

## SYSTEMIC RISK CONTRIBUTION OF INDIVIDUAL BANKS

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*ABSTRACT* In this study, we measure systemic importance of individual banks that are listed in the Istanbul Stock Exchange. Regarding the whole system as a portfolio of individual banks, we calculate the system-wide risk via contingent claims analysis. Using Shapley values, we assess the systemic importance of each bank according to its marginal contribution to the calculated system wide risk measure, expected shortfall of the system. Our calculations reveal that market participants perceived 2000 and 2001 banking crises to be devastating for the Turkish banking sector. Since 2002, the banking sector seems to do a good job in eliminating idiosyncratic shocks within the system.

*JEL* G10, G13, C71

*Keywords* Systemic risk, Contingent claims analysis, Shapley value

*ÖZ* Bu çalışmada, İMKB’de işlem gören bankaların sistemik riske katkıları ölçülmektedir. Bankacılık sistemine bankaların oluşturduğu bir portföy gibi yaklaşarak, Koşullu Alacak Analizi yoluyla sistemik risk hesaplanmaktadır. Shapley değerleri kullanılarak, her bankanın hesaplanan sistemik risk ölçütü olan beklenen kayba marjinal katkısı değerlendirilmektedir. Hesaplamalar, piyasa katılımcıları tarafından, 2000 ve 2001 bankacılık krizlerinin Türk Bankacılık Sektörü için oldukça tahripkar olarak değerlendirildiğini ortaya koymaktadır. 2002 sonrası için ise, bankacılık sektörünün sistem içindeki bireysel şokları bertaraf etmekte başarılı olduğu görülmektedir.

*BANKALARIN SİSTEMİK RİSKE KATKISI*

*JEL* G10, G13, C71

*Anahtar Kelimeler* Sistemik risk, Koşullu alacak analizi, Shapley değeri

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## 1. Introduction

In this study, we try to find out systemic importance of each individual bank within the banking system. Using a single portfolio of all available banks and all other sub-portfolios formed by individual banks, we calculate the system-wide risk via contingent claims analysis, hereafter CCA. Hence, we are able to derive systemic importance for only those banks that are listed in the Istanbul Stock Exchange (ISE). We implement expected shortfall as a measure of systemic risk. By using Shapley values, a solution concept for cooperative games, we derive the systemic importance of each bank according to its marginal contribution to the calculated expected shortfall of the system.

Bisias et al. (2012) present 31 different quantitative ways of measuring systemic risk in the literature. Among these measures, we use CCA, which has its roots in the option pricing theory. A contingent claim is an asset whose payment is made when a certain specified state of the economy is realized. In other words, the CCA deals with the pricing of risky debt. The CCA is an application of the Merton Model, a corporate debt-valuation model initially developed by Merton (1973), on systemic risk measurement literature.

We employ CCA mainly due to its ability to represent the forward-looking expectations of the market. The CCA makes use of equity prices of the banks observed in the market and book level data simultaneously. Moreover, high correlation of CCA, both with the credit ratings observed in the market and historic probabilities of default, encourages us to use CCA as a measure of riskiness. CCA enables to tell how market judges the probability that an entity will default on its obligations. CCA also enables us to derive loss given default, expected loss and expected shortfall of that entity under consideration.

Following the derivation of systemic risk in the market, we make use of game theory to distribute this risk among the players, i.e. individual banks, in a systemic risk allocation game. Payoffs of this game are defined as the negative of expected shortfall of entities. Within our framework, the entity is represented either by each individual bank or all possible subsets of the banks in the system. In other words, we treat all possible subsets of individual banks as interconnected portfolios and by this way we try to figure out interactions within each coalition. These interactions can be attributed to either being exposed to common risk factors or being interconnected.

We calculate risk associated to each and every possible such sub-portfolios of the whole system. Then, the increment in the risk of a sub-portfolio when a specific bank is included gives us the marginal risk contribution of that bank. Combining all such possible marginal contributions yields the overall risk contribution of that specific bank to the system. The marginal contribution of a bank is named as the Shapley value, after Shapley (1953), of that bank in cooperative games and represents the systemic importance of that bank. One important property of that method is that the sum of the marginal risk contributions of individual banks is equal to the risk measure of the whole system.

We run analysis for two samples due to data constraints. In the first sample, we have four banks and data for these banks spans the period June 2000-March 2012. For the second sample, we have eight banks and data range is from June 2007 to March 2012. In four bank case, results show that the banking crisis for 2000 and 2001 are more devastating than the global financial crisis for these banks. In eight bank case, we observe peaks in probability of defaults and expected shortfalls during the global turmoil in 2008. Also, Shapley value analysis helps us to allocate the system wide risk to individual banks. We observe changes in the contributions of banks through time.

The paper is structured as follows. In the Next Section, we review the relevant literature and in Section 3 we introduce our model, which uses both the CCA and the Shapley values. Section 4 summarizes the data. In Section 5 we present our results and we conclude in Section 6.

