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PUBLIC DEBT STOCK SUSTAINABILITY IN SELECTED OECD COUNTRIES

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ABSTRACT Among the different criteria used for testing sustainability of public debt, the econometric approach determines whether a government is able to sustain its budget deficits without defaulting on its debt. In this contribution, by linking three different motives proposed respectively by Trehan and Walsh (1991), Hakkio and Rush (1991) and Bohn (1995), we develop a compact step-wise test algorithm and apply that to the European economies, including the PIIGS countries and Turkey. Secondly, we introduce phase-space reconstruction methodology in order to locate the path for debt dynamics, which enables us to observe fiscal policy implications in short and medium-term. We conclude that most economies are characterized by sustainable debt policies with the exception of Greece, Ireland, Portugal, Spain, and possibly United Kingdom. For the latter country, while the outcome of test algorithm indicates the sustainability of debt policy, phase-space examination shows that the reaction of the government to diverging debt stock GDP ratio cannot be sufficient to stabilize the path for debt dynamics.
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Keywords Public debt sustainability, Intertemporal budget constraint, Time series analysis, Cointegration, Chaotic process, Phase-space reconstruction, OECD countries

ÖZ Kamu borcu sürdürülebilirliğini test etmekte kullanılan farklı kriterler arasında yer alan ekonometrik yaklaşım, hükümetlerin borç ödeme güçlüğüne düşmeden bütçe açıklarını sürdürüp sürdüremeyeceğini araştırır. Bu çerçevede, Trehan ve Walsh (1991), Hakkio ve Rush (1991) ve Bohn (1995) tarafından literatüre kazandırılmış üç farklı ekonometrik analiz yöntemini temel alan bütünleşik bir test algoritması meydana getirdik. Bu algoritmayı kullanarak avro-bölgesi ekonomileri, PIIGS grubu ülkeleri, İngiltere ve Türkiye'nin de aralarında bulunduğu bazı OECD ülkelerinin kamu borcu sürdürülebilirliğini inceledik. Araştırmamızı güçlendirmek maksadı ile bütünleşik ekonometrik yaklaşıma ek olarak, borç dinamiklerinin kısa ve orta-dönem seyrini belirleyebilmek için faz-uzayı yapılandırılması metodunu önerdik ve uyguladık. İki farklı yaklaşım neticesinde Yunanistan, İrlanda, Portekiz ve İspanya haricindeki ülkelerin kamu borcu sürdürülebilir olarak belirlendi. İngiltere için ise açık bir neticeye ulaşamadık. Şöyle ki; İngiltere için bütünleşik ekonometrik yaklaşım sürdürülebilir bir kamu borcuna sahip olduğunu gösterirken, faz-uzayı yaklaşımı ise denge kümesinden sapmakta olan kamu borcu GSYİH oranına karşı mali hamlelerin borç dinamiğinin bozunum seyrini önleyemediğini ortaya koymaktadır.
SEÇİLMİŞ OECD ÜLKELERİNDE KAMU BORÇ STOĞU SÜRDÜRÜLEBİLİRLİĞİ
JEL C02, C22, C32, E60, H63
Anahtar Kelimeler Kamu borcu sürdürülebilirliği, Bütçe süreci kısıtı, Zaman serisi analizi, Eşbütünleşme, Kaotik süreçler, Faz-uzayı yapılandırılması, OECD ülkeleri

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1. Introduction

Fiscal policy, sustainability, and solvency have come to the forefront of stabilization policy in the recent years. The questions of whether a given level of public debt is "sustainable" and/or whether large and persistent deficits will lead a government to default became common concerns for both developed and less-developed countries.[1] Large public deficits and high levels of public debt over GDP are important factors that affect the budget process of public sector. Without balanced budgets the ensuing deficits accumulate and lead to a rise of public debt in individual countries. Over the last decade a lot of European countries have suffered from steadily high public deficits.[2] This trend still represents a serious problem from the economic and political point of view, especially for members of the European Monetary Union.

The related theoretical literature emphasizes the intertemporal budget constraint as well as the flow-budget constraint of the government, and focuses on whether current fiscal policy can be followed without disturbing government solvency. At the level of empirical policy analysis, the "fiscal sustainability" remains highly ambiguous, and nearly each empirical study attempts to develop its own definition of the concept and derive its conclusions accordingly. Mainly, three different motives for debt repayment are mentioned in the literature (Eaton, 1989; Dreher et al., 2006): "Illiquidity"; "unwillingness-to-pay" (insufficient incentive to repay), and "insolvency" (inability-to-pay).

In perspective of empirical analysis, the applied literature with regard to the ability-to-pay approach can be considered to have mainly followed two main paths. First, according to the country risk (or debt servicing capacity) models, a country's prospects of repaying the funds borrowed are evaluated by a variety of economic indicators, usually in the form of aggregate ratios such as primary surplus/GDP, debt/GDP, debt/exports and debt-service/exports (see Blanchard, 1990; Cuddington, 1997; Croce and Juan-Ramon, 2003).

[1] Corsetti and Roubini (1991) and Chalk and Hemming (2000) focus on fiscal sustainability in the OECD economies and announce rather mixed results. Following the Stability Growth Pact and the Maastricht Treaty fixing maximum values for the deficit (3% of GDP) and net public debt (60% of GDP), the budgetary policy in Europe has been a matter of concern, see Buiter (2003). The sustainability of the fiscal policy as well as the solvency of the governments in the less developed countries' have received great attention from the IMF and the World Bank. Among others, see Buiter and Patel (1992) for India, Gerson and Nellor (1997) for Philippines, Bascard and Razin (1997) for Indonesia, and Agénor (2001) on Ghana and Turkey.

