The Dynamic Relationship Between Stock, Bond and Foreign Exchange Markets

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In this paper, we investigate whether deviation of a currency from its fundamentally determined rate of return affects its interaction with interest rates and stock market yields. A time varying transition probability Markov-Switching Vector Autoregressive (MS-VAR) model is utilized for this purpose. Wald and Likelihood ratio tests are used as model adequacy measures. In order to analyse the link among the variables, impulse-response functions are employed. States are defined as overvalued state and undervalued state depending on the position of the observed exchange rate to its fundamentally determined rate which is computed by sticky price exchange rate model. The model is implemented to four major currencies: Australian dollar, the Canadian dollar, the Japanese yen, and the British pound. Transition between the states are linked to risk adjusted excess return (the Sharpe ratio) of debt market and equity market returns of respected currencies in order to understand whether overvaluation and undervaluation is connected to the returns in these markets. The results provide evidence that the relationship between economic fundamentals and the nominal exchange rates are subject to change depending on the overvaluation or undervaluation of the currencies relative to their fundamentally determined rate of return. As an extension of the model, we found that the Sharpe ratios of debt and equity investments in the currencies influence the evolution of transitional dynamics of the exchange rates’ deviation from their fundamental values.

**Keywords:** Bond Price, Stock Price, Exchange Rate, Sharpe Ratio, Wald Ratio Test, Likelihood Test, Impulse-Response Functions, Markov-Switching Vector Autoregressive Model

**JEL:** C32, C58, E44, F31, G15

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1. Introduction

The linkages between stock markets, bond markets and exchange rates in this age of integrated international financial markets is important not only as an academic endeavor, but also because it has significant implications for investors, who constantly monitor these markets to take advantage of excess returns. Huge quantities of capital flows between the international markets in search of an extra yield\(^1\) have the potential to cause undervaluation or overvaluation of currencies by pushing them away from their fundamental values. Since the rate of appreciation and depreciation of the currencies is also an important part of investment strategies in international markets, the dynamic relationship between exchange rates, stock and bond markets depends on overvaluation and undervaluation of the domestic currencies. In this context, we investigate two subjects: First, does the relationship between bond markets, stock markets and exchange rates vary depending on overvaluation and undervaluation of the domestic currencies? Second, does risk adjusted excess return in bond and stock markets have any impact on overvaluation and undervaluation of the domestic currencies?

There are two approaches to explain the relationship between stock prices and exchange rates relationship: First one is the traditional approach. This approach (Goods Market Approach) argues that currency depreciation affects stock prices through export-import channels by making domestically produced goods cheaper and foreign produced goods more expensive and increases the domestic stock prices (Solnik, 1987). The second approach is the portfolio adjustment approach (Asset Approach). According to this approach, portfolio adjustments occur through foreign capital inflows and outflows as a result of changes in stock prices. A persistent upward trend in stock prices will attract foreign capital (inflow) and cause appreciation of the currency, while decrease in stock prices will decrease stock holders wealth causing a fall in money demand and will lower interest rates causing capital outflows which will lead to depreciation of the currency.

\(^1\) The total value of the world’s financial stock, comprising equity market capitalization and outstanding bonds and loans, has increased from $175 trillion in 2008 to $212 trillion at the end of 2010 (Roxburgh et al. 2011).
The relationship between exchange rates and interest rates is an area where the different approaches are deeply rooted and distinctive between economic schools of thought. Under the flexible prices approach (Classical School), the relationship between the exchange rates and interest rates is positive. While under the sticky prices approach (Keynesian School), this relationship is negative.

There are also different views for the interest rate-stock price relationship. Higher interest rates increase the opportunity cost of money, decreasing the return and stock prices of the companies. On the other hand, lower interest rates do not have the opposite impact on the stock prices. Higher money supply pushes the interest rates down causing higher inflation, ceteris paribus. Higher inflation lowers the real value of stocks and may reduce the demand for stocks.

Different approaches and diversity of opinion about interaction between exchange rates, interest rates and stock market yields among economists, reflect the fact that these economic variables may interact differently under different states. As a matter of fact, Krolzig (2000) uses MS-VAR model to capture the time-varying behavior and interactions of stock returns and exchange rates and concludes that taking the regime-switching feature advantage of MS-VAR fits better to economic relationships and leads to better forecasting than time-invariant linear models. Also, recently available evidence suggests that in the foreign exchange markets, traders attribute to economic fundamentals such as interest rates and stock prices time varying importance in valuation of the exchange rates (Cheung et al. 2001). This sort of state dependent relationship may exist between the exchange rates, interest rates, stock prices, gold prices and also oil prices as evidence provided by Kal et al. 2013a and 2013b. Therefore, Markov-switching specifications have been utilized to investigate regime-switching behavior among economic variables in literature.