## 2. Literature Review

In this part, we summarize the related literature on both measuring systemic risk<sup>1</sup> with CCA and on distributing the calculated system-wide risk among individual banks.

As summarizing the related literature on measuring systemic risk can be a topic of another paper, we will solely deal with the literature on measuring systemic risk with the CCA. The CCA enables researcher to derive a probability of default<sup>2</sup> for the entity under consideration by using the most current market and book data. This entity can be a firm, a bank, a sector in the economy or a government. Kealhofer et al. (2001) provide an application of the CCA model to calculate expected default frequencies for firms and financial institutions by measuring their implied asset values and volatilities. They derive actual default probabilities by mapping their risk neutral default

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<sup>1</sup> An exhaustive literature review on various systemic risk measurement techniques can be found in Bisias et al. (2012).

<sup>2</sup> The derived probability of default is the risk neutral probability of default.

probabilities by means of a database of historic default probabilities for firms. Gray et al. (2007), on the other hand, use CCA approach to derive the riskiness of a national economy. They consider the risk-adjusted balance sheets and interlinkages for four key sectors (sovereign, financial, corporate, and household) in a national economy to derive the default risk of that economy. By using the CCA approach, Gray and Walsh (2008) derive various risk measures for the Chilean financial sector. They also attribute measured riskiness of the system to the selected macroeconomic and financial variables.

Gray and Jobst (2010) coin the term “Systemic CCA” to refer to the measurement of systemic risk of the financial sector as the implicit contingent liabilities of the government. The expected losses of the financial sector under distress are implicit contingent liabilities of the government arising from its role of the lender of last resort. They derive the value of these liabilities from market implied government support.

Our next strand of the literature deals with assigning each separate entity with a portion of the measured system-wide risk. Jorion (2007a) evaluates the beta of the losses of each bank with respect to the losses of a portfolio as the contribution of each bank to overall risk. In component VaR setting of Jorion (2007b), sum of the marginal risk contributions of individual banks gives the total risk of the whole system. Adrian and Brunnermeier (2011) propose a conditional value at risk model to evaluate the co-dependence of institutions on each other. Then, contribution of a bank to the systemic risk is the difference between the value at risk of the financial system under two states; one being the bank under consideration is in distress and the other being that bank is in its median state. They use market prices at the bank-level to derive systemic importance of the bank and quantify the importance of leverage, size, and maturity mismatch on systemic risk contribution. One important note to mention here is that the sum of conditional value at risk over all institutions does not necessarily give the value at risk of the whole system.

Denault (2001) uses the marginal risk of a sub-portfolio, calculated by Shapley values over all possible sub-portfolios, to determine the per unit risk contribution of that sub-portfolio in a portfolio risk management problem. Adopting portfolio risk management methodology to the problem of attributing the systemic risk among banks, Staum (2011) proposes a Shapley value mechanism to distribute the systemic risk, which is measured as the premium required to insure all of the deposits in the banking system, across banks. Huang et al. (2011) also use price of insurance against systemic financial distress as the measure of systemic risk and calculates the systemic contribution as the conditional expected losses of possible sub-portfolios, given a large loss for the full portfolio. They found that a bank’s

contribution to the systemic risk is a linear function of its size and a non-linear function of its individual probability of default and its asset correlation with other banks in the system. On the other hand, Tarashev et al. (2009) find that the systemic importance of a bank is convex function of its size i.e. systemic importance grows faster than size.

The methodology that we use to distribute systemic risk among banks is similar to that used by Tarashev et al. (2009). They borrow the notion of Shapley value from game theory to distribute systemic risk among individual banks. The contribution of a bank to the systemic risk is not limited to the expected losses that it incurs on other entities in the event of default. One bank's riskiness has impact on the probability of default of other banks and quantity of the losses in the system as well. Shapley value enables to derive risk contribution of an individual bank as the average of marginal changes in the risk of all possible subsets. This contribution mainly depends on the size, individual probability of default, and exposure of the bank to common risk factors. Drehmann and Tarashev (2011) add interbank connections to the previous paper to account for the role of bank in propagation of exogenous shocks through the system.

### 3. The Model

Our model has two dimensions; one involves the calculation of systemic risk and the other deals with the distribution of the calculated risk across banks. In the first subsection, we introduce the CCA approach, on which our systemic risk calculation is based, and then we present the notion of Shapley value that we use for attributing the risk across banks.

#### 3.1. CCA

In this subsection, we briefly give a picture of CCA, which we employ to derive the credit risk profiles of banks from the observed market variables. The methodology applied originates from the seminal paper of Merton (1973), in which Merton extends the general idea of option pricing theory to corporate liabilities.

