is 2 percent, whereas in the US, interest rates were as high as 15 percent in the early 80's and as low as 1.5 percent in 2005. With these values I can approximate the 1982-2005 period. Figure 6 plots the housing prices and transaction volume implied by the model.

The median housing price in the model increases around 50 percent. Remember that the decline in interest rates that I feed into the model is smaller than the one observed in the US. If I had the original history, the model would predict around 70 percent increase in the housing prices. I must caution that the way I calculated the real interest rates is the simplest way and may give different results than more sophisticated techniques. On the other hand, one important parameter in which I am more interested is the elasticity of housing prices with respect to interest rates and it does not change much with the specific interest rates.

Interestingly, the model predicts a price decline in the 90s. This prediction is also consistent with the US data. In the 90s, housing prices in most big cities as well as the US declined.[18] Transaction volume increases by around 25 percent in the model, which is lower than the observed increase in housing transactions presented in Figure 3. The increase in the transaction volume in Figure 3 includes first time homebuyers who switch from renting because of lower down payments and the attractiveness of the housing market, which I do not have in the model. Similar to housing prices, the model predicts the decline in transaction volume in the 90s. Finally, even in this short time period, it can be seen that housing prices and transaction volume move together.

6.1 Housing Prices

In this section, I show the relation between housing prices and interest rates. Figure 7 depicts housing price as a function of current interest rates where I assume that the last period interest rate is 6 percent (the results with other histories look very similar). On the y axis I have *Log* of the price and on the x axis I have *Log* of the current interest rate. Unit prices are higher than median prices because the size of the median house is almost always smaller than 1.

The slope of the graph gives the interest elasticity of housing prices. The elasticity is around -5, which means a 1-percentage-point decrease in the interest rates will cause prices to increase by 5 percent. Real interest rates decreased more than 10 percentage points during the last two decades.[19] In this case, the model predicts that housing prices would appreciate approximately

[18] I explain the reason for this prediction in the next section.

[19] The real interest rates in the model corresponds to the real interest rates on a 15-year FRM contract.

Figure 6: Time Series of the Interest Rates, Housing Prices, and Transaction Volume. The history of the interest rates is aproximated to the data. Median price is calculted by multiplying unit price by the young's housing. Transaction volume and housing price series are rescaled to approximately match their values at the starting date, 1982.

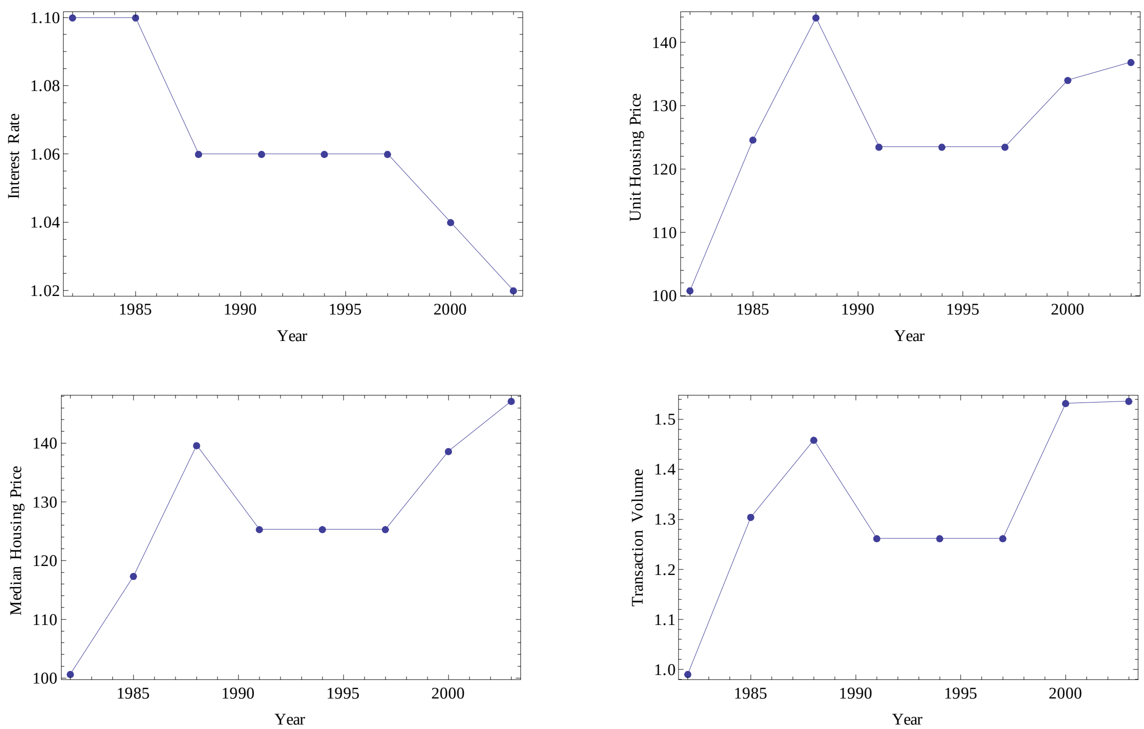
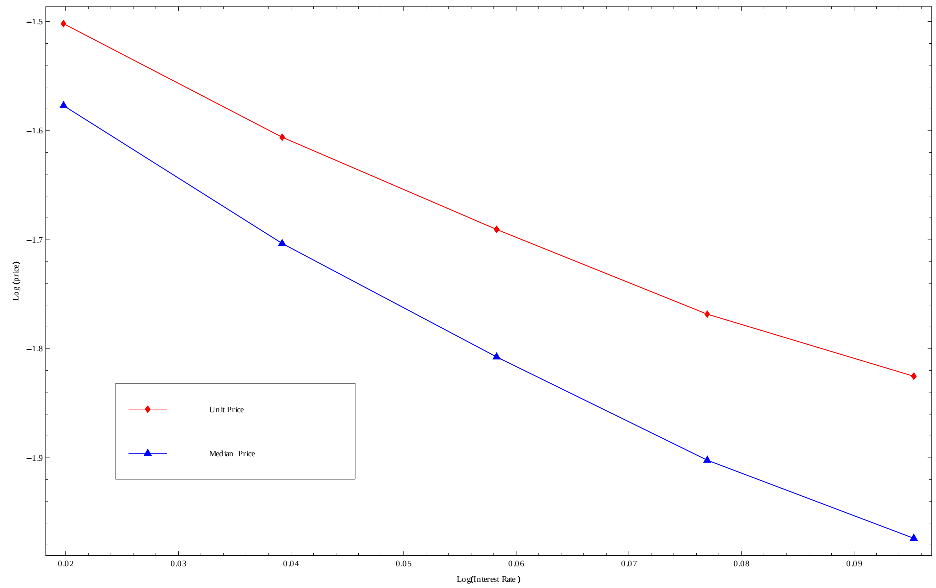


