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Wealth Distribution and Social Security Reform in an Economy with Entrepreneurs

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Abstract

This paper analyzes the effects of removing the unfunded social security in an economy where entrepreneurs play a critical role in replicating the dispersion of wealth observed in the U.S. data. The quantitative findings reveal that the wealth Gini falls from 0.804 to 0.521 indicating a more equal distribution of wealth after privatization. Unlike the models without entrepreneurs, the fall in entrepreneurial profits further contributes to the reduction in the wealth inequality. The wealth distribution among the entrepreneurs is initially more equal than the one among the non-entrepreneurs, and privatization leads to exactly the opposite case in the long run. Furthermore, the occupational composition is reported to change in favor of entrepreneurship. Even though the fraction of entrepreneurs increases by 29%, their share in total wealth and income falls by 39%, and 7%, respectively. Finally, the increase in aggregate consumption as a percentage of the rise in aggregate output is always significantly lower with entrepreneurs than without them underlining the importance of additional saving motive introduced by the entrepreneurship.

JEL Codes: E1, E2, E6

Keywords: Entrepreneurship, Social Security Reform, Wealth Distribution, Consumption Distribution, Income Distribution

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1 Introduction

The unfunded social security system is a mechanism that provides households with partial insurance against the income shocks that are related to old age and longevity risks. Households, especially those who are too impatient or lack the resources to save for their retirement, mostly benefit from this system which guarantees a certain flow of income during their retirement period. Besides its benefits, the public pension system causes distortions in the optimal saving and work decisions of the households via the payroll taxes levied on the labor income and the transfer payments to the retired. This two-sided nature of the unfunded social security system raises questions about whether its costs or benefits outweigh the other. Hence, the effects of eliminating the unfunded social security on the stock of physical capital, aggregate output, and the welfare of the households have been studied in different frameworks by many researchers.

The previous papers in the literature on the unfunded pension system mostly focus on the effects of privatization on the accumulation of physical capital and the welfare of households. They pay little or no attention to its impacts on the wealth, income and consumption distributions. Principally, the privatization affects the amount of total net worth accumulated in the economy and its dispersion across households via the saving channel. Unlike the previous works, this paper mostly concentrates on the later, the effects of privatization on the distribution of wealth. There are two main reasons why entrepreneurs, a special group of households who own and manage their own businesses, are embedded into the model while examining the effects of privatization on the economy. First of all, they hold 40% of the total net worth in the U.S. economy even though they constitute a small fraction of the total population. Second, models of entrepreneurship are very successful in replicating the distribution of wealth observed in the U.S. data. Examples of the papers that model entrepreneurship in a general equilibrium framework include Quadrini (2000), Cagetti and DeNardi (2006), and Kitao (2008) among the others. A model that incorporates entrepreneurship and fruitfully matches both tails of the U.S. wealth distribution is indispensable in a detailed analysis of the effects of the unfunded social security on the distribution of wealth.

Of all the papers about privatization, only a couple of them partially reports how the wealth distribution reshapes after the policy change by providing coefficients of wealth Gini. One of those few studies, Abel (1985), analyzes the effects of the social security on the wealth inequality in a model with lifespan uncertainty. According to this work, a less concentrated

wealth distribution is observed after the introduction of the unfunded social security. On the contrary, Fuster (1999) reports that wealth Gini decreases from 0.58 to 0.46 in the steady state without the social security in a model that excludes the life-time uncertainty. Similarly, Fuster et al. (2007) shows that privatization decreases the wealth inequality in an economy in which both parents and children are altruistic towards each other. Even though these studies reach some results concerning the wealth distribution, they fail to match the skewness of the U.S. wealth distribution observed in the data. Even if a more equal wealth distribution is reported without social security, it still needs to be explained whether a rise in the assets of the households at the bottom tail or a decrease in the share of those at the top tail of the distribution or both bring about a more equal distribution. Unlike the previous works, this paper also presents quantitative results regarding the effects of privatization on the distribution of income and consumption expenditures besides being the first to study its effects in an economy that closely matches the U.S. wealth distribution.

This paper mainly studies the effects of eliminating the unfunded social security on the wealth distribution, entrepreneurial sector, and key macroeconomic aggregates in an infinite horizon life-cycle model in which households face both mortality and income risks. It differs from the previous papers in the literature in several aspects. First, it explicitly models entrepreneurship in a general equilibrium framework and successfully matches both tails of the U.S. wealth distribution. The presence of entrepreneurship creates occupational heterogeneity and additional incentive for saving by offering possibly huge amounts of profit income. There are some other models that match the distribution of wealth in the U.S. data by using other techniques, but they do not investigate the effects of privatization on the economy[1]. Second, households, contrary to the models of two-sided altruism[2], are only altruistic towards their descendants. This allows households to transfer wealth to their descendants in the form bequests. Finally, the household income is assumed to be taxed progressively by using the formula developed and estimated by Gouveia and Strauss (1994). Since the progressive taxation creates saving disincentives for the households who earn high interest income and somewhat discourages them from accumulating huge amounts of wealth, its inclusion strengthens the reliability of the model to match the wealth distribution observed in the data. The model with entrepreneurs is named as the benchmark model throughout

[1]For example, Krusell and Smith (1998) assumes stochastic time discount factors in replicating the U.S. wealth distribution. Another is Casteneda et al. (2003) who calibrate the model economy to the Lorenz curves of U.S. earnings and wealth.

[2]Fuster (1999) and Fuster et al. (2003) are the examples of the papers assuming two-sided altruism.

the paper[3].

The ability of the model to match the skewness of the U.S. wealth distribution is essential and strengthens its findings when studying the effects of privatization on wealth distribution. The benchmark model successfully generates a wealth Gini of 0.804 that is very close to 0.803, the one reported by Rodriguez et al. (2002) for the U.S. data. Nonetheless, having a Gini coefficient of wealth that is very close the one in the data does not necessarily mean that the model successfully replicate the wealth distribution observed in the data. A very high coefficient of wealth Gini may also be obtained by a distribution in which a big mass of households are concentrated in the bottom tail. Here, the benchmark model successfully generates a distribution where wealth is highly concentrated in its top tail. The share of the wealthiest 1% is found to be 30.05% which is very close to the one in the data, 34.70%, and the ratio of households with zero wealth is 14.94%. Even though it is not intended, the model is also very successful in replicating the income distribution observed in the U.S. data. The income Gini in the model economy is reported to be 0.500 which is slightly lower than the U.S. income Gini of 0.553. Not only the income Gini but also the shape of income dispersion is closely matched by the benchmark model. On the other hand, the model economy generates a distribution of consumption expenditures that is more concentrated than the one in the U.S. data. The coefficient of consumption Gini is found to be 0.406 while the one in the data is reported to be 0.320 by Casteneda et al. (2003).

The elimination of unfunded social security produces a more equal wealth distribution in the economies with or without entrepreneurs indiscriminately. In the economies without entrepreneurs- the standard and alternative economies, households optimally raise their asset holdings in order to insure themselves against future uncertainties in the absence of partial insurance that used to be provided by the public pension system. This reaction of households to privatization mainly increases (decreases) the concentration of wealth at the bottom (top) of the distribution leading to a more equal distribution of wealth. Unlike the economies without entrepreneurs, the wealth share of the households at the top of the distribution is additionally cut down by the losses in the profits of the most productive entrepreneurs further reducing the wealth inequality in the benchmark economy[4]. So, any model that

[3]In addition to this *benchmark model*, two other models- *the standard and alternative models* are developed for comparison purposes. The standard model is exactly the same as the benchmark model except having entrepreneurship. The alternative model is obtained by modifying the standard model to match the wealth distribution in the data without introducing entrepreneurship. Here, the alternative model is built upon the method developed by Casteneda et al. (2003).

[4]The wealth Gini decreases from 0.804 to 0.521 in the benchmark model, from 0.492 to 0.388 in the

ignores entrepreneurship underestimates the reduction in the wealth inequality associated with privatization even though it successfully replicates the dispersion of wealth in the data. Moreover, the wealth distribution among the young is less concentrated than the one among the old in both initial and final steady states, and both distributions become more equal after privatization. If the households are categorized with respect to occupation, the wealth distribution among non-entrepreneurs is found to be more (less) concentrated than that of the entrepreneurs in the initial (final) steady state. Again, both distributions among the entrepreneurs and the non-entrepreneurs become more equal after the removal of unfunded social security.

The removal of the pension transfers to the retired greatly exacerbates the income and consumption inequality irrespective of the presence of entrepreneurs. For instance, the income Gini is reported to increase from 0.500 to 0.560 in the benchmark economy after privatization while the consumption Gini jumps from 0.406 to 0.426[5]. However, the profit losses that the most productive entrepreneurs experience after the policy reform are observed to restrict the deteriorations in both income and consumption distributions by reducing the share of the households at the top of the corresponding distribution. In this sense, this observation seems very important when coupled with the amplifying effect of the profit losses on the decline in the wealth concentration. Furthermore, a more detailed analysis leads to an interesting observation that privatization reduces both income and consumption inequality among the young and among the entrepreneurs while it hugely deteriorates those among the old and among the non-entrepreneurs according to the corresponding Gini coefficients obtained for each group independently. For example, the income Gini for the young falls from 0.479 to 0.436, while the one for the old climbs from 0.419 to 0.905 following the removal of unfunded social security in the benchmark model.

In case of occupational choice or fraction of entrepreneurs, more households are observed to choose to be entrepreneur in the new steady state without the unfunded social security as they optimally respond to privatization. Numerically, the ratio of entrepreneurs in the whole population increases by 29% after the elimination of the social security. Unlike the ratio of entrepreneurs, their share in the total wealth and income are observed to jump down by 39%, and 7%, respectively. Moreover, the average and maximum sizes of the firms

standard model, and from 0.803 to 0.674 in the alternative model.

[5]The income Gini is raised from 0.370 to 0.468 in the standard model and from 0.629 to 0.674 in the alternative model. On the other hand, the consumption Gini also increases from 0.256 to 0.343 in the standard model and from 0.562 to 0.598 in the alternative model.

owned and managed by the most efficient entrepreneurs are observed to fall down considerably while those of the firms operated by the entrepreneurs with relatively lower productivity become significantly larger after privatization. In the final calculation, the overall average size of entrepreneurial firms is reported to fall by 20% as the fraction of the low-efficiency entrepreneurs greatly increases in the absence of social security. When the non-corporate sector[6] is investigated as a whole, the amount of capital employed by entrepreneurial firms is observed to increase by 9% while the total labor employed and total output produced go down by 22% and 10%, respectively. This implies a significant amount of income loss for entrepreneurs.

The quantitative results also reveal that the aggregate output, consumption, and capital are shoved up in the new steady state of the economy after the policy change confirming most of the previous results in the literature on the privatization of the unfunded social security[7]. This strengthens the robustness of the relevant results that have been documented by the previous works. These aggregates are also reported to increase when the same experiment is conducted in an economy without entrepreneurs for comparison purposes. Most importantly, the increase in aggregate consumption (investment) as a percentage of the change in aggregate output is always reported to be lower (higher) in the benchmark model than those implied by the standard and alternative models underlining the importance of additional saving motive introduced by the entrepreneurship. Numerically, aggregate consumption accounts for 39% of the rise in aggregate output in the benchmark model while it accounts for 56% and 59% in the standard and alternative models, respectively. Similarly, the contribution of aggregate investment roughly amounts to 61% in the benchmark economy, while it is just 44% and 41% in the standard and alternative economies, respectively.

Finally, the welfare implications of privatizing the public pension system are analyzed from the viewpoint of an unborn household, and it is found that he prefers to be born into

[6]Following Quadrini (2000), the two sectors of production in the economy are named as corporate and non-corporate sectors. The non-corporate sector refers to the entrepreneurial sector which consists of the small and medium size firms of entrepreneurs.

[7]Barro (1974) shows that the social security reform has no effect on the capital accumulation when the bequests are included. On the contrary, Auerbach and Kotlikoff (1973) and Huang et al. (1997) report the capital stock to increase by 24% and 40%, respectively, following the privatization. Conesa and Krueger (1999) not only confirms an increase in the capital stock but also shows that a household would prefer to be born into an economy without social security. In a similar way, Hubbard and Judd (1987) shows that the social security reduces the capital stock and welfare. According to Fuster (1999), the physical capital accumulation with the unfunded social security is 8% less than what it would be otherwise. Finally, Fuster et al. (2003) reports 6% rise in the capital stock.

an economy without the unfunded social security in the economies without entrepreneurs. However, the introduction of entrepreneurship into the standard economy makes him to choose the economy with the public pension system over the other. This reflects the loss of entrepreneurial profit due to the changes in the relative factor prices in the new steady state. According to the results of a sensitivity analysis in which the restrictions on the parameters of relative risk aversion and income share of capital in both corporate and non-corporate sectors are relaxed, the associated robustness test is observed to fail. Since the model assumes an unrealistically high longevity risk[8], an OLG model would be more appropriate to study the welfare effects of privatization. However, this choice of model is adopted in this paper in order to embed in the bequest mechanism through which intergenerational transfer of resources are made possible much easier than an OLG model in a computational sense. Furthermore, the analysis of the economy during the transition between two steady states shows that none of the households at the time of announcement supports the elimination of the unfunded social security without any compensation irrespective of the presence of entrepreneurship as an alternative occupation.

The rest of the paper is organized in the following way. Section 2 talks about the benchmark model in detail. Section 3 and 4 explain the competitive equilibrium and the calibration methods, respectively. Section 5 reports the benchmark results. A sensitivity analysis is conducted in section 6. Finally, section 7 concludes the paper.

2 The Model

The model economy is similar to those developed by Quadrini (2000), Cagetti and DeNardi (2006), and Kitao (2008) in certain aspects. Unlike overlapping generations models that are preferred by most of the papers studying the effects of the privatization of the unfunded social security, it abstracts from specifying the age of a household.

2.1 Households

The economy is occupied by a continuum of households of measure one. A household is assumed to go through two phases of lifetime or demographic states- namely, young age and old age. At the end of every period, each household realizes a demographic shock that reveals his demographic state in the next period. There is a positive probability, denoted by

[8]Theoretically, there is a positive probability that a household works a very short period of time, a couple of periods at most, and lives an extremely long retirement period.

$p_o \in (0,1)$, that a young household ages at the end of a given period while an old household faces the risk of death, probability of which is denoted by $p_d \in (0,1)$[9]. In this setup, a young household is assumed to be immune to the mortality shock and only realizes aging shock. If an old household realizes the mortality shock, he is replaced with his descendant, a young household, and his assets are transferred to his descendant in the form of bequests. Moreover, the population is assumed to be constant over time abstracting from population growth.

