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ON THE FRIEDMAN RULE WITH LABOR MARKET FRICTIONS

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ABSTRACT This paper uses a cash-in-advance model to study whether the optimality of a nominal interest rate equal to zero holds in the presence of labor market frictions. Results show that labor market frictions may break the equality between the marginal rate of substitution and the marginal rate of transformation, thereby inducing households to supply a suboptimal amount of labor. A non-zero nominal interest rate may correct the inefficient labor market decisions and improve efficiency. Numerical evaluations of the model quantify the inefficiencies generated by labor market frictions. Finally, the paper shows that an appropriate fiscal policy stance may restore the optimality of a nominal interest rate equal to zero.
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ÖZ Bu çalışmada sürtünmeli bir iş gücü piyasasının varlığı altında optimal nominal faiz oranının sıfıra eşit olup olmadığı bir peşin ödeme modeli çerçevesinde ele alınmaktadır. Sonuçlar, iş gücü piyasası sürtünmelerinin marjinal ikame ve marjinal dönüştürme oranları arasındaki eşitliği bozarak hanehalkını optimalin altında emek arz etmeye ittiğine işaret etmektedir. Sıfırdan farklı bir nominal faiz oranı iş gücü arzı kararlarındaki etkinsizliği düzeltebilmekte ve etkinliği artırabilmektedir. İş gücü piyasası sürtünmelerinin ortaya çıkardığı etkinsizliklerin sayısal değerlendirmelerle ölçüldüğü makalede, uygun maliye politikası duruşu ile nominal faiz oranının optimal değeri olan sıfıra eşitlenebileceği gösterilmektedir.
İŞ GÜCÜ PİYASASI SÜRTÜNMELERİ ALTINDA FRIEDMAN KURALI ÜZERİNE
JEL E52
Anahtar Kelimeler Emek piyasaları, Friedman kuralı

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1. Introduction

Milton Friedman's (1969) influential essay argues that the optimal quantity of money is associated with a nominal interest rate equal to zero. This has become known as the Friedman rule. Friedman argues that only monetary policies that follow this prescription equate the private opportunity cost of holding money (the nominal interest rate) to its social opportunity cost (the cost of printing money), and therefore lead to the optimal allocation of resources. To formalize this intuition, Wilson (1979), Cooley and Hansen (1989), Cole and Kocherlakota (1998), and Ireland (2003) use models with cash-in-advance constraints, in which households demand money in order to buy consumption goods. They find that any policy that deviates from the Friedman rule introduces distortions to households' consumption-leisure decisions, thereby generating a sub-optimal equilibrium. In this framework the optimality of the Friedman rule is tightly linked with the structure of the labor market: once money is a valuable commodity to buy goods, monetary policy affects the households' labor supply decisions by changes in the nominal interest rate. The approach to modelling labor supply decisions in this framework is based on frictionless labor markets. However, in practice, labor markets are characterized by frictions that prevent the competitive market mechanism from determining labor market equilibrium allocations.[1] If this is the case, does the optimality of the Friedman rule continue to hold?

To answer this question, Section 3 sets up a cash-in-advance model, as in Lucas (1980) and Ireland (2003), enriched with labor market frictions based on the Diamond-Mortensen-Pissarides model of search and matching. Results show that search and matching frictions alter the ways in which households form their optimal labor supply decisions, and, hence, generate suboptimal allocations. Monetary policy is able to neutralize these distortions, by setting a nominal interest rate different from zero. In particular, in a decentralized economy with labor market frictions, workers and firms share the surplus from working by engaging in wage bargaining. When the household's relative bargaining power is lower than the elasticity of hiring costs relative to labor market tightness, the equilibrium allocations are suboptimal, similarly to the Hosios' (1990) condition. In this instance, a monetary policy characterized by a positive interest rate, is able to, in effect, correct these labor market distortions. The intuition of this result is straightforward. When the household's bargaining power is lower than the

[1] For a review on the topic see the survey by Rogerson and Shimer (2010) and references therein.

elasticity of hiring costs with respect to labor market tightness, the household supplies units of labor at a higher than optimal level. A positive nominal interest rate increases the return on bonds, whose effect is to generate a higher income in the next period, which induces the household to supply fewer labor units, thereby correcting for the inefficiency introduced by a lower bargaining power.

Section 4 provides numerical evaluations of the theoretical model in order to quantify the inefficiencies generated by labor market frictions. Labor market frictions generate a reduction of at least 0.5 percent in the household's consumption compared to a perfectly competitive labor market. A positive interest rate enables the decentralized economy with labor market frictions to produce the Pareto optimum allocations of the centralized economy. The analysis establishes that a 5 percent nominal interest rate requires a permanent 0.12 percent increase in consumption to make the household of a decentralized economy with labor market frictions as well off as under a centralized economy with frictions. The analysis also shows that the non-negativity condition on the nominal interest rate generates welfare losses when the household's bargaining power is higher that the elasticity of hiring costs relative to labor market tightness (i.e. the level that guarantees efficient allocations). For instance, the household's consumption reduces by approximately 0.2 percent compared to a centralized economy with labor market frictions when the household retains all the bargaining power.

Section 5 shows that the optimality of the Friedman rule is restored when an appropriate fiscal policy stance is used to neutralize the distortions that wage bargaining introduces into a frictional labor market economy, inducing the household to supply units of labor at a higher than optimal level. In this instance, a tax on labor income would impose a cost on the supply of labor units, decreasing the household's incentive to work and neutralizing the distortions that labor market search frictions and wage bargaining bring about. Before proceeding with the analysis, the following section relates the paper to the literature.