Figure 7: Housing Prices and Interest Rates. Median housing price is found by multiplying the young's housing by the unit price. Unit price is the price of 1 unit of housing. For clarity, last period interest rate is assumed to be 6%.

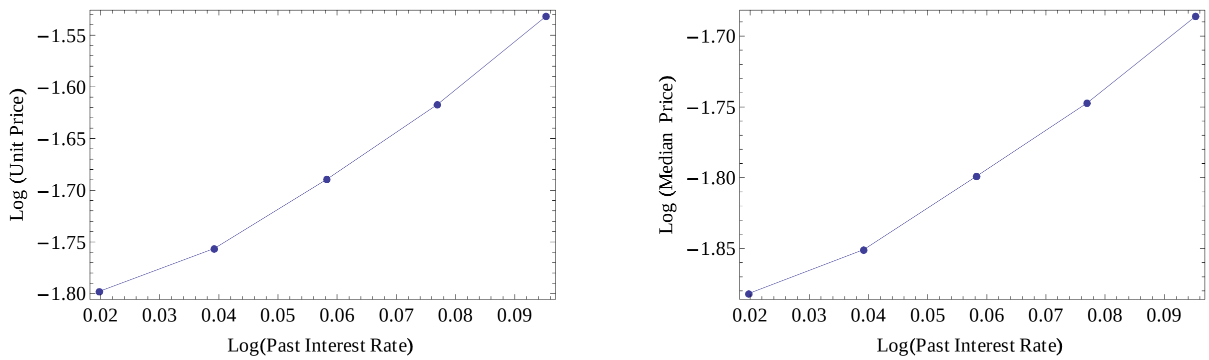


50 percent. This amount of housing price appreciation explains a large part of the housing price appreciation in big cities (it explains almost all of the increase in median national housing prices).

There are other factors that affect the current period housing prices. The biggest effect comes from the last period's interest rate, as it affects the savings of agents in the last period. If the interest rate during the last period was high, it means that the old agents will have more assets in the current period, which increases their willingness to pay higher housing prices. By this reasoning, high interest rates in the past will increase housing prices today. In this case, the interest (last period) elasticity of *unit* housing prices is 4.8. For median housing prices, the elasticity is smaller, 2.5. This is due to the change in the size of the median house as a response to an increase in prices. If housing prices increase because of the last period's interest rate, the young agent's mortgage payments increase linearly with prices, which decreases the housing demand of the young agent. As in the model, the median house is always bought by a young agent, the size of the median house will decrease, pushing down the median housing price.

It is this *last period's interest rate effect* that enabled the model to match the decrease in housing prices in 1990 in Figure 6. In the approximate interest rate history, during the 1982-1985 period, interest rates are 10 percent. During this period, because of the high interest rates, agents in the model save more, bringing extra assets to the next period (1988-1992.) In 1988, the interest rate decreases to 6 percent. The low interest rates together with the extra assets of old agents cause housing prices to increase. Since this period is a low interest rate period, young agents in this

Figure 8: The Effect of Last Period's Interest Rate on Housing Prices.



period save less compared to the young agents of 1985. When the next period (1991-1994) comes, although interest rates are still 6 percent, in this period there will be less asset available, which will cause a price decline.

In the literature, when questioning the existence of a housing price bubble, as one measure researchers used the housing price-income ratio. Especially for cities with high house price appreciations such as New York and San Francisco, looking at the housing price-income ratio, they find evidence of a housing price bubble, since that ratio became unusually high compared to its historical values. In the model, I do not have any income change. The average housing price to income ratio is 2.6. As housing prices fluctuate because of interest rates, the ratio becomes more than 3.5 when housing price reaches its maximum, and it becomes less than 1.5 when housing price reaches its minimum. My conclusion is that the housing price-income ratio cannot be a real measure of the real cost of housing, since that measure does not take into account the effect of interest rates on housing prices.