Recently, there have been some new studies published analyzing the linkages between stock markets, bond markets and exchange rates. Andersen et al. (2007) studied how high frequency US, German, and British stock, bond and exchange rate dynamics were linked to economic fundamentals. They found that high-frequency stock, bond and exchange rate dynamics are linked to fundamentals.
Pavlova and Rigobon (2007) also address the relationship between three asset classes for the US and the UK, but they do not use regime-switching models. Nieh and Li (2001) found that there is no significant long run relationship between exchange rates and stock prices for G7 countries, their findings however is mixed in the short run. Tabak (2006) investigated the relationship between Brazilian stock market and exchange rates and he found that there is no longer a relationship between them. Phylaktis and Ravazzolo (2005), on the other hand show that stock prices and FX markets are positively linked using cointegration and multivariate Granger causality tests for some Pacific Basin Countries.

Using a Markov Switching approach to markets in the East Asian region Flavin et al. (2008) find that shocks that originate in either equity or FX markets have spillover effects on the other markets during “turbulent” market conditions. Henry (2009) employs a regime switching MS-EGARCH model in order to investigate the relationship between short term interest rates and the UK equity market. In the low mean-high volatility regime, the conditional variance of equity returns responds persistently but, symmetrically to equity return innovations. In high return-low volatility regime, equity volatility responds asymmetrically and without persistence to shocks to equity returns. Ning (2010) found significant and positive tail dependence between the foreign exchange market movements and the stock market in G5 countries (US, UK, Germany, Japan, and France) for the period pre- and post-euro using a copula based approach.

Employing a two regime Markov switching-EGARCH model Chkili et al. (2011) found strong evidence of regime switching behavior in volatility on emerging stock markets in “calm” and “turbulent” periods. They reveal the presence of two volatility regimes for Hong Kong, Singapore, Malaysia and Mexico. They also provide strong evidence that the relationship between stock and foreign exchange markets is regime dependent and stock price volatility responds asymmetrically to events in the foreign exchange market. Their results show that foreign exchange rate changes have a significant impact on the probability of transition across regimes. In another paper, Chkili and Nguyen
(2014) discovered that stocks markets have more influence on exchange rates during both calm and turbulent periods. Ehrmann el al. (2011) studied the international transmission mechanism between stocks, bonds and exchange rates in US and Euro area. They find that asset prices react strongest to other domestic asset price shocks, and that there are also substantial international spillovers, both within and across asset classes.

In international perspective where final returns are always counted in terms of US dollar, exchange rate returns is naturally part of returns on both equity (stock) markets and debt (treasury) markets. Therefore, an overvalued or undervalued currency with respect to its fundamentally determined value may be instrumental to the interaction between these markets. Yet, this interaction under these circumstances (overvaluation and undervaluation) between exchange rate, debt and equity markets has not been investigated before to our best knowledge. So, this study will give a new perspective to the interrelationship between stocks, bonds and exchange rates by releasing one state limitation and answering the question as to whether the overvaluation and undervaluation of the domestic currencies has a potential influence on this interrelationship. Also, the model we will employ here will give us additional tool to investigate whether overvaluation and undervaluation of the currencies is caused by risk adjusted returns in these markets. This approach is unique in its integration of capital flows to exchange rate, stock and bond dynamics in a setting where currency overvaluation and undervaluation matters. To achieve this goal, a two state time varying transition probability Markov regime switching process is implemented to vector auto regression between exchange rates, interest rates and stock market yields. In this model, the transition between the overvalued and undervalued states of the exchange rates is governed by the risk adjusted excess return of debt and equity markets (the Sharpe ratio). Here we used the Sharpe ratio because, in the international foreign exchange markets as a benchmark for investments among different currencies. This model is implemented quarterly in the dollar exchange rates of the most active four currencies between 1972 and 2010: the AUD, the CAD, the JPY and the GBP.
Our results provide further evidence that the relationship between special exchange rates, interest rates and stock index yields are time varying and depend on overvaluation and undervaluation of currencies with respect to their fundamentally determined values. We also found that the Sharpe ratios of debt and equity markets have meaningful influence on overvaluation and undervaluation exchange rates.