2. Related Literature

The literature on this topic has principally focused, with the exception of a few notable works, as detailed below, on perfectly competitive labor markets. Wilson (1979), Cooley and Hansen (1989), Cole and Kocherlakota (1998), and Ireland (2003) use a cash-in-advance framework, similar to that used here, to study whether the Friedman rule is optimal. Recently, Schmitt-Groh'e and Uribe (2010) access the Friedman rule using an array of macrofounded models. All these authors reach the conclusion that Friedman's prescription allows the economy to achieve Pareto optimal allocations, building their argument in a framework characterized by perfectly competitive labor markets.

Another approach used to establish the optimality of the Friedman rule is to use models which embed heterogeneous agents. Heterogeneity has been introduced through population growth, as in Ireland (2005b), or through an overlapping generations framework, as in Abel (1987), Gahvari (1988), and Bhattacharya, Haslag, and Russell (2005), or through idiosyncratic risk, as in Akyol (2004). In general, these authors show that once heterogeneity is introduced, the Friedman rule fails to hold since a positive nominal interest rate redistributes resources to heterogeneous agents and improves efficiency. Heterogeneity has also been introduced in other ways. For instance, Bhattacharya, Haslag, and Martin (2005) incorporate heterogeneity in money holdings among agents in the context of a random-matching model of money, the turnpike model, and an overlapping generation model. They find that the Friedman rule is not the welfare maximizing monetary policy in all these economies. Ireland (2005a) extends these results to show that deviations from the Friedman rule may achieve re-distributional policy objectives. Bhattacharya, Haslag, Martin, and Singh (2008) use a money-in-the-utility function modified with agents' heterogeneity in their utilities for real money balances to show that a deviation from the Friedman rule is also optimal in this instance. More recently, da Costa and Werning (2008) incorporate agents with heterogeneous productivity in a regime of non-linear labor income taxation. They find that the Friedman rule is Pareto efficient when it is combined with a non-decreasing labor income tax, since a positive taxation of income enables a redistribution of resources from high- to low-utility individuals. Again, these contributions find that heterogeneity plays an important role when determining the optimality of the Friedman rule, but none of them focuses explicitly on the structure of the labor markets. As mentioned, relatively few theoretical studies consider optimal monetary policy in the presence of labor market frictions. Shi (1998), studying the monetary propagation mechanism, suggests that the Friedman rule may be inefficient in the presence of search frictions, since it discourages the unemployed workers from searching for jobs, thereby preventing employment and output from being on the Pareto efficient frontier. Cooley and Quadrini (2004), in a search model that focuses on the cyclical properties of optimal monetary policy, find that a policy that credibly commits to its future choices could lead to a higher inflation rate in the presence of labor market frictions than a policy that is set on a period-by-period basis, thereby implying a positive nominal interest rate. Blanchard and Gali (2010) explicitly investigate optimal monetary policy in the presence of labor market frictions and staggered price setting and find that the optimal policy should respond to unemployment. Along the same lines, Thomas (2008) quantifies the structure of unconditional optimal policy and Ravenna and Walsh (2012) investigate the welfare consequences of monetary policy and the role of tax policies. Although these works use labor market frictions in the analysis, the focus is on either the monetary transmission mechanism, the cyclicality of monetary policy, the fluctuations of macroeconomic aggregates, or the optimality of monetary policy rules. In contrast, the contribution of this paper is to investigate explicitly whether the optimality of the Friedman rule holds under different structures of the labor market.

The results of this paper also relate to the literature on search models of money. For instance, Head and Kumar (2005), Rocheteau and Wright (2005) and Craig and Rocheteau (2008) develop models in which money is a means of exchange; frictions in the exchange process then give a role for money as part of an equilibrium arrangement. Similar to this paper, they find that, depending on the extent of search frictions, optimal monetary policy might or might not correspond to the Friedman rule. However, the mechanism that generates these findings is radically different: while their results are driven by frictions in the bilateral meetings of agents in the process of exchange, here the structure of the labor market is uniquely responsible for the results.

3. A Cash-in-Advance Model

The theoretical framework is based on the cash-in-advance model as developed by Lucas (1980) and Ireland (2003), and is enriched to allow for labor market frictions of the Diamond-Mortensen-Pissarides model of search and matching as in Blanchard and Gali (2010) and Thomas (2008). The model economy consists of a representative household made up of a continuum of members, a representative firm, and a government.

During each period $t = 0,1,2,\ldots$, the representative household maximizes the utility function

$$\sum_{0}^{\infty} \beta^t \, \mathrm{U}(\mathrm{C_t}, 1 - \mathrm{N_t}) \tag{1}$$

where the variable $\mathrm{C_t}$ is consumption, N_t is the fraction of household members who are employed, and beta is the discount factor, $0<\beta<1$. This utility function is strictly concave and satisfies the Inada conditions for both its arguments.[2]

The representative household enters period t with bonds B_t and money M_t carried over from the previous period. The representative household is required to use these previously-acquired money balances to purchase

[2] Formally, $\lim_{C\to 0} U(C, 1 - N) = \infty$, $\lim_{C\to\infty} U(C, 1 - N) = 0$, $\lim_{(1-N)\to 0} U(C, 1 - N) = \infty$ and $\lim_{(1-N)\to\infty} U(C, 1 - N) = 0$.