Households are assumed to be perfectly altruistic towards their descendants in the sense that a household values the consumption of his offspring as if his own. In other words, he puts the same weight on the utility from his own consumption and his offspring's. Hence, the bequest mechanism creates an additional incentive for saving and leads to accumulation of a higher level of personal wealth than what would be otherwise. Given that a household values his own consumption and his descendant's equally, his preferences over the consumption sequence $\{c_t\}_{t=1}^{\infty}$ can generally be represented by the following expected discounted lifetime utility:

$$E_0 \left\{ \sum_{t=1}^{\infty} \beta^t u(c_t) \right\} \tag{2.1}$$

where $\beta \in (0,1)$ is the subjective time discount factor, E_0 is the expectation operator, and $u(.)$ is the period utility function which is identical across households and time. To be more specific, $u(.)$ takes the following Constant Elasticity of Substitution (CES) form:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{2.2}$$

where $\sigma > 0$ is the coefficient of relative risk aversion[10].

Additionally, households are assumed to be heterogenous with respect to their labor and entrepreneurial abilities. Labor efficiency or labor productivity per unit of work time of a household, l, is assumed to follow N_l-state Markov process. The state space of the labor efficiency process is given by the set, $\mathbb{L} = \{l_1, l_2, ..., l_{N_l}\}$. A household's labor productivity is picked from $\mathbb{L}$ according to the conditional probabilities defined as $p(l_t, l_{t+1}) = prob(l_{t+1}/l_t)$

[9] Constant probabilities of aging and death create a stationary distribution of households across demographic states.

[10] $\frac{1}{\sigma}$ is the elasticity of intertemporal substitution.

before he makes any occupational or consumption-saving decision. However, a newborn household draws a value from the same set according to the corresponding unconditional probabilities, and so does an old entrepreneur. Unlike an old entrepreneur, a retired household is assumed to have no labor productivity and, thus, can not supply any labor into the market in exchange for the prevailing wage rate. An old entrepreneurs is allowed to supply his labor endowments into his own business only.

In a similar manner, the values of the entrepreneurial factor productivity, θ, come from the set, $\mathbb{T} = \{\theta_1, \theta_2, ..., \theta_{N_\theta}\}$. The process that governs the entrepreneurial ability is assumed to be an N_θ-state Markov process and completely independent from the labor ability process. The random variable θ takes values from the set, Θ, according to the conditional probabilities denoted by $p(\theta_t, \theta_{t+1}) = prob(\theta_{t+1}/\theta_t)$. When an old entrepreneur dies, his descendant is assumed to draw a value from Θ conditional on his parent's entrepreneurial state so as to exploit the fact that children can benefit from their parents' experiences and knowledge. If an old entrepreneur chooses to get retired, he foregoes entrepreneurship forever and never realizes another entrepreneurial shock again in the remainder of his lifetime. Therefore, his entrepreneurial state is automatically set to θ_1, the state in which households have zero entrepreneurial productivity upon retirement.

A worker is assumed to supply his labor inelastically into the market in return for a wage per unit of efficiency labor. Since the workers only care about the market wage rate, and it does not vary across sectors, they arbitrarily supply their labor into both corporate and non-corporate sectors. The market wage rate does not vary across sectors. On the other hand, an entrepreneur can only supply his labor into his own business and does simultaneously manage its productive activities. He can always hire additional labor at the prevailing wage rate when his labor supply falls short of the demand by his own firm. This assumption makes entrepreneurship extremely attractive especially when the entrepreneurial idea possessed by a household is so productive that he can get his full labor income as well as the entrepreneurial profit.

2.2 Technology

The production is conducted by the aforementioned two sectors- corporate and non-corporate sectors. Corporate sector consists of very large competitive firms utilizing a constant-returns-to-scale technology while the non-corporate sector consists of relatively small firms operated by entrepreneurs.

Corporate Sector : All corporate firms are assumed to have access to the same production technology. This common technology is represented by a standard Cobb-Douglas production function:

$$F(K_c, H_c) = AK_c^{\alpha} H_c^{(1-\alpha)} \tag{2.3}$$

where α is the income share of capital, A is the sector specific total factor productivity, K_c and H_c are the total capital and labor employed in the corporate sector, respectively. Capital is assumed to depreciate at the constant rate of δ. The market rate of return on physical capital and labor wage rate are settled in the corporate sector by the interaction of supply and demand. Since this specific technology exhibits constant returns to scale, both industry and firm level profits in the corporate sector become zero.

Non-corporate Sector : Each entrepreneurial state has its own production efficiency denoted by θ. Given the value of θ and the production technology, an entrepreneur combines labor and capital in the production process so as to maximize the profit of his firm in a given period. The associated production technology is similar to its corporate counterpart and characterized by

$$f(k, h, \theta) = \theta k^{u_1} h^{u_2} \tag{2.4}$$

where $u_1 > 0$ and $u_2 > 0$ are the income shares of capital and labor, respectively. u_1 and u_2 are constant fractions of the income shares of the capital and labor in the corporate sector[11]. Then, the profit or the income share of the entrepreneur is given by $(1 - \nu)$ where $\nu = (u_1 + u_2)$. $(1 - \nu)$ is assumed to be strictly positive in order for the technology to exhibit decreasing returns to scale so that the entrepreneurs can enjoy positive amounts of profit. Even though the entrepreneurial firms are homogenous with respect to the share parameters of capital and labor, there is still a great degree of heterogeneity among themselves in terms of entrepreneurial productivity and strictness of of borrowing constraints. Finally, the capital used in the entrepreneurial sector [12] depreciates at the same constant rate of δ as the corporate capital does.

[11] $u_1 = \nu\alpha$ and $u_2 = \nu(1 - \alpha)$ where $\nu \in (0, 1)$ is the constant of proportionality.

[12] Entrepreneurial and non-corporate sector are interchangeably used throughout the paper

2.3 Intermediation

The intermediary sector consists of financial institutions that finance the investment projects in both corporate and non-corporate sectors by transferring the necessary funds from households to firms. Households earn interest income per unit of funds that are transferred to the production sectors via the intermediary sector. The rate of return on savings of the households is assumed to be equal to the risk free rate of return on capital, r, such that financial institutions make zero profit. In case of borrowing, households are assumed to have no access to the funds provided by the financial institutions with the exception of entrepreneurs. More precisely, entrepreneurs can borrow from the intermediary sector only to finance their investment projects. On the other hand, corporate firms are allowed to borrow as much as they need at the current interest rate.

Unlike corporate firms, an entrepreneur can only borrow up to a certain amount which is a multiple of his current asset holdings. Here and henceforth, the multiplication factor, χ, is called as leverage ratio. Moreover, he pays the risk-free interest rate plus a fixed cost of intermediation, η, per unit of resources borrowed in excess of his current wealth. This extra cost is neither consumed nor invested by the financial institutions. It is just taken away from the economy. Therefore, an entrepreneur with asset holdings, a, pays the rate, $r + \eta$, to the financial institutions per unit of resources borrowed in excess of a and can borrow up to χa. The parameters, χ and η, are also assumed to be invariant across entrepreneurs.

2.4 Government

The first task of the government in this setup is to guarantee the continuation of the pay-as-you-go social security system. Second, the government purchases a constant fraction of the aggregate output for its own purposes, and its cost is financed by the revenue from the taxes on the consumption expenditures and personal income[13].

2.4.1 Social Security System

Social security system functions as an insurance mechanism against the age-dependent income shocks and longevity risks. So, it ensures that households with low income who are less likely to save enough for their retirement obtain a certain flow of income in their elderly

[13]Here, the definition of personal income encompasses the labor earnings, profits, interest income, and pension payments.

years. In this system, households pay payroll taxes when working and collects benefits during their retirement. It is the government's responsibility to collect payroll taxes from working population and to disburse the earned benefit payments to the retired. In compliant with the U.S. social security system, it is assumed that only the labor income is subject to the social security taxation excluding all other sources of income. Hence, entrepreneurs also pay the payroll taxes levied on their labor income ruling out the profit and capital income.

All workers and entrepreneurs are obliged to pay social security taxes as long as they actively work irrespective of their current demographic state. Only the part of the labor income that is below y_s^m falls within the scope of social security taxation, not all of the labor income of a household. In other words, y_s^m is the maximum amount of labor income that is subject to the social security taxation. The amount of social security tax that a household with the labor income y_l pays is computed by

$$T_s(y_l) = \tau_s \min\{y_l, y_s^m\} \tag{2.5}$$

where τ_s is the flat rate of payroll taxes that is set by the government to balance the social security budget. On the other hand, the benefit that a retiree or an old entrepreneur gets is a fixed fraction of his average lifetime labor income. Since the model has the lack of ability to keep track of the average labor earnings over the life cycle, the heterogeneity in the social security benefits is assumed away. Thus, the benefit a retiree receives is assumed to be a fixed fraction of the average of the individual averages of lifetime labor earnings which is denoted by $\bar{I}$ [14]. Then, the amount of the benefit, b_s, is computed by

$$b_s = \gamma_s \bar{I} \tag{2.6}$$

where γ_s is the replacement ratio[15].

2.4.2 Government Purchases and Taxation

The level of government purchases, G, is assumed to be given exogenously as a constant fraction of the aggregate output. The government finances its purchases by the revenue generated from taxes on consumption expenditures and income from all sources. While the

[14] This average is calculated within simulations. $\bar{I} = \int_{\tilde{\mathbb{S}}} y_{ly}(\tilde{s}) d\mu(\tilde{s})$ where $y_{ly}(\tilde{s}) = \min\{y_l(\tilde{s}), y_s^m\}$ See Section 3 for an explanation of the state variable, $\tilde{s}$.

[15] This formulation abstracts from progressive calculation of benefits in the U.S. social security system, which has certain redistributive effects over the economy.

consumption expenditures of a household is taxed proportionally at the constant rate of τ_c, his income is taxed both proportionally and progressively. In case of income taxation, labor earnings, entrepreneurial profit, and interest income are subject to taxation. Social security transfer payments are considered partially taxable conditional on a certain criterion. Only a fraction of the social security benefits, f_b^o, is taxable if the total income of the household from all sources exceeds y_b^o. Otherwise, pension payments are exempt from income taxation.

The progressiveness of income taxes is likely to reduce the inequality in the wealth and income distribution by imposing higher rates of tax as the income gets higher[16]. Inclusion of progressive taxation reinforces the reliability of the model in replicating the dispersion of wealth in the U.S. data. Therefore, a progressive income taxation is incorporated into the model so as to capture the progressive nature of the U.S. income taxation. The income tax schedule proposed by Gouveia and Strauss (1994) is used when calculating the amount of income tax that is levied on a household with a taxable income of y_{tx}:

$$t(y_{tx}) = \tau_0 \left[y_{tx} - (\tau_1 + y_{tx}^{-\tau_2})^{-\frac{1}{\tau_2}} \right] \tag{2.7}$$

where $\tau_0, \tau_1, \tau_2 > 0$ are the constant parameters of the tax formula. Since the government receives tax revenue from sources other than income such as real estate and corporate income which are ignored in this specific model, household income is also assumed to be taxed proportionally in order to represent the missing part of taxes. As a result, the total income tax, $T(.)$, is computed by the following generalized progressive income tax formula:

$$T_y(y_{tx}) = t(y_{tx}) + \tau_3 y_{tx} \tag{2.8}$$

where τ_3 is the constant tax rate on the income that signifies the non-progressive portion of the income taxation.

2.5 Profit Maximization

In this section, the profit maximization problems of a corporate firm and an entrepreneurial firm are comprehensively explained. There are two inputs of production; capital and labor in both sectors of production.

[16]For a further discussion of this issue, see Casteneda et al. (1999).

2.5.1 Corporate Firms

A typical firm in the corporate sector solves the following maximization problem taking the gross interest rate, (R) and the wage rate (w) as granted:

$$\max_{K_c, H_c > 0} \left\{ A K_c^{\alpha} H_c^{1-\alpha} + (1-\delta) K_c - R K_c - w H_c \right\} \tag{2.9}$$

where A represents the total factor productivity, K_c and H_c are the capital and labor demand by the firm, respectively. The gross output of the firm is the sum of the newly produced goods and the part of the capital remaining after production. A constant fraction, δ, of the capital depreciates during the process of production. Furthermore, aggregate corporate capital evolves according to the following rule

$$K_c' = (1-\delta) K_c + I_c \tag{2.10}$$

where I_c is the gross investment in the corporate sector, and K_c' is the next period's capital. As mentioned before, the factor prices are determined by the aggregate capital and labor employed in the corporate sector.

2.5.2 Non-Corporate Firms

Non-corporate firms exploit a decreasing returns to scale technology through which entrepreneurs collect positive amounts of profit. In this sector of production, an entrepreneur is characterized by his asset holdings at the beginning of period, a, and entrepreneurial productivity, θ. The maximization problem of an entrepreneur with (a, θ) is depicted by

$$\pi(a, \theta) = \max_{\left\{ \begin{array}{r} 0 \leq k \leq (1+\chi)a \\ h \geq 0 \end{array} \right\}} \left\{ \theta k^{u_1} h^{u_2} - \tilde{R}(k-a) - Ra - wh + (1-\delta)k \right\} \tag{2.11}$$

and

$$\tilde{R} = \left\{ \begin{array}{ll} R + \eta & if \;\; k > a \\ R & if \;\; k \leq a \end{array} \right\} \tag{2.12}$$

where η is the extra cost of borrowing for entrepreneurs, k is the entrepreneur's capital demand and h is the entrepreneur's labor demand. Here, the parameters, χ and η, establish the degree of strictness of borrowing constraints. In the face of very tight borrowing conditions,

a household with a high θ may optimally choose to be a worker if he does have very little or no wealth wealth to start off. Therefore, strict borrowing constraints create an additional saving motive for the households when combined with the possibility of high entrepreneurial profits.

2.6 Utility Maximization

Six different value functions are defined and utilized in order to represent the preferences of households over the consumption bundles. These are denoted by $V_r(.)$, $V_{oe}(.,.,.)$, $V_o(.,.,.)$, $V_w(.,.,.)$, $V_{ye}(.,.,.)$, and $V_y(.,.,.)$- explicitly, the value function of the retired, the old entrepreneur, the old before occupational choice, the worker, the young entrepreneur, and the young before occupational choice, respectively. So, the expected discounted lifetime utility of a household can obviously be written as a convex combination of the relevant value functions by using the weights computed by the probabilities assigned to the various possible states of the nature. In the following two subsections, the corresponding utility maximization problems are presented in more detail.

2.6.1 The Old

A household may always choose between being an entrepreneur and getting retired regardless of his occupational status when he reaches the retirement age. An old entrepreneur always makes an occupational choice in the beginning of every following period until he prefers to enjoy benefits of retirement. Once he does get retired, he never realizes an entrepreneurial shock and hence is not allowed to return to entrepreneurship in the rest of his life. On the other hand, all old households including actively working entrepreneurs are assumed to qualify to receive pension payments. In other words, entrepreneurship does not preclude an old household from collecting social security benefits.