The rest of the paper is organized as follows: Section 2 explains the Markov Switching models and details the specifications of our models. Section 3 and 4 discuss the data, the estimation and the adequacy tests of the MS-VAR model. Section 5 presents and interprets the empirical results obtained including impulse-response functions. Section 6 summarizes and concludes the paper.

2. The Model

In this study, we use two-state Markov Regime Switching Vector Autoregressive model with time varying transition probabilities. Quandt (1958), and Godlfeld and Quandt (1973) who use the Markov regime switching models in econometrics are pioneer in this literature. However, Markov regime switching method was popularized by Hamilton’s (1990) groundbreaking study in empirical economics and improvement in the computer technology. Since switching algorithm is computationally demanding and requires high computer capacity, popularity of the model is peaked as the computing power and speed of the computers multi folded in last decade.

In an MS-VAR model, we observe variable \( y_t \) depending on unobservable state variable. We use \( s_t \) to define state variable and our two states are state 0 and state 1. We do not observe state (regime) variables. The unobserved state variable evolve according to Markov chain. State \( s_t \) variables determine the distribution of each period. Therefore, mean and variance of \( y_t \) depend on the state. We define state 0 as an overvalued state and state 1 as an undervalued state. We do not directly

\footnote{State variables are latent variable.}
observe whether we are in overvalued state or undervalued state. Nonetheless, we observe variable \( y_t \) whose behavior depends on state variable as follow:

\[
(y_t/s_t) \sim N(\mu_t/\sigma^2_t)
\]  

(1)

In a two-state case; when \( s_t=0 \) the observed changes of \( y_t \) is a random draw from distribution \((y_t/s_t) \sim N(\mu_0/\sigma^2_0)\) and when \( s_t=1 \), the observed changes of \( y_t \) is a random draw from distribution \((y_t/s_t) \sim N(\mu_1/\sigma^2_1)\). The probability density of \( y_t \) conditional on state variable is formulated as follows:

\[
(f(y_t|s_t=i)) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{(y_t - \mu_i)^2}{2\sigma^2_t} \right\}
\]  

(2)

Switching in Markov models between two states depends only on the previous state. We define transition probability as the probability of switching from one state to another state. For instance, switching from State \((t-1)\) to State \((t)\) is shown by the following notation:

\[
P\{s_t = i|s_{t-1} = j\} = p_{ij}, \text{ where } P \text{ is probability, } i=0, 1 \text{ and } j=0, 1
\]  

(3)

In a two-state switching model, we write transition probabilities as follow:

\[
P\{s_t = 1|s_{t-1} = 1\} = p_{11}
\]  

(4)

\[
P\{s_t = 0|s_{t-1} = 1\} = p_{01} = 1 - p_{11}
\]  

(5)

\[
P\{s_t = 0|s_{t-1} = 0\} = p_{00}
\]  

(6)

\[
P\{s_t = 1|s_{t-1} = 0\} = p_{10} = 1 - p_{00}
\]  

(7)

We collect the transition probabilities in a \((2 \times 2)\) transition matrix \( \Gamma \). We write \( \Gamma \) as follow:

\[
\Gamma = \begin{bmatrix}
p_{00} & p_{10} \\
p_{01} & p_{11}
\end{bmatrix} \text{ and } \sum_{j=0}^{1} p_{ij} = 1, 0 \leq p_{ij} \leq 1
\]  

(8)

Transition probabilities can be constant, as Hamilton suggested in his very well cited work (Hamilton, 1989), or it can be time varying, as developed by Diebold et al. (1994). In Hamiltonian...
framework, the probabilities of switching between the states are fixed, exogenous and do not vary over the time. These features of the model limit the explanatory power of the Markov process. Allowing transition probabilities to change over time, depending on a vector of variables, enriches the Markov process by enabling it to model the underlying process of transitional dynamics explicitly. Thereby, Diebold et al. (1994) extend the Hamilton’s framework by adding time-varying transition probabilities that are estimated as logistic functions of vector of Sharpe ratios \((x_{t-1})\) as shown in the following equations:

\[
p_{t}^{11} = P(s_{t} = 1 | s_{t-1} = 1, x_{t-1}; \beta_{1}) = \frac{\exp x_{t-1} \beta_{1}}{1 + \exp x_{t-1} \beta_{1}}
\]