One interesting implication of the model is that the last period interest rate has predictive power for the current period's price. If the last period's interest rate was high, people expect (on average) higher prices. This is because young agents in the last period saved more (more precisely, borrowed less), making them wealthier in the current period, which then increases their willingness to pay higher prices (see Figure 9). Figure 8 showed the relation between housing prices and the last period's interest rate. The elasticity of prices with respect to the last period's interest rate is approximately 4. To show the importance of savings, I solve the model with the same parameters, but I assume that agents cannot save. Figure 10 shows that there is almost no predictability anymore and the new elasticity is very close to 0. One may think that the predictability can be coming from fixed transaction costs. To check that claim, I solve the model with no transaction cost ($\Delta = 0$.) The new interest rate (last period) elasticity of housing is around 5. So transaction

Figure 9: Young's Saving/Borrowing (as a share of income) and Interest Rates

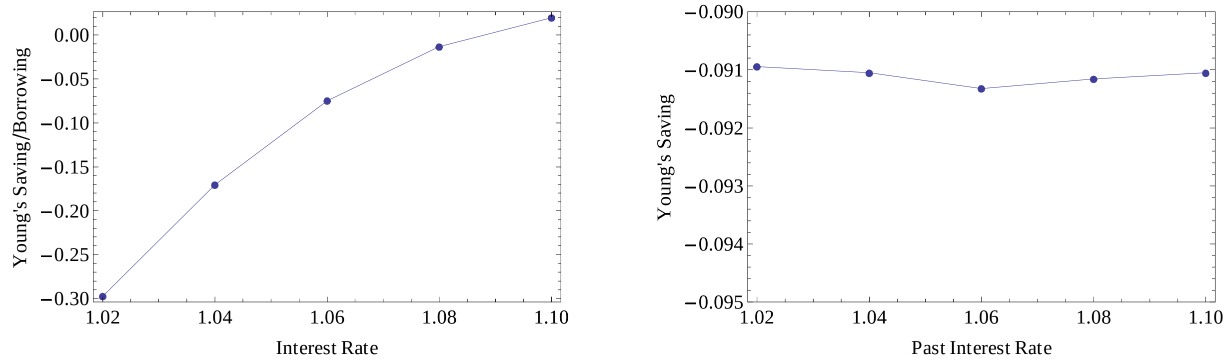
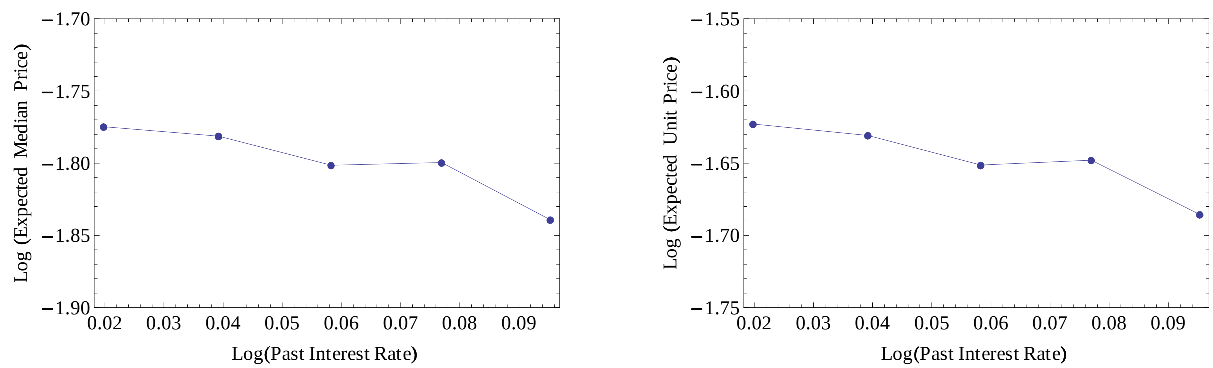


Figure 10: Housing Price Predictability without Saving.



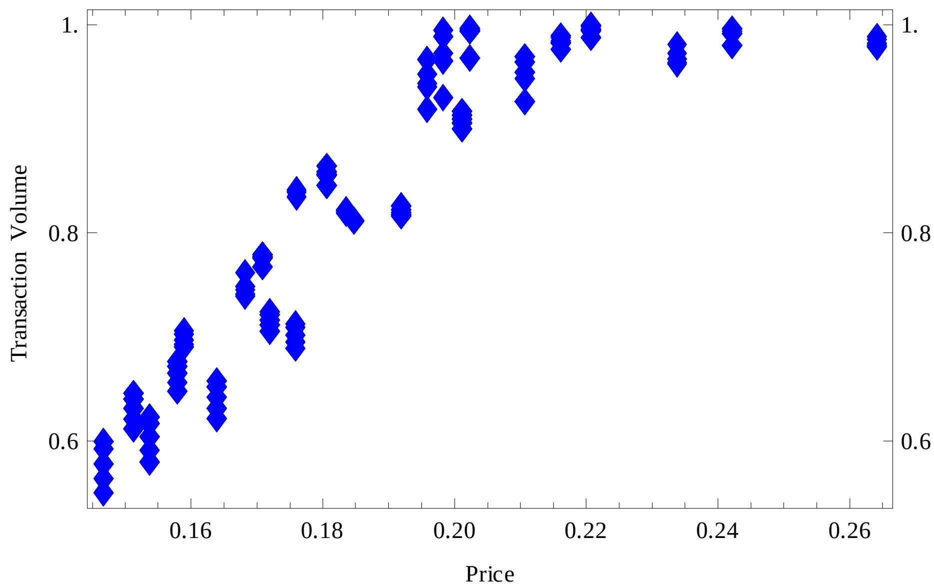
costs do not cause the predictability in the model.