The assumptions made in CCA are as follows; perfect capital markets with no transaction costs, taxes and equal access to information for all investors, continuous trading, stochastic movement of the value of the firm with Ito dynamics, constant volatility of assets, non-stochastic term structure for interest rates, shareholder wealth maximization, perfect bankruptcy and anti-dilution protections, and perfect liquidity (Jones et al., 1984).

This setting assumes that a firm issues two types of securities; equity and debt. Equity,  $E$ , does not pay any dividends and the firm needs to pay its debt, which is amount of  $D$ , at the end of a time period  $T$ . At the end of  $T$ , if

assets,  $V$  exceed debt, the payment of the debt is realized and the remaining belongs to the shareholders. If the value of assets is less than the debt, then the firm defaults and shareholders receive nothing. Thus, the equity at the time of  $T$  can be characterized as following:

$$E_T = \max[V_T - D; 0] \quad (1)$$

This payoff is same as the payoff of a European call option on the assets of the firm where strike price is the debt of the firm ( $D$ ) to be repaid. Assuming that the value of the assets moves throughout time with Geometric Brownian Motion having a constant volatility ( $\sigma_V$ ) and the risk free interest rate ( $r$ ) has a constant term structure, Black-Scholes-Merton option pricing model is an available tool for establishing the relationship between equity price and assets.

$$E_0 = V_0 N(d_1) - D e^{-rT} N(d_2) \quad (2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{V_0}{D}\right) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}} \text{ and } d_2 = d_1 - \sigma_V \sqrt{T}$$

In this framework, since the value of the equity is a function of the value of assets, using Ito's lemma it can be shown that the volatility of the equity is related with the volatility of assets according to the following Equation:

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0 \quad (3)$$

Equations 2 and 3 enable us to determine the unobserved value of assets and volatility of assets from the observed equity price, volatility of equity, maturity of debt, and level of debt. Having obtained these values, the probability of default (PD) can be determined since it is simply the probability of assets being under the debt level at the end of the period and given by  $N(-d_2)$ .

Expected loss for the creditors of the firm can also be assessed in the same manner. Market price of the debt today equals the value of assets over the value of equity. On the other hand present value of debt is simply the discounted debt amount to be paid at time  $T$ . Thus expected loss for the debt can be expressed as:

$$EL = 1 - \frac{(V_0 - E_0)}{D e^{-rT}} \quad (4)$$

After having probability of default and expected loss for the debt, since exposure at the default is the whole amount of the debt, loss given default, can be evaluated as:

$$LGD = \frac{EL}{PD} \quad (5)$$

These three variables, namely probability of default, loss given default and expected loss, draw a complete picture of the credit risk profile for the

firm. It should be strictly underlined that since Merton’s model uses market prices, this credit risk profile reflects the perceptions of equity market participants for the firm in a risk-neutral world.

CCA framework gives us the distribution of the asset value at time  $T$ . We can calculate the expected shortfall (ES) which corresponds to the tail risk by using this distribution. We use 1% threshold for the tail risk and derive the expected shortfall as the expected loss for the worst 1% cases. Then expected shortfall for worst  $\alpha\%$  cases can be expressed as:

$$ES = 1 - \frac{V_0 (1 - N(\sigma_V \sqrt{T} - N^{-1}(\alpha)))}{\alpha D e^{-rT}} \quad (6)$$

where  $N^{-1}$  is inverse cumulative normal distribution.<sup>3</sup> Note that we calculate expected shortfall for the all possible sub-portfolios including entire banking system. We treat each sub-portfolio as a single entity. Thus, we use the total market capitalizations and debts of the individual banks in the sub-portfolio as the equity and debt of that entity, respectively. Also, we calculate the return of each sub-portfolio by weighting the returns of the banks in that portfolio by their market capitalization. Thus, the above formulation implicitly takes into account relative bank sizes and asset correlations.

### 3.2. Shapley Value

In this subsection, first we define cooperative games and introduce Shapley value. After that, we discuss the Shapley value in terms of attribution of systemic risk to individual institutions in the system. Finally, we assess the use of Shapley value to distribute the risk calculated by the Merton model.

Cooperative games in game theory deal with the games that players can generate a worth by combining their skills. A cooperative game  $(N; v)$  characterized by set of players  $N$  and its characteristic function  $v : 2^N \rightarrow R$ , which assigns a real value to any subset of  $N$ . In all cooperative games, value of empty subset is assumed to be zero,  $v(\emptyset) = 0$ , and coalition formed by the participation of all players is called as grand coalition.

The main goal in a cooperative game is to distribute the value of the grand coalition among all players. A value operator takes a cooperative game  $(N; v)$  as input and produces  $|N|$ -dimensional pay-off vector. Value operators can be interpreted as solution concepts for cooperative games.