\[
p_{t}^{01} = P(s_{t} = 0 | s_{t-1} = 1, x_{t-1}; \beta_{1}) = 1 - \frac{\exp x_{t-1} \beta_{1}}{1 + \exp x_{t-1} \beta_{1}}
\]

\[
p_{t}^{00} = P(s_{t} = 0 | s_{t-1} = 0, x_{t-1}; \beta_{0}) = \frac{\exp x_{t-1} \beta_{0}}{1 + \exp x_{t-1} \beta_{0}}
\]

\[
p_{t}^{10} = P(s_{t} = 1 | s_{t-1} = 0, x_{t-1}; \beta_{0}) = 1 - \frac{\exp x_{t-1} \beta_{0}}{1 + \exp x_{t-1} \beta_{0}}
\]

Equations (9) to (12) are transition probabilities, which are time varying logistic functions.

Since Sims’ (1980) study, vector autoregressive model has become one of the major tools of empirical studies. Krolzig (1997) introduces the regime changes to VAR models. An MS-VAR model provides framework of modeling multivariate representation of related variables non-linearly.

The MS-VAR model of exchange rate, interest rate differential and stock yield differential is formulated as follows:

\[
\Delta e_{t} = c^e_{st} + \alpha^e_{st}[\Delta e_{t-1}] + \theta^e_{st}[id_{t-1}] + \lambda^e_{st}[smyd_{t-1}] + \epsilon_{t} 
\]

\[
id_{t} = c^i_{st} + \alpha^i_{st}[\Delta e_{t-1}] + \theta^i_{st}[id_{t-1}] + \lambda^i_{st}[smyd_{t-1}] + \pi_{t} 
\]

\[
smyd_{t} = c^p_{st} + \alpha^p_{st}[\Delta e_{t-1}] + \theta^p_{st}[id_{t-1}] + \lambda^p_{st}[smyd_{t-1}] + \omega_{t} 
\]
\[ \Delta e_i = \frac{e_i - e_{i-1}}{e_{i-1}} \]  

(16)

\[ id_i = i^{dm} - i^{us} \]  

(17)

\[ smyd_i = smy^{dm} - smy^{us} \]  

(18)

\[ \Delta e_i \] is percentage change in exchange rate defined as price foreign currency (US dollar) in terms of domestic currency. In accordance with this definition, higher (lower) \( \Delta e_i \) states appreciation (depreciation) of domestic currency. “id” is interest rate differential; here in this model, treasury rate differentials are used for this purpose. “smyd” is stock market yield differential and we used the difference between S&P 500 quarterly yield and yield of respected domestic stock markets for this purpose.

The model posits that the risk adjusted excess return in each period changes the expectations of the global investors and through capital flows effects the position of the exchange rate relative to its fundamentally determined rate of return. In this time varying transition MS-VAR model, model risk adjusted is used as the driving factor of transition probabilities. As the transition probabilities vary depending on Sharpe ratio (risk adjusted excess return), exchange rates go above or below their fundamentally determined rate of return. Fundamentally determined rate of return of each exchange rate on the other hand is found by using sticky price exchange rate model. So, we constructed fundamentally determined rate of return for each currency by using the formula above:

\[
\Delta f_i = a_0 + a_1(\Delta m_i^{dm} - \Delta m_i^*) - a_2(\Delta y_i^{dm} - \Delta y_i^*) + a_3(i_i^{dm} - i_i^*) + a_4(\Delta p_i^{dm} - \Delta p_i^*) \\
+ a_5(smyd_i^{dm} - smyd_i^*)
\]  

(19)

According to this, fundamentally determined percentage change in each currency (\( \Delta f \)) is a function of differential in money supply (M2) growth rates (\( \Delta m^{dm} - \Delta m^* \)) between each country and US, differential in GDP growth rates (\( \Delta y^{dm} - \Delta y^* \)), differential in inflation rates (\( \Delta p^{dm} - \Delta p^* \)),


differential in short term treasury rates \( i^{dm} - i^* \) and stock market yields differential \( smyd^{dm} - smyd^* \).

As a second step to overvaluation and undervaluation periods, we constructed \( d_i \) series from the difference between fundamentally determined rate of return and observed rate of return for each exchange rate (Equation 20).

\[
d_i = \Delta e_i - \Delta f_i \tag{20}
\]

\( \Delta e_i \) is the observed rate of return, and \( \Delta f_i \) is the fundamentally determined change in exchange rate following the sticky price exchange rate model with stock prices.