The value function of a retired household is the simplest of all six value functions. The retired have only one state variable, $\{a\}$, that represents wealth transferred from the previous period. Given the state vectors $\zeta_r = \{a\}$ and $\zeta = \{a, \theta, l\}$, the decision problem

that a retiree faces can be formulated as follows

$$V_r(\zeta_r) = \max_{c,a'\geq 0}\left\{\frac{c^{1-\sigma}}{1-\sigma} + \beta\left\{(1-p_d)V_r(\zeta_r') + p_d E_{l'}E_{\theta'/\theta=\theta_1}V_y(\zeta')\right\}\right\}$$

$$s.t.$$

$$\begin{aligned}
(1+\tau_c)c + a' &= a + y - T_y(y_{tx}) \\
y &= (R-1)a + b_s \\
y_{tx} &= \left\{\begin{array}{ll} y-(1-f_b^o)b_s & \text{if} \quad y-(1-f_b^o)b_s \geq y_b^o \\ y-b_s & \qquad\qquad\qquad\qquad otherwise \end{array}\right\}
\end{aligned}$$

where y_b^o is the maximum amount of taxable income below which the pension payments are free from income taxation, f_b^o is the fraction of pension transfers that is included in the calculation of taxable income, $\zeta' = \{a', \theta', l'\}$ denotes the state vector in the next period, b_s is the amount of social security benefits received, $E_{\theta'/\theta}$ is the expectation operator over θ' conditional on current θ, and $E_{l'}$ is the unconditional expectation operator with respect to labor efficiency, l. It is straightforward to conclude from the objective function of the retired that he does not have an occupational choice in the next period.

In case of old entrepreneurs, there are three state variables, $\{a, \theta, l\}$; asset holdings at the beginning of period, entrepreneurial efficiency, and labor productivity, respectively. Then, his maximization problem is formalized below,

$$V_{oe}(\zeta) = \max_{c,a'\geq 0}\left\{\frac{c^{1-\sigma}}{1-\sigma} + \beta\left\{(1-p_d)E_{l'}E_{\theta'/\theta}V_o(\zeta',\theta_\prime) + p_d E_{l'}E_{\theta'/\theta}V_y(\zeta')\right\}\right\}$$

$$s.t.$$

$$\begin{aligned}
(1+\tau_c)c + a' &= a + y - T_y(y_{tx}) - T_s(y^e) \\
y &= (R-1)a + b_s + \pi(a,\theta) + y^e \\
y^e &= w\min\{h^e, l\bar{t}\} \\
y_{tx} &= \left\{\begin{array}{ll} y-(1-f_b^o)b_s & \text{if} \quad y-(1-f_b^o)b_s \geq y_b^o \\ y-b_s & \qquad\qquad\qquad\qquad otherwise \end{array}\right\}
\end{aligned}$$

where y^e is the labor income of the household from entrepreneurial activities, h^e is the labor demand by the entrepreneur's firm, and $\bar{t}$ is the time endowment of the household. There are only two decision variables- consumption and wealth transferred to the next period.

The occupational decision of an old household always takes place before the optimal allocation of his resources between consumption and intertemporal transfer. He chooses

between entrepreneurship and retirement at the very beginning of each period just after observing his labor and entrepreneurial abilities. Hence, the household has three state variables, $\{a, \theta, l\}$. The value function of an old household is given by

$$V_o(\zeta) = \left\{ \begin{array}{lll} V_r(\zeta_r) & \text{if} & \theta = \theta_1 \\ \max_{i\in\{0,1\}} \{(1-i)V_r(\zeta_r) + iV_{oe}(\zeta)\} & & otherwise \end{array} \right\} \tag{2.13}$$

where i is the occupational indicator. If an entrepreneur realizes the lowest efficiency shock, θ_1, he automatically gets retired and his state space reduces to ζ_r.

2.6.2 The Young

Given the knowledge of labor and entrepreneurial efficiencies of a young household chooses his occupation at the very beginning of a period. The value of the current labor efficiency is picked from the set $\mathbb{L}$ conditional on its value on the previous period unless he is just replacing an old who realized the mortality shock in which case l is drawn from unconditional distribution. Likewise, new entrepreneurial shock is drawn from the set $\mathbb{T}$ conditional on his entrepreneurial state in the preceding period but independent of his occupational status. In other words, an entrepreneur and a worker with the same level of entrepreneurial productivity have identical probabilities of drawing a particular θ in the next period.

A worker has three state variables, $\{a, \theta, l\}$. The decision problem and the corresponding value function of a worker are represented by

$$V_w(\zeta) = \max_{c,a'\geq 0} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \left\{ (1-p_o)E_{l'/l}E_{\theta'/\theta}V_y(\zeta') + p_oE_{l'}E_{\theta'/\theta}V_o(\zeta') \right\} \right\}$$

$$\begin{array}{rcl} & & s.t. \\ (1+\tau_c)c + a' & = & a + y - T_y(y_{tx}) - T_s(y^w) \\ y & = & (R-1)a + y^w \\ y^w & = & wl\bar{t} \\ y_{tx} & = & y \end{array}$$

where $E_{l'/l}$ is the conditional expectation operator with respect to labor efficiency, and y^w is the labor income.

Similarly, the state space of a young entrepreneur also consists of three variables,

$\{a, \theta, l\}$. Then, his value function is given by

$$V_{ye}(\zeta) = \max_{c,a'\geq 0} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \left\{ (1-p_o) E_{\theta'/\theta} E_{l'/l} V_y(\zeta') + p_o E_{l'} E_{\theta'/\theta} V_o(\zeta') \right\} \right\}$$

$$\begin{aligned} & s.t. \\ (1+\tau_c)c + a' &= a + y - T_y(y_{tx}) - T_s(y^e) \\ y &= (R-1)a + \pi(a,\theta) + y^e \\ y^e &= w \min\{h^e, l\bar{t}\} \\ y_{tx} &= y \, . \end{aligned}$$

where y^e is the labor income of the entrepreneur as stated before.

A young household chooses either to run his own business or to supply his labor into the market knowing his state variables, $\{a, \theta, l\}$. The corresponding value function is given by

$$V_y(\zeta) = \left\{ \begin{array}{lll} V_w(\zeta) & \text{if} & \theta = \theta_1 \\ \max_{i\in\{0,1\}} \{(1-i)V_w(\zeta) + iV_{ye}(\zeta)\} & & otherwise \end{array} \right\}. \tag{2.14}$$

If he chooses to be entrepreneur, the indicator, i, is set to unity.

3 Competitive Equilibrium

A state vector, $s = (a, \theta, l, \epsilon)$, specifies the asset holdings, labor efficiency, entrepreneurial efficiency, and age of a household at the very beginning of every period where $a \in \mathbb{R}$ and $\mathbb{R} = \mathbb{R}_+ \cup \{0\}$, $\theta \in \mathbb{T}$, $l \in \mathbb{L}$, $\epsilon \in \mathbb{E} = \{young,\ old\}$, and $s \in \mathbb{S} = \mathbb{R} \times \mathbb{T} \times \mathbb{L} \times \mathbb{E}$. $\tilde{s} = (a, \theta, l, o) \in \tilde{\mathbb{S}}$ also qualifies as a state vector just after the occupational choice where $o \in \mathbb{O} = \{young\ entrepreneur,\ worker,\ old\ entrepreneur,\ retired\}$, and $\tilde{\mathbb{S}} = \mathbb{R} \times \mathbb{T} \times \mathbb{L} \times \mathbb{O}$. Then, a stationary competitive equilibrium is defined as a set of prices $\{R, w\}$, value functions $\{V(s),\ \tilde{V}(\tilde{s})\}$, allocations $\{a(\tilde{s}), c(\tilde{s}), k(\tilde{s}), h(\tilde{s}), o(s)\}$, a government tax system, $\{\tau_s, \tau_1, \tau_3\}$, social security transfers, $\{b_s(\tilde{s})\}$, aggregate demands $\{K_c, H_c, K_e, H_e\}$, the stationary distribution of the households over the state space, $\mu(\tilde{s})$ such that[17]:

1. Given the prices, government tax system and transfer payments, allocations solve the maximization problem of the household;

[17] e and c refer to entrepreneurial and corporate sectors, respectively.

2. Capital and labor markets clear

$$K_c + \int_{\tilde{\mathbb{S}}} k(\tilde{s}) d\mu(\tilde{s}) = \int_{\tilde{\mathbb{S}}} a(\tilde{s}) d\mu(\tilde{s}) = K$$
$$H_c + \int_{\tilde{\mathbb{S}}} h(\tilde{s}) d\mu(\tilde{s}) = \int_{\tilde{\mathbb{S}}} n(\tilde{s}) d\mu(\tilde{s}) = H$$

where $n(\tilde{s})$ is the labor supply of the household;

3. The factor prices are competitively determined by

$$R = \alpha A \left(\frac{K_c}{H_c}\right)^{\alpha-1} + (1-\delta),$$
$$w = A(1-\alpha)\left(\frac{K_c}{H_c}\right)^{\alpha};$$

4. The intermediary sector is perfectly competitive and financial institutions make zero profit[18];

5. The social security system is self-financing

$$\int_{\tilde{\mathbb{S}}} b_s(\tilde{s}) d\mu(\tilde{s}) = \int_{\tilde{\mathbb{S}}} T_s(y_l(\tilde{s})) d\mu(\tilde{s})$$

where $\{y_l\}$ refers to the labor income;

6. The government budget is balanced

$$\begin{aligned} G &= \int_{\tilde{\mathbb{S}}} \{\tau_c c(\tilde{s}) + T_y(y_{tx}(\tilde{s}))\} d\mu(\tilde{s}) \\ &= \tau_c C + \int_{\tilde{\mathbb{S}}} \{T_y(y_{tx}(\tilde{s}))\} d\mu(\tilde{s}) \end{aligned}$$

where C is the aggregate consumption;

7. The distribution of households over the state $\tilde{s}$ is time-invariant

$$\mu(\tilde{s}) = \mathbb{P}_\mu \mu(\tilde{s})$$

where $\mathbb{P}_\mu$ is the transition matrix from one period to the next;

[18] Banks charge the risk-free interest rate to the firms, $r = R - 1$, per unit of funds provided and transfer this exactly the same amount to the households. The extra cost, η, per unit of funds intermediated to the non-corporate sector is paid by entrepreneurs, and it is never used in any productive activity.

8. The goods market clears

$$C + G + \delta K + \Gamma = AK_c^{\alpha} H_c^{1-\alpha} + \int_{\tilde{\mathbb{S}}} y_e(\tilde{s}) d\mu(\tilde{s})$$

where Γ is the total cost of excessive borrowing by entrepreneurs[19].

4 Benchmark Calibration

This section explains the methods used in the process of calibrating the five sets of parameters; demographic and preference, intermediation sector, production technology, labor productivity, and government sector parameters. Table 4.5.1 displays a list of calibrated parameter values at the end of the section. One model period is taken to be one year throughout the calibrations.

4.1 Demographics and Preferences

There are two preference parameters to be calibrated; the relative risk aversion coefficient, σ, and the subjective time discount factor, β. The relative risk aversion coefficient is set to 1.5, a value that is commonly used in the literature. Next, the subjective time discount factor is calibrated in such a way that the ratio of capital to output becomes 2.66 in the steady state equilibrium of the economy. The definition of capital used in the calibration excludes consumer durables and public capital. Within the set of demographic parameters, probability of aging, p_o, is set to a value such that the average duration of working age is 46 model periods. In a similar way, probability of death, p_d, is chosen such that the average duration of retirement is 14 periods.

4.2 Intermediation

There are two parameters that characterize the intermediation sector and need to be calibrated. The first parameter is the leverage ratio, χ, the maximum amount up to which entrepreneurs can borrow, and it is set to 40% following Evans and Jovanovic (1994). The operational or extra cost, η, of intermediating funds to the non-corporate firms is fixed at 5% following Diaz-Gimenez et al. (1992).

[19] $\Gamma = \eta \int_{\tilde{\mathbb{S}}} \max\{(k(\tilde{s}) - a(\tilde{s})), 0\} d\mu(\tilde{s})$.

4.3 Technology

4.3.1 Corporate Sector

The production technology in the corporate sector is represented by a Cobb-Douglas production function, $F(K, H) = AK^{\alpha}H^{1-\alpha}$. The productivity parameter is normalized to unity implying a zero rate of growth of the total factor productivity. Capital's share of output takes the value of 0.36, a number that is consistent with the data. The depreciation rate, δ, is obtained from the following expression

$$\delta = \frac{\left(\frac{I}{Y}\right)}{\left(\frac{K}{Y}\right)} \tag{4.1}$$

where $\frac{K}{Y}$ and $\frac{I}{Y}$ is the steady state ratios of capital to output and investment to output, respectively. The latter is fixed at 0.133 in line with the definition of capital and the U.S. data. The definition of capital in this model does not include the public capital.

4.3.2 Non-Corporate Sector

The share parameters, u_1 and u_2, of the production technology at the disposable of entrepreneurs are assumed to be proportional to the shares of capital and labor in the corporate sector. Thus, they can be expressed as $u_1 = \nu\alpha_e$ and $u_2 = \nu(1 - \alpha_e)$ where $\alpha_e = \alpha$. The constant of proportionality, $\nu \in (0,\ 1)$, determines the degree of decreasing returns to scale. Furthermore, the entrepreneurial ability, θ, is assumed to take $N_\theta = 4$ values for computational simplicity. The first value corresponds to the case in which the household has no entrepreneurial project to implement and θ_1 is set to zero. Then, the vector of entrepreneurial efficiency grids can be written as

$$\mathbb{T} = [0 \quad \theta_2 \qquad \theta_3 \quad \theta_4]\,. \tag{4.2}$$

Within the same framework, the associated probabilities of transition from one entrepreneurial grid to another is given by a transition matrix, $\mathbb{P}_\theta$. In order to reduce the number of unknown parameters, the evolution of entrepreneurial state of a household is assumed to occur gradually in the sense that a household with θ_i may only move to $\theta_{i\pm1}$ or remain as θ_i in the next period. For the special cases of $i = 1$ and $i = 4$, the transition is only possible to the states θ_2 and θ_3, respectively, if it happens. For all other possibilities transition

probabilities are set to zero in an effort to reduce the number of parameters to be calibrated. The transition probability from state i to state j is denoted by $p_{ij} = prob(\theta_{t+1} = \theta_j / \theta_t = \theta_i)$. It is further assumed that $p_{1,2} = p_{23} = p_{34},\ p_{43} = \frac{1}{2}p_{32}$ and $p_{12} = p_1,\ p_{21} = p_2,\ ,p_{32} = p_3$. Then, the matrix of transition probabilities becomes

$$\mathbb{P}_\theta = \begin{bmatrix} 1-p_1 & p_1 & 0 & 0 \\ p_2 & 1-p_1-p_2 & p_1 & 0 \\ 0 & p_3 & 1-p_1-p_3 & p_1 \\ 0 & 0 & \frac{1}{2}p_3 & 1-\frac{1}{2}p_3 \end{bmatrix}. \tag{4.3}$$

Above assumptions that reduce the number of unknown parameters are standard in the literature on the entrepreneurship.