6.2 Transaction Volume

Stein (1995) and Ortalo-Magne and Rady (2004) use credit constraints and large down payments to explain the positive relation. Wheaton (1990) and Berkovec and Goodman (1996) use information imperfections to explain the comovement. Genesove and Mayer (2001) claim that transaction volume-price correlation is consistent with the loss aversion of sellers.

The scatter plot in Figure 11 shows that housing prices and transaction volume are positively related in the model as in the US data. But the mechanism that generates the positive relation

Figure 11: Comovement of Housing Prices and Transaction Volume.



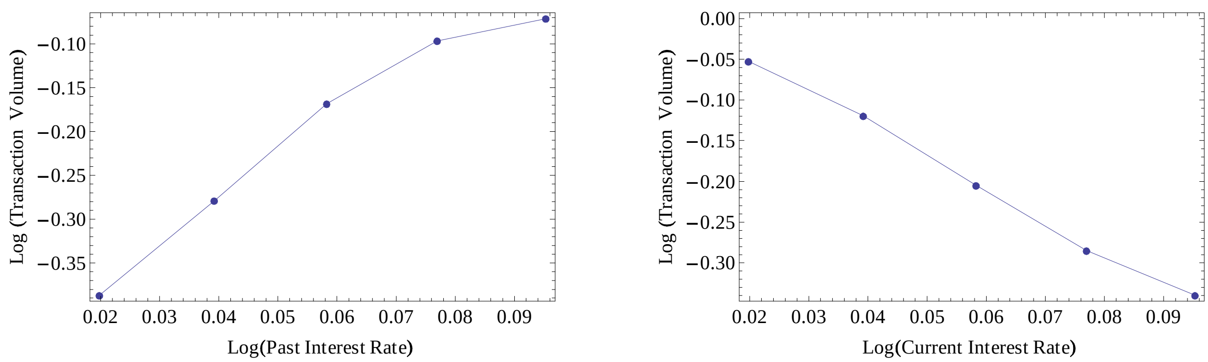
is different from the mechanisms used in Stein (1995) and Ortalo-Magne, and Rady (2004).[20] To check the effect of down payment on transaction volume, I solve the model with no down payment requirement. The results look very similar, meaning the down payment requirement is not an important part of the mechanism of the model that generates the comovement of prices and transaction volume (see appendix Figure 17.)

Figure 12 shows the effect of the current and the last period's interest rate on transaction volume. The graph in the right panel of Figure 12 shows that the current period interest rate and transaction volume are negatively related where I assumed that the last period's interest rate is 6 percent (it looks very similar to the other histories.) The interest (current) elasticity of transaction volume is around -4, which implies a 10 percent decrease in interest rates, similar to the U.S. data, will cause a 40 percent increase in transaction volume, explaining a significant part of the increase in transaction volume. In the left panel of Figure 12, I plot the expected transaction volume as a function of the last period's interest rate. The last period's interest elasticity of transaction volume is around 3.5. Both effects combined help the model to predict the large increase in transaction volume observed in the US data.

To see the intuition clearly, I write the costs and returns of a transaction. To make it simpler, I assume that the down payment requirement, λ, is 0 (I already showed that it is not crucial in the model). If an agent sells his house, the return from the transaction is

[20] The mechanism is obviuosly different from Wheaton (1990) and Berkovec and Goodman (1996), as the model does not have any information frictions.

Figure 12: Transaction Volume and Interest Rates. For clarity, the last period's interest rate is assumed to be 6% for the graph in the left panel.



$$P_t h_{t-1} - P_t h_{t,o,m} - P_{t-1} h_{t-1}(1 - D_{t-1} + d_{t-1}).$$

The return the agent gets when he sells his old house is $P_t h_{t-1}$. The agent will pay $P_t h_{t,o,m}$ for his new house. The last term, $P_{t-1}h_{t-1}(1 - D_{t-1} + d_{t-1})$, is the payment that the agent has to make to the lender for the debt arising from the house he sold. When the agent stays in his house his cost of housing will be,

$$\frac{P_{t-1}h_{t-1}D_{t-1}}{1 + r_t}.$$

I discount the second mortgage payment $P_{t-1}h_{t-1}D_{t-1}$ with $1 + r_t$ to find the discounted value of the payment.