The overvalued regime \( (R_0) \) is identified as being when the deviation \( (d_i) \) parameter is below zero; in other words, when the fundamentally determined change is greater than the observed change in the exchange rate. The undervalued regime \( (R_1) \) is identified as being when deviation \( (d_i) \) parameter is above zero; in other words when the fundamentally determined change is smaller than the observed change in the exchange rate.

As explained above (Equations 9-12), in this particular version of the Markov process, transition probabilities are not constant, but is time varying and that is determined by logistic function (Equations 8-11) which governed by vector of \( x_{-1} \) which will be explained below.

In this study, we will utilize debt and equity Sharpe ratios of each currency as the vector \( x_{-1} \) which governs transition probabilities. In this context, the Sharpe ratios will be used as proxy variables to measure the capital flows, since the risk adjusted rate of excess return has been considered as a criterion to invest in currencies by the Foreign Exchange Trade Desks. We denote the Sharpe ratio with \( x_{-1} \). We formulate the Sharpe ratios \( (x_{-1}) \) as follow:

\[
x_{-1} = \frac{\bar{R}_{x} - R_{y}}{\sigma_{k}} \quad k= \text{Domestic debt market or equity market} \tag{21}
\]
$E[R_k]$ is the expected rate of return from investments in the domestic debt market or the equity market. $R_f$ is risk-free interest rate and $\sigma_k$ is the standard deviation of expected return of the investment strategy. Here the standard deviation of the last ten observations is used. We describe these two markets as follow:

Debt Market (DM): $E[R_{DM}] = -E\left[\frac{e_{t+1} - e_t}{e_t}\right] + \left[i_t - i_t^*\right]$ \hspace{1cm} (22)

The first expression on the right hand side of equation (22), $\{-E\left[\frac{e_{t+1} - e_t}{e_t}\right]\}$, is the expected return due to the appreciation of domestic currency and the second expression, $[i_t - i_t^*]$, is the interest rate differential between domestic and foreign nominal interest rates.

Equity Market (EM): $E[R_{EM}] = -E\left[\frac{e_{t+1} - e_t}{e_t}\right] + \left[\frac{smyd_{t+1} - smyd_t}{smyd_t}\right]$ \hspace{1cm} (23)

The first expression on the right hand side of equation (23), $\{-E\left[\frac{e_{t+1} - e_t}{e_t}\right]\}$, is the expected return due to appreciation of domestic currency and the second expression, $\left[\frac{smyd_{t+1} - smyd_t}{smyd_t}\right]$, is the stock market yield differential between domestic and foreign currencies (US Dollar).

As indicated above, the Sharpe ratios ($\chi_{\cdot\cdot}$) of equity and debt markets governs the transition between the overvalued state and undervalued state. With the expected returns defined above, respected Sharpe ratios for equity and debt markets are defined as follows.

Equity Market Sharpe Ratio: $\frac{E[R_{EM}] - R_f}{\sigma_{EM}}$ \hspace{1cm} (24)

Debt Market Sharpe Ratio: $\frac{E[R_{DM}] - R_f}{\sigma_{DM}}$ \hspace{1cm} (25)
As the relative position of the observed exchange rate to its fundamentally determined value (Equation 19) changes by the global investors' expectations which are formed by the risk adjusted return (Sharpe ratio), the relationship between observed exchange rate and economic fundamentals are subject to vary. According to this, in the overvalued regime ($R_1$) when the Sharpe ratio of debt or equity market increases and if beta (the coefficient of the $x_t$) is positive then, the likelihood of staying in the overvalued state is formulated by a logistic function and approaches 1 (one hundred percent) and decreases the probability of switching to the undervalued state, since some of these probabilities are equal to 1. If the beta is negative however, the opposite of what is described above will occur.

3. The Data and Estimation

The data set used in this paper covers quarterly observations of four bilateral nominal exchange rates (price of the US dollar in terms of each currency): the Australian dollar (AUD), the Canadian dollar (CAD), the Japanese Yen (JPY), and the British Pound (GBP). In addition to this, five macroeconomic measurements from these four countries and the US are used: money supply, income, inflation rates, short-term interest rates, and share prices. The data are acquired from the web site of International Financial Statistics of the IMF and the Bloomberg database.

