

**THE STABILITY OF MONEY MULTIPLIER:  
A TEST FOR COINTEGRATION**

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## **THE STABILITY OF MONEY MULTIPLIER: A TEST FOR COINTEGRATION**

This paper applies integration and cointegration tests to assess the non-stationary characteristics of money multipliers using monthly unadjusted monetary data of Turkey. Money multipliers derived by using broadly defined money supply aggregates are clearly non-stationary on the basis of unit root tests. The non-stationarity of money multiplier relating to M2 is confirmed by the non-rejection of the hypothesis of non-cointegration. For the others, test results indicate that there is a long-run relationship between M1, M2Y and monetary base measured by reserve money and central bank money. These results could be interpreted as currency substitution effect exploiting the stable relationship between M2 and monetary base. For M2Y, comprising both domestic currency and foreign exchange deposits, this effect does not count so that a long-run relationship between monetary base and M2Y exists.

### **I. INTRODUCTION**

The multiplier model of the money supply, originally developed by Brunner (1961) and Brunner and Meltzer (1964), has become the standard paradigm in macroeconomics and money and banking textbooks to explain how the policy actions influence the money stock. It has also been used in empirical analyses of money stock control and the impact of monetary policy actions on other economic variables. In monetarist analysis, the interaction of the supply of money - or its inverse, the velocity of circulation of money, forms the basis of models of price and/or exchange rate determination.

The multiplier model of the money supply argues that the control of the money supply relies on the authorities being able to control the base and to predict behavioral relations of the banks and the non-bank private sector.

The question of the stability of the money supply processes arises because the components, in the words of Friedman and

Shwartz, are "proximate determinants" in the sense that they are in turn governed by fundamental factors. Since stability is a prerequisite of predictability, stability of the money multipliers is a contentious matter. For this ratio to exhibit stability, it should be stationary; also both monetary base and money supply should be cointegrated.

This paper applies integration and cointegration tests to assess the non-stationary characteristics of money multipliers using nine years of monthly money supply and monetary base data of Turkey. Examination period covers January 1986 and December 1994. The non-stationarity of money multipliers is confirmed by the acceptance of the hypothesis of non-cointegration between money supply and monetary base.

## II. AN OUTLINE OF THE MONEY MULTIPLIER MODEL OF MONEY SUPPLY

The salient characteristics of these models are the key roles of certain fixed ratios describing the portfolio behavior of the banks and the public, in determining the relation between the quantity of money and the monetary base or other reserve aggregates controllable by the central bank.

High powered money or the monetary base  $B$  is net liabilities of the Central Bank held either by the non-bank private sector  $RP$  or the banks  $RB$ . Given the definition of money supply  $MS$  as cash held by the non-bank private sector  $C$  and deposits of the banking system  $D$ , related identities are:

$$MS = C + D \quad (1)$$

$$B = RP + RB \quad (2)$$

By multiplying both sides of the identity (1) by  $B/(RP+RB)$ ;

$$MS = [(C+D)/(RP+RB)]*B \quad (3)$$

and further by multiplying both the numerator and denominator of the term in square brackets by  $1/D$ ;

$$MS = [(1+(C/D))/((RP/D)+(RB/D))]*B$$

$$MS = [(1+c)/(p+b)]*B$$

$$MS = k * B \quad (4)$$

By the fourth identity money supply may be described by the cash to deposits ratio  $c$ , and reserves to deposits ratio of the non-bank private sector  $p$ , and reserve assets to deposits ratio of the commercial banks  $b$ . The term in square brackets is referred to as the money multiplier  $k$ , so that changes in the money supply are the product of changes in the base  $B$  and in the value of the multiplier  $k$ .

#### II.a. An Outline of Tests of Stationarity

From the fourth identity,  $MS = k * B$ , where  $MS$  is the money supply,  $B$  is the monetary base and  $k$  is the money multiplier, money multiplier is stationary if and only if the ratio  $MS/B$  is stationary, that is to say it is stochastic in nature and thus predictable.

By rearranging the money multiplier identity (4), a monetary aggregate  $MS_i$  can be defined as:

$$MS_i = k_i + B \quad (5)$$

where  $MS_i$  is the log of money supply,  $B$  is the log of monetary base and  $k_i$  is the log of money multiplier. The necessary condition for a stable long-run money multiplier is not the constancy but the stationarity of  $k_i$ . The stationarity of  $k_i$  refers to a case that a) both  $MS_i$  and  $B$  in (5) are stationary or b)  $MS_i$  and  $B$  are cointegrated with cointegrating parameter equal to one, if each of them are non-

stationary with the same order of integration. In this context, a starting point to test stability of a money multiplier may be investigating the integration properties of the variables.