perishable consumption goods at a nominal price P_t. That is, the household must satisfy the cash-in-advance constraint

$$M_t \geq P_t C_t$$

At the end of each period, the representative household receives a lump-sum nominal transfer T_t from the government and the household's bonds mature, providing B_t additional units of money. The household acquires $B_{t+1}/(1+i_t)$ new bonds, where i_t is the net nominal interest rate, carries M_{t+1} units of money into the next period, and supplies N_t units of labor to the representative firm at the nominal wage rate W_t. Therefore, the household faces the budget constraint

$$M_t + B_t + T_t + W_t N_t \geq P_t C_t + B_{t+1}/(1+i_t) + M_{t+1}$$

In addition, the household must satisfy

$$C_t \geq 0, 1 \geq N_t \geq 0, M_{t+1} \geq 0$$

and is not permitted to borrow more than it can repay, such that Ponzi schemes are ruled out.

During each period t=0,1,2,..., the representative firm employs N_t units of labor from the representative household, in order to produce Y_t units of goods according to the constant return to scale technology

$$Y_t = F(N_t) \quad (2)$$

During each period t=0,1,2,..., the government specifies the sequence of monetary lump-sum transfer according to

$$T_t = M_{t+1} - M_t$$

and, in equilibrium, imposes the condition $B_t = B_{t+1} = 0$.

The following subsections introduce labor market frictions into the model and analyze to what extent the Friedman rule holds. First, the analysis focuses on the centralized equilibrium to establish the Pareto optimal allocations in the economy, and then on the decentralized equilibrium.

3.1. Labor Market Frictions in a Centralized Economy

The labor market frictions are based on the Diamond-Mortensen-Pissarides model of search and matching. This framework relies on the assumption that the processes of job searching and hiring are costly. Job creation takes place when a firm and a searching worker meet and agree to form a match at a negotiated wage. The match continues until the parties exogenously terminate their relationship. When this occurs, job destruction takes place and the worker moves from employment to unemployment. During each period t=0,1,2,..., the level of employment is given by the number of workers who survive the exogenous separation, and the number of new hires, H_t. Hence, employment evolves according to

$$N_t = (1 - \delta)N_{t-1} + H_t \tag{3}$$

where delta is the exogenous separation rate, and 0<δ<1. Consequently, unemployment at the beginning of the period, before hiring takes place, evolves according to

$$U_t = 1 - (1 - \delta)N_{t-1} \tag{4}$$

It is convenient to introduce the variable for labor market tightness,

$$x_t = H_t/U_t \tag{5}$$

for which 0<x_t<1, since new hires are made from the pool of unemployed workers at the beginning of the period. Hence, x_t also represents the probability that an unemployed worker finds a job. Hiring labor is costly, the cost per hire G_t, as in Blanchard and Gali (2010), is a function of labor market tightness of the form

$$G_t = Bx_t^{\alpha} \tag{6}$$

where $\alpha \geq 0$ and $B \geq 0$. The aggregate resource constraint is $Y_t = C_t + G_t H_t$, where $G_t H_t$ represents the aggregate cost of hiring.

Thus, in a centralized economy, during each period t=0,1,2,..., the planner chooses $\{C_t, N_t\}_{t=0}^{\infty}$ to maximize the household's utility subject to the structure of the labor market, represented by equations 3-6, the aggregate resource constraint, and the production technology. By substituting equations 3-5 into 6, and using the production technology 2, the aggregate resource constraint can be written as

$$F(N_t) = C_t + B\frac{[N_t - (1 - \delta)N_{t-1}]^{1+\alpha}}{[1 - (1 - \delta)N_{t-1}]^{\alpha}} \tag{7}$$

Hence, the planner maximizes the household's utility subject to the aggregate resource constraint 7. The first order conditions are

$$U_C(C_t, 1 - N_t) = \Lambda_t \tag{8}$$

and

$$-\frac{U_N(C_t, 1 - N_t)}{\Lambda_t} = F_N(N_t) - (1 + \alpha)G_t + \beta(1 - \delta)[1 + (1 - x_{t+1})\alpha]\frac{\Lambda_{t+1}}{\Lambda_t}G_{t+1} \tag{9}$$

where Λ_t is the Lagrange multiplier on the aggregate resource constraint and variables $U_C(C_t, 1 - N_t)$, $U_N(C_t, 1 - N_t)$ and $F_N(N_t)$ represent the marginal utility of consumption and labor, and the marginal product of labor respectively. Note that $U_N(C_t, 1 - N_t) < 0$. Equation 8 states that the Lagrange multiplier equals the marginal utility of consumption. Equation 9 states that the marginal rate of substitution between consumption and leisure, the left-hand side, must equal the marginal rate of transformation, the right-hand side. Since the labor market is characterized by frictions, the marginal rate of transformation comprises the marginal productivity of labor $F_N(N_t)$, as in the perfectly competitive economy, as well as the contribution of hiring costs to output, represented by the two right-hand side terms. Of these, the first term represents the increase in output generated by an additional employed worker, net of hiring costs, and the second term represents the output from saving in hiring costs generated by the decrease in hiring in the next period. The equilibrium can be defined as a set of sequences $\{C_t, N_t, x_t, H_t, G_t, U_t, \Lambda_t\}_{t=0}^{\infty}$ that satisfy equations 8, 3-9. To simplify the system, by substituting Equation 8 into 9 for Λ_t, the steady-state can be derived imposing $\mathrm{C_t = C}$, $\mathrm{N_t = N}$, $\mathrm{x_t = x}$, $\mathrm{H_t = H}$, $\mathrm{G_t = G}$ and $\mathrm{U_t = U}$ on the equilibrium system. Consequently, the equilibrium optimal labor market condition can be written as

$$F_N(N) = -\frac{U_N(C, 1-N)}{U_C(C, 1-N)} + (1+\alpha)G - \beta(1-\delta)[1 + (1-x)\alpha]G \tag{10}$$