Table 4.3.1: Target Moments and Model Results

Moment Name	**Model Value**	**Target Value***
Fraction of entrepreneurs	7.6%	7.6%
Overall exit rate	20.8%	20.0%
New entrants' exit rate	41.9%	40.0%
Entrepreneurs share in income	28.1%	27.0%
Entrepreneurs share in wealth	42.8%	40.0%
Ratio of median wealth of entrepreneurs to the rest	7.97	8.00
Wealth Gini	0.804	0.803

* The numbers are taken from Cagetti and DeNardi (2006), Rodriguez et al. (2002), and Quadrini (2000).
** The percentage deviations from the target values are less than or equal to 7%

There are seven non-corporate parameters to be calibrated. The values of ν, θ_2, θ_3, θ_4, p_1, p_2, and p_3, are calibrated to match the seven moments from the U.S. data- fraction of entrepreneurs, income share of entrepreneurs, wealth held by the entrepreneurs, ratio of median assets of entrepreneurs to that of workers, overall exit rate from entrepreneurship, new entrepreneurs' exit rate, and coefficient of the wealth Gini. Table 4.3.1 compares the target values and those generated by the model after the benchmark calibration. The resulting vector of efficiency points and the corresponding transition matrix are given in Appendix B.

4.4 Labor Efficiency

The logarithm of labor earnings is assumed to follow an AR(1) process. The persistence parameter of the process is taken to be 0.95, a number that is consistent with the estimates

in the literature such as Hubbard et al. (1994), and Storesletten et al. (2004). The variance of the process is chosen so that the Gini coefficient of the labor earnings in the steady state is 0.38 as in Cagetti and DeNardi (2006). Then, the continuous AR(1) process is transformed into a $N_l = 5$-state discrete process by using the quadrature-based method of Tauchen and Hussey (1991). See Appendix C for the vector of grid points and the transition matrix.

As an alternative calibration strategy, the labor grid points and transitional probabilities are calibrated to the Lorenz curves of income and wealth from the U.S. data following Casteneda et al. (2003). In this case, the number of grids is reduced to $N_l = 4$ in order for the number of parameters to be equal to the number of targets. This calibration strategy enables the model to generate a wealth distribution that is as concentrated as the one in the data. Again, see Appendix C for the resulting vector of grids and matrix of transition probabilities.

4.5 Government

4.5.1 Social Security System

The public pension system in the model economy is assumed to be self financing. Hence, the government determines the constant rate of social security payroll tax that balances the budget of the social security system. Of the parameters characterizing the social security system, the replacement rate, γ_s, is taken to be 0.44, the replacement rate that is found for a household with an average life-time earnings that is equal to the average labor earnings in the U.S. economy. Average labor earnings, $\bar{y}_l$ per year in the U.S. economy is $\$38,000$ in 2006. In the calibration of y_s^m, the maximum level of labor income that is subject to the social security taxation, first its ratio to the average labor earnings in the U.S. data is computed and then multiplied with the average labor earnings from the model. The maximum taxable labor income in the year of 2006 is reported to be $\$94,000$ by the Social Security Administration(SSA).

4.5.2 Income Taxation

The progressive income taxation formula depends on three parameters; τ_0, τ_1, and τ_2. Gouveia and Strauss (1994) estimates these parameters by using the U.S. data. Their estimates for τ_0, τ_1, and τ_2 are 0.258, 0.031, and 0.768, respectively for the year 1989. Since the second estimate, τ_1, is unit dependent, its value is found within the simulations. So, τ_1 of the model

is given by

$$\tau_1^{model} = \tau_1 \left(\frac{\bar{y}_{US_{1989}}}{\bar{y}_{model}} \right)^{\tau_2} \tag{4.4}$$

where $\bar{y}_{US_{1989}}$ is the average total household income in the U.S. in 1989, and $\bar{y}_{model}$ is the same average from the model. In this way, the average income tax rate that is applicable to a household with average income in the model is equated to the one that is implied by the U.S. data in 1989. Following Erosa and Koreshkova (2007), $\bar{y}_{US_{1989}}$ is fixed at 50 which corresponds to an annual income of $\$50,000$.

On the other hand, social security benefits are assumed to be partially subject to or exempt from income taxation depending on certain conditions. The fraction of social security benefit, f_b^o, that is conditionally subject to income taxation is taken to be as 0.5 as in the U.S. economy. The maximum level of income, y_b^o, below which the benefit payments are exempt from income taxation is reported to be $\$32,000$ by the SSA. Similar to the upper limit of labor earnings in social security taxation, the corresponding number in the model economy is obtained by multiplying the model generated average labor earnings with the corresponding ratio from the U.S. economy.

Moreover, Mendoza et al. (1989) finds the average tax rate on consumption expenditures to be 5.5%. Now, the only remaining tax parameter is the non-progressive income tax rate, τ_3. Its value is set within simulations so that the government budget always remains balanced. Finally, Government purchases are assumed to be equal to 21.5% of the total output as in Fuster et al. (2007).

Table 4.5.1: List of Calibrated Parameters

Parameter	**Value**	**Explanation**
Household		
p_o	0.02	probability of aging
p_d	0.07	probability of death
σ	1.50	coefficient of relative risk aversion
β	0.92	subjective discount factor
Intermediation		
χ	0.40	leverage ratio
η	0.05	extra cost of borrowing
Corporate Technology		
A	1.00	total factor productivity
α	0.36	corporate share of capital
δ	0.05	depreciation rate
Non-Corporate Technology		
ν	0.88	constant of proportionality
$[u_1 \;\; u_2]$	$[0.32 \;\; 0.56]$	non-corporate factor shares
$[\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4]$	in text	vector of entrepreneurial efficiencies
$\mathbb{P}_\theta$	in text	transition matrix for θ's
Labor Efficiency		
$\bar{t}$	1.00	time endowment
$[l_1 \;\; l_2 \;\; l_3 \;\; l_4 \;\; l_5]$	in text	vector of labor efficiency points
$\mathbb{P}_l$	in text	transition matrix for labor efficiencies
Social Security System		
γ_s	0.44	average replacement rate
$\left(\frac{y_s^m}{\bar{y}_l}\right)$	2.47	the level above which no SS* tax applies
Taxation		
$\left(\frac{G}{Y}\right)$	0.22	ratio of government purchases to output
τ_c	0.06	average consumption tax rate
$[\tau_0, \tau_1, \tau_2]$	$[0.26, 0.40, 0.77]$	parameters of progressive income tax formula
τ_3	0.06	proportional income tax rate
$\left(\frac{y_b^o}{\bar{y}_l}\right)$	0.84	threshold when taxing the pension benefits
f_b^o	0.50	fraction of pension benefits that can be taxed

*SS stands for Social Security.

4.6 Economies without entrepreneurs

Two additional economies are also considered and calibrated in order to isolate the role of entrepreneurship in the policy experiment conducted in this paper. The first of these economies is named as the standard economy and has the same features as the benchmark economy but assumes away entrepreneurship. It has exactly the same labor efficiency process as the benchmark model, and the parameter values for the benchmark calibration are allowed to remain unchanged except for the subjective discount factor, depreciation rate, and the parameters of income tax formula which are recalibrated to match the ratio of capital to output, the ratio of investment to output, and the degree of progressiveness of income taxation in the U.S. economy. The other framework, the alternative economy, is the same as the standard economy in all aspects but the labor process is calibrated to match the labor earnings and wealth distributions in the U.S. data following Casteneda et al. (2003) [20]. The subjective discount factor, depreciation rate, and the parameters of income tax formula are calibrated once more in order for the alternative model replicate the relevant moments in the data. All other parameters are assumed to take the benchmark values.

5 Benchmark Results

In this section, the quantitative results that characterize the effects of privatization in the benchmark economy[21] are presented and compared to those obtained by the two additional models; the standard and alternative models. Moreover, the policy experiment, uncompensated elimination of the unfunded public pension system, is assumed to be revenue neutral in the sense that the level of government purchases in the initial steady state remains unaltered throughout the transition and in the final steady state. Yet, the relevant income tax parameters are deliberately allowed to adjust in the process so as to preserve the degree of progressiveness of the income tax formula so as to focus only on the impacts of the pertinent policy reform. In case of the benchmark economy, the results not only focus on the two steady states but also encompasses the figures that depict the transition between them.

[20] This model helps us to further understand the role and importance of entrepreneurship in the characterization of the effects of privatization besides matching the wealth distribution in the data.

[21] Note that this name does not refer to the benchmark calibration but just distinguishes the economy with entrepreneurs from the others.

5.1 Macroeconomic Aggregates and Non-Corporate Sector

The presence of an entrepreneurial sector may permanently change the way key macroeconomic aggregates react to the policy reform or introduce new channels through which the behavior of households is affected and altered. In this respect, the results obtained in an economy populated by both entrepreneurs and workers serve as a robustness test of the previous findings regarding the implications of the removal of the unfunded social security. In such an economy, entrepreneurs can accumulate huge amounts of wealth engaging in highly profitable business activities in relatively shorter periods of time than the workers. Hence, the availability of profit channel makes entrepreneurship very attractive for households, especially those endowed with high entrepreneurial ability, θ. The possibility of high entrepreneurial profit creates an extra saving incentive coupling with the fact that entrepreneurs can only borrow up to a particular fraction of their asset holdings. Accordingly, this unique feature of entrepreneurship helps the benchmark model achieve a dispersion of wealth that very closely matches data. Before a detailed analysis of the consequences of privatization on the wealth distribution, its effects on some macroeconomic aggregates such as aggregate capital stock, consumption, and output are studied concisely.

Table 5.1.1: The long-run aggregate effects of the policy change

	Benchmark			Standard			Alternative		
	SSS*	No SSS	%Δ	SSS	No SSS	%Δ	SSS	No SSS	%Δ
τ_s	13%	0%		13%	0%		13%	0%	
r	3.28%	1.40%	−57%	8.54%	4.62%	−46%	8.53%	6.03%	−29%
w	1.46	1.69	16%	1.11	1.35	21%	1.11	1.25	11%
K	5.13	8.38	64%	3.53	6.02	71%	3.52	4.84	38%
Y	1.93	2.19	14%	1.33	1.61	21%	1.32	1.49	12%
C	1.23	1.34	9%	0.87	1.02	18%	0.86	0.96	11%

* SSS stands for Social Security System.

The steady state values of the aforementioned aggregates within the frameworks of the three model economies are reported in Table 5.1.1 along with the social security tax rate, wage, and interest rate. Since the social security replacement rate is not permitted to vary across economies, the corresponding payroll tax rate, τ_s, is found to be roughly 13% in all three economies. Moreover, the numbers exposed in Table 5.1.1 show that aggregate capital, output, and consumption are considerably higher in the new steady state regardless of the presence of entrepreneurs and the success of the model in generating a dispersion of wealth

as concentrated as the one in the data[22]. On the other hand, the factor prices, interest rate and wage rate, adjust in the long run in accordance with the changes in the aggregate variables. Therefore, the results obtained within benchmark and alternative models confirm most of the previous findings in the literature as serving two separate robustness tests[23].

At this point, it is also worth to mention the relative contributions of investment and consumption to the rise in aggregate output, and how they differ across three different economies if they do[24]. In the benchmark case, consumption accounts for 39% of the increase in aggregate output, and the remaining 61% is attributed to investment ignoring the small fall the extra cost of entrepreneurial borrowing[25]. On the other hand, consumption accounts for 56% and 59% of the expansion in aggregate output in the standard and alternative models, respectively. So, the remaining 44% and 41% stem from the increase in investment. Most importantly, the contribution of consumption (investment) to the rise in aggregate output is relatively lower (higher) in the benchmark model than those in the other two models highlighting the importance of additional saving motive created by the introduction of entrepreneurship. In other words, most of the extra resources released and produced after privatization is poured into investment in the benchmark model and into consumption in both the standard and the alternative economies. Therefore, the presence of entrepreneurs, not the model's ability to produce highly concentrated wealth distribution changes the way key macroeconomic aggregates react to the policy reform.

Additionally, Figure 5.1.1 illustrates how these key macroeconomic variables behave along the transition path between two steady states of the benchmark economy. The transition to the new steady state takes 57 periods after new policy is announced and implemented. During this special period, interest rate gradually falls down to its new equilibrium value while physical capital stock, output, and wage rate smoothly move up to their new values in the final steady state. Notice that the figure also includes the time path of the aggregate saving rate. It is observed to make a huge initial upward jump as a sign of households' early

[22]The level of wealth held by workers, entrepreneurs, and retirees changes by 116%, −0%, and 87%, respectively in the benchmark model.

[23]None of the papers in the relevant literature has studied the effects of privatization in a model that replicates the wealth distribution in the U.S. data. In this sense, not only the benchmark model but also the alternative model contributes to the literature.

[24]For instance, the contribution of the increase in aggregate consumption to the increase in the aggregate output can be expressed as $\frac{\Delta C}{\Delta Y}$. In other words, this is the increase in aggregate consumption as a fraction of the rise in aggregate output.

[25]This extra cost of borrowing is a deadweight loss and reported to decrease by 0.43% after the policy reform.

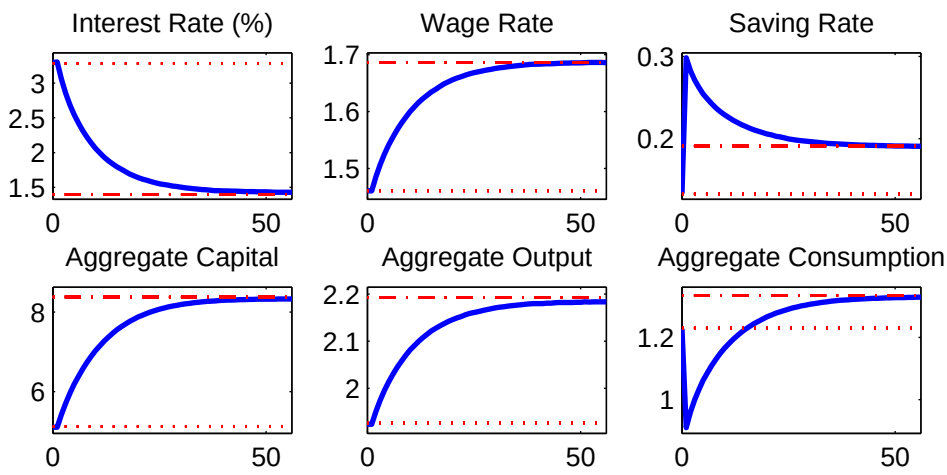

Figure 5.1.1: Benchmark economy: Macroeconomic indicators during transition.

attempts to compensate for the partial insurance that used to be provided by the unfunded social security. Clearly, the saving rate just after the policy reform is much higher than the one in the final steady state to which it slowly converges in the course of time. This initial jump occurs because the announcement is unexpected and all households immediately increase their asset holdings in order to compensate for the insurance that used to be provided by the social security. For the same reasons, the aggregate consumption suddenly jumps down at the beginning and then gradually moves up to its new steady state value. The initial sharp decrease in aggregate consumption is due to not only the increase in the savings but also the removal of pension payments disbursed to the retired.