[2] See a recent survey by Arghyrou and Tsoukalas (2011).

The second path in the applied literature, the "standard approach" or the econometric approach for sustainability, focuses on the sustainability of debt policies over the long-run. All that is required (i.e., the solvency condition) is that the present value of the flow of future primary surplus is not less than the net current indebtedness. Specifically, the non-stationarity analysis provides useful tools in gaining deep insight into the long-run implications of a government or a country's intertemporal solvency. By using intertemporal budget constraint the tests determine whether a government is able to sustain its budget or external deficits without defaulting on its debt. Such tests are first found in the literature regarding government's solvency (or the sustainability of fiscal deficits), beginning with the contribution by Hamilton and Flavin (1986), and developed by Wilcox (1989), Trehan and Walsh (1991), Hakkio and Rush (1991), Buiter and Patel (1992), Tanner and Liu (1994), Bohn (1995), Wu (1998), Makrydakis et.al., (1999). In this paper we resort to the studies by Trehan and Walsh (1991), Hakkio and Rush (1991) and Bohn (1995) which have received considerable attention in economics. Firstly, these three different motives on public debt and fiscal policy sustainability are re-examined, and linked to form a compact step-wise test algorithm. This step-wise econometric test procedure enables us to perform a deeper analysis for public debt sustainability. Secondly, we observe that the period under consideration covers the recent global financial crisis and especially its effects on European countries' public finance policies, which may generate possible nonlinearities on the observed data. Thus, we aim to strengthen our findings for sustainability of debt policy and also we intend to make a small contribution to the literature that goes beyond linear estimation techniques in the associated area and that tries to find structural breaks, thresholds or possible nonlinearities (see e.g. Bajo-Rubio et al., 2004; Martin, 2000; Payne et al., 2008; Westerlund and Prohl, 2010; Fincke and Greiner 2011). We present a different simple method, namely the "non-standard approach" for sustainability, which enables us to visualize the path for debt dynamics i.e., moving from some initial stable "equilibria" and diverging through unsustainable values. To do this, we revisit dynamical systems literature, more specifically Takens' theorem for phase-space reconstruction (Takens, 1981 and Eckmann et al., 1987). Our motive to introduce phase-space reconstruction in sustainability analysis is that the fiscal policy may exhibit deterministic character in most of its duration (see Fincke and Greiner, 2011) and that "standard approach" focuses on long-run implications, having possible drawbacks to distinguish between more vulnerable[3] countries in the short and medium-term. The

[3] We refer to the term "vulnerable" in the sense of deterioration of both fiscal policy and budget process.

phase-space examination of time-evolution characteristics of debt dynamics has an economic intuition: If the government's debt policy remains stable, depending on some pre-determined benchmark scenario then we are able to observe ex-ante that public debt GDP ratio values move closer to some "attracting[4]" set. This movement shows us whether the path for debt dynamics is sustainable or not, respectively corresponding to convergence or divergence of those values on phase-space. In its policy approach in determining whether a fiscal position is sustainable, the IMF is observed to apply the following steps[5]: (i) based on the available data, a five-year projection is made assuming that the current fiscal policy is to be continued. This is regarded as the benchmark scenario. (ii) From this projection, a path for debt dynamics is generated and its sustainability is assessed. It is possible that different criteria are used for sustainability, but an increasing debt ratio is usually regarded as a cause for concern. (iii) If the path for debt dynamics is indicated as "unsustainable", an alternative scenario is prepared, making necessary corrections on fiscal policy variables, which will typically define a "stable path" over the medium-term. Thus, the "non-standard approach" enables us to observe the consistency of such benchmark scenarios in ex-ante sense. We have to note that we do not consider the central bank when dealing with the question of under which conditions a given path of public debt is sustainable. We do this because central banks are independent and governments should not rely on central banks to reduce public debt when deciding about debt and deficits.

The rest of the paper is organized as follows. Section 2 presents the data. Section 3 develops a step-wise algorithm for econometric analysis. Section 4 presents the results of econometric analysis performed for selected countries. Section 5 introduces the phase-space reconstruction methodology and presents the results. Section 6, finally, summarizes the central findings.

2. Data

The countries[6] considered in our study are France, UK, Greece, Ireland, Italy, Portugal, Poland, Turkey, and Spain. UK, France, and Italy are included because they are among largest economies in Europe. We also include Greece, Ireland, Portugal, and Spain because they belong to the so-

[4] Attracting set definitely refers to the attractor on phase-space, corresponding to unique or multiple "equilibria".

[5] See, e.g., Chalk and Hemming (2000) and Agénor and Montiel (1999, chap.13). The IMF's official programming model, known as the Polak model, see Polak (1997).