Suppose that the current interest rate is low. In this case the second mortgage payment will be more expensive (compared to a higher interest rate state) for an agent who borrowed in the last period, since the denominator in $\frac{P_{t-1}h_{t-1}D_{t-1}}{1+r_t}$ will be smaller. As the cost of the second mortgage payment increases, more people find it optimal to move, which increases transaction volume. I also showed in the previous section that low interest rates cause high prices. If I combine both findings, it is the interest rates that cause the comovement of housing prices and transaction volume. But this is only half of the story. I showed in Figure 12 and Figure 8 that the last period's interest rate positively affect transaction volume and housing prices. If the last period's interest rate was high, it means that the agents who borrowed in the mortgage market during the last period have higher interest payments in the current period. If the cost of staying is high, then more people choose to move. This is the other channel that causes the comovement of housing prices and transaction volume.

The key to the model's success in generating the comovement of housing prices and transaction volume is the mortgage contract. To quantify the effect of the mortgage contract in the model, I eliminate the mortgage market from the model by assuming that the down payment requirement, λ, is 1. In this case the agents will have to pay for their purchase entirely in cash, so they will not borrow in the mortgage market. I calibrate the model using the same targets that I used to calibrate the baseline model. The results show that: (i) a model without the mortgage contract has a similar interest elasticity of housing prices (about -5,) (ii) without the mortgage contract the model is incapable of explaining the negative correlation between transaction volume and interest rates, and (iii) transaction volume has a much smaller coefficient of variation (around 0.03.) These findings show that the mortgage market is a key component of the baseline model. (See Figures 15 and 16 in the appendix.) Even after eliminating the mortgage contract the model still generates high housing price volatility since housing is not a liquid asset in the model.

To simplify the analysis I assumed that the consumers did not have a refinancing option. If I had a refinancing option, some of the implications of the model for transaction volume would change since consumers would have another option to decrease the cost of their mortgages. But, first, a refinancing option would not be used when interest rates increase, which implies that with rising interest rates, housing prices and transaction volume will decrease together. Moreover, the consumers will only refinance when the decrease in interest rates exceeds some threshold amount, as refinancing is costly. If the decrease in interest rates stays below the threshold, housing prices and transaction volume increase and the model's implications would not change. Only if the drop in interest rates is sufficiently large that consumers choose to refinance will it change the transaction volume implications of the model.

6.3 The Dispersion of Interest Rates, Housing Prices and Transaction Volume

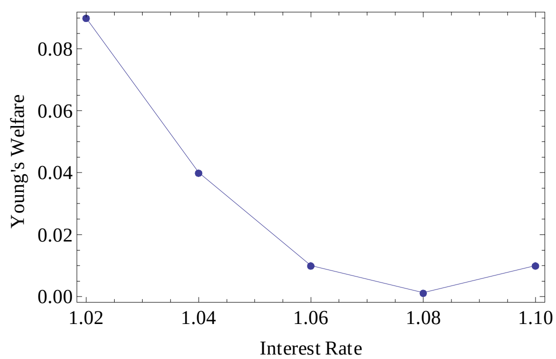
In Table 1 I reported three important statistics of the housing market. In this section I compare the statistics reported in Table 1 with their model generated counterparts. The findings on the elasticities of prices and transaction volume with respect to interest rates imply large volatilities of the two variables in response to reasonable interest rate movements. In this part, I simulate the model to obtain the coefficient of variation for both variables to compare with the data. From the simulations I obtain that the coefficient of variation of housing prices is 0.14. This is very close to the observed value for big cities. Since I assumed that housing supply is fixed in the model, the model's results are comparable the big cities. For the transaction volume, the model predicts a coefficient of variation of 0.16, which is more than the observed value. Compared to the correlation of 0.72 in the data, the model implies a correlation of 0.82 between housing prices and transaction volume.

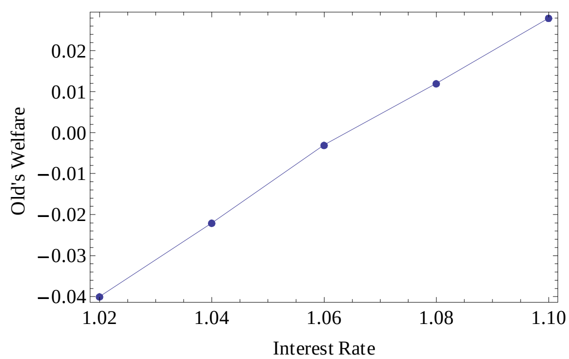
Table 6: Housing Market Statistics

	Data	Model
CV(real 15 year mortgage rate)	0.5	0.43
CV(housing price, big cities)	0.18	0.14
CV(transaction volume)	0.15	0.16
Correlation (transaction volume, housing prices)	0.72	0.82

Note. –CV is the coefficient of variation calculated as the standard deviation divided by the mean.

Figure 13: Old's and Young's Welfare. Welfare is measured in terms of consumption equivalents. Uniform weights are used to calculate the welfare of a generation. For clarity, recent past interest rates are assumed to be 6%.