The period covers the 145 end of the period observations of the post Bretton Wood period beginning with the first quarter of 1972 and ending with the fourth quarter of 2009 which also includes the global financial crisis. As a money supply measure; the seasonally adjusted M2+CDS is used for Japan, the seasonally adjusted M2 is used for the US, the seasonally adjusted gross M2 is used for Canada, and the seasonally adjusted M3 is used for Australia. Since there are some discontinuities for the UK monetary aggregate data in IFS, we acquired M4 for the UK from the Statistical Data Base of Bank of England. The GDP chain volume with 2002 reference prices is used as income measure for Australia, the GDP chain volume with 2002 prices is used as an income measure for Canada, nominal GDP is used as an income measure for Japan, GDP chain volumes with 2000 prices used are used as income measures for the UK and the US. 15 Year Government Bond Yield is used as the long-term
interest rate for all the currencies. The price level in each economy is measured by CPI. The inflation is calculated by using CPI for each quarter. Share prices are used as the bases for stock market yield for each currency. The end of quarter share prices is used for each country.

We used the Expected Maximum Likelihood algorithm developed by Diebold et al. (1994) to estimate the model. This algorithm depends on maximizing the incomplete log likelihood function by iterating the expected complete data log likelihood for $\Delta e_t$, $id_t$, and $smyd_t$ conditioned on the data observed. Using the observed data smoothed state probabilities which can be defined as the unconditional probability of being in a particular state are calculated. Then the expected complete likelihood function is found by using the transition probabilities. This step is called the expectation step. The next step is the maximization step when the expected likelihood function is maximized to find updated estimates of the parameters. These updated parameter estimates are used to find new smoothed probabilities, which in turn are used as inputs to expected likelihood function. Finally, this expected likelihood function is maximized till convergence is reached. In this paper, a VAR (1) model is utilized.

4. Model Adequacy Tests

The adequacy of the MS-VAR model utilized in this paper implicitly assumes that exchange rates of the currencies used follow the two states and the relationships between exchange rates, interest rates and prices are defined by two states instead of a single state. We used two tests in this context, one Wald test as in Frommel et al. (2005) and De Grauwe (2001). Regime switching properties is also tested by Likelihood ratio test.

4.1. Wald Test

Table 1-3 displays the results of the Wald Test. According to this, for the exchange rate equation (Table 1), the states for all the currencies differ mainly due to the difference in the coefficient of lag of interest rate differential and volatility except for Canadian dollar. For the interest rate differential equation (Table 2), only the coefficient of volatility for Australia is different. There is no other
coefficient that is statistically significant. For the stock market yield differential equation (Table 3), all the coefficients are statistically insignificant.

Table 1: Wald Test Results for Exchange Rate Equation

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1013</td>
<td>0.0011</td>
<td>0.0000</td>
<td>-0.0012</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.1599</td>
<td>0.0289</td>
<td>1.8708</td>
<td>0.3223</td>
</tr>
<tr>
<td>$id_{t-1}$</td>
<td><strong>4.7964</strong></td>
<td>0.646</td>
<td><strong>10.9077</strong></td>
<td><strong>5.4278</strong></td>
</tr>
<tr>
<td>$smyd_{t-1}$</td>
<td>0.5752</td>
<td>0.0203</td>
<td>0.0157</td>
<td>0.0008</td>
</tr>
<tr>
<td>Volatility</td>
<td><strong>6.7312</strong></td>
<td>0.4296</td>
<td><strong>5.6573</strong></td>
<td><strong>4.6773</strong></td>
</tr>
</tbody>
</table>

Note: Bold numbers indicate that a variable is significant at 5% significance level according to the $\chi^2$-distribution.

Table 2: Wald Test Results for Interest Rate Differential Equation

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0029</td>
<td>0.0011</td>
<td>0.0153</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.4472</td>
<td>0.0008</td>
<td>0.0038</td>
<td>0.6638</td>
</tr>
<tr>
<td>$id_{t-1}$</td>
<td>1.2624</td>
<td>0.0466</td>
<td>0.6621</td>
<td>0.7272</td>
</tr>
<tr>
<td>$smyd_{t-1}$</td>
<td>0.0086</td>
<td>0.0036</td>
<td>0.0643</td>
<td>0.0336</td>
</tr>
<tr>
<td>Volatility</td>
<td><strong>4.4520</strong></td>
<td>0.0479</td>
<td>0.1017</td>
<td>0.2406</td>
</tr>
</tbody>
</table>

Note: Bold numbers indicate that a variable is significant at 5% significance level according to the $\chi^2$-distribution.