[6] Except Turkey those countries have to stick to the Convergence Criteria of the Maastricht Treaty of the European Union and to the Stability and Growth Pact, which limits public deficits to 3% of GDP and public debt to 60% of GDP.

called PIIGS countries that have been characterized by large deficits and debt to GDP ratios recently that have raised questions about sustainability of their debt policies respectively. Among European economies, we take Turkey into account, because following from its greatest financial crisis in 2000-2001, Turkey is steadily respecting Criteria of the Maastricht Treaty. The common period chosen for quarterly data is 1999Q1-2010Q1. The data is obtained from Eurostat Public Finance database. Turkish data is obtained from Turkish Statistical Institute. The debt stock data begin from 1999Q1 for all EU countries. On the other hand, for some other variables, the data of some countries go back through 1989Q1, i.e., expenditures and revenues of UK, France, and Italy. However, the analysis including debt stock as a variable begins from 1999Q1. In order to account for the size of the different countries in the EU, public debt is usually expressed in terms of ratios, mostly relative to GDP. This measurement is also resorted in the Convergence Criteria of the Maastricht Treaty of the EU.

3. Theoretical Background and Methodology for Econometric Approach

In empirical debt stock sustainability literature the "standard approach" is based on the equation of the time evolution of public debt and hence on general budget identity. We consider Equation 1,

$$B_T = (1 + i_T).B_{T-1} + G_T - T_T \tag{1}$$

where B_T is the volume of public debt at the end of period *T*, i_T the nominal ex-post interest rate during period *T*, G_T the volume of public expenditures excluding interest payments and T_T volume of taxes. To simplify the argument, we have not taken into account explicitly seignorage revenues, so that they find themselves in the ex-post interest rate. Let P_T denote the price-level during period *T* and $GDPA_T$ denote the nominal annual GDP, which are defined by Equations 2 and 3 respectively. Throughout the study the upper index *r* denotes the variables in their real terms.

$$P_T = (1 + \pi_T).P_{T-1} \tag{2}$$

where π_T denotes the rate of change in price-level.

$$GDPA_{T+1} = (1 + \eta_{T+1}).GDPA_T \tag{3}$$

where η_T is the growth rate.

By forward induction and as period goes infinity, we can rewrite Equation 1 as shown by Equation 4.

$$B_T = \lim_{k\to\infty} B_{T+k}.\rho_{T,k} - \sum_{n=1}^{k=\infty}(G_{T+n} - T_{T+n}).\rho_{T,n} \quad (4)$$

where $\rho_{T,k} = \prod_{j=1}^{k}\left(1+i_{T+j}\right)^{-1}$ is *k*-period ahead discount factor. By using Equations 2 and 3, Equations 1 and 4 can also be arranged in real terms, and in real terms adjusted by the real rate of growth as well. The second term on RHS of the Equation 4 represents the sum of the present values of expected future deficits. The first term should tend to zero in order to assure the stability of the debt. This transversality condition is also known under the name of "Ponzi game". The econometric tests of sustainability are based on Equation 4 and the "sustainability" corresponds to the "stationarity". Thus, public debt stock will be accepted to be sustainable if it exhibits stationary behavior. Otherwise, we should consider the causes of nonstationarity.[7] Two possibilities are reported in the literature: Either the transversality condition does not hold or the sum of the present values of anticipated deficits is non-stationary. Trehan and Walsh (1991) show that the convergence of the present value of the deficit signifies the satisfaction of the transversality condition. For the present value of the deficit to converge, the total budget balance has to be stationary. Secondly, Hakkio and Rush (1991) suggest that a necessary and sufficient condition for sustainability is the existence of at least one co-integration relation between total revenues and total expenditures with co-integrating vector of a particular type. Finally, Bohn (1995) proposed a test for sustainability based on a co-integration relation between primary balance and debt stock. On the other hand we may also consider the role that the maturity may play. If the average maturity is assumed to directly affect the interest rates, then the model of Bohn captures it. Bohn (1995) states that the government has to satisfy an intertemporal budget constraint and associated transversality condition regardless of the level of the safe interest rate. Secondly, Hakkio and Rush (1991) assume that all government bonds have one period maturity.

3.1. Test Algorithm

The test algorithm that we follow is given in this section. Figure 1 depicts the diagrammatic representation of the algorithm. The variables which take place in algorithm are in real terms over real annual GDP.

[7] "Nonstationarity" covers the integrated process of order $n \geq 1$ and nonlinear process as well, i.e. the chaotic process.

Step 1: Stationarity (unit root) test is applied for debt stock (B) over GDP annual (GDPA) in real terms. If it is estimated to be stationary, the algorithm ends. If it is estimated to be non-stationary then the algorithm goes to step 2.

Step 2: *a.* First, we test the stationarity of primary surplus (PS) over GDP and if it is stationary then the debt stock is not sustainable and algorithm goes to the Step 3.

b. If primary surplus over GDP is non-stationary, we seek at least one linear combination of the primary surplus over GDP and the debt stock (1-period lagged) over GDP that is stationary. If there is at least one, then debt stock is sustainable. Particularly, if primary surplus over GDP series is I(1) then algorithm goes to the Step 4.

c. If there is no linear combination, then we test the stationarity of budget balance (BB) over GDP. If it is stationary then the sustainability exists. Otherwise, the algorithm goes to the Step 3.

Step 3: If there is at least one co-integration relation between total expenditures over GDP and total revenues over GDP, then the debt stock may be sustainable, depending on the value of estimated co-integrating vector. That is with a vector [1,-c] under the condition that $0 < c < 1$.

Step 4: *a.* If there is at least one co-integration relation between primary balance (G-T) over GDP and debt stock over GDP, then we conclude that the debt stock is sustainable, implying both intertemporal budget constraint and transversality condition hold.