Shapley value (1953) is a value operator that takes into account the marginal contributions of each player in each coalition. Let  $N = \{1, 2, \dots, n\}$  denotes the set of players in a cooperative with characteristic function  $v : 2^N \rightarrow R$ . The Shapley value for player  $i$ :

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<sup>3</sup> The derivation for expected shortfall is provided in the Appendix.

$$\varphi_i(v) = \sum_{S \subset N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S)) \quad (7)$$

where the fraction term corresponds to the probability of player  $i$  entering into coalition  $S$  when players are ordered randomly and the remaining term corresponds to marginal contribution of player  $i$  to coalition  $S \subset N \setminus \{i\}$ . The probability is calculated under the assumption that all permutations are equally likely. Since Shapley value is a value operator, it satisfies  $\sum_{i \in N} \varphi_i(v) = v(N)$ .

**Example 1:** Let  $N = \{1,2,3\}$ . The characteristic function:  $v(\emptyset) = 0$ ,  $v(\{1\}) = v(\{2\}) = 3$ ,  $v(\{3\}) = 6$ ,  $v(\{1,2\}) = 6$ ,  $v(\{1,3\}) = v(\{2,3\}) = 15$ ,  $v(\{1,2,3\}) = 18$ . Players 1 and 2 are symmetric in this game.

**Table 1. Calculation of Shapley Value**

Perm.	Player 1	Player 2	Player 3
123	$v(\{1\}) - v(\emptyset)$	$v(\{1,2\}) - v(\{1\})$	$v(\{1,2,3\}) - v(\{1,2\})$
132	$v(\{1\}) - v(\emptyset)$	$v(\{1,2,3\}) - v(\{1,3\})$	$v(\{1,3\}) - v(\{1\})$
213	$v(\{1,2\}) - v(\{2\})$	$v(\{2\}) - v(\emptyset)$	$v(\{1,2,3\}) - v(\{1,2\})$
231	$v(\{1,2,3\}) - v(\{2,3\})$	$v(\{2\}) - v(\emptyset)$	$v(\{2,3\}) - v(\{2\})$
312	$v(\{1,3\}) - v(\{3\})$	$v(\{1,2,3\}) - v(\{1,3\})$	$v(\{3\}) - v(\emptyset)$
321	$v(\{1,2,3\}) - v(\{2,3\})$	$v(\{2,3\}) - v(\{3\})$	$v(\{3\}) - v(\emptyset)$

The first column includes the all possible permutations. The permutation 123 corresponds the ordering started by player 1 and after that player 2 and player 3 enter, respectively. All probabilities assigned to each permutation are equal,  $1/6$ , and add up to 1. The second column shows the marginal contribution of player 1 to subsets associated with the permutations. The remaining columns display the marginal contributions of player 2 and player 3 respectively.

The Shapley value for players can be calculated as:

$$\varphi_1(v) = \varphi_2(v) = \frac{1}{6} (3 + 3 + 3 + 3 + 9 + 3) = 4$$

$$\varphi_3(v) = \frac{1}{6} (12 + 12 + 12 + 12 + 6 + 6) = 10$$

Since players 1 and 2 are symmetric, their Shapley values are the same. Also the sum of Shapley values equal to the value of grand coalition  $N = \{1,2,3\}$ ,  $\sum_{i \in N} \varphi_i(v) = 18 = v(N)$ .

Shapley value satisfies the properties given below:

**Symmetry:** If player  $i$  and  $j$  are symmetric in the game  $(N; v)$ , the Shapley values are equal for  $i$  and  $j$  i.e.  $\varphi_i(v) = \varphi_j(v)$ .



**Dummy:** If player  $i$  is dummy player in the game  $(N; v)$ ,  $v(S \cup \{i\}) - v(S) = 0$ ,  $\forall S \subset N \setminus \{i\}$ , the Shapley value for player  $i$  equals to zero i.e.  $\varphi_i(v) = 0$ .

**Additivity:** Let  $(N; v_1)$  and  $(N; v_2)$  are any two coalitional games. If  $(N; w)$  is the sum of two games:  $\forall S \in 2^N, w(S) = v_1(S) + v_2(S)$ , then  $\forall i \in N, \varphi_i(w) = \varphi_i(v_1) + \varphi_i(v_2)$ .

**Linearity:** Let  $(N; v)$  and  $(N; w)$  are two coalitional games s.t.  $\forall S \in 2^N, v(S) = \lambda w(S)$  for some  $\lambda \in R$ . Then  $\forall i \in N: \varphi_i(v) = \lambda \varphi_i(w)$ .

Shapley (1953) shows that Shapley value is the unique value operator that satisfies symmetry, dummy, and additivity properties. Tarashev et al. (2009) discuss the appealing implications of linearity.

As mentioned in previous subsection, Merton model prices the risk in the market by using the asset prices, debt, and risk free return. We calculate the risk of all subsets as expected shortfall of these portfolios and use the Shapley values in order to distribute the system wide risk.