In the study, first Augmented Dickey-Fuller (ADF)<sup>1</sup> unit root tests are carried out on money multipliers (K1, K2, K2Y, K11, K22, K22Y) as one method for establishing multipliers' non-stationarity, then money supply and monetary base. In the derivations of the money multipliers, for the money supply aggregate three definitions of money supply (narrow-M1, broad-M2, and broader-M2Y) are taken into account, and for the monetary base both reserve money (RM) and Central Bank Money (CBM) are considered.

Table I lists the results of the ADF tests for the monetary aggregates and the corresponding money multipliers. The test results suggest that the unit root null hypothesis cannot be rejected at 5 percent significance level for the level of money supply (M1, M2, M2Y) and monetary base (RM, CBM). The same is valid for the money multipliers except the level of money multiplier K1 is found to be stationary. In contrast to the levels, the unit root null is strongly rejected at 5 percent significance level for the first difference of all the variables.

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<sup>1</sup>Dickey and Fuller, 1981

**TABLE 1**  
**Unit Root Tests**

Variables	t-ratio of the level ( $X_t$ ) <sup>1</sup>	t-ratio of the first difference of ( $X_t$ ) <sup>2</sup>
Money Supply (M1)	0.76(1)	-7.75*(1)
Money Supply (M2)	2.95(4)	-5.59*(4)
Money Supply (M2Y)	1.32(3)	-3.80*(3)
Reserve Money (RM)	1.65(1)	-6.63*(1)
Central Bank Money (CBM)	3.06(5)	-3.72*(4)
Money Multiplier (K1) <sup>3</sup>	-3.05*(1)	-9.29*(1)
Money Multiplier (K2) <sup>3</sup>	0.83(4)	-8.15*(3)
Money multiplier (K2Y) <sup>3</sup>	1.94(1)	-8.92*(1)
Money multiplier (K11) <sup>4</sup>	-2.02(3)	-7.34*(1)
Money multiplier (K22) <sup>4</sup>	0.28(2)	-6.87*(1)
Money multiplier (K22Y) <sup>4</sup>	0.20(3)	-10.50*(1)

Note: The values in parentheses are the lag lengths of the independent variables making residuals white noise.

$$(1) \text{ Estimated model is: } \Delta X_t = \alpha + \beta t + \delta X_{t-1} + \sum_{i=1}^k d_i \Delta X_{t-i}$$

$$(2) \text{ Estimated model is: } \Delta \Delta X_t = \alpha + \beta \Delta X_{t-1} + \sum_{i=1}^k d_i \Delta \Delta X_{t-i}$$

(3) K1= M1/RM, K2= M2/RM, K2Y= M2Y/RM

(4) K11=M1/CBM, K22= M2/CBM, K22Y= M2Y/CBM

(\*) Significant at 5 percent level.

### II.b. Choice of Lag Length

Both Johansen test statistics for cointegration vectors and the values of the vector parameters are sensitive to the choice of lag length ( $k$ ). Table II shows the Chi-Square values of the VARs with the lag lengths 8 versus 7, 7 versus 6, 6 versus 5, and 5 versus 4. The estimation results suggest that for the systems related to the narrow

definition of money M1, k=5 appears to be valid. For the systems related to broader definition of money (M2 and M2Y) and central bank money, k=8 gives the appropriate lag length. For the remaining two cases, k=5 is taken to be valid.

**TABLE 2**  
**Bivariate System Analysis**

$\chi^2$ Statistics				
System	k=8/7	k=7/6	k=6/5	k=5/4
$\Delta M1$ & $\Delta RM$	1.90	1.12	2.83	12.19*
$\Delta M1$ & $\Delta CBM$	5.42	9.00	6.02	35.43*
$\Delta M2$ & $\Delta RM$	1.27	4.72	0.58	19.49*
$\Delta M2$ & $\Delta CBM$	10.01*	7.67	2.38	-
$\Delta M2Y$ & $\Delta RM$	1.16	4.39	1.30	18.76*
$\Delta M2Y$ & $\Delta CBM$	11.46*	6.95	3.34	-