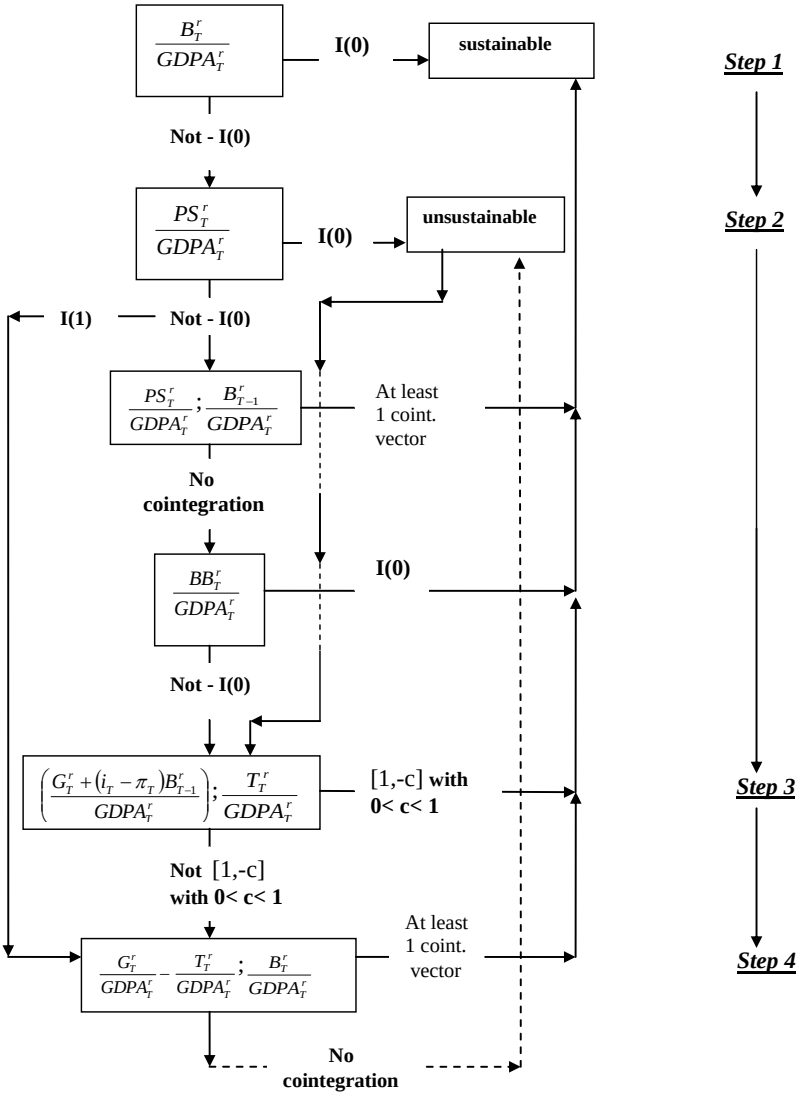
b. Otherwise we decide that the debt stock is not sustainable.

To analyze stochastic behavior[8] of a given series, we use three different test methods: Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test, and KPSS[9] test. To analyze the multivariate process generated by revenues and expenditures, we have chosen to use the method proposed by Johansen (1991). The advantages of this method compared with Engle and Granger's (1987) is that it allows us to test for the number of cointegration relations, it does not impose an arbitrary normalization on cointegrating vector and it permits us to test for constraints on the coefficients of the cointegration relation.

[8] The estimation of lags are performed by both SIC, Hannan-Quinn and Akaike criteria. Where two methods give different lag values than each other, we choose the one estimated by SIC. Both E-views 5.00 and Stata 10 are used for the analysis. In order to conserve space, we do not locate here the details of estimation results. They can be supplied upon request.

[9] Kawiatowski, D., Phillips, P.J.B., Schmidt, P., Shin, Y. (1992)

Figure 1. Diagrammatic Representation of the Step-wise Algorithm



4. Results of Econometric Analysis

We employ the test algorithm that we introduce in Section 3.1. Three types of unit-root tests are deployed: ADF, PP, and KPSS test statistics. The results of the analyses are depicted in Table 1. Each column consists of three inputs; ADF, PP, and KPSS test results respectively. The variables shown in

Table 1 and Table 2 are in their real terms. The overall results are given by Table 2.

Table 1. Sustainability Analysis of Public Debt Stock GDP Ratio for 9 Countries

Country	*Debt stock/GDP*	*PS/GDP*	*BB/GDP*	*Exp/GDP*	*Rev/GDP*
Ireland	I(1);I(1);I(1)	I(1);I(0);I(1)	I(1);I(0);I(1)	I(1);I(1);I(1)	I(0);I(0);I(0)
Greece	I(1);I(1);I(1)	I(1);I(1);I(0)	I(1);I(0);I(0)	I(1);I(1);I(0)	I(1);I(0);I(1)
Spain	I(1);I(1);I(1)	I(1);I(0);I(1)	I(0);I(0);I(1)	I(0);I(0);I(0)	I(1);I(0);I(0)
France	I(1);I(1);I(1)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(0);I(0);I(0)
Italy	I(1);I(1);I(0)	I(1);I(0);I(1)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(1);I(0);I(0)
Poland	I(1);I(1);I(0)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(0);I(0);I(0)
Portugal	I(1);I(1);I(1)	I(0);I(0);I(1)	I(1);I(0);I(1)	I(1);I(0);I(1)	I(1);I(0);I(0)
UK	I(1);I(1);I(1)	I(1);I(0);I(0)	I(1);I(0);I(0)	I(1);I(1);I(1)	I(1);I(0);I(1)
Turkey	I(1);I(1);I(1)	I(0);I(0);I(0)	I(0);I(0);I(1)	I(1);I(1);I(0)	I(1);I(1);I(0)