In order to highlight the role of new dynamics introduced by entrepreneurship, the effects of the policy change on the non-corporate sector are also analyzed in various dimensions. It is very intuitive to enlighten the role of borrowing restrictions before presenting the relevant quantitative results. In the presence of constraints on entrepreneurial borrowing, a household needs not only to be endowed with high entrepreneurial productivity but also to have accumulated enough wealth to start off his own business. Similarly, an entrepreneur can still increase his profit and income by simply accumulating greater amounts of wealth given that his firm is operating below profit maximizing levels without borrowing. Hence, those constraints give a special role to the individual wealth and provide households of any occupation with additional saving motives playing a key role to attain a wealth distribution that is as concentrated as the one in the data. In this context, privatization seriously affects the entrepreneurial firm size, occupational choice, labor and capital demand by entrepreneurs,

and output in the non corporate sector through the changes in factor prices.

Table 5.1.2: The long-run effects of the policy change on the non-corporate firm size

	SSS	No SSS	%Δ
Average Firm Size			
$\theta_2 = 1.20$	4.5	5.6	24%
$\theta_3 = 1.46$	13.4	14.9	11%
$\theta_4 = 2.45$	59.7	56.4	−6%
Overall	29.3	23.6	−20%
Maximum Firm Size			
$\theta_2 = 1.20$	6.7	8.7	29%
$\theta_3 = 1.46$	35.9	46.5	29%
$\theta_4 = 2.45$	560.2	418.6	−25%

The long-run response of entrepreneurial firm size to the policy reform towards full privatization is studied by means of the changes in both average and maximum sizes of those firms. Here, the size of a firm is defined as the amount of capital employed by that firm during the process of production. In this context, Table 5.1.2 presents the average and maximum firm sizes corresponding to each efficiency level in the two steady states separately. In the new long-run equilibrium, average firm size is observed to augment notably for the firms with θ_2 and θ_3 and to shrink for the most productive entrepreneurs who are endowed with θ_4[26]. Although the absolute cost of borrowing falls after the reform, the extra cost of borrowing relative to the cost of capital becomes higher in the final steady state, which basically brings about the observed result for θ_4. Furthermore, the overall average of the firm sizes in the non-corporate sector falls by 20%, which can mainly be attributed to the change in the distribution of entrepreneurs over the entrepreneurial efficiency points[27].

Likewise, the maximum firm size is reported to climb up considerably for the firms characterized by the second and third entrepreneurial states, and to fall down for those with the highest entrepreneurial productivity[28]. The observation that the maximum firm size is

[26]Since none of the most productive entrepreneurs is, irrespective of the presence of the social security, reported to operate at profit maximizing levels without borrowing, privatization leads to a reduction in profit when those with identical asset holdings in both steady states are compared. Hence, this partially explains why entrepreneurs can not reach the firm sizes of the initial steady state in the absence of social security.

[27]In the initial steady state, 30% of entrepreneurs have entrepreneurial efficiency of θ_2, 33% have θ_3, and 37% of them are endowed with θ_4. In the final steady state, these percentages become 38%, 33%, and 29%, respectively.

[28]The maximum firm size is observed to be the profit maximizing level of capital except for the firms with θ_4 all of which are observed to operate below the profit maximizing levels without borrowing.

reduced in the final steady state is mostly derived by the constancy of the extra cost of borrowing, η, throughout the reform. Even though the level of η remains unchanged in the new steady state, borrowing becomes more expensive for entrepreneurs with respect to the real interest rate that considerably drops in the post-privatization equilibrium. So, a separate robustness test is conducted regarding the value of η, but its results are not presented in the paper in order to save space. In this experiment, the ratio of extra cost of capital to the real interest rate in the initial steady state is preserved after the policy change, and both average and maximum firm sizes are observed to increase for all entrepreneurial efficiency points in the final steady state without the pension system[29]. This clearly indicates that the ratio of the value of η to the real interest rate has a decisive impact on the response of entrepreneurial capital demand to the reform.

One of the other possible explanations for the rise in the average and maximum of the entrepreneurial firm sizes is the change in the composition of inputs in favor of capital if they are operating at profit maximizing levels without resorting to borrowing. Notice that the profit maximizing level of capital without borrowing is higher in the new steady state for any given given level of efficiency as the firms optimally switch from labor to capital in order to take advantage of new factor prices. Alternatively, the rise in the savings of households for precautionary purposes also contributes to the accumulation of greater individual wealth than before, and as a result entrepreneurs gain access to larger amounts of capital. Additionally, the increase in the minimum firm size somewhat pulls up the average firm sizes in all efficiency levels as households need a larger firm in order to choose entrepreneurship optimally in the face of increasing wage rates. It is also important to mention that entrepreneurial firms generate lower profits at all efficiency levels in the new equilibrium provided that they operate below profit maximizing levels without borrowing, and their owners have the same level of wealth as before.

Further moments from the entrepreneurial sector are shown in Table 5.1.3 for a better and thorough understanding of the entrepreneurial sector. One of the most striking findings of this paper is related to the fraction of entrepreneurs in the whole population. The ratio of entrepreneurs is documented to increase by a very significant percentage, 29%, mostly because of the huge fall in the number of households with zero wealth and the rise in the average asset holdings after privatization. In other words, more households prefer to be

[29]Numerically, the average and maximum firm sizes associated with the highest efficiency level rise by 2% and 7%, respectively.

Table 5.1.3: The long-run effects of the policy change on non-corporate sector

	SSS*	No SSS	%Δ
Ratio of Entrepreneurs	7.60%	9.84%	29%
Share of Entrepreneurs' Wealth in Total	42.79%	26.11%	−39%
Share of Entrepreneurs' Income in Total	28.09%	26.00%	−7%
Wealth held by Entrepreneurs	2.19	2.19	−0%
Non-Corporate Capital (K_e)	2.22	2.41	9%
Non-Corporate Labor (H_e)	0.48	0.38	−22%
Non-Corporate Output (Y_e)	1.26	1.13	−10%

* SSS stands for Social Security System

entrepreneur in the absence of social security. Additionally, the share of entrepreneurs in total wealth is reduced from 43% to 26% although the level of wealth held by entrepreneurs remains roughly unchanged[30]. Similarly, the share of entrepreneurs in total income experiences a fall that approximately amounts to 7%[31]. Moreover, the amount of capital employed in the non-corporate sector increases by 9% while the labor demand and output decrease by 22% and 10%, respectively implying a significant loss of income for entrepreneurs[32].

Finally, the behavior of entrepreneurs along the transition path has to be mentioned at this point to complete the illustration of the effects of privatization on the non-corporate sector. The evolution of the moments listed in Table 5.1.3 is an integral part of the transitional analysis of the economy. In this context, Figure 5.1.2 plots the paths followed by these preselected moments during the transition. According to these figures, fraction of entrepreneurs is observed to increase over time with a glitch in period 8, while their share in total wealth is being reduced slowly as the economy moves to the new steady state. This glitch occurs because the ratio of entrepreneurs characterized by θ_2 and l_3 becomes zero at this date and onwards due to the increase in the wage rate. On the other hand, the share of entrepreneurs in total income suddenly jumps up to 30% upon the announcement of new policy and progressively goes down to 26% in the passage of time. In the non-corporate

[30]The level of wealth held by entrepreneurs expands by 7% in the experiment in which $\frac{\eta}{r}$ is kept constant. Nevertheless, the share of entrepreneurs in total wealth falls to 27% in the same experiment in support of the benchmark result.

[31]More precisely, it falls by 7.44%. Additionally, the share of entrepreneurs in total consumption goes down by 6.95%.

[32]In the benchmark model, the average income is observed to decrease by 32% and 24% for the young and old entrepreneurs, respectively. Similarly and supposedly, the mean income of the retired falls by 92% while that of the workers increases by 17%. The economy-wide average income is reported to go down by approximately 3%.

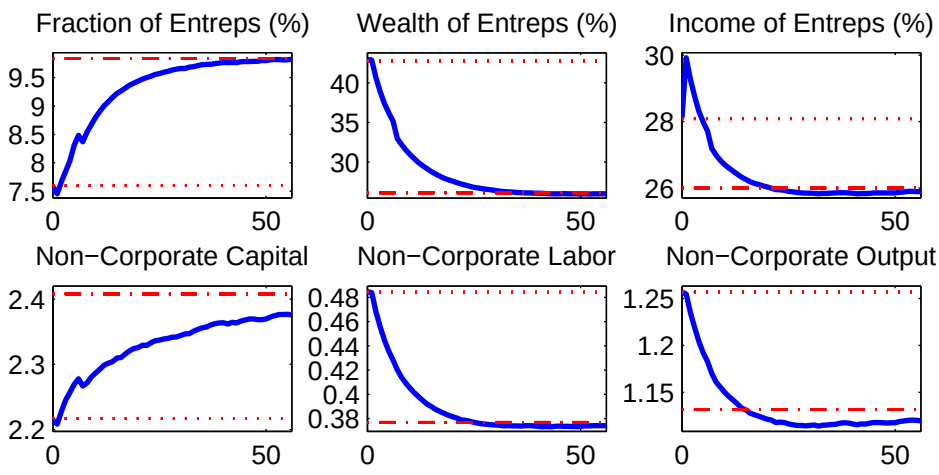

Figure 5.1.2: Entrepreneurs along the transition path.

sector, the amount of capital employed arises sluggishly with a glitch that reflects the one in the ratio of entrepreneurs. On the contrary, the non-corporate labor demand and output are reported to go down continuously along the transition path.

5.2 Wealth Distribution

The policy experiments that are conducted within benchmark, alternative, and standard frameworks reveal that the economy with social security has a more unequal wealth distribution than the one without it, and the relevant quantitative findings are exposed in Table 5.2.1. Unlike the standard model, both benchmark and alternative models successfully replicate not only the Gini coefficient but also the dispersion of wealth observed in the data, especially in the top tail. In the final steady state, the wealth Gini is documented to fall down in all three models but at different rates implying a more equal distribution of wealth. Here, there are mainly two central observations that notably contribute to the decrease in the Gini coefficient of wealth or in the degree of wealth inequality. First, the fraction of households who have very little or no wealth dramatically drops in the new long-run equilibrium following the policy reform. Second, the reform leads to comparatively significant losses in the wealth share of the wealthiest households thereby reducing the concentration of wealth in the top tail.

When the bottom tail of the wealth distribution is taken into account, the fraction of households without any wealth is observed to become zero in the post-privatization era

Table 5.2.1: The long-run effects of the policy change on wealth distribution

	% of the wealth owned by the top						% with	Wealth
	1%	5%	10%	20%	40%	60%	a≤ 0	Gini
U.S. Data	34.70	57.80	69.10	81.70	93.90	98.90	6.9-12.9	0.803*
Benchmark								
SSS	30.05	60.38	70.33	81.77	94.26	98.99	14.94	0.804
No SSS	13.16	30.07	39.97	55.18	76.20	89.24	0.00	0.521
Standard								
SSS	4.17	16.88	29.52	49.45	76.63	92.04	1.71	0.492
No SSS	3.18	13.61	24.55	42.57	68.59	85.40	0.00	0.388
Alternative								
SSS	36.50	52.48	66.73	83.28	95.27	99.50	3.45	0.803
No SSS	25.88	41.02	54.83	71.21	86.71	94.98	0.00	0.674

* Wealth shares and wealth Gini are taken from Rodriguez et al. (2002).

undiscriminatingly in all three economies as they optimally expand their personal stock of assets so as to insure themselves against income and lifespan uncertainties in the absence of the public pension system. Notice that the households with zero wealth constitute a noteworthy fraction of the whole population in the benchmark economy that roughly amounts to 15%. This is a fairly big mass of households gathered at the bottom of the wealth distribution when compared to, 4% and 2%, in the alternative and standard models, respectively[33]. Moreover, the wealth shares of the households in the bottom quintiles are also observed to increase drastically as they optimally update their saving decisions in response to the policy change[34]. All these changes in the bottom of the wealth distribution greatly contribute to the reduction in the wealth inequality as a consequence of privatization.

Even though all households are optimally forced to increase their savings for insurance purposes, shares of the wealthiest households are substantially lowered further reducing the wealth concentration in all three models according to Table 5.2.1. Nonetheless, the rate of change in the share of the wealthiest households is significantly higher in the benchmark economy than those obtained for the standard and alternative models as shown in the table. Although the maximum and average level of individual wealth turn out to be higher in the steady state after privatization, the share of the wealthiest is observed to go down in the economies without entrepreneurs[35]. Hence, it leads to the conclusion that the share of the

[33]The wealth Gini is found to be 0.769, 0.800, and 0.484 in the benchmark, alternative, and standard economies, respectively in the presence of social security when computed for the households who have non-zero wealth.

[34]For instance, the share of the lowest 40% increases from 1% to 11%, from 1% to 5%, and from 8% to 15% in the benchmark, alternative, and standard economies, respectively.

[35]The maximum level of wealth increases by 16% and 5% in standard and alternative models, respectively.

wealthiest households falls mostly because of the relatively higher increase in the shares of the poorest households in those economies. Since the households with little or no wealth are more vulnerable to the future risks, they aggressively increase their savings for insurance purposes. On the other hand, the maximum level of asset holdings is reduced by 39% in the benchmark framework indicating a direct and additional contraction in the share of the wealthiest since the most productive entrepreneurs suffer considerable losses in business profits after the removal of social security[36]. Since the profits of entrepreneurs with identical asset holdings and efficiency either fall or remain the same after privatization, business profits are influential in the observed reduction in the dispersion of wealth[37]. In this respect, it is the introduction of entrepreneurship that further reduces the share of the wealthiest in total wealth not the ability of the model to match the distribution of wealth in the data.

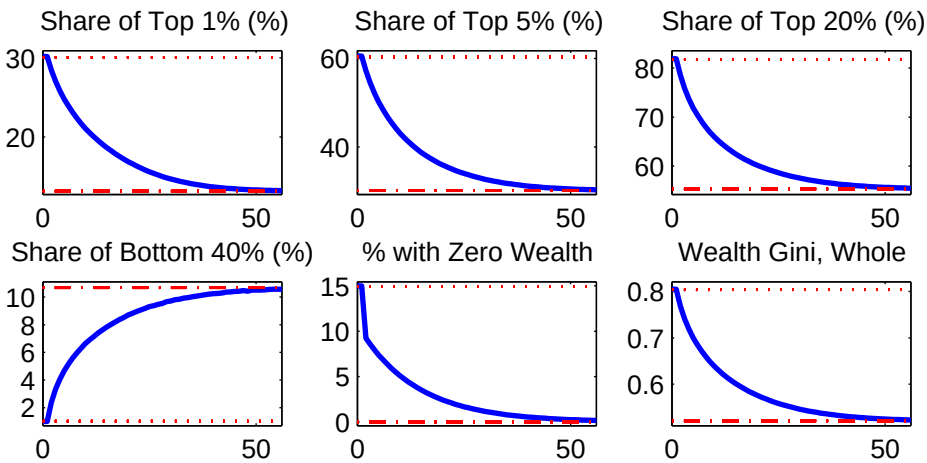

Figure 5.2.1: Wealth distribution along the transition path, whole population. The dotted line represents the initial steady state while the dashed line stands for the final steady state in this figure and the others related to the transition.