Table 3: Wald Test Results for Stock Market Yield Rate Differential Equation

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0000</td>
<td>0.0962</td>
<td>0.0000</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.3665</td>
<td>0.0013</td>
<td>0.0115</td>
<td>0.0220</td>
</tr>
<tr>
<td>$id_{t-1}$</td>
<td>0.0028</td>
<td>0.4727</td>
<td>0.0004</td>
<td>0.0432</td>
</tr>
<tr>
<td>$smyd_{t-1}$</td>
<td>0.0029</td>
<td>0.0418</td>
<td>0.0809</td>
<td>0.0664</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.1063</td>
<td>0.1997</td>
<td>0.0152</td>
<td>0.3685</td>
</tr>
</tbody>
</table>

Note: Bold numbers indicate that a variable is significant at 5% significance level according to the $\chi^2$-distribution.
4.2. Likelihood Ratio Test

Second adequacy test conducted to see whether VAR relationship between these three variables are better described by nonlinear Markov model is Likelihood Ratio test: \( LR = -2 \times (L_{\text{Null}} - L_{\text{Alternative}}) \).

Table 4 shows results of this test. According to Table 4, except Canadian dollar all other currencies show regime switching properties between the states.

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Test</td>
<td>69.7291</td>
<td>-3.9907</td>
<td>86.5983</td>
<td>110.8363</td>
</tr>
</tbody>
</table>

Notes: Bold numbers indicate that a variable is significant at 1% significance level to the \( \chi^2 \)-distribution.

5. Results

Impulse response functions (IRF) of a VAR model will be very useful tool to gain a deeper understanding of the relationship between variables. Yet drawing such relationship in MS-VAR model is rather tricky, since there is more than one state and in each state impulse and response of the variables are different from each other. In order to develop an impulse response model for MS-VAR, one needs to make some assumptions about the sequence of the states. In this study, there are 4 possible IRF for each currency, but due to difficulty in interpretation we will present only first two, namely IRF with assumption that states continue without switching:

1. IRF for State 0, assuming that State 0 is followed by State 0,
2. IRF for State 1, assuming that State 0 is followed by State 1,
3. IRF with the assumption of sequence of the states are as in the last n observations
4. IRF with the assumption that sequence of states is random with an average probability of smoothed state probability.
As explained above states are defined depending on the position of change in exchange rate of currency to its fundamentally determined change. According to this if change of an exchange rate is greater than its fundamentally determined change, it is assumed to be in overvalued state.

Figures 1-4 display IRF of each equation of each currency in overvalued and undervalued states. Figure 1 displays IRF for UK pound. It is noteworthy that while for a one unit shock in stock market returns have similar impacts on exchange rate and interest rate, a one unit shock to interest rates has opposite effect on exchange rate between the states. Similarly, shock to exchange rate has different impact on stock returns between the states.

Figure 2 shows IRF for AUD. For AUD, response of exchange rate to a stock market yield shock is opposite between the states and response of exchange rate to interest rate shock is just opposite of that of UK pound. It is also interesting to see that stock market returns show different response to interest rate shock across the states. Again, stock market returns display opposite response to exchange rate shock.

Figure 3 depicts IRF for CAD. For CAD responses to stock market return are very similar across the states, response of exchange rate and stock returns and exchange rate are very different across the states and again responses of stock market yield and interest rate to exchange rate shock are very similar.

IRF’s of JPY are shown in Figure 4. Similar to the other currencies, the difference across the states for JPY in terms of responses to the shocks is most distinguishable for interest rate shock. In this regard, JPY shows similar results to that of AUD and opposite of GBP in State 1.

Although we prefer not to report coefficients of VAR systems, we will still interpret coefficients of transition equation in order to see the effects of excess returns in the stock and the debt market on deviation of currencies from their fundamentally determined changes. Table 5 shows estimated coefficients of time varying transition probabilities of debt market (DM) and equity market (EM). According to this, debt market returns for AUD increases the probability of switching between
the states. Yet returns in equity market have different impact on transition between the states, as such during overvalued state higher debt market returns decrease the probability of staying in the undervalued state and opposite in equity market.