Shapley value ignores the network effects which may distort the risk attribution. However, direct liabilities of Turkish banks to each other are not in considerable terms and interactions between banks are mainly caused by being exposed to common risk factors.

#### 4. Data

Operating in a high inflationary and fiscal dominant macroeconomic environment in the late 1990s, Turkish banking system could be characterized with low level of financial intermediation, high share of government bonds in assets, a result of financing government deficits, low credit to deposit ratios, insufficient risk management practices, and low levels of foreign investments. By means of a comprehensive reform package, which took place in the aftermath of the crises and enabled public banks to strengthen their capital structures, the banks with problems of rolling over their liabilities were taken under the oversight of Savings Deposit Insurance Fund (SDIF), measures were taken to support financial soundness of private banks and regulatory and supervisory framework was improved. Turkish Banking System responded positively to these policies and, with the help of diminishing fiscal dominance and enhancing macroeconomic stability, has started conventional banking practices of lending to real sector on a sound basis.

Taking into account the structural break that Turkish banking system experienced in the aftermath of the devastating 2000 and 2001 crises, we do not extend the scope of our study to pre-2000 period.

To be able to apply Merton model, we use only those banks whose stocks are traded on the Istanbul Stock Exchange. Among the ISE listed banks, we focus on those with total asset size not less than 1% of the whole sector. Our sample period covers three important periods; one being the Turkish balance of payment crisis of November 2000, banking sector crisis of February 2001 and finally the global financial crisis that began in summer of 2007. However, we have to split the sample period in order to have the maximum number of banks that qualify our selection criteria.

For the whole time period between June 2000 and March 2012, we have only 4 banks that are both traded on ISE and satisfy minimum 1% asset size qualification.<sup>4</sup> As of June of 2000, total share of assets of this sample is around 25%. In order to have a better representation of the system and to shed more light on the global crisis period, we extend our sample to include four more banks that satisfy our data selection criteria during the recent period between June 2007 and March 2012. Share of assets of this enlarged sample is almost 70% of that of the whole system as of June 2007.

Our data source for the equity, debt, and risk free interest rate is Bloomberg. We have used market capitalizations of each bank as the market value of equity and total liabilities excluding shareholders' equity as the amount of debt to be paid. For possible subsets of banks, we treat each subset as a single entity. Thus, we use the total market capitalizations of the individual banks in the subset as the equity of that entity. The same treatment also applies for the amount of debt. We have used the benchmark interest rate in ISE Bonds and Bills Market, which is the annual compound yield of the on the run zero coupon government security, as the risk free interest rate in our estimations.

Considering the short duration of the total liabilities of the Turkish banking system, we assumed that the total debt matures within 3 months (we take  $T=0.25$  year in the Merton model). When we merge individual banks under the roof of larger portfolios, we calculate the return of each portfolio by weighting the returns of the banks in that portfolio by their market capitalization. We calculate yearly volatility of equity by annualizing the daily volatility of the most recent 21 trading days.

## 5. Results

As discussed in the previous section, for the whole time period between June 2000 and March 2012, we have only 4 banks that are both traded on ISE and satisfy minimum 1% asset size qualification. And for the global crisis period, beginning with June 2007 till March 2012, we have 8 banks

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<sup>4</sup> We exclude one of the banks that passes our qualification criteria since it underwent a merger operation.

that satisfy our criteria. We report the results for both time periods separately.

For the whole time period, we first report the risk neutral probability of default rates (Figure 1) and expected shortfall values (Figure 2) for individual banks and for the system, as if it were only composed of these four private banks which constituted approximately a quarter of the total assets of the system at that time. Individual probability of default rates exhibit simultaneous peaks especially in December 2000, February 2001, November 2002, March 2003, and October 2008. Probability of default hikes in other periods may be regarded as idiosyncratic rather than systemic shocks to the banks. The levels of probability of default rates imply that 2000 and 2001 banking crises were perceived to be more devastating than the global financial crises of 2008 by the market participants. Expected shortfalls, calculated by the risk neutral probabilities, of this four-bank system also exhibit a similar pattern. Although expected shortfalls of individual banks display more severe hikes at different time episodes, the expected shortfall of the four-bank system, shown by solid black line, has never reached levels that it attained during December 2000 and February 2001 crises. This four-bank system even performed well during the global financial crisis of 2008. The most severe hike in the expected shortfall of an individual bank takes place in May 2002, when that bank had hard times due to rumors of merging with its defaulted group bank.