This equilibrium condition describes the Pareto optimal allocations in the economy characterized by labor market frictions regardless of the particular cash-in-advance trading environment that the households use to allocate resources. It states that the marginal rate of substitution between consumption and leisure, in equilibrium, must equal the marginal productivity of labor adjusted for the inefficiencies generated by labor market frictions. Note that if the labor market were frictionless, such that there is no cost of hiring ($B = 0$ such that $G = 0$), Equation 10 would simplify to $\mathrm{F_N(N) = -U_N(C, 1-N)/U_C(C, 1-N)}$, the standard equilibrium condition in a perfectly competitive labor market.[3]

3.2. Labor Market Frictions in a Decentralized Economy

In a decentralized economy the allocation of resources depends on the cash-in-advance trading environment. Moreover, in the presence of labor market frictions a realized job match yields some economic surplus, whose share between the representative household and the firm is determined by the wage level. This section derives the equilibrium allocations of the economy.

During each period t=0,1,2,..., the representative household chooses $\{C_t, m_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the cash-in-advance constraint, the budget constraint, the non-negativity constraints, and the no

[3] The Appendix derives the equilibrium of the model with a perfectively competitive labor market.

Ponzi scheme constraint. The first order conditions for this problem can be written as

$$\frac{\Lambda_t}{P_t} = \beta \frac{(\Lambda_{t+1} + \Xi_{t+1})}{P_{t+1}} \tag{11}$$

$$\frac{\Lambda_t}{(1 + i_t)P_t} = \beta \frac{\Lambda_{t+1}}{P_{t+1}} \tag{12}$$

$$U_C(C_t, 1 - N_t) = \Lambda_t + \Xi_t \tag{13}$$

and

$$\Xi_t \geq 0, M_t \geq P_t C_t, \Xi_t(M_t \geq P_t C_t) \tag{14}$$

for all t=0,1,2,..., and

$$\lim_{t \to \infty} \beta \frac{\Lambda_t}{P_t} M_{t+1} = 0 \tag{15}$$

where Λ_t and Ξ_t are Lagrange multipliers on the budget and cash-in-advance constraints respectively.

During each period t=0,1,2,..., the representative household sets the wage according to the Nash bargaining solution, as in Pissarides (2000). Let W_t^N and W_t^U denote the marginal value of the expected income of an employed, and unemployed worker respectively. The employed worker earns a wage, suffers disutility from work, and faces the probability δ of losing its job. Hence, the marginal value of a new match is:

$$W_t^N = \frac{W_t}{P_t} + \frac{U_N(C_t, 1 - N_t)}{\Lambda_t} + \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{[1 - \delta(1 - x_{t+1})]W_{t+1}^N + \delta(1 - x_{t+1})W_{t+1}^U\} \tag{16}$$

This equation states that the marginal value of a job for a worker is given by the wage, accounting for the marginal disutility that the job produces to the worker, and the expected-discounted net gain from either working or not working during the next period.

The unemployed worker expects to move into employment with probability x_t. Hence, the marginal value of unemployment is:

$$W_t^U = \beta \frac{\Lambda_{t+1}}{\Lambda_t}[x_{t+1} W_{t+1}^N + (1 - x_{t+1})W_{t+1}^U \tag{17}$$

This equation states that the marginal value of unemployment is made up of the expected-discounted capital gain from either working or not working during the next period.

The worker and the firm split the surplus from the match using Nash bargaining, with the worker's bargaining power $0<\eta<1$. The difference between equations 16 and 17 determines the worker's share of the economic surplus. The firm's surplus is simply given by the real cost per hire, G_t, since any current worker can be replaced with an unemployed worker by paying the hiring cost. Hence, the total surplus from a match is the sum of the worker and firm surpluses. Consequently, the Nash wage bargaining rule for a match is

$$\eta G_t = (1-\eta)(W_t^N - W_t^U)$$

Substituting equations 16 and 17 into this last equation produces the agreed real wage:

$$\frac{W_t}{P_t} = -\frac{U_N(C_t, 1-N_t)}{\Lambda_t} + \frac{\eta}{(1-\eta)} G_t - \beta(1-\delta)\frac{\eta}{1-\eta}(1-x_{t+1})\frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1} \tag{18}$$

Equation 18 shows that, if the match continues, the real wage equates the marginal disutility of working as well as the present cost of hiring, net of the savings in terms of the future hiring costs.