For CAD dollar, only statistically significant effect is equity market return in undervalued state which increases the probability of transition. For JPY, these variables are only statistically significant for overvalued state which increases the probability of staying in the same regime. For GBP except equity returns in overvalued state all cases are statistically significant with positive sign indicating increase in the probability of staying in the same regime. Moreover, all the coefficients are highly statistically significant with negative sign except equity market in overvalued state.

Table 5: Estimated Coefficients of Time Varying Transition Probabilities of Debt Market (DM) and Equity Market (EM)

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>t-stat</th>
<th>Canada</th>
<th>t-stat</th>
<th>Japan</th>
<th>t-stat</th>
<th>UK</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B₀</strong></td>
<td>DM</td>
<td>-3.562</td>
<td>0.0633</td>
<td>0.0668</td>
<td>1.3601</td>
<td>6.8676</td>
<td>0.9755</td>
<td>3.8698</td>
</tr>
<tr>
<td><strong>B₀</strong></td>
<td>EM</td>
<td>0.297</td>
<td>5.8662</td>
<td>-0.0780</td>
<td>-0.0556</td>
<td>0.6580</td>
<td>1.5411</td>
<td>-0.0621</td>
</tr>
<tr>
<td><strong>B₁</strong></td>
<td>DM</td>
<td>-1.731</td>
<td>-15.6558</td>
<td>-0.5520</td>
<td>-1.1318</td>
<td>0.1068</td>
<td>0.1354</td>
<td>1.9344</td>
</tr>
<tr>
<td><strong>B₁</strong></td>
<td>EM</td>
<td>-2.217</td>
<td>-29.5079</td>
<td>0.3660</td>
<td>79.2403</td>
<td>0.5049</td>
<td>0.5914</td>
<td>0.0933</td>
</tr>
</tbody>
</table>

**Note:** Bold numbers (between 1.660 and 1.984) indicate that variable is significant at 5% significance level according to t-test result. Numbers with bold t-values lower than 1.66 has 10% level of significance. The rest of them have 99% confidence interval. Betas ($β$) are the coefficients of governing variables ($x_{t-1}=$Sharpe ratio ($SR_{t-1}=E[R_{t-1}]-R_{p})/\sigma$) of time varying transition probability which is formulated as a logistic function ($p_{t}^{0}=\frac{exp[x_{t}\beta]}{1+exp[x_{t}\beta]}$). According to this, if the beta is positive, higher SR increases the likelihood of staying in the same regime and decreases the probability of transition to the other regime ($p_{t}^{0}=1-p_{t}^{1}$).

Additionally, Figure 5 in the appendix shows the smoothed state transition probability of the AUD, the CAD, and the JPY and the GBP staying in State 1 respectively.
6. Conclusion

In this paper, an analysis of the relationship between exchange rates, interest rates and stock market yield differentials in the Markovian VAR framework where the states are identified as overvalued exchange rates and undervalued exchange rates with respect to the fundamentally determined value of exchange rates. In this model, the transition dynamics between the states are governed by Sharpe ratios of equity and debt markets of each currency. This structure permits not only an analysis of the relationship of these important financial series under different exchange rate conditions (overvaluation and undervaluation) but it also provides a deeper understanding of exchange rate dynamics between undervaluation and overvaluation depending on risk-adjusted returns of debt and equity markets in each currency.

Regarding the impulse-response functions, shock to stock market yield, interest rate and exchange rate equations has different impacts on different variables in debt and equity markets between the states across the countries although there are a few exceptions in which similar effects are observed.

The results of this study show that the relationship between exchange rates, interest rate differentials and stock market yield differentials and their lags as specified in the VAR model are subject to change as each currency moves above or below their fundamentally determined value. We also find evidence that the risk-adjusted returns in each currency influence the transition dynamics of them. That can be attributed to global capital inflows and outflows among other possible sources.
Appendix: Figures

**Figure 1:** Impulse Responses Shock to Stock Market Yield, Interest Rate and Exchange Rate Equations for the UK pound
Figure 2: Impulse Responses Shock to Stock Market Yield, Interest Rate and Exchange Rate Equations for Australia
Figure 3: Impulse Responses Shock to Stock Market Yield, Interest Rate and Exchange Rate Equations for Canada
Figure 4: Impulse Responses Shock to Stock Market Yield, Interest Rate and Exchange Rate Equations for Japan
Figure 5: Smoothed State Transition Probability of Being in State 1

AUD


CAD


JPY


GBP

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