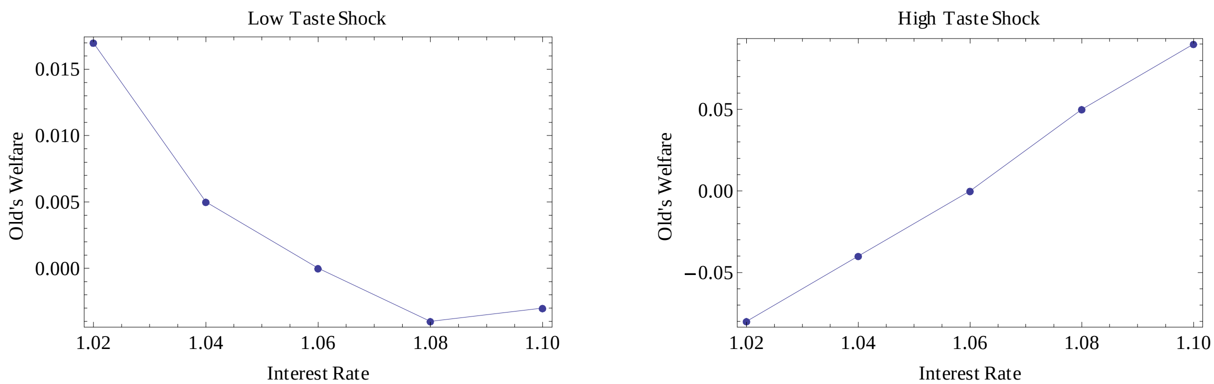


7 A Welfare Analysis

In this section, I analyze the effect of interest rate movements on the welfare of consumers. I measure welfare similar to Lucas (1987), where I calculate how much consumption consumers would give up to avoid a change in interest rates. I first look at the welfare of each generation (Figure 13 .) Rising interest rates decrease the welfare of young agents. The first reason is the higher cost of borrowing in the bond market when interest rates increase. The second reason is the higher cost of borrowing in the mortgage market. For old agents, rising interest rates are welfare improving since most of the agents upgrade their houses. With higher interest rates, they pay less for the larger houses they purchase. I find that for young agents, going from a 2 percent interest rate to a 10 percent interest rate decreases their welfare by a factor equivalent to more than 8 percent of their average consumption. For old agents, the welfare gain is equivalent to around 6 percent of consumption. In the aggregate, there is welfare loss.

I go further and ask whether all old agents who move are affected in the same way by the interest rate movements. In this part, I separate the old agents into two groups. First, I plot the

Figure 14: Differential Welfare Effects of Interest Rate Movements. $\theta_h = 2$ for High Taste Shock. $\theta_h = 0.1$ for Low Taste Shock. Welfare is measured in terms of consumption equivalents. For clarity, recent past interest rates are assumed to be 6%.



welfare of the old agents who receive a low taste shock (low θ_h) and then I plot the welfare of the old agents who receive a high taste shock (high θ_h). I find that while the old agents with low taste shocks are better off if the economy is hit by a low interest rate shock causing high housing prices, the old agents with high taste shocks are worse off. For the high taste shock ($\theta_h = 2$) the benefit from going from the lowest interest rate (2 percent) to the highest interest rate (10 percent) is around 18 percent of their average consumption level. For $\theta_h = 0.1$ the welfare loss resulting from moving from the lowest interest rate to an 8 percent interest rate is around 2.5 percent of their average consumption.

8 Conclusion

In this paper, I examined the role of interest rate movements in determining the fluctuations in housing prices and in transaction volume. First, I found that housing prices react strongly to interest rates. Next , I analyzed the model's implications for transaction volume. The model can account for both the comovement of housing prices and transaction volume and the high volatility of transaction volume.

The present model abstracts from some important aspects of the housing market to isolate the effect of interest rates. First, I assumed that income does not fluctuate. It is straightforward to write and solve a very similar model with income uncertainty instead of interest rate uncertainty. On the other hand, there are some ingredients in the model that mimic the behavior of income, such as savings. I showed that the effect of interest rates on housing prices are similar to the effect of savings on the housing prices. I also assumed that housing supply is fixed. This assumption makes the model work better for places where land is scarce or zoning restrictions are more effective. It would be interesting to include home production in the model and see the effect of zoning restrictions on the housing market.

One restrictive assumption I made is that there is no rental market in the economy. I made this assumption to make the computation of the model simpler. However, I should also note that there is no straightforward way to model rental market. For example, it will matter for the results whether the renters are renting because of a greater taste for renting rather than being a homeowner, borrowing constraints or income risk. In addition, the technology of transformation of rental units to housing units is critical and it should be studied in detail. This technology (or cost of transformation) will affect the total housing supply in the market and will be important for equilibrium levels of housing prices and rents. As a result, there is no straightforward quantitatively relevant way yo implement a rental market in the model of this paper is not straightforward. Depending on the modeling strategy, the quantitative effects of the model presented in this paper may become larger or smaller.

For computational reasons, I assumed that the agents can transact only twice during their lifetime. Adding more transaction choices to the model would impose large computational burden. The difficulty is that distribution will be a state variable. It is possible to solve the model with distribution as a state variable by using the algorithm developed Krusell and Smith (1998). On the other hand, the existence of fixed transaction costs in the housing market makes the computation of the model more complicated. Transaction costs are important since one contribution of the paper is matching the price-transaction volume correlation in the data. In addition, one reason that the model implies a large volatility in housing prices is the rigidity in the housing market. In the current formulation I have the solutions in closed form which makes the computation feasible. With transaction choice every period, dealing with fixed transaction costs will be a very challenging problem since other approximation methods will be needed.

References

[1] Berkovec, J. and J. Goodman, (1996), "Turnover as a Measure of Demand for Existing Homes", Real Estate Economics, 24, 421–40.