During each period t=0,1,2,..., the representative firm chooses $\{N_t, H_t\}_{t=0}^{\infty}$ to maximize its total market value given by

$$\sum_{t=0}^{\infty} \beta^t \Lambda_t [F(N_t) - N_t \frac{W_t}{P_t} - H_t G_t] \tag{19}$$

subject to the law of employment 3, where $\beta_t \Lambda_t$ measures the marginal utility value to the representative household of an additional dollar in profits received during period t. Solving Equation 3 for H_t, and substituting it into Equation 19, yields the first order condition

$$\frac{W_t}{P_t} = F_N(N_t) - G_t + \beta(1-\delta)\frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1} \tag{20}$$

By substituting Equation 18 into 20 for W_t/P_t, the optimal labor equilibrium under Nash bargaining can be written as

$$F_N(N_t) = -\frac{U_N(C_t, 1-N_t)}{\Lambda_t} + \left(1 + \frac{\eta}{1-\eta}\right) G_t - \beta(1-\delta)\left[1 + (1-x_{t+1})\frac{\eta}{1-\eta}\right]\frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1} \tag{21}$$

Consequently, the equilibrium is defined as a set of sequences $\{C_t, m_{t+1}, N_t, i_t, x_t, H_t, G_t, U_t, \Lambda_t, \Xi_t\}_{t=0}^{\infty}$ that satisfy equations 3-6, 11-13, 14-15, and 21. The steady-state can be derived by imposing $C_t = C$, $m_{t+1} = m$, $N_t = N$, $i_t = i$, $x_t = x$, $H_t = H$, $G_t = G$, $U_t = U$, $\Lambda_t = \Lambda$, and $\Xi_t = \Xi$ on the equilibrium system. After imposing these conditions, substituting Equation 13 into Equation 11, and substituting the outcome into Equation 12, the Lagrange multiplier on the budget constraint can be expressed as

$$\Lambda = \frac{U_C(C, 1-N)}{(1+i)} \tag{22}$$

Substituting Equation 22 into 21 for Λ, the equilibrium optimal labor market condition can be written as

$$F_N(N) = -\frac{U_N(C, 1-N)}{U_C(C,\ 1-N)}(1+i) + \left(1+\frac{\eta}{1-\eta}\right)G - \beta(1-\delta)\left[1+(1-x)\frac{\eta}{1-\eta}\right]G \qquad (23)$$

Proposition 1 *In a decentralized economy characterized by labor market frictions, equilibrium allocations are Pareto efficient for either* $i = 0$, *or* $i > 0$. *If the household's relative bargaining power is equal to the elasticity of hiring costs with respect to labor market tightness, such that* $\eta/(1-\eta) = \alpha$ *, then* $i = 0$. *If the household's relative bargaining power is lower than the elasticity of hiring costs with respect to labor market tightness, such that* $\eta/(1-\eta) < \alpha$, *then* $i > 0$.

Proof Given the same underlying structure between the centralized and decentralized economy, the Pareto optimal allocations of the centralized economy C, N, x, G, H and U also represent the Pareto optimal allocations of the decentralized economy. By equating the condition that generates Pareto optimal allocations in the centralized economy, expressed by Equation 10, with the equilibrium optimal labor market condition in the decentralized economy, expressed by Equation 23, the nominal interest rate that guarantees Pareto optimal allocations in the decentralized economy is

$$i = -\frac{U_C(C,\ 1-N)}{U_N(C, 1-N)}[1-\beta(1-\delta)(1-x)]\left(\alpha - \frac{\eta}{1-\eta}\right)G \qquad \blacksquare \qquad (24)$$

Similar to Hosios (1990), in a model without money, the decentralized economy is at its Pareto optimum when the household's relative bargaining power, $\eta/(1-\eta)$, coincides with the elasticity of hiring costs relative to labor market tightness, α. If this condition is satisfied, the household receives a wage that is the same as that the planner would choose. Under this condition, the structure of the decentralized economy generates the same equilibrium allocations as the centralized economy. Therefore, once money is incorporated through a cash-in-advance constraint, the nominal interest rate must be equal to zero, as any other value introduces distortions into the household's choice of labor units and generates suboptimal allocations. On the other hand, when the household's relative bargaining power is different from the elasticity of hiring costs, a positive nominal interest rate can correct the labor market distortions, and induce the decentralized economy to produce Pareto optimal allocations. In particular, when the household's relative bargaining power is less than the elasticity of hiring costs (i.e. $\eta/(1-\eta) < \alpha$) the household supplies a higher-than-optimal quantity of labor units for any given wage level. A positive net interest rate would correct for this by increasing the contribution of an additional unit of labor to the household's utility. In fact, since working would generate higher disutility, due to a positive net interest rate, the household is induced to supply fewer units of labor, which would counteract the distortions generated by a lower than optimal relative bargaining power. In particular, a net nominal interest rate that satisfies Equation 24 would induce the decentralized economy to produce the same Pareto optimal equilibrium allocations as the centralized economy.

4. Numerical Evaluation and the Welfare Cost of Inflation

This section calibrates the models with different labor market structures to quantify the finding of proposition 1 and assess the welfare cost of inflation. The evaluation concerns the long-run equilibrium and, to implement it, explicit functional forms for the utility and the production technology need to be assumed. As in Cooley and Hansen (1989), the household's utility function has the form

$$\sum_{t=0}^{\infty} \beta^t \left[logC_t + Alog(1-N_t)\right] \tag{25}$$

and the production technology has the linear form

$$Y_t = N_t$$

To derive a welfare measure for different interest rates, this section uses the method advocated by Lucas (2000). This measure is based on the increase in consumption that the representative household would require to be as well off as under a reference allocation. More formally, let $\bar{U}$ denote the level of utility attained under a reference allocation, and let C and N denote the level of consumption and employment associated with the labor market considered. Finally, let ω be defined by

$$\beta\{\log[(1+\omega/100)C] - \phi N\} = \bar{U}$$

Hence, ω measures the percentage increase in consumption that makes the representative household as well off in the scenario under consideration as it is under the reference allocation, which measures the long-run cost of inflation.[4]