Unlike the standard and alternative models, the concentration of wealth is further reduced in the benchmark framework after the policy change because the presence of entrepreneurs opens a new channel through which the share of the wealthiest households in total net worth is lowered to a greater extent. Therefore, the wealth Gini falls by 35% in the benchmark model coupled with the increase in the share of the households at the bottom of

[36]The maximum level of wealth falls by 24% even in the experiment in which the ratio, $\frac{\eta}{r}$, is kept constant, and so the average and maximum firm sizes of the most productive entrepreneurs are observed to increase after the reform.

[37]In the initial steady state, entrepreneurs constitute 71%, 53%, and 38% of the households in the group of the wealthiest 1%, 5%, and 10%, respectively.

the distribution[38]. On the other hand, the Gini coefficient of wealth decreases by smaller percentages, 21% and 16% in the standard and alternative models, respectively. A model that does not incorporate entrepreneurship ignores this new mechanism and underestimates the reduction in the wealth inequality even if it successfully matches the distribution of wealth observed in the data. In order to shed some light on the benchmark economy during the transition, the time paths of wealth Gini, the fraction with zero wealth, the shares of the top 1%, 5%, 20%, and bottom 40% are also displayed in Figure 5.2.1. They all are observed to gradually converge to their corresponding values in the steady state without social security.

Table 5.2.2: The long-run effects on the age-dependent wealth distribution

	% of the wealth owned by the top						% with	Wealth
	1%	5%	10%	20%	40%	60%	a$\leq$ 0	Gini
Young (1)	28.42	57.34	67.07	78.60	92.16	98.14	8.12	0.773
Young (2)	12.08	27.89	37.29	52.04	73.22	87.03	0.00	0.476
Old (1)	36.99	73.28	83.51	92.54	99.09	100.0	42.60	0.894
Old (2)	18.62	41.31	52.69	67.67	65.09	94.20	0.00	0.652

* (1) refers to the case with unfunded social security and (2) refers to case without it.

In addition to the study of wealth distribution among the whole population, it is also important to investigate the effects of privatization on the age-specific wealth distribution, the wealth distribution among the households belonging to the same demographic state. In doing so, Table 5.2.2 displays some key statistics describing the wealth distribution among the young and old individually. However, the analysis solely focuses on the benchmark economy excluding both the standard and alternative models. According to those findings, the distribution of wealth among the young households seems more equal than the one for the old households regardless of the presence of unfunded social security. Notice that the ratio of old households without any assets among the old is extremely high, and this leads to a very low concentration of wealth in the bottom tail of the corresponding distribution. For the same reasons explained in the previous paragraphs, the wealth distribution becomes less concentrated after privatization within each demographic group. Supplementarily, Figure 5.2.2 shows the evolution of the wealth shares of top 1% and 20%, bottom 40%, and wealth Gini within each individual group throughout the transition. The paths followed by these statistics during the transition are rather similar to those displayed for the whole population in Figure 5.2.1.

[38]The fall in the wealth Gini turns into 32% in the benchmark model after accounting for the households with zero wealth.

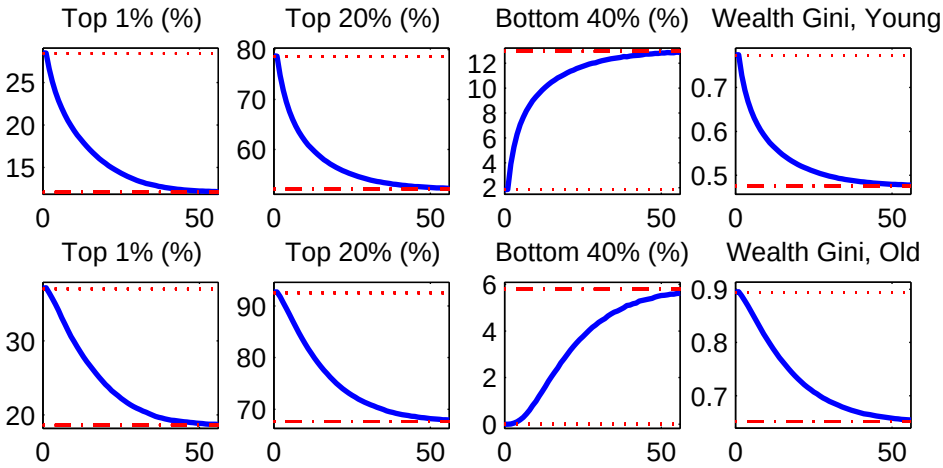

Figure 5.2.2: Age-specific wealth distribution along the transition path: upper half for the young and lower half for the old. First three figures on each row are the shares of corresponding groups of households in total wealth.

Table 5.2.3: The long-run effects on the occupation-specific wealth distribution

	% of the wealth owned by the top						% with	Wealth
	1%	5%	10%	20%	40%	60%	a≤ 0	Gini
Entrepreneurs (1)	11.06	35.88	54.02	74.35	90.89	96.47	0.00	0.701
Entrepreneurs (2)	8.91	30.06	46.47	66.05	83.70	92.32	0.00	0.605
Non-Entrepreneurs (1)	26.62	48.15	59.46	75.19	92.51	98.76	16.17	0.749
Non-Entrepreneurs (2)	8.74	20.61	31.37	48.67	72.81	87.74	0.00	0.455

* (1) refers to the case with unfunded social security and (2) refers to case without it.

Furthermore, the effect of privatization on the distribution of wealth among the households who are categorized with respect to their occupation is also considered, and the resulting findings are summarized in Table 5.2.3. More overtly, the relevant descriptive statistics about the wealth distribution among entrepreneurs and non-entrepreneurs are computed separately within each group. Those results indicate that wealth is dispersed more equally within entrepreneurs than non-entrepreneurs in the presence of unfunded social security. After the new policy is implemented, wealth distribution becomes more equal as the concentration of wealth decreases in the top tail and increases in the bottom tail of the distribution within both groups of households. Unlike the initial steady state, the wealth turns out to be distributed more equally among non-entrepreneurs than entrepreneurs in the final steady state. In other words, wealth inequality is observed to decline among each group of households but it is larger among entrepreneurs than non-entrepreneurs in the absence of unfunded

social security as measured by the corresponding Gini coefficients.

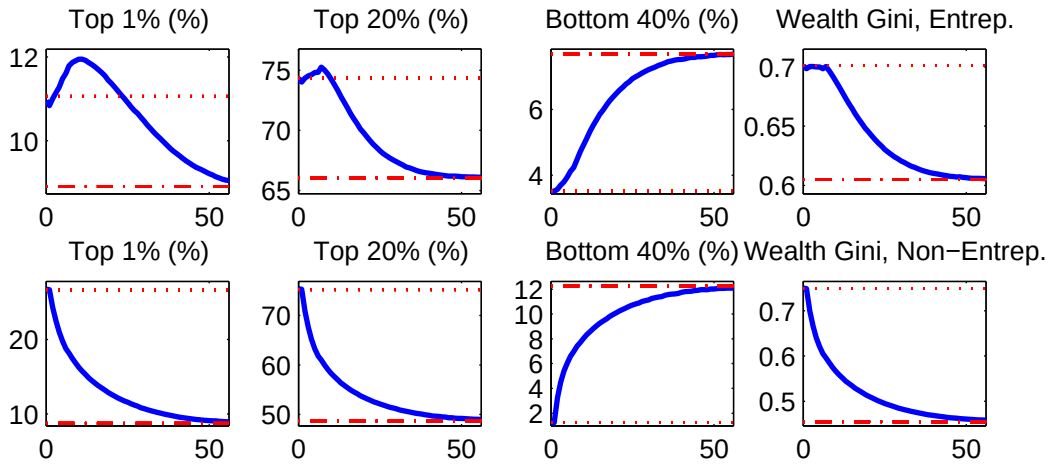

Figure 5.2.3: Occupation-specific wealth distribution along the transition path: upper half for entrepreneurs and lower half for the rest. First three figures on each row are the shares of corresponding groups of households in total wealth.

To complete the discussion about the occupation-specific wealth distribution, Figure 5.2.3 shows the evolution of the shares of the top 1% and 20%, bottom 40%, and the wealth Gini among entrepreneurs and non-entrepreneurs discretely. In case of non-entrepreneurs, all these shares and the Gini coefficient of wealth follow paths that are very similar to those for the whole population[39]. On the other hand, the paths obtained for the entrepreneurs exhibit different patterns during transition. The share of bottom 40% in total entrepreneurial wealth constantly increases as the most productive entrepreneurs suffer big losses in their profits after privatization, and all households optimally increase their wealth in the absence of pension payments. In the other tail of the distribution, the shares of top 1% and 20% slowly increase in the early periods and then constantly decrease until the new steady state is reached[40]. So, the coefficient of wealth Gini stays almost the same for the first nine periods including the date of announcement and then gradually descends to its value in the new steady state.

[39]See Figure 5.2.1.

[40]This can be explained by the increase in the ratio of entrepreneurs with θ_2 and θ_3, while the ratio of those with θ_4 remains almost unchanged.

5.3 Further Results

The benchmark model unintendedly generates an income distribution that is very close to the one observed in the U.S. data[41]. Given the uncompensated elimination of the unfunded social security, it is obvious that income distribution becomes more unequal after privatization in all three models because the retired are deprived of pension payments, a significant and reliable source of income. There are of course other factors such as the fall in the rate of return on capital, the rise in the wage rate, and the abolishment of payroll taxes that somewhat affect the distribution of income after the policy change. Nevertheless, the removal of social security may have different impacts on the age-specific and occupation-specific income distributions. So, the effects of privatization on income distribution, similar to the case of wealth distribution, are first analyzed within the whole population and then among the households that belong to either the same demographic or occupational state.

Table 5.3.1: The long-run effects of the policy change on income distribution

	% of the income of the top						Income
	1%	5%	10%	20%	40%	60%	Gini
U.S. Data	17.50	32.80	43.10	58.00	78.00	90.50	0.553*
Benchmark							
SSS	14.91	28.69	38.85	56.09	75.95	87.11	0.500
No SSS	11.32	25.54	37.50	57.02	79.98	92.94	0.560
Standard							
SSS	2.97	13.56	25.55	44.03	67.87	82.89	0.370
No SSS	3.07	14.67	28.36	48.61	74.74	89.86	0.468
Alternative							
SSS	18.73	35.70	54.30	68.45	85.63	92.97	0.629
No SSS	17.43	35.41	55.92	70.71	89.28	95.09	0.674

* Income distribution data is taken from Rodriguez et al. (2002).

The distribution of income and the corresponding Gini coefficients for the whole population are displayed in Table 5.3.1 for each economy. The Gini coefficient of income is reported to increase in the absence of social security mainly because of the contraction in the shares of the households with the lowest income in all three models. At the top of the income distribution, the effect of privatization varies across models. In the standard setup, the shares of the households in the top percentiles are observed to go up by a small amount.

[41] As a reminder, the definition of income includes the labor earnings, profits, interest income, and pension payments.

Unlike the standard model, the shares of the top 1% and 5% decrease slightly in the alternative model thereby limiting the increase in the Gini coefficient of income[42]. Similarly but more strongly, the shares of those in the top 1%, 5%, and 10% are reported to go down in the benchmark economy bounding the extent to which the income inequality deteriorates[43]. In this sense, the presence of entrepreneurs can somewhat lessens the deterioration in the income inequality following the removal of social security. Next, the evolution of the income

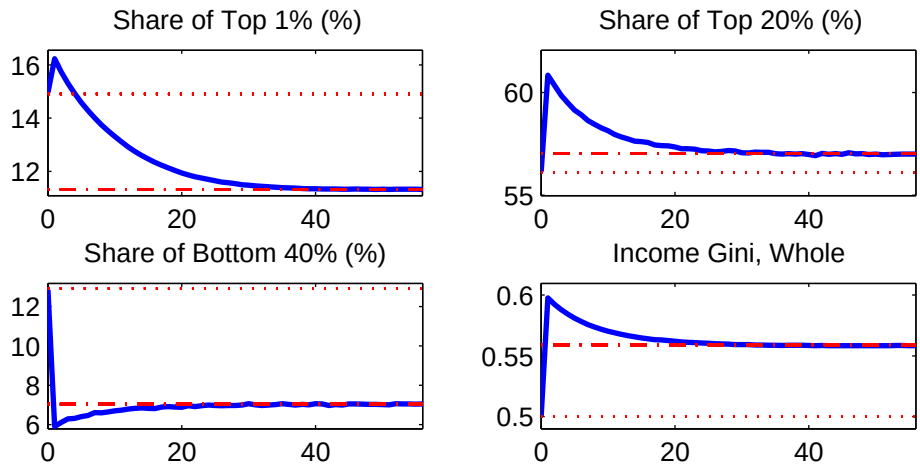

Figure 5.3.1: Income distribution along the transition path, whole population.

shares and the Gini coefficient along the transition path are presented in Figure 5.3.1 for the benchmark model. The income Gini, the shares of the top 1% and 20% gradually move to their corresponding values in the new steady state after an initial sharp increase upon the implementation of the new policy. On the other hand, the share of the bottom 40% initially experiences a very sharp fall and slowly rises up to its new long-run value which is still below the one in the initial steady state.

In case of age-specific income distribution, Table 5.3.2 shows the relevant quantitative findings from the benchmark model. According to those results, the income distribution is becoming more equal among the young and more unequal among the old after privatization as indicated by the changes in the corresponding Gini coefficients. For the young, there

[42]Since the distribution of labor earnings remain unchanged after the policy reform, the fall in those shares can be attributed to the changes in the interest income and thus in the distribution of wealth.

[43]In this case, the shares of the households at the top of the income distribution are lowered mostly because of the losses in business profits of entrepreneurs after privatization. Entrepreneurs constitute almost 100%, 57%, and 29% of the households in the top 1%, 5%, and 10%, respectively, with respect to income in the final steady state.

Table 5.3.2: The long-run effects on the age-specific income distribution

	% of the income owned by the top						Income
	1%	5%	10%	20%	40%	60%	Gini
Young (1)	13.40	26.16	35.43	53.29	74.33	87.46	0.479
Young (2)	9.25	21.21	31.01	49.94	71.96	85.83	0.436
Old (1)	23.47	41.35	47.44	54.46	66.66	100.0	0.419
Old (2)	44.81	82.40	90.22	93.65	97.16	98.92	0.905

* (1) refers to the case with unfunded social security and (2) refers to case without it.

are mainly two channels through which the income distribution is getting less concentrated. First, the fall in the wealth inequality acts to reduce the variability in the capital income. Second and most importantly, the lower entrepreneurial profits reduce the income shares of the households in the top percentiles which is absent in the economies without entrepreneurs. On the other hand, the removal of the pension payments makes the retired solely depend on interest income and largely deteriorates the income inequality among the old.