[2] Campbell, J. and J.F. Cocco (2003), "Household Risk Management and Optimal Mortgage Choice," Quarterly Journal of Economics (2003), Volume 118, Issue 4, Pp. 1449-1494.

[3] Genesove, D. and Mayer, C. (2001), "Loss Aversion and Seller Behavior: Evidence from Housing Market ," Quarterly Journal of Economics, Vol 116, No 4, 1233-1260

[4] Hanushek, E. A., and J. M. Quigley (1980), "What is the Price Elasticity of Housing Demand," The Review of Economics and Statistics,62(3), 449-454

[5] Heathcote, J and M. Davis (2005), "Housing and the Business Cycle," International Economic Review, August 2005, 46/3, p. 751-784

[6] Himmelberg, C., C. Mayer, and T. Sinai (2005), "Assesing High House Prices: Bubbles, Fundamentals, and Misperceptions," Journal of Economic Perspectives 19, 67-92

[7] Erik Hurst, Ming Ching Luoh, Frank P. Stafford and William G. Gale (1998), "The Wealth Dynamics of American Families, 1984-94," Brookings Papers on Economic Activity, 1998, 1998(1), pp. 267-337.

[8] Hussey, R. and Tauchen, G. (1992), "Quadrature-Based Methods For Obtaining Approximate Solutions for Nonlinear Asset Pricing Models," Econometrica, 59, 371-396

[9] Iacoviello, M.(2005), "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," American Economic Review, 95(3)

[10] Iacoviello, Matteo, and Stefano Neri (2010), "Housing Market Spillovers: Evidence from an Estimated DSGE Model," American Economic Journal: Macroeconomics, 2 (April): pp. 125–164.

[11] Krusell, P. and Smith, A.(1998), "Income and Wealth Heterogeneity in the Macroeconomy," Journal of Political Economy, 1998, vol. 106, no5.

[12] Lucas, R., (1978), "Asset Prices in an Exchange Economy," Econometrica, 46, 1426–45.

[13] Lucas, R., E., Jr. (1987), "Models of Business Cycles," New York, Blacwell.

[14] Lustig, H. and Yi-Li Chien (2010), "The Market Price of Aggregate Risk and theWealth Distribution," Review of Financial Studies, 2010, Volume 23, Issue 4, Pp. 1596-1650.

[15] Ortalo-Magne, R. and S. Rady (1999), "Boom In, Bust Out: Young Households and the Housing Price Cycle," European Economic Review, 43, 755-766.

[16] Rios-Rull, J. and Sanchez-Marcos, V.(2008), "An Aggregate Economy with Different Size Houses," Journal of the European Economic Association, Volume 6, Issue 2-3, pages 705–714, April-May 2008

[17] Stein, C.J. (1995), "Prices and Trading Volume in the Housing Market: A Model with Down-Payment Effects," The Quarterly Journal of Economics, Vol. 110, No.2. (1995), pp.379-406

[18] Veracierto, Marcelo (2002), "Plant Level Irreversible Investment and Equilibrium Business Cycles", American Economic Review, March 2002, Vol 92, pp. 181-97.

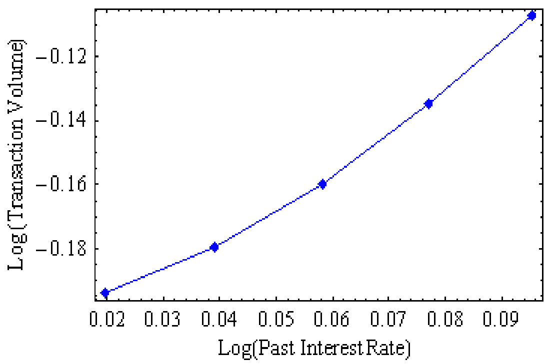
[19] Veracierto,Marcelo (2008), "Firing Costs and Business Cycle Fluctuations", International Economic Review, February 2008, 49:1, 1-39.

[20] Wheaton, W. (1990), "Vacancy, Search, and Prices in a Housing Market Matching Model", Journal of Political Economy, 98, 1270–92.

Appendix A The Effect of Mortgage Contract

As could be seen from Figure 15, interest rates have a very small effect on the transaction volume if the model does not have a mortgage contract. The housing price implications of a model without mortgage contracts are similar to those of model with mortgage contracts (Figure 16.)

Figure 15: Transaction Volume without Mortgage Contracts.