The calibration of the structural parameters, on a quarterly frequency, is as follows. The intertemporal discount factor $\beta = 0.99$, as is standard in the literature. The exogenous separation rate $\delta = 0.1$ so that, in the long-run, 10 percent of jobs are destroyed every quarter, as suggested by Hall (1995). The disutility generated by an additional unit of work $\phi = 2.8$, as in Cooley and Hansen (1989). The scaling parameter B in the definition of cost per hire, Equation 6, equals 1, such that the fraction of hiring costs is 0.5 percent of output, similarly to Blanchard and Gali (2010). The baseline calibration

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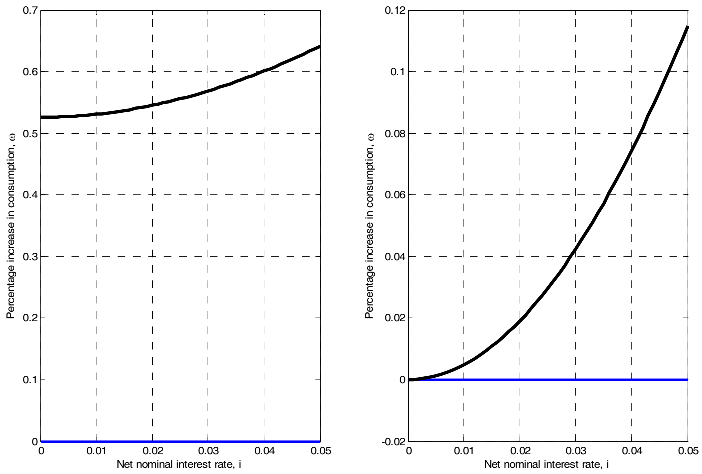
[4] As shown in Cooley and Hansen (1989), the household's utility function 25 can be conveniently written as $\sum_{t=0}^{\infty} \beta^t (logC_t - \phi N_t)$.

of the elasticity of cost per hire with respect to labor market tightness α is set equal to 1, and the baseline calibration of the household's bargaining power η is equal to 0.5, as in Petrongolo and Pissarides (2001), such that household and firm share the same bargaining power, and the decentralized economy produces the same equilibrium allocations as the centralized one. Given the relevance of these last two parameters for the determination of the results, the sensitivity of the results to alternative calibrations, described in Figure 1, is considered.

Figure 1. Labor Markets, Percentage Increase in Consumption, and Nominal Interest Rates



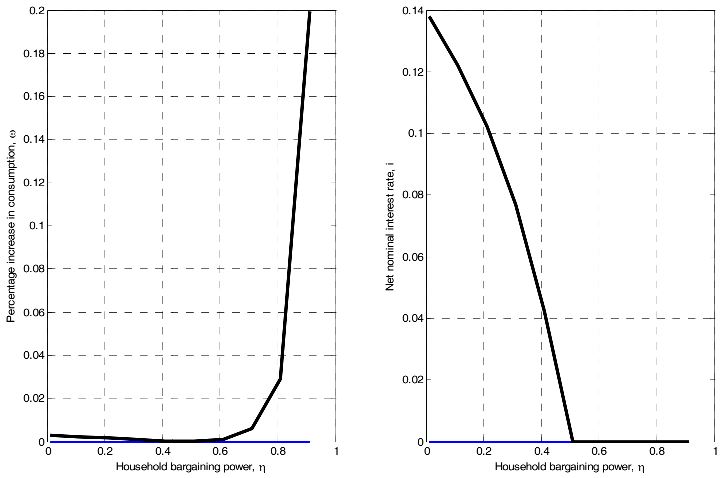
Note: The left-hand panel shows how changes in the net nominal interest rate affect the percentage increase in consumption that makes the household as well off under a centralized economy characterized by labor market frictions as in a centralized economy with a perfectly competitive labor market. The right-hand panel reports how the net nominal interest rate affects the percentage increase in consumption that would make a household in a decentralized economy characterized by labor market frictions as well off as under a centralized economy with frictions. Parameters are calibrated with their baseline values.

Figure 1 shows the findings for the baseline calibration of the model. The left-hand panel shows how different levels in the net nominal interest rate affect the percentage increase in consumption that makes the household as well off under a centralized economy characterized by labor market frictions as in a centralized economy with a perfectly competitive labor market.[5] Since the centralized competitive economy is not characterized by any deadweight loss, this comparison helps to evaluate the degree of inefficiency that labor market frictions introduce in a centralized economy, and, also, how changes in the nominal interest rate affect the overall magnitude of that inefficiency. As expected, $i = 0$ makes the equilibrium of a frictional labor

[5] The Appendix derives the equilibrium of the model with a perfectively competitive labor market.

market economy closer to that of a perfectly competitive labor market economy, but, nevertheless, the presence of labor market frictions generates a reduction of at least 0.5 percent in the household's consumption. The loss in consumption increases where net nominal interest rates are different from zero. A 5 percent net nominal interest rate needs approximately a permanent 0.65 percent increase in consumption to make the household as well off under a centralized economy characterized by labor market frictions as in a centralized economy with a perfectly competitive labor market.

Figure 2. Different Calibrations of the Household's Bargaining Power, η.



