

## Estimating Value-at-Risk for the Turkish Stock Index Futures in the Presence of Long Memory Volatility

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### Abstract

This paper examines the long memory properties for closing prices of the Turkish stock index futures market using the FIGARCH(1, $d$ ,1) model with three different distributions: Normal, Student- $t$ , and skewed Student- $t$ . The value-at-risk (VaR) values are calculated using the estimated models. The results indicate strong evidence of long memory in volatility. The evidence of long memory in volatility shows that uncertainty or risk is an important determinant of the behavior of daily futures prices in the Turkish futures market. The empirical results further indicate that based on the Kupiec LR failure rate test the FIGARCH(1, $d$ ,1) models with skewed Student- $t$  distribution perform better than those of generated by normal distribution.

*JEL Classification:* C53, G15.

*Keywords:* Value-at-Risk; FIGARCH; Long memory.

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## 1. Introduction

Value-at-risk (VaR) has become one of the most popular risk measures for quantifying and controlling the market risk of a portfolio by institutions including banks, portfolio managers and regulators in recent years (see Jorion, 1996, 2000).<sup>1</sup> The market crash in October 1987, recent financial crises in emerging markets, and trading losses of well known financial institutions have led regulators and supervisory committees to favor using reliable quantitative techniques to appraise possible losses. Hence, modeling VaR has become an appealing research area. In the last few years, great efforts have been put to develop the best model for VaR computation.<sup>2</sup> The results of recent empirical papers have shown that the estimated VaR can be sensitive to the assumed model (see Huang and Lin, 2004; Tang and Shieh, 2006; Wu and Shieh, 2007). This is an important problem because of the increasing demand on relying VaR for risk management decisions by the market agents and regulators.

Several methods have been developed for measuring VaR. The RiskMetrics model developed by the risk management group at J.P. Morgan in 1994 has become a benchmark for measuring market risk. This model assumes that asset returns follow a conditional normal distribution with zero mean and variance is an exponentially weighted moving average of historical squared returns. The main drawback of this model is that a return distribution generally has a fatter tail than a normal distribution. A normally distributed return series may produce significant bias in VaR estimation that mainly concerns the tail properties of the return distribution. The other drawback with this model is that recent empirical studies have been showing that many financial return series exhibit long memory on market volatility (see Ding *et al.*, 1993). The presence of long memory in return and volatility implies that there exist dependencies between distant observations. Hence, the market does not immediately respond to information flowing into the financial markets, but reacts to it gradually over time. Long memory in returns and volatility are also found to have significant effect on the pricing of financial derivatives as well as forecasting market volatility.

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<sup>1</sup> The VaR tries to answer the following question: What is the predicted financial loss over a given time period, with a given level of confidence? The VaR of a portfolio is the maximum loss it may suffer in the course of a certain holding period, which is usually one day or ten days. Hence, the VaR of an investor's portfolio is the maximum amount of money that can be lost in the short-term.

<sup>2</sup> Beder (1995), Hendricks (1996), and Marshall and Siegel (1997) discuss the importance of the underlying models for estimating VaR.

Time varying nature of the volatility of returns has been extensively modeled by the GARCH and its many extensions with high frequency data.<sup>3</sup> However, when financial return series exhibit long memory behavior in volatility, the GARCH models, which only capture the short-run dependencies, could have poor performance (see Baillie *et al.*, 1996 and 2000). Several models were proposed to incorporate the long memory property of volatility in financial time series in recent years. To allow for fractional integrated processes of the conditional variance and therefore, provide a useful model for series in which the conditional variance is persistent, Baillie *et al.* (1996) proposed the fractionally integrated generalized autoregressive conditional