For Ireland, Greece, Spain, and Italy "Step 2.b" and "Step 4" are performed. Steps 2.b and 4 consider that given primary surplus/deficit over GDP is non-stationary and particularly is I(1) then we should seek at least one co-integration between primary surplus/deficit and debt stock. The results are presented by Table 2. For France, Portugal, Poland, Turkey, and UK "Step 2.a" and "Step 3" are performed. Step 2.a and 3 basically say that in case where primary balance over GDP exhibits I(0) behavior and if there is at least one co-integration relation between total expenditures over GDP and total revenues over GDP, indicating one-unit increase in revenues results with less-than-one unit increase in expenditures then debt policy is sustainable.

In Table 2, the nonstationarity of some data is determined according to two affirmative and one negative test results while some other's by three affirmative test results. The "steps" column in Table 2 defines the steps proposed in the Section 3.1. The second columns in Tables 1 and 2 indicate debt stock over GDP, while the third, fourth and fifth columns refer to primary surplus/deficit, budget balance, expenditures and revenues over GDP, respectively. The "No of Coint" column depicts the estimated co-integration relationships based on Johansen and Juselius (1990) values. The sign shown in "No of Coint" column indicates the sign of estimated coefficient vector. The last column in Table 2 shows the state determined: "Sus" refers to sustainable while "Unsus" refers to unsustainability of debt stock over GDP data.

Table 2. Results of the Step-wise Algorithm for the Sustainability of Debt Stock over GDP

Country	*Debt stock/GDP*	*PS/GDP*	*BB/GDP*	*Exp/GDP*	*Rev/GDP*	*Steps*	*No of Coint*	*State*
Ireland	I(1)	I(1)	I(1)	I(1)	I(0)	1;2.b;4	1(+)	Unsus
Greece	I(1)	I(1)	I(0)	I(1)	I(1)	1;2.b;4	No	Unsus
Spain	I(1)	I(1)	I(0)	I(0)	I(0)	1;2.b;4	1(+)	Sus
France	I(1)	I(0)	I(0)	I(0)	I(0)	1;2.a;3	No need	Sus
Italy	I(1)	I(1)	I(0)	I(0)	I(0)	1;2.b;4	1(+)	Sus
Poland	I(1)	I(0)	I(0)	I(0)	I(0)	1;2.a;3	No need	Sus
Portugal	I(1)	I(0)	I(1)	I(1)	I(0)	1;2.a;3	No need	Unsus
UK	I(1)	I(0)	I(0)	I(1)	I(1)	1;2.a;3	1(+)	Sus
Turkey	I(1)	I(0)	I(0)	I(1)	I(1)	1;2.a;3	1(+)	Sus

For Ireland, we look for co-integration between primary surplus/deficit and debt stock. Both Trace test and max-eigenvalue test results indicate one co-integration relation with positive coefficient vector. Even though there exists co-integration relation, expenditures are I(1) and revenues are estimated to be I(0). It is straightforward that Ireland is bubble-financing its expenditures, in which old debt that matures is financed by issuing new debt, implying Ponzi scheme. Therefore we conclude that the debt stock is not sustainable.

For Greece, Spain, and Italy we look for co-integration relation between primary surplus/deficit and debt stock. Both the Trace test and max-eigenvalue test results indicate 1 co-integration relation with positive coefficient vector. We conclude for Italy that debt stock is sustainable. We have to note that, ADF and KPSS statistics values for both budget balance and expenditures of Spain is very close to critical values and if we work with 90% confidence interval, then we accept that both budget balance and expenditures of Spain is non-stationary, implying unsustainability of debt stock GDP ratio. For Greece, since primary surplus/deficit over GDP exhibits nonstationary character and particularly it is I(1) we do not look for whether budget balance is stationary or not, instead we seek co-integration relation between primary surplus/deficit over GDP and debt stock over GDP. We found that neither Trace test nor max-eigenvalue test indicates co-integrating relation between primary surplus/deficit and debt stock. We conclude that debt policy of Greece is certainly unsustainable.

For UK, we look for co-integration relation between total expenditures and total revenues. Both Trace and max-eigenvalue tests indicate one co-integration relation with positive coefficient vector, indicating that one-unit increase in revenues results with less-than-one unit increase in expenditures. Therefore, we conclude that debt stock is sustainable. The budget balance, expenditures and revenues of France are all I(0).Therefore, we conclude that debt stock over GDP is sustainable. The Polish budget balance, expenditures and revenues are all I(0), implying that the debt stock over GDP is sustainable. For Portugal neither budget balance nor expenditures is I(0). Thus debt stock over GDP is not sustainable and Portugal is bubble-financing its expenditures, in which old debt that matures is financed by issuing new debt, implying violation of transversality condition. For Turkey, we look for co-integration relation between expenditures and revenues. We found that there exist one co-integration relation with a co-integrating vector [1,-c] under the condition that $0 < c < 1$, implying that debt policy is sustainable for the given period.