Table 5.3.3: The long-run effects on the occupation-specific income distribution

	% of the income owned by the top						Income
	1%	5%	10%	20%	40%	60%	Gini
Entrepreneurs (1)	7.53	27.97	45.02	66.03	85.54	93.42	0.614
Entrepreneurs (2)	6.67	25.83	43.06	66.10	85.45	92.73	0.605
Non-Entrepreneurs (1)	3.49	14.08	27.00	46.42	70.26	83.63	0.394
Non-Entrepreneurs (2)	3.06	14.81	29.22	50.62	77.55	92.68	0.505

* (1) refers to the case with unfunded social security and (2) refers to case without it.

In order to partially isolate the effect of the fall in the earnings of the entrepreneurs on the income distribution, an analysis that distinguishes between the two occupational states is also conducted. As shown in Table 5.3.3, income distribution is more concentrated among entrepreneurs than non-entrepreneurs in both steady states of the benchmark economy. A more important and interesting result arises when the impact of privatization is considered within each occupational group separately. The removal of unfunded pension system leads to a more equal distribution of income among entrepreneurs since the ratio of the most productive entrepreneurs is lowered in the new steady state[44]. On the contrary, the income inequality is notably worsened among the non-entrepreneurs as the share of the households in the bottom of the distribution is reduced. The income Gini for the non-entrepreneurs still

[44]The fraction with θ_4 among the entrepreneurs falls from 37% to 29% mostly because of the increase in the number of entrepreneurs with θ_2 and θ_3.

remains below the one for the entrepreneurs.

Table 5.3.4: The long-run effects of the policy change on the distribution of consumption expenditures

	% of the consumption by the top						Consumption
	1%	5%	10%	20%	40%	60%	Gini
U.S. Data	4.83	11.13	20.84	40.27	63.60	80.87	0.320*
Benchmark							
SSS	11.03	25.38	33.78	47.44	68.23	82.63	0.406
No SSS	7.69	20.06	29.66	45.76	69.99	86.27	0.426
Standard							
SSS	2.35	10.26	18.99	34.16	58.53	77.12	0.256
No SSS	2.56	11.48	21.38	38.45	64.73	83.14	0.343
Alternative							
SSS	17.61	32.38	45.88	61.52	79.25	90.28	0.562
No SSS	15.44	31.70	46.62	62.76	82.86	92.93	0.598

* Consumption distribution data is taken from Casteneda et al. (2003).

The elimination of unfunded social security has also serious impacts on the distribution of consumption expenditures. Table 5.3.4 depicts the distribution of consumption in both initial and final steady states of all three models along with the U.S. data. Notice that the distribution of consumption expenditures in the data is not as concentrated as the U.S. income and wealth distributions. The consumption is dispersed more unequally in both the benchmark and alternative models and less unequally in the standard model than the data. The policy experiments reveal that the privatization worsens the consumption inequality, and the share of the bottom 40% is significantly reduced after the removal of pension payments in all three models. Although the fall in the profits of most productive entrepreneurs reduces the concentration in the top tail of distribution, the consumption Gini still increases in the benchmark model[45]. Additionally, the evolution of the consumption shares and the Gini coefficient are displayed in Figure 5.3.2 for the benchmark economy. All of them are reported to converge constantly to their equilibrium values in the final steady state after an initial sudden jump. Those paths initially make either a peak or trough due to the unexpected removal of pension payments.

Similar to the analysis of wealth and income distribution, the effects of privatization on the age-specific consumption distribution are also investigated within the benchmark

[45]Entrepreneurs constitute almost 90%, 61%, 36%, 21%, and 14% of the households in the top 1%, 5%, 10%, 20%, and 40%, respectively, with respect to consumption expenditures in the final economy.

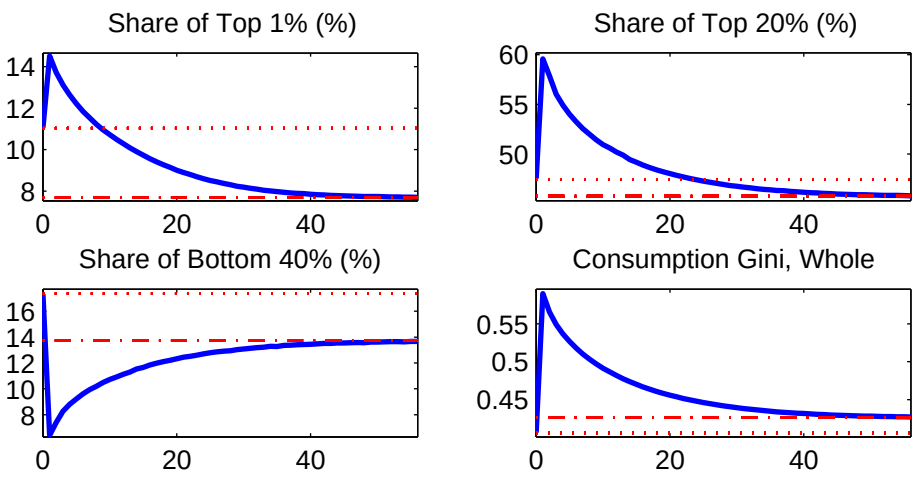

Figure 5.3.2: Consumption expenditures distribution along the transition path, whole population.

Table 5.3.5: The long-run effects on the age-specific distribution of consumption expenditures

	% of the consumption by the top						Consumption
	1%	5%	10%	20%	40%	60%	Gini
Young (1)	10.54	24.41	32.64	46.23	67.96	83.59	0.406
Young (2)	6.58	17.46	25.90	40.32	63.35	80.22	0.330
Old (1)	13.02	29.19	37.01	47.73	64.01	76.84	0.352
Old (2)	17.88	41.94	53.01	67.84	85.33	94.44	0.656

* (1) refers to the case with unfunded social security and (2) refers to case without it.

framework, and the results are shown in Table 5.3.5. Notice that the share of the young households at the top of the distribution noticeably falls down in the final steady state while that of the households at the bottom oppositely increases. Therefore, the distribution of consumption among the young becomes less concentrated after the policy change. In case of the old, exactly the opposite takes place- the shares of the top percentiles increase, the share of the bottom 40% is reduced, and the Gini coefficient accordingly grows indicating the deterioration in the consumption inequality among the old. Moreover, the distribution of consumption among the young is more concentrated than the one among the old in the initial steady state while it becomes less concentrated in the final steady state.

Additionally, the occupation-specific distribution of consumption expenditures in the benchmark model is displayed in Figure 5.3.6 to further comprehend the impact of the unfunded social security. The consumption distribution among the entrepreneurs is reported to be more concentrated than the one among the non-entrepreneurs as indicated by the

Table 5.3.6: The long-run effects on the age-specific distribution of consumption expenditures

	% of the consumption by the top						Consumption
	1%	5%	10%	20%	40%	60%	Gini
Entrepreneurs (1)	7.42	25.58	40.25	59.11	79.43	89.71	0.537
Entrepreneurs (2)	6.16	22.18	36.04	55.12	76.63	87.99	0.494
Non-Entrepreneurs (1)	6.42	15.86	24.72	40.09	63.45	79.83	0.325
Non-Entrepreneurs (2)	4.29	13.71	23.77	41.19	67.68	85.48	0.384

* (1) refers to the case with unfunded social security and (2) refers to case without it.

corresponding Gini coefficients irrespective of the unfunded social security. The removal of unfunded social security lessens the consumption inequality among the entrepreneurs by raising the share of the lowest 40% and reducing that of the households at the top of the distribution. On the contrary, the consumption distribution among the non-entrepreneurs becomes more concentrated in the absence of the public pension system despite the decline in the shares of the top 1%, 5%, and 10%, as the share of the households at the bottom of the distribution becomes smaller.

Finally, the welfare implications of the policy reform within the frameworks of all three economies are analyzed by comparing the long-run equilibria before and after the reform from the viewpoint of an unborn household. The expected utility of this household is used as a unit of measure in comparisons. The corresponding consumption compensations are computed within simulations by the help of corresponding expected utilities[46]. The compensation in the absence of entrepreneurship is found to be approximately 3% and 2% for the standard and alternative models, respectively, while it is reported to be -1% in the benchmark setting with entrepreneurs. The negative sign means that the utility value of partial insurance provided by the unfunded social security outperforms the one associated with the increase in the stock of productive capital. Therefore, an unborn household would be better off if he is born into an economy without the unfunded social security in the absence of entrepreneurship. Nevertheless, the introduction of entrepreneurship into this economy as an alternative occupation reverses this conclusion for an unborn household. Furthermore, the study of the economy in transition reveals that none of the existing household would

[46] $CC = \left[\left(\frac{V^{NS}}{V^{S}}\right)^{\frac{1}{1-\sigma}} - 1\right]$ where V^S and V^{NS} are the expected value functions of the young with and without the social security, respectively. Consumption compensation(CC) or consumption equivalent variation(CEV) is the necessary percentage change in the amount of consumption of a household and his descendants in all periods and contingencies of the initial equilibrium in order for this household to be indifferent between the two equilibria of the same economy.

vote in favor of the privatization without any compensation regardless of the presence of entrepreneurship. In other words, the removal of the unfunded social security would lack the support of the majority of the households in all three model economies.

6 Sensitivity Analysis

This section documents the results of the robustness test regarding the main findings from the benchmark calibration. Here, a sensitivity analysis is performed by relaxing the restrictions on the coefficient of relative risk aversion, σ, the share of capital in the non-corporate sector, $\nu\alpha_e$, and corporate sector, α.

6.1 Coefficient of Relative Risk Aversion

As stated before, privatization triggers a limited boom in the savings of households, the extent of which partially depends on the strength of their risk averseness. In order to analyze the persistence of the main results, the coefficient of relative risk aversion is allowed to deviate from its benchmark specification and to take two additional values; 2.0 and 1.0, in case of the latter period utility function reduces to the logarithmic form. A value of σ greater than 2.0 is assumed to lead to an interest rate that is either negative or too close to zero in the final steady state of the benchmark economy[47]. Additionally, some parameters of the model are allowed to adjust within each experiment in order to maintain the consistency of the model with data, and quantitative results are obtained for each value of σ. The discount factor, entrepreneurial efficiencies and transition probabilities, and the income tax parameters are among those parameters.

The results of the sensitivity analysis concerning the wealth distribution are listed in Table 6.1.1. They all indicate that the benchmark findings with regard to the wealth distribution are very robust to the changes in σ. At the bottom of the distribution, both the share of the lowest 40% and the fraction with zero wealth adjust after the reform as in the benchmark specification. Despite small differences, the share of the poorest 40% hugely expands, and the fraction with zero wealth becomes almost zero. Moreover, the shares of the wealthiest 1% and 20% fall by considerable amounts leading to a more equal distribution of wealth regardless of the value of σ. In conjunction with these observations, the coefficient of wealth Gini is observed to decline meaningfully amount of which is proportional to the

[47]The maximum value of σ at which the interest rate still stays positive in the final steady state of the benchmark model is 2.15.

Table 6.1.1: Wealth distribution in the benchmark setup

	$\sigma = 1.0$		$\sigma = 1.5^*$		$\sigma = 2.0$	
	SSS	No SSS	SSS	No SSS	SSS	No SSS
Wealth						
Gini	0.800	0.574	0.804	0.521	0.803	0.477
Share of top 1%	26.92%	14.89%	30.05%	13.16%	30.57%	10.91%
Share of top 20%	81.62%	59.87%	81.77%	55.34%	81.73%	51.58%
Share of lowest 40%	0.91%	8.42%	1.01%	10.68%	1.12%	12.50%
% with zero wealth	14.55%	0.00%	14.94%	0.00%	10.21%	0.00%

* Benchmark calibration.

value of the coefficient of risk aversion. On the other hand, the results concerning the effects of privatization on the age- and occupation-specific wealth distributions in the benchmark economy do not vary much across different values of σ, but they are not included in the table and neither are those about the income and consumption distributions in order to save space[48]. For the same reason, the results of the robustness test in the standard and alternative economies are not included either[49].

Table 6.1.2: The long-run aggregate effects

	Percentage Increases after the Removal of the Unfunded Social Security								
	$\sigma = 1.0$			$\sigma = 1.5^*$			$\sigma = 2.0$		
	(Bnch	Stnd	Altr)	(Bnch	Stnd	Altr)	(Bnch	Stnd	Altr)
K	37%	37%	22%	64%	71%	38%	95%	115%	54%
Y	9%	12%	7%	14%	21%	12%	19%	32%	17%
C	6%	11%	7%	9%	18%	11%	10%	25%	15%

* Benchmark calibration.
* Bnch, Stnd, and Altr refer to benchmark, standard, and alternative models.

Next, Table 6.1.2 shows how aggregate capital stock, output, and consumption respond to privatization in all three economies under each specification of σ. For each value of σ, all

[48] Additionally, the findings about the income and consumption distributions are robust across the additional values of σ with a single exception. Unlike benchmark observation, the income distribution among the entrepreneurs becomes somewhat more unequal for $\sigma = 1$ as indicated by a 1.12% increase in the Gini coefficient.

[49] The findings about the wealth distribution in alternative and standard models are also found to be robust to the changes in σ with the exception that the wealth distribution among the old of the standard model becomes slightly unequal for $\sigma = 1$ as measured by a 0.39% rise in the wealth Gini. In case of the income and consumption distributions, the results from both standard and alternative models are persistent.

these aggregates are observed to increase significantly after the reform in all three economies, the magnitudes being proportional to the value of σ. As mentioned before, the rise in aggregate consumption accounts for 39% of the increase in aggregate output in the presence of entrepreneurs while it turns out to be 56% and 59% in the standard and alternative models, respectively. After allowing the coefficient to diverge from its benchmark value, the contribution of consumption to the increase in aggregate output is observed to be inversely proportional to the value of σ. Moreover, the contribution of consumption (investment) to the increase in output is reported to be at least 27% lower (38% higher) in the benchmark model than the standard and alternative models for the additional values of σ strengthening the relevant benchmark findings.

Table 6.1.3: The long-run effects on the non-corporate sector and welfare

	Percentage Change after the Reform		
	$\sigma = 1.0$	$\sigma = 1.5^*$	$\sigma = 2.0$
Ratio of Entrepreneurs	33%	29%	35%
Share of Entr. in Total Wealth	−28%	−39%	−44%
Non-Corporate Capital (K_e)	4%	9%	17%
Non-Corporate Labor (H_e)	−16%	−22%	−26%
Non-Corporate Output (Y_e)	−8%	−10%	−11%
CC with Entrepreneurs	2.58%	−0.95%	−7.74%

* Benchmark calibration.
* CC refers to consumption compensation.

Further sensitivity analysis is conducted regarding the benchmark findings about the effects of policy reform on the aforementioned moments in the non-corporate sector and the welfare. The fraction of entrepreneurs, following the elimination of the unfunded social security, is significantly raised for each choice of σ, and the percentage changes are not too different from the benchmark value as displayed in Table 6.1.3. Even though the ratio of entrepreneurs increases, their share in total wealth notably falls after the reform regardless of the value of σ. The results also reveal that the physical capital demanded by the non-corporate sector increases while the non-corporate labor demand and output decline in the new steady state for each value of σ. In case of the welfare, the consumption compensation is reported to be positive for logarithmic utility but negative for $\sigma = 2.0$ with a larger magnitude than the one in the benchmark case. Therefore, the benchmark sign of the consumption compensation is not robust to the strength of relative risk aversion.