Figure 3 shows the attribution of risk, expected shortfall of the system, across banks. We present overall expected shortfall of the system by the solid black line and contribution of each bank to the expected shortfall of the system by colored bars. According to our calculations, expected shortfall of the whole system wanders around zero except 2000 and 2001 banking crises and March 2003. Regarding the attribution of this expected shortfall to individual banks, both 2000 and 2001 crises point out the existence of a common shock that hit the system. None of the banks in the system were able to escape from the devastation brought about by the two crises. Towards the end of 2001 crisis, the existence of two banks eliminates the risk created by the other two in the system. More interestingly, the idiosyncratic hike, caused by the aforementioned merger rumors, in the expected shortfall of an individual bank in June 2002 is totally dampened by the other three banks in the system. Thanks to the gradual amelioration that took place in the aftermath of the 2000 and 2001 crises, the system of these four banks have not confronted a systemic shock as serious as those occurred during the 2000 and 2001 crises. In March 2003, possible negative outcomes caused by the Iraq war increased the tension in the stock exchange and created another systemic shock to the banking system. Accordingly, expected shortfall of the four bank system shows another peak in this period.

During 2008 global financial crisis, the expected shortfall of the system exhibits a negligible amount of systemic risk.

In order to have a better representation of the system during global financial crisis, we expand our sample to include four more banks. These additional four banks are again the only ones that both qualify for our asset size criterion and are listed in the ISE. Among these eight banks, two are state banks and remaining six are private banks, of which two are owned by foreign partners. Our system of eight banks constitutes almost 70% of the total assets in the banking system as of June 2007. With this new sample, we investigate the systematic importance of the individual banks and associated risk pattern of the system during the global financial crisis.

We again report the risk neutral probability of default rates (Figure 4) and expected shortfall values (Figure 5) for individual banks and for the system. In Figure 4, we observe two time episodes in which probability of default rates of some banks, but not all, exhibit significant hikes. In the first episode, the two private banks experience a simultaneous increase in their probability of defaults by July 2008. These two banks are those with foreign partners. With their main partners being in the center of the crisis, market participants perceived these to be more risky than others by the summer of 2008. However, by October 2008 the focus of the anxiety was tilted towards a state bank with specialization on loans to small and medium enterprises. With the collapse of Lehman Brothers in September 2008, the concerns about the adverse effect of the crisis on the global economy increased by the autumn of 2008. With intensifying global recession woes, market participants seem to regard this state bank more risky than others as it is more prone to adverse changes in the business cycle than others. These patterns in probability of defaults are also reflected in expected shortfalls that are shown in Figure 5. Due to its high probability of default, state bank H exhibits a very high level of expected shortfall. Expected shortfalls of the above mentioned two private banks are not high due to their relatively small sizes.

Finally, Figure 6 shows the attribution of risk across banks within this eight-bank system. Inclusion of four more banks into the system seems to significantly alter the overall riskiness of the system. The solid black line represents our calculation of systemic risk within this eight-bank system. And again, we show the contribution of each bank to the systemic risk with colored bars. Excluding the four months of crisis period, the total expected shortfall of the system is slightly above zero. However, at the peak of the crisis, in October 2008, the total expected shortfall of the eight-bank system becomes significantly larger than that of the system with four banks. In the simplest term, this shows that four-bank system is far from being a representative for the whole system or it is financially more sound than the

remaining banks in the system. Hence, simply considering the four-bank system and making comparison among global financial crisis and previous crises may not be appropriate.

Taking a detailed look at the beginning, July of 2008, and the peak, October 2008, of the crisis period indicates that the existence of other banks in the system contributed to the elimination of the risk brought about by the two private banks with foreign partners and one specific state bank, respectively. In our view, this indicates the overall resilience of the system during the global financial crisis period.

## 6. Conclusion

In order to find out systemic importance of each individual bank within the Turkish banking system, we jointly make use of two distinct models; contingent claims analysis and Shapley value in game theory. By using the expected shortfall derived from the Merton model as a measure of systemic risk, we are able to derive systemic importance for those banks that are listed in the Istanbul Stock Exchange and have asset size not less than 1% of the whole system.

Our calculations reveal that the expected shortfall of the four bank system has reached to considerably high levels during December 2000, February 2001, March 2003, whereas this four bank system shows resilience during the global financial crisis of 2008. On the other hand, when we take into account four other banks and enlarge our sample, we see that Turkish banking system has been prone to systemic shock in October 2008 since both probability of default rates and expected shortfall levels increase.

In eight bank system, the existence of healthy banks seems to eliminate idiosyncratic shocks within the system and idiosyncratic hikes have not turned into severe systemic events. During the global financial crisis of 2008, we see that the distribution of systemic risk to the banks show a considerable variety. For instance bank F increases the systemic risk in September, October, and November 2008 whereas bank C acts in the other direction during the same period.

For future research, the volatility of equity can be modeled with GARCH models and the instantaneous changes in volatilities could be captured. Also, with the accumulation of data as the new banks are listed in ISE, the sample could be enlarged in the near future.