The drawback in most of the studies focusing on public debt sustainability in the aftermath of the global finance crisis is that, they omit the possibility

of non-linearity: During the crisis period the fiscal stimulus packages put heavy burden on the public finance while the low economic growth rates lead to further disruption of the debt dynamics. In this sense, debt dynamics may follow a non-linear path, even a chaotic behavior. Therefore, we present a different simple method which enables us to visualize the path for debt dynamics i.e., moving from some initial stable "equilibria" and diverging through unsustainable values. Our motive to introduce phase-space reconstruction in sustainability analysis is that the fiscal policy may exhibit deterministic character in most of its duration (see Fincke and Greiner 2011) and that the "standard approach" focuses on long-run implications, having possible drawbacks to distinguish between more vulnerable[10] countries in the short and medium-term.

5. Theoretical Background for Phase-space Examination of Debt Stock GDP Ratio

Let us denote the dynamical system, $f : R^n \to R^n$, with the trajectory,

$$x_{t+1} = f(x_t) + \varepsilon_t \quad , \quad t = 0,1,2,...., \tag{5}$$

The dynamical system itself may be assumed to convey noise, or the observed time series z_t given in the Equation 7 does.[11] The Lyapunov exponents (LE) for such a dynamical system are measures of the average rate of divergence or convergence of a typical trajectory or orbit. The trajectory is also written in terms of the iterations of f. For a *n*-dimensional system as above, there are *n* exponents which are customarily ranked from largest to smallest:

$$\lambda_1 \geq \lambda_2 \geq ..... \geq \lambda_n \tag{6}$$

One rarely has the advantage of observing the state of the system at any period *t*, x_t, and that the actual functional form, f, that generates the dynamics. The model that is widely used is the following: Associated with the dynamical system in the Equation 5 there is a measurement function $h : R^n \to R$ which generates the time series,

$$z_t = h(x_t) \tag{7}$$

It is assumed that all that is available to observer is the sequence $\{z_t\}$. Assume that the observed time series is generated by the Equation 5 and is

[10] We refer to the term "vulnerable" in sense of deterioration of both fiscal policy and budgeting process.
[11] See Nychka et al. (1992) and Kantz (1994).

observed through a measurement function as given in the Equation 7. Then, following Takens' famous theorem (Takens, 1981), the reconstructed trajectory is an embedding of the original trajectory when the value of embedding dimension *m*, is sufficiently large. If any attractor has appeared in the original dynamical system, another attractor, which retains the phase structure of the first attractor, will appear in the reconstructed phase-space. In order that such reconstruction achieves embedding, it has been proven that the dimension *m* should satisfy $m \geq 2n+1$. However, this is a sufficient condition and upper-worst case.[12] Depending on the data, embedding can be established even when *m* is less than 2*n*+1. In the embedding method, there are two parameters to be chosen, embedding dimension and time delay (see Abarbanel, 1995). From observed time series $\{z_t\}$, we can generate the data vector, $y_t = \left(z_{t+(m-1).d}, z_{t+(m-2).d}, ..., z_t\right)$ where *d* is the time delay; this vector indicates a point of *m*-dimensional reconstructed phase-space R^m, and *m* is embedding dimension. Therefore a trajectory can be drawn in the *m*-dimensional reconstructed phase-space by altering *t*. Let us rewrite the observed time series as $z_1, z_2, ..., z_N$, where *N* is the length of the observed sequence $\{z_t\}$. While embedding those data to *m*-dimensional phase-space, the time delay *d* can be used to obtain $\left(N-(m-1).d\right)$ points in phase-space. Therefore a point on the orbit shown by y_i is obtained as given in the Equation 8.

$$y_i = \left(z_i, z_{i+d}, ..., z_{i+(m-1).d}\right) \text{ for all } \quad i \in \left(N-(m-1).d\right) \tag{8}$$

If this is applied for all *i*, such that $i \in \left(N-(m-1).d\right)$, then we obtain the orbit. From now on, the time delay *d* is taken to be equal to 1, corresponding to our observation interval on time-domain.

5.1. Results of the Phase-space Analysis

In order to perform the analysis, we employ the Equation 8 to debt stock-GDP ratio series of each country. The public gross debt stock GDP ratio data of Poland, Portugal, Ireland, Greece, UK, and Turkey are embedded in 3-dimensional[13] phase-space. The reason why we choose those countries is that in the period under examination the primary balances of Poland, Portugal, Ireland, Greece, UK economies are deteriorated and enjoy

[12] Gencay and Dechert (1992)

[13] The method of recurrence plots (RP) was firstly introduced by Eckmannn et al., (1987) to visualize the time dependent behavior of the dynamics of systems (the recurrence of states in a phase-space) which can be pictured as a trajectory in the *m*-dimensional phase-space.

negative trend, while debt stock GDP ratio of each is increasing. This observation clearly indicates that the reaction of each government to increasing debt stock GDP ratio does not constitute to increase primary surplus. Turkey is taken into account since it steadily reduces the public debt stock over GDP ratio and simultaneously increases primary surplus GDP ratio. The results are depicted in Figure 2 to Figure 7. In each figure; *x-axis* shows the quarterly debt stock over GDP series, *y-axis* shows one-quarter lagged debt stock over GDP series, and *z-axis* shows two-quarter lagged series. The similarity of debt stock GDP ratio phase-evolution of Greece, Portugal, UK, and Ireland is interesting. Greece, Ireland, and UK debt stock over GDP exhibit nearly same movement path. The initial states of debt stock over GDP of these countries seem stable around some equilibria, implying stable public finance policy in 1999Q1-2003Q1 period. However, the values diverging on phase-space show that such equilibria have not been sustainable. For Portugal the movement is slightly slow with respect to that of Greece, implying slower divergence.