6.2 Share of Capital

The income share of capital in the corporate sector is denoted by α, and the one in the non-corporate sector is given by $u_1 = \nu\alpha_e$. In the benchmark calibration, the parameter, α_e, is set equal to α following the literature on entrepreneurship so that the non-corporate share of capital becomes a constant fraction of the corporate share. This assumption is relaxed in two directions in order to test the robustness of the benchmark results to the variations in the share parameters, α and α_e. First, the corporate share is fixed at its benchmark value, and α_e is allowed to take two additional values, 0.30 and 0.42. Second, the assumption of both α's having the same value remains unchanged, and α is allowed to take values from the set, $\{0.33, 0.42\}$. Since the numerical experiments with $\alpha < 0.33$ yield non-positive interest rates in the initial steady state, those values are simply ignored. For each specification of the parameters, the benchmark model is accordingly calibrated and solved with and without the unfunded social security and so are the standard and alternative models[50].

Table 6.2.1: Wealth distribution in the benchmark setup

	$\alpha_e = 0.30$		$\alpha_e = 0.36^*$		$\alpha_e = 0.42$	
	SSS	No SSS	SSS	No SSS	SSS	No SSS
Wealth						
Gini	0.804	0.502	0.804	0.521	0.807	0.542
Share of the top 1%	29.23%	11.56%	30.05%	13.16%	31.07%	14.95%
Share of the top 20%	81.82%	53.53%	81.77%	55.34%	82.11%	57.37%
Share of lowest 40%	0.95%	11.25%	1.01%	10.68%	1.01%	10.08%
% with zero wealth	15.41%	0.00%	14.94%	0.00%	11.31%	0.00%
	$\alpha = 0.33$		$\alpha = 0.42$			
	SSS	No SSS	SSS	No SSS		
Wealth						
Gini	0.803	0.506	0.806	0.545		
Share of the top 1%	29.87%	12.28%	30.40%	15.04%		
Share of the top 20%	81.66%	53.96%	82.12%	57.69%		
Share of lowest 40%	0.96%	11.15%	1.10%	10.01%		
% with zero wealth	15.33%	0.00%	11.24%	0.00%		

* Benchmark calibration.

The distribution of wealth is presented in Table 6.2.1 under various specifications of the share parameters, α and α_e. For the values displayed in the table, the share of the wealthiest

[50] As in the case of σ, some parameters such as the discount factor, entrepreneurial efficiencies and transition probabilities are adjusted to match the certain characteristics of the data.

1% in total wealth is at least halved, and that of the wealthiest 20% is also observed to fall down considerably contributing to the reduction in the wealth inequality after the removal of the unfunded social security. On the other hand, the share of the poorest 40% hugely expands, and the fraction of the households with zero wealth becomes nearly zero in the new steady state regardless of the values assigned to α's. Therefore, the coefficient of wealth Gini indiscriminately decreases after privatization indicating a more equal dispersion of wealth in the benchmark model, and this is highly robust to the changes in the aforementioned share parameters. The results of the same experiments regarding the wealth distributions in the standard and alternative models are not displayed in the table so as to save space, and neither are those about the age- and occupation-specific wealth distributions in the benchmark economy[51].

Table 6.2.2: The long-run aggregate effects

	Percentage Change after the Removal of the Unfunded Social Security										
	$\alpha_e = 0.30$	$\alpha = 0.36^*$			$\alpha_e = 0.42$	$\alpha = 0.33$			$\alpha = 0.42$		
	Bnch	(Bnch	Stnd	Altr)	Bnch	(Bnch	Stnd	Altr)	(Bnch	Stnd	Altr)
K	75%	64%	71%	38%	58%	71%	78%	39%	52%	59%	32%
Y	14%	14%	21%	12%	15%	14%	21%	11%	15%	21%	13%
C	6%	9%	18%	11%	11%	6%	16%	10%	12%	21%	13%

* Benchmark calibration.
* Bnch, Stnd, and Altr refer to benchmark, standard, and alternative models.

For the given values of share parameters, Table 6.2.2 shows the percentage changes in aggregate capital, output, and consumption as the economy gradually moves from the initial to the final steady state. In addition to the benchmark economy, the table documents the results from the standard and alternative economies for comparison purposes. These aggregates are observed to increase persistently in all three economies for all values of the share parameters listed in the table. Most importantly, the increase in aggregate consumption (investment) as a fraction of the rise in aggregate output is at least 16% lower (19% higher) in the benchmark economy than the economies without entrepreneurs for $\alpha \in \{0.33, 0.42\}$ sustaining the equality of corporate and non-corporate shares and $\alpha_e \in \{0.30, 0.42\}$. Also, notice that the contribution of consumption is proportional to the value of α and α_e in all models while the opposite holds for the investment. The findings listed in the table

[51] According to the results that are not displayed in the table, the benchmark findings regarding the wealth, income and consumption distributions in all three economies are robust to the changes in the share parameters, α and α_e.

apparently proves the robustness of the benchmark results regarding the macroeconomic aggregates to the variations in α and α_e.

Table 6.2.3: The long-run effects of the policy change on non-corporate sector and welfare

	Percentage Change after the Reform				
	$\alpha_e = 0.30$	$\alpha_e = 0.36^*$	$\alpha_e = 0.42$	$\alpha = 0.33$	$\alpha = 0.42$
Ratio of Entrepreneurs	31%	30%	36%	43%	27%
Share of Entr. in Total Wealth	−43%	−39%	−32%	−38%	−35%
Non-Corporate Capital (K_e)	6%	9%	16%	14%	7%
Non-Corporate Labor (H_e)	−24%	−22%	−18%	−22%	−20%
Non-Corporate Output (Y_e)	−13%	−10%	−4%	−10%	−8%
CC with Entrepreneurs	−6.15%	−0.95%	2.80%	−5.47%	7.59%

* Benchmark calibration.

Finally, the robustness of the effects of eliminating the unfunded social security on the non-corporate sector is also tested, and the findings are summarized in Table 6.2.3 for the aforesaid values of α and α_e. According to those numbers, the fraction of entrepreneurs turns out to be considerably higher in the final steady state of the economy regardless of the choice of parametrization. In a similar way, the fall in the share of entrepreneurs in total wealth also seems very robust. The amount of capital employed is shoved up at significantly non-zero rates while the total labor employed and output produced in the non-corporate sector are lowered regardless of the values of α's. Furthermore, Table 6.2.3 presents the effects of the policy reform upon the welfare as measured by the consumption compensations. The required consumption compensation is reported to take positive values when $\alpha_e = 0.42$ and both α's are 0.42. In all other cases, it even takes smaller values than the one in the benchmark model.

7 Concluding Remarks

This paper contributes to the literature by investigating the quantitative effects of eliminating the unfunded social security in an economy with a wealth distribution that is as concentrated as the one in the U.S. data. For this purpose, an infinite horizon life-cycle model[52] is altered to include an unfunded social security system and entrepreneurship as an alternative

[52]This paper also deviates from the previous literature by adopting an infinite horizon model given the fact that almost all of the previous papers in the literature utilize overlapping generations models in their quantitative analysis.

occupation. So, the results concerning the effects of the policy reform on macroeconomic aggregates can be treated as a robustness test of the previous findings in the literature. In this sense, the results from all three economies- benchmark, standard, and alternative economies disclose that aggregate capital, output and consumption considerably increase after privatization supporting most of the findings in the literature. However, the presence of entrepreneurs, not the model's ability to replicate U.S. wealth distribution, radically alters the way the increase in output is divided between consumption and investment. A bigger (smaller) portion of the rise in output goes to investment (consumption) in the benchmark model than the standard and alternative models underlining the significance of additional saving motive created by entrepreneurship.

The analysis of the effects of privatization on the non-corporate sector is new to the literature. This sector consists of relatively small firms owned and managed by entrepreneurs. Although the fraction of entrepreneurs in total population, 7.6%, is comparatively small, they own 43% of the total net worth in the model economy. Their steady state ratio is found to increase by almost 29% after the removal of unfunded social security. Even though the occupational distribution changes in favor of entrepreneurship, the share of entrepreneurs in total wealth and income is observed to decline significantly in the new steady state. Moreover, the average and maximum firm sizes, measured as the amount of capital employed, are reported to contract for the most productive entrepreneurs and to expand remarkably for the other entrepreneurs. On the other hand, labor demand and total output in the non-corporate sector are shoved down considerably while the capital demand increases at a relatively lower rate than the one in the corporate sector.

Most importantly, the effect of privatization on wealth distribution is extensively studied in the benchmark economy along with the standard and alternative economies. Both benchmark and alternative economies successfully match the wealth dispersion observed in the U.S. data. The quantitative findings indicate that the elimination of the unfunded social security results in a less concentrated distribution of wealth in the long run regardless of the presence of entrepreneurs. In both standard and alternative economies, the wealth distribution becomes more equal as the households optimally react to privatization accumulating larger amounts of wealth in order to insure themselves against the growing income and longevity risks in the absence of social security. Since the households with little or no wealth are more vulnerable to those risks, the wealth share of the households at the bottom of the distribution is observed to increase substantially while the share of those at the top

is observed to fall down accordingly. Unlike the economies without entrepreneurs, the losses in the profits of the most productive entrepreneurs associated with the removal of social security significantly cut down the share of the wealthiest households further reducing the wealth inequality.

Additionally, the age-specific and occupation-specific wealth distributions are investigated independently. According to the findings, the young has a less concentrated distribution than that of the old but both of them become less concentrated after the policy change. Similarly, the privatization reduces the wealth inequality among the entrepreneurs and non-entrepreneurs individually. Even though the wealth is dispersed more equally among entrepreneurs than non-entrepreneurs in the initial steady state, the opposite is observed in the final steady state.

Furthermore, the model generates an income distribution that is very close to the one in the U.S. data, though it is not intended. The elimination of the unfunded social security directly takes away the pension payments from the old thereby worsening the income inequality quite apart from the presence of entrepreneurs. However, its impact is limited in the benchmark economy because of the profit losses of the most productive entrepreneurs. In a closer analysis of the benchmark model, the income distribution is seen to become more equal for the young and more unequal for the old after privatization. Similarly, the income Gini is reported to decline slightly for the entrepreneurs and to increase considerably for the non-entrepreneurs. Additionally, the effects of privatization on the distribution of consumption expenditures is analyzed even though the benchmark model produces a more concentrated distribution than the data. Similar to the income distribution, consumption distribution turns out to be more concentrated after the policy reform in all three economies. Referring to the same reasons as in the income distribution, the negative effect of privatization on the consumption distribution stays limited in the presence of entrepreneurs.

In case of welfare, an unborn household would like to be born in an economy with social security when the two steady states of the benchmark economy are compared from the viewpoint of an unborn household. This finding reverses when the economies without entrepreneurs are taken into account. Nevertheless, these findings from the benchmark calibration of all three models are not robust to the variation in the coefficient of relative risk aversion and income shares of capital[53]. Finally, a transitional analysis reveals that none

[53] An overlapping generations model is obviously a better choice when analyzing the welfare implications of privatization when the unrealistically high lifespan uncertainty in this model is considered.

of the existing households supports the elimination of unfunded social security at the time of its announcement regardless of the presence of entrepreneurs.

Appendices

A Solution Algorithm

A.1 Steady State

The maximization problem of the household is solved by iterating on the value functions over a discrete state space because of the incompleteness of the markets and resulting non-linearities.

- ***Step 1:*** Guess the factor price R, average labor earnings, and fiscal policy as well as the value functions at each state.
- ***Step 2:*** Obtain the optimal decision rules for the households.
- ***Step 3:*** Derive the invariant distribution over the state space by using the policy rules obtained in Step 2.
- ***Step 4:*** Compute the factor prices, average labor earnings, and the fiscal policy that balances the government budgets.
- ***Step 5:*** If the factor price, average labor earnings, and fiscal policy are not the same as those in Step 1, go to Step 1, and update your guess accordingly.

A.2 Transition Path

The transition path between two steady states is also computed by an iterative method. The economy is assumed to be in the initial steady state before period 0 inclusively. The new policy is announced in period 1 and the economy is assumed to reach the final steady state in period T. It is also important to choose T long enough so that the transition path is not altered by increasing T.

- ***Step 1:*** Guess the paths of gross interest rate, average labor earnings, and fiscal policy.
- ***Step 2:*** Solve the maximization problem of the household backwards by using the value functions in the final steady state for the last period, T.

- ***Step 3:*** Starting with the distribution in the initial steady state, obtain a path for it throughout the transition.
- ***Step 4:*** Compute the paths of gross interest rate, average labor earnings, and fiscal policy by utilizing the distribution generated in Step 3. If they are not the same as those in Step 1, go to Step 1, and update your guess. Also make sure that the market clearing conditions hold in every period.

B Parameters of Entrepreneurial Productivity

The resulting vector of entrepreneurial productivity and transition matrix for the benchmark calibration are given by

$$\mathbb{T} = [0 \quad 1.20 \quad 1.46 \quad 2.45]\,, \tag{B.1}$$

$$\mathbb{P}_\theta = \begin{bmatrix} 0.92 & 0.08 & 0.00 & 0.00 \\ 0.38 & 0.54 & 0.08 & 0.00 \\ 0.00 & 0.23 & 0.69 & 0.08 \\ 0.00 & 0.00 & 0.11 & 0.89 \end{bmatrix} \tag{B.2}$$

respectively.

C Parameters of Labor Efficiency

By using the method of Tauchen and Hussey (1991), the vector of grid points for labor productivity in the standard model are obtained as following

$$\mathbb{L} = [0.25 \quad 0.45 \quad 0.77 \quad 1.31 \quad 2.36]\,. \tag{C.1}$$

Additionally, the following transition matrix for the labor productivity is produced by applying the same method;

$$\mathbb{P}_l = \begin{bmatrix} 0.7376 & 0.2473 & 0.0149 & 0.0002 & 0.0000 \\ 0.1947 & 0.5555 & 0.2328 & 0.0169 & 0.0001 \\ 0.0112 & 0.2221 & 0.5334 & 0.2221 & 0.0112 \\ 0.0001 & 0.0169 & 0.2328 & 0.5555 & 0.1947 \\ 0.0000 & 0.0002 & 0.0149 & 0.2473 & 0.7376 \end{bmatrix}. \tag{C.2}$$

The labor efficiency grid points for the alternative calibration method that matches the dispersion of wealth and the labor earnings in the data is given by

$$\tilde{\mathbb{L}} = \begin{bmatrix} 0.18 & 0.90 & 3.65 & 301.4 \end{bmatrix}. \tag{C.3}$$

Then, the corresponding transition probabilities are given by

$$\tilde{\mathbb{P}}_l = \begin{bmatrix} 0.969 & 0.024 & 0.007 & 0.000 \\ 0.041 & 0.949 & 0.010 & 0.000 \\ 0.025 & 0.026 & 0.949 & 0.000 \\ 0.100 & 0.015 & 0.140 & 0.745 \end{bmatrix}. \tag{C.4}$$

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