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## Appendix

This part shows the derivation of the expected shortfall we use in our calculations. Under the assumptions of CCA, it can be shown that asset value at time T,  $V_T$ , is log-normally distributed and natural logarithm of the asset value is normally distributed (with mean  $\ln(V_0) + \left(r - \frac{\sigma_v^2}{2}\right)T$  and standard deviation  $\sigma_v\sqrt{T}$ ). We want to obtain the expected shortfall, which is the expected loss in the tail of this distribution, i.e. the expected loss in the worst  $\alpha$  cases. For this, we first derive the threshold for the asset value at time T,  $V_{threshold}$ , in the worst  $\alpha$  cases:

$$\begin{aligned} \Pr(V_T \leq V_{threshold}) &= \alpha \\ \Rightarrow \Pr(\ln V_T \leq \ln V_{threshold}) &= \alpha \\ \Rightarrow \Pr\left(z \leq \frac{\ln V_{threshold} - \ln V_0 - \left(r - \frac{\sigma_v^2}{2}\right)T}{\sigma_v \sqrt{T}}\right) &= \alpha \\ \Rightarrow V_{threshold} &= \exp(N^{-1}(\alpha)\sigma_v \sqrt{T} + \ln V_0 + \left(r - \frac{\sigma_v^2}{2}\right)T) \end{aligned}$$

After finding this threshold, we want to obtain the expected shortfall, which equals expected loss for the cases that asset value falls below this threshold.

$$\begin{aligned} ES &= \frac{\left[ D - \frac{\int_0^{V_{threshold}} V_T f(V_T) dV_T}{\alpha} \right] e^{-rT}}{D e^{-rT}} \\ \Rightarrow ES &= \frac{\left[ D - \frac{\left[ \int_0^{\infty} V_T f(V_T) dV_T - \int_{V_{threshold}}^{\infty} V_T f(V_T) dV_T \right]}{\alpha} \right] e^{-rT}}{D e^{-rT}} \\ \Rightarrow ES &= \frac{\left[ D - \frac{\left[ E[V_T] - \int_{V_{threshold}}^{\infty} V_T f(V_T) dV_T \right]}{\alpha} \right] e^{-rT}}{D e^{-rT}} \end{aligned}$$

By taking the partial expectation and taking into account the log-normal distribution of asset value at time T, this yields us to:

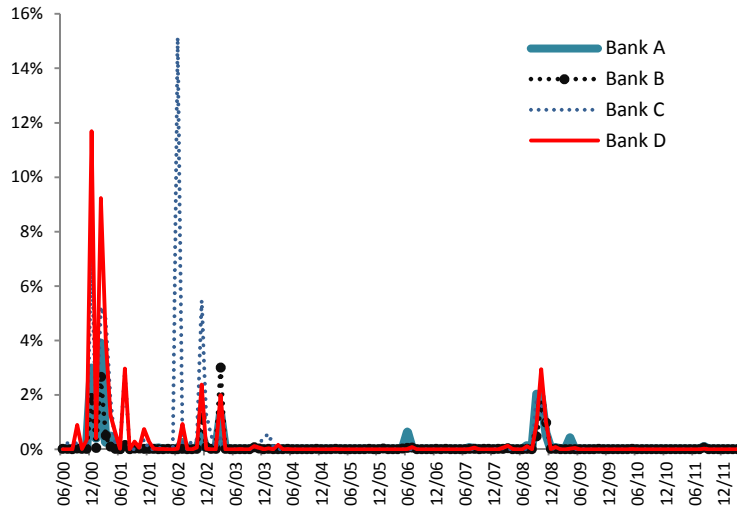
$$ES = \frac{D - \frac{e^{\ln V_0 + rT} \left( 1 - N \left( \frac{\ln V_0 + \left( r + \frac{\sigma_v^2}{2} \right) T - \ln V_{threshold}}{\sigma_v \sqrt{T}} \right) \right)}{\alpha}}{D e^{-rT}} e^{-rT}$$

If we replace the threshold we found, we reach the final result:

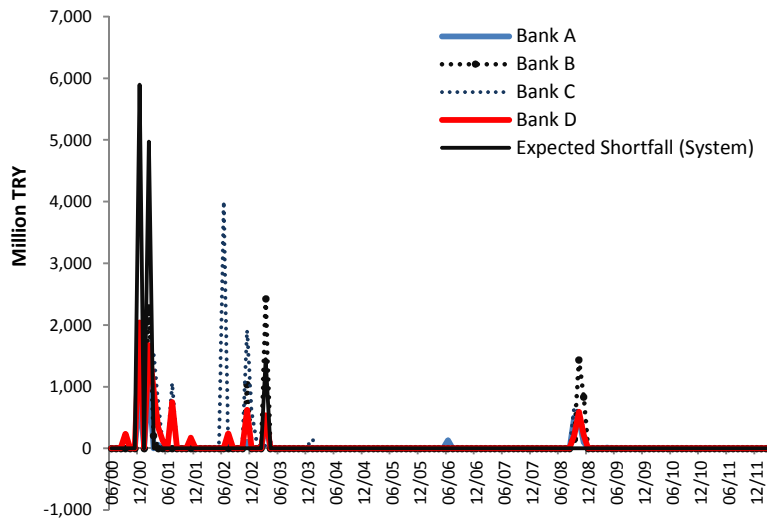
$$ES = 1 - \frac{V_0 \left( 1 - N(\sigma_v \sqrt{T}) - N^{-1}(\alpha) \right)}{\alpha D e^{-rT}}$$