The Poland's debt stock GDP ratio exhibits distributive behavior on phase-space. In Figure 2, it is seen that there is no equilibria set for the dynamic values of debt stock GDP ratio, implying steadily changing public finance policy. Moreover, the data show us that the movement is not cyclic, but mostly linear and ascending. It reaches 60%.

Figure 2. Poland - Gross Debt Stock/GDP

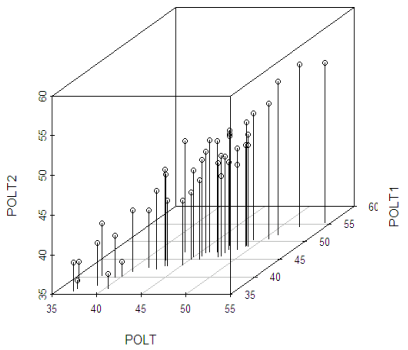
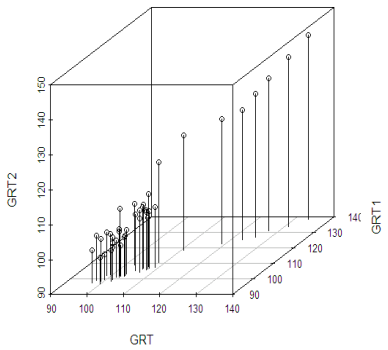


Figure 3. Greece - Gross Debt Stock/GDP



The Greek debt stock to GDP ratio wastes most of the space-time on two equilibria sets shown on Figure 3. The initial equilibria is settled at approximately 100-110%. However, there is second equilibria set which is around 110-115%, implying the non-synchronization of the coordination of expectations on 100-110%. This can be due to a type of market correction

on interest rates. Figure 3 enables us to see a major shift from 110-115% to 145%. To sharpen the results, one should take into account the phase-space reconstruction for the real ex-post interest rates.

The Portugal's structure of multiple equilibria slightly differs from that of Greece. In Figure 4 we observe that until 80% is reached, there exists minor shifts and debt stock GDP ratio exhibits high instability. Market expectations seem partially coordinate on some equilibria but it is not permanent and quickly disrupted. The minor shifts toward 100% imply that "common knowledge" of disruption may enable traders to expect debt repudiation, however as Detragiache (1996) points it out, instead of not-lending the traders prefer to lend to government to roll over its debt.

Figure 4. Portugal - Gross Debt Stock/GDP

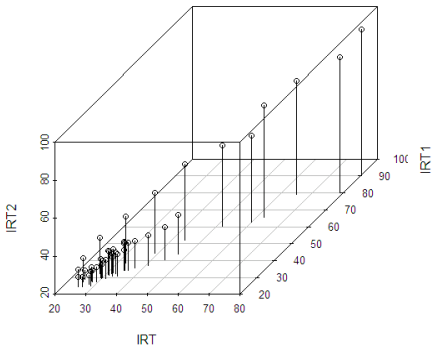
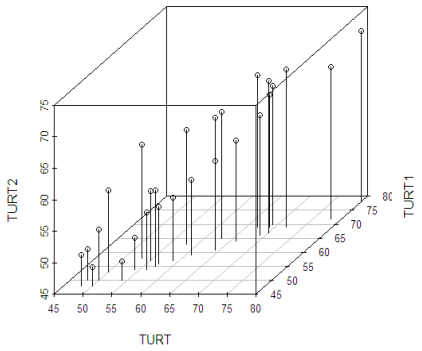


Figure 5. UK - Gross Debt Stock/GDP



The outcome of econometric test algorithm for UK's debt stock to GDP ratio leads us to decide that public debt is sustainable. On the other hand, from Figure 5 we observe that the debt stock exhibits divergent character, implying a major shift from the initial equilibria. Moreover it slightly slows down around 78%, other equilibria, to which public finance policy seems to be settled down. When its divergence is compared with that of Poland, UK's public finance seems to be disturbed more quickly than that of Poland's possibly because of its steadily increasing primary deficit. We know that the international credit rating institutions do not anticipate public debt crisis for UK (see pg.6, Fitch 2011). Figure 8 shows the time evolution of UK primary surplus over GDP: Beginning from 2001Q2, there is negative trend. The disrupted structure of UK primary surplus shown in Figure 8 explains us why the debt stock over GDP series enjoys quick shift through instability. According to phase-space analysis, UK will probably have liquidity crisis.

Figure 6 depicts that Ireland's debt stock over GDP series quickly escape from initial equilibria (1999Q1-2003Q1). We see that the financial policy loses its stability. The initial equilibria of debt stock over GDP is settled approximately around 40-50%. The quick shift probably shows us the insufficient repayment capacity of the economy for the value 50%. However, we need more detailed analysis to explain why Ireland economy jumped into debt crisis and UK's has not yet.