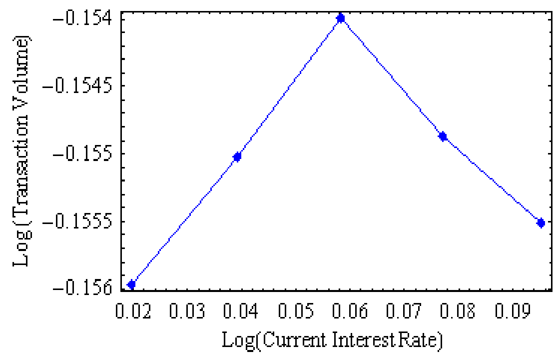
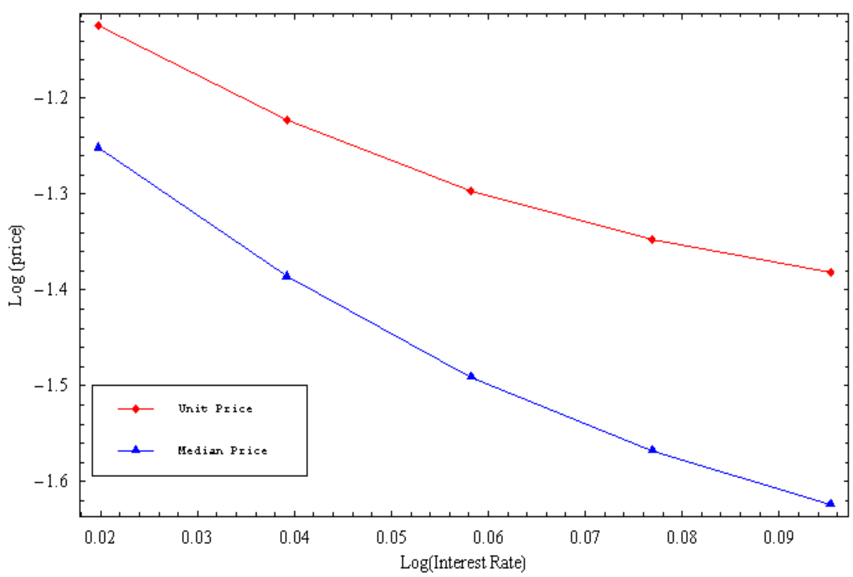


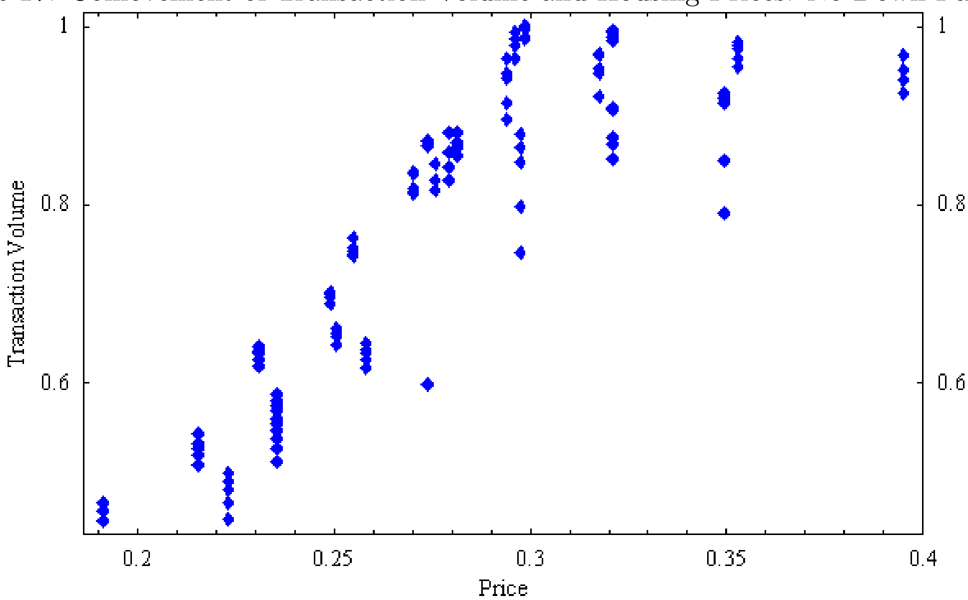
Figure 16: Housing Prices without Mortgage Contracts.



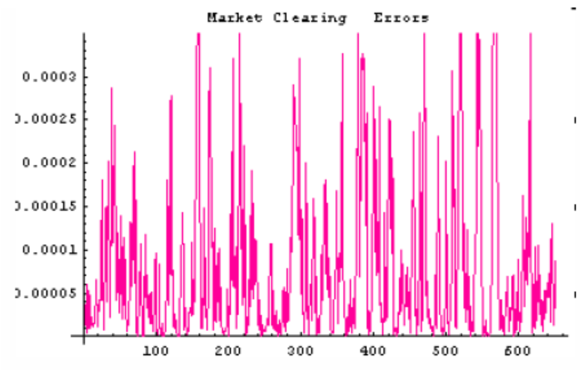
Appendix B The Effect of Down Payment Requirements

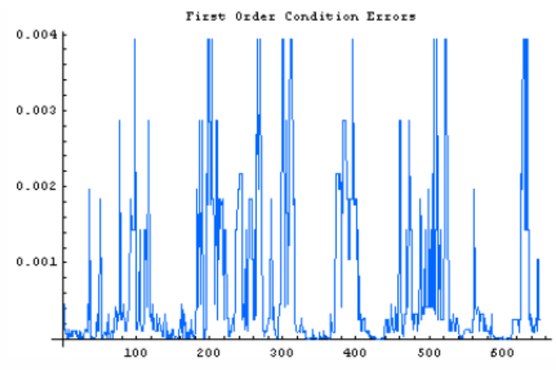
Several papers used large down payment requirements to explain the comovement of transaction volume and housing prices. In Figure 17, I show that the existence of the down payments is not the driving force in the model.

Figure 17: Comovement of Transaction Volume and Housing Prices. No Down Payment



Appendix C First-Order Condition and Market Clearing Errors





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