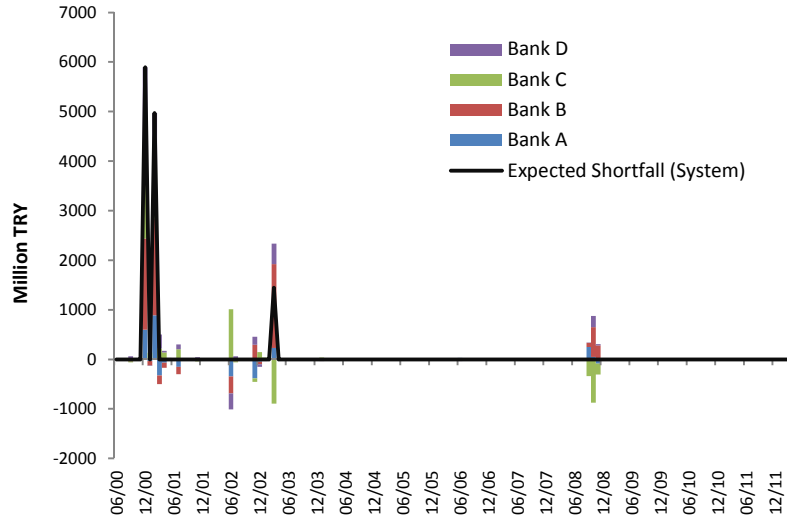
**Figure 1. Probability of Default (4 Banks)**



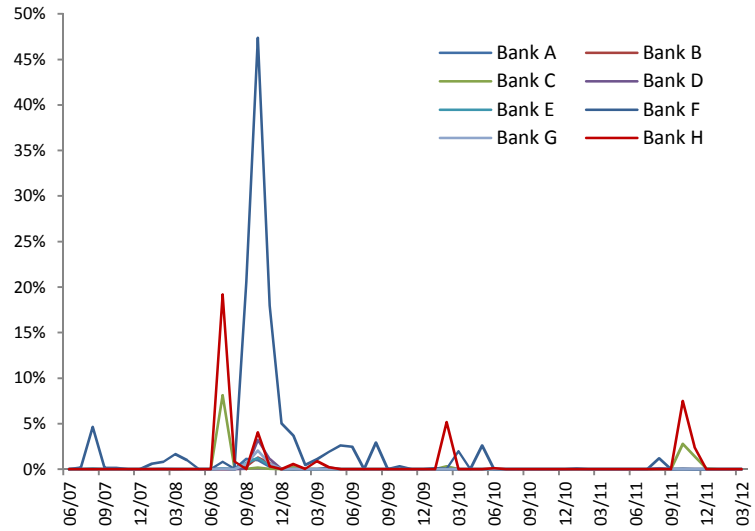
**Figure 2. Expected Shortfall (4 Banks)**



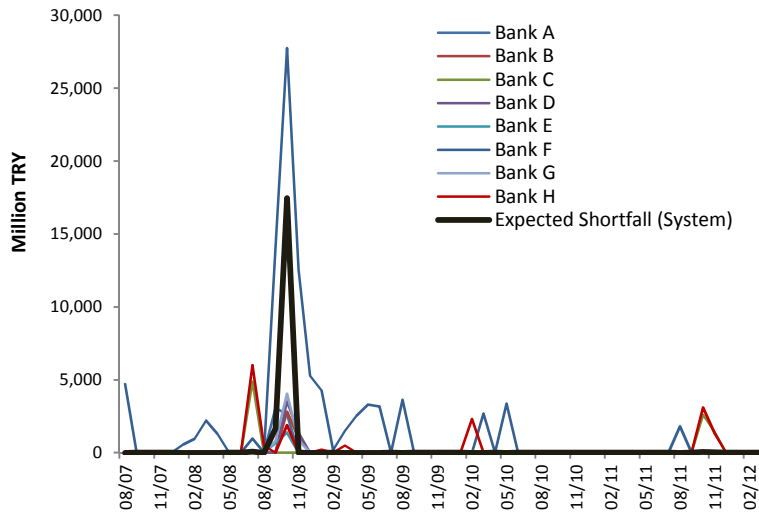
**Figure 3. Systematic Importance-Shapley Values (4 Banks)**



**Figure 4. Probability of Default (8 Banks)**



**Figure 5. Expected Shortfall (8 Banks)**



**Figure 6. Systematic Importance-Shapley Values (8 Banks)**