Figure 6. Ireland - Gross Debt Stock/GDP

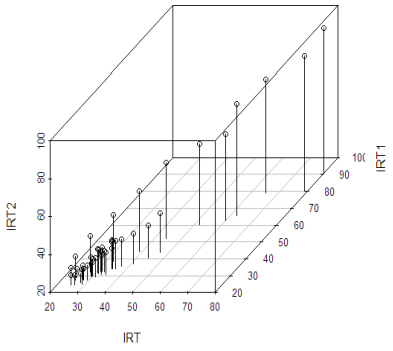
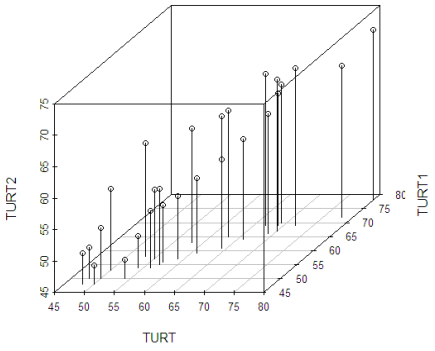
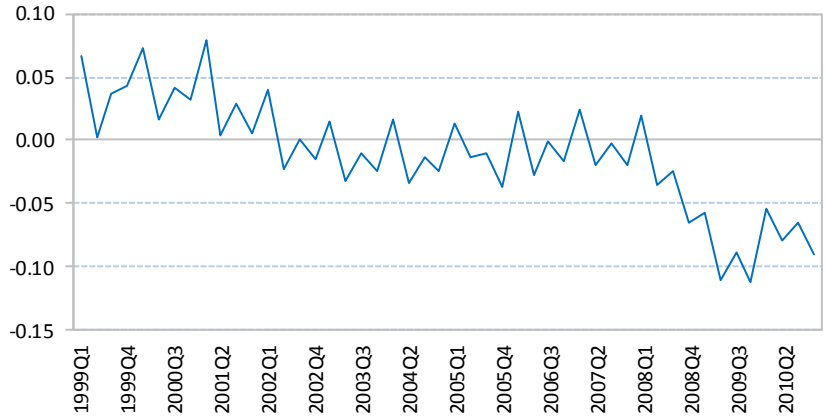


Figure 7. Turkey - Gross Debt Stock/GDP



Beginning from 2002Q1 Turkish debt stock GDP ratio exhibits deterministic character on phase-space. In Figure 7 there is no equilibria set for debt stock GDP values. Moreover, the data show us that the movement is not cyclic but descending, which is the difference between Polish and Turkish cases. We observe the divergence from 2001-2002 debt stock values: It reaches 45-50% and is not settled around pre-determined equilibria. It seems that debt management and public finance policy of Turkish government, indeed focus on a maximum decrease in debt stock over GDP (see Medium Term Macroeconomic Program 2012-2014).

Figure 8. UK -Primary Surplus/GDP



6. Concluding Remarks

In this paper we have analyzed whether selected European countries and Turkey have followed sustainable debt policies over the last 12 years. For this, we use "standard approach" and "non-standard approach" for sustainability. The results of first approach suggest that Ireland and Portugal have pursued unsustainable debt policies over the period considered in our estimation. Besides, Greece clearly has conducted an unsustainable debt policy that culminated in the Greek debt crisis started in early 2010. Spanish fiscal policy can be said to be unsustainable if we consider 90% confidence intervals in our estimations. We also found out that France, Italy, Poland, and Turkey have pursued sustainable debt policies. On the other hand, we observe during the most of the period under consideration that the primary balances of Poland, Portugal, Ireland, Greece, and UK economies are deteriorated and exhibit negative trend, while debt stock GDP ratio of each is increasing. This observation clearly indicates that in short and medium-term to the increasing debt stock to GDP ratios, the governments do not react with increasing the primary surplus. Therefore we found it more useful to introduce phase-space representations of debt stock GDP ratio of Poland, Portugal, Ireland, Greece, UK, and Turkey. We included Turkey into our analysis since Turkish government follows sharper debt policy and reacts to decreasing debt stock GDP ratio with increasing primary surplus. We found that Greece's, Ireland's and UK's debt stock over GDP series exhibit nearly same movement path: The initial states of debt stock GDP ratio of these countries seem stable around some equilibria, implying stable public finance policy in 1999Q1-2003Q1 period. This may be due to the result of stabilization policies in Europe in the late 1990s with the upcoming

Monetary Union. However, according to the phase-space analysis such equilibria set stayed no more sustainable. Given the contrast between the results of two approaches, we consider not drawing a clear-cut conclusion in the case of UK.

The earlier studies (for a survey see Afonso, 2005) that performed "standard approach" to the series of public debt or discounted debt rather favored the conclusion that debt policies in Europe are not sustainable. More recent studies, in particular those that test how the primary surplus reacts to public debt, tend to conclude that debt policies are sustainable. However these studies suffer from not taking into account non-linearity: The period under consideration covers the recent global financial crisis process and especially its effects on European countries' public finance policies, which may generate possible nonlinearities on the debt dynamics through the imbalanced linkage between the high public debt and deteriorated economic growth rates. Finally, from a methodological point of view the novelty of our paper consists in implementing a compound sustainability analysis. Developing a step-wise econometric test procedure leads us to perform long-term sustainability analysis in broader perspective and resorting to phase-space analysis enables us to obtain short and medium-term behavior of debt policy in the countries under consideration.

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