Note: The left-hand panel shows how calibrations of η ϵ [0,1] affect the percentage increase in consumption required to make the allocations of a decentralized economy with labor market frictions the same as those of a centralized economy. The right-hand panel shows how calibrations of η ϵ [0,1] affect the net nominal interest rate that causes a decentralized economy characterized by labor market frictions to produce the Pareto optimum allocations of a centralized economy.

Another interesting exercise is to investigate how far the welfare of the decentralized economy deviates from that of a centralized one in the presence of a labor market with frictions in both cases. This would shed light on what values of the nominal interest rate these two settings would require to produce the same allocations. The right-hand panel reports how the nominal interest rate affects the percentage increase in consumption that would make the household of a decentralized economy characterized by labor market frictions as well off as under a centralized economy with frictions. As in Proposition 1, since the baseline calibration is such that $\alpha = \eta/(1-\eta)$, a net nominal interest rate $i = 0$ guarantees that the decentralized economy produces Pareto efficient allocations, such that the level of consumption is the same in the two economies. A 5 percent net nominal interest rate needs a permanent 0.12 percent increase in consumption to make the household of a decentralized economy characterized by labor market frictions as well off as under a centralized economy with frictions. As shown below, this is not always the case for alternative calibrations where $\alpha > \eta/(1-\eta)$. In these cases, the decentralized economy with labor market frictions requires that the net nominal interest rate is strictly positive in order to produce the Pareto optimum allocations.

Figure 2 analyzes different calibrations of the household's bargaining power, η. This exercise sheds light on how changes in the redistribution of rents affect the equilibrium allocations of the economy. The left-hand panel shows how calibrations of $\eta \in [0,1]$ affect the percentage increase in consumption required to make the allocations of a decentralized economy with labor market frictions the same as those of a centralized economy. As before, all the other parameters are calibrated with their baseline values, and $i = 0$. If the household's bargaining power differs from 0.5, which is the value that makes $\alpha = \eta/(1-\eta)$, the equilibrium of the decentralized economy is not Pareto optimal so that a net nominal interest rate equal to zero leaves the economy on a suboptimal equilibrium and, therefore, the percentage increase in consumption, ω, is positive. In particular, if $\alpha < \eta/(1-\eta)$ the percentage increase in consumption rises and when the household retains all the bargaining power (i.e. $\eta = 1$) it reaches about 0.2 percent. On the other hand, if $\alpha > \eta/(1-\eta)$, the percentage increase in consumption remains close to zero and reaches approximately 0.005 percent when the firm retains all the bargaining power, (i.e. $\eta = 0$).[6] A positive net nominal interest rate can act to correct this inefficiency and may establish optimality. The right-hand panel shows how calibrations of $\eta \in [0,1]$ affect the net nominal interest rate that makes a decentralized economy characterized by labor market frictions produce the allocations of a centralized economy. Once again, for values of the household's relative bargaining power $\alpha > \eta/(1-\eta)$ the households would supply a larger number of labor units such that a positive net nominal interest rate is required to correct this inefficiency, inducing the household to draw more income from bonds, which are now more profitable, and this reduces labor units and consumption. If the firm retains all the bargaining power, such that $\eta = 0$, a net nominal interest rate of 14 percent is needed to make the decentralized economy characterized by labor market frictions produce the Pareto optimum allocations of the centralized economy. As the household's bargaining power reaches $\alpha = \eta/(1-\eta)$, the net nominal interest rate needed drops to zero.

[6] Note that, in principle, the reduction of consumption might be driven by the resources lost in the search process. In this setting, however, the proportion of resources lost in the search process, $G_t H_t / Y_t$, is between 0.4 and 0.9 percent of output for values of η between 0 and 1 respectively, suggesting that their role is limited.

This analysis reveals that a positive interest rate is able to correct a small fraction of inefficiencies, since the welfare gain of a positive interest rate reaches approximately 0.005 percent when the firm retains all the bargaining power. On the other hand, the non-negativity condition on the interest rate generates welfare losses of about 0.2 percent when the household retains all the bargaining power. It would certainly be a useful task for future research to investigate what alternative policy actions could restore efficiency when the non-negativity condition on the nominal interest rate holds.

5. The Friedman Rule Restored

The previous sections show how the Friedman rule fails to hold in a setting where the labor market is characterized by search frictions and the economy is decentralized. This section shows how an appropriate fiscal policy regime can offset the inefficiency that the wage bargaining power introduces, so that the optimality of the Friedman rule is restored. This result is similar to the findings of Abel (1987) and Gahvari (1988) in an overlapping generation setting, where a net nominal interest rate equal to zero becomes optimal only when an appropriate fiscal policy regime rebalances the intergenerational transfers. Similar results are echoed in more recent studies by Ireland (2005b), and Bhattacharya, Haslag, and Russell (2005), who argue that the Friedman rule is indeed valid if an appropriate fiscal policy stance counterbalances negative redistributional effects.