(\*) Significant at 5 percent level

### **II.c. Cointegration Between Money Supply and Monetary Base**

After integration properties and the lag structure are specified, cointegration properties of the time series are analyzed using Johansen ML estimation procedure<sup>2</sup>. The Johansen estimation method is based on the error correction representation of the VAR(p) model;

$$\Delta Z_t = \mu + \sum_{i=1}^k \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-p} + B w_t + u_t \quad (6)$$

where  $Z_t$  is an  $m \times 1$  vector of  $I(1)$  variables,  $w_t$  is an  $s \times 1$  vector of  $I(0)$  variables,  $\Gamma_i$  and  $\Pi$  are  $m \times m$  matrices of parameters,  $B$  is an  $m \times s$

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<sup>2</sup>Johansen (1988)

matrix, and  $u_t \sim N(0, \Sigma)$ . The Johansen ML procedure estimates subject to the hypothesis that  $\Pi$  has a reduced rank,  $r < m$ . This hypothesis can be written as:

$$H(r): \quad \Pi = \alpha\beta' \quad (7)$$

where  $\alpha$  and  $\beta$  are  $m \times r$  matrices. Under certain conditions, the reduced rank condition (6) implies that the process  $\Delta X_t$  is stationary,  $X_t$  is non-stationary, and  $\beta'X_t$  is stationary and these stationary relations are referred to as the cointegrating relations.

In the study, trace and maximum eigenvalue statistics are computed proposed by Johansen to test different hypothesis regarding the rank of  $\Pi$ , namely  $r$ . Table III exhibits the test statistics for  $r=0,1$  concerning the cointegrating relations between money supply and monetary base aggregates.

Cointegration test results indicate that there is a long run relation between narrow definition of money (M1) and monetary base (RM and CBM). In case of broader definition of money supply; evidence in favour of a "parallel" movement between M2 aggregate (including time deposits and currency in circulation) and monetary base is very weak. There is not a cointegrating relation between M2 and reserve money. For central bank money, maximum eigenvalue and trace tests yield conflicting results; maximum eigenvalue statistics indicating of no-cointegration conflicts with the trace test results of  $r=1$ . Finally, test results for the case of broader definition of money supply M2Y which is the aggregate comprising the foreign exchange deposits also indicate a "parallel" movement.

### III. CONCLUSION

The stationary nature of money multiplier was rejected based on Augmented Dickey Fuller unit root tests. Test results indicate that M1 money multiplier exhibits stationarity, while money multipliers derived using broadly defined money supply aggregates (M2 and M2Y) are non-stationary.

Secondly, the cointegration technique developed by Johansen (1988) was applied to money supply and monetary base aggregates to test for cointegration and the null of non-cointegration could not be accepted except for M2 and reserve money relationship. The rejection of no cointegration reflects that there is a long-run relationship between money supply and monetary base indicating the stability of money multiplier.

The broad conclusion from the above results is that the non-cointegration of money supply M2 and reserve money can reflect the currency substitution effect. Since the broadly defined money supply M2Y, comprising both domestic currency and foreign exchange deposits, and monetary base have a “parallel” movement in the long-run.

**TABLE 3**  
**Cointegration Analysis**

System	Max. Eigenvalue		Trace		$\beta^1$		$\alpha^1$	
	r=0	r=1	r=0	r=1	MS <sup>2</sup>	B <sup>2</sup>	MS <sup>2</sup>	B <sup>2</sup>
M1 & RM	32.63*	7.54	40.17*	7.54	-1.63 (-1.00)	1.69 (1.03)	0.24 (0.39)	0.07 (0.11)
M1 & CBM	26.51*	4.24	30.75*	4.24	0.82 (-1.00)	-0.83 (1.02)	-0.18 (0.15)	0.018 (-0.02)
M2 & RM	12.08	5.34	17.42	5.34	-	-	-	-
M2 & CBM	12.96.	7.03	19.99*	7.03	0.87 (-1.00)	-0.90 (1.03)	-1.41 (1.22)	0.05 (-0.04)
M2Y & RM	17.96*	5.54	23.49*	5.54	-1.03 (-1.00)	1.22 (1.19)	0.06 (0.06)	-0.04 (-0.04)
M2Y & CBM	15.79*	4.07	19.86*	4.07	-0.06 (-1.00)	0.04 (-0.73)	0.07 (-0.01)	-0.05 (0.003)

Note: Critical values at 5 percent significance levels are: For r=0; maximum eigenvalue is 15.67, trace test is 19.96, and for r=1; maximum eigenvalue is 9.24, trace test is 9.24.

(1) The values in the parenthesis are the normalized values.

(2) MS refers to money supply, B refers to monetary base.

(\*) Significant at 5 percent level.

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