Here, suppose that the government levies labor income taxation τ_n for all t=0,1,2,..., such that the representative household's budget constraint becomes

$$M_t + B_t + T_t + W_t N_t(1 - \tau_{nt}) \geq P_t C_t + B_{t+1}/(1 + i_t) + M_{t+1}$$

The introduction of labor income taxation alters the optimal labor market decisions such that Equation 21 becomes

$$F_N(N_t) = -\frac{U_N(C_t, 1 - N_t)}{\Lambda_t} + \left[1 + \frac{\eta}{(1-\eta)(1-\tau_n)}\right] G_t - \beta(1-\delta)[1 + \frac{(1 - x_{t+1})\eta}{(1-\eta)(1-\tau_n)]} \frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1} \quad (26)$$

leaving the other equilibrium conditions unchanged. Hence, the equilibrium can be defined as a set of sequences $\{C_t, m_{t+1}, N_t, i_t, x_t, H_t, G_t, U_t, \Lambda_t \Xi_t\}_{t=0}^{\infty}$ that satisfy equations 3-6, 11-13, 14-15, and 26. Once again, the steady-state is defined as $C_t = C$, $m_{t+1} = m$, $N_t = N$, $i_t = i$, $x_t = x$, $H_t = H$, $G_t = G$, $U_t = U$, $\Lambda_t = \Lambda$, and $\Xi_t = \Xi$. An appropriate fiscal policy regime, such that

$$\tau_n = 1 - \frac{\eta}{\alpha(1-\eta)} \quad (27)$$

can counterbalance the inefficiency generated by the redistribution of rents in a frictional labor market. In fact, when Equation 27 is satisfied, the decentralized economy characterized by labor market frictions produces the same equilibrium allocations as a centralized economy with labor market frictions. In that instance, the government taxes labor in order to counterbalance the labor market distortions introduced by the redistribution of rents that may generate a different wage from what the planner would choose. By levying an appropriate tax on labor, the government can neutralize the distortionary effects that wage bargaining brings about, and in that case the Friedman prescription always holds. For instance, as detailed above, when the household's relative bargaining power is less than the elasticity of hiring costs, such that $\eta/(1-\eta) < \alpha$, the household supplies a higher-than-optimal quantity of labor units for any given wage level. In this case, a tax on labor income would correct for this by imposing a cost on the supply of labor units, thereby decreasing the household's incentives to work. Therefore, by setting the labor tax in accordance with Equation 27, a monetary policy that follows the Friedman's prescription would lead to optimal resource allocations.

Note that here labor market income taxation is just one possible fiscal policy action that would neutralize the inefficiency produced by labor market frictions in a decentralized economy. For instance, in this framework, a fiscal policy regime that levies taxes on firms for firing workers would have the same effect. Similarly, the presence of unemployment benefits, not accounted for in this model, would decrease the distortions generated by a suboptimal wage bargaining power, thereby requiring a lower positive nominal interest rate. Extending the analysis with a more elaborate model that includes these potential alternative reforms would certainly be a very useful task for future research.

6. Concluding Remarks

This paper studied whether the optimality of the Friedman rule holds in the presence of labor market frictions. The theoretical framework embeds labor market frictions of the Diamond-Mortensen-Pissarides type into the Lucas (1980) cash-in-advance model.

The findings show that the Friedman rule is not always optimal. In particular, in a decentralized economy characterized by a frictional labor market agents bargain over the wage in order to split the surplus from working. If the wage bargaining is lower than the elasticity of hiring costs relative to labor market tightness, the equilibrium allocations are suboptimal, similarly to the Hosios' (1990) condition. In such circumstances, the household supplies units of labor at a higher than optimal level. A positive nominal interest rate increases the return on bonds, whose effect is to generate a higher income in the next period, which induces the household to supply fewer labor units. Numerical results quantify the importance of wage bargaining power for the validity of the Friedman rule. Finally, even when the Friedman rule is suboptimal, an appropriate fiscal policy stance can offset the inefficiencies that labor market frictions generate and restore optimality.

The analysis of this paper is conducted using a cash-in-advance model. It would be interesting to establish whether the same results carry over into other environments, such as those where money is directly embedded in the household's utility function, or as a means of intergenerational transfers. In addition, the numerical results point out that the non-negativity condition on the interest rate generates high welfare losses. It would be interesting to study what policy interventions could correct this inefficiency. These investigations remain outstanding tasks for future research.

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Appendix: A Perfectly Competitive Labor Market in a Centralized Economy

In a centralized economy characterized by a perfectly competitive labor market, during each period t=0,1,2,..., the planner chooses $\{C_t, N_t\}_{t=0}^{\infty}$ to maximize the household's utility subject to the aggregate resource constraint and the production technology. The optimal conditions for this problem can be written as Equation 8 and

$$F_N(N_t) = -\frac{U_N(C_t, 1 - N_t)}{\Lambda_t} \tag{28}$$

where Λ_t is the Lagrange multiplier on the aggregate resource constraint and variables $U_C(C_t, 1 - N_t)$, $U_N(C_t, 1 - N_t)$ and $F_N(N_t)$ represent the marginal utility of consumption and labor, and the marginal product of labor respectively. Note that $U_N(C_t, 1 - N_t) < 0$. Steady-state equilibria exist in which each variable of the two aggregates is constant over time. In particular, steady-state equilibria can be derived by imposing $C_t = C$, $N_t = N$ and $\Lambda_t = \Lambda$ on the system of Equation 28 and the aggregate resource constraint $\mathrm{Y_t} = \mathrm{C_t}$. After imposing these conditions, substituting Equation 8 into 28 for Λ, the optimal equilibrium labor market decision can be written as

$$F_N(N) = -\frac{U_N(C, 1 - N)}{U_C(C, 1 - N)} \tag{29}$$

This familiar equilibrium condition describes the Pareto allocations in the economy regardless of the particular cash-in-advance trading environment that the households use to allocate resources. It states that the marginal productivity of labor, the left-hand side, in equilibrium, must equal the marginal rate of substitution between consumption and leisure, the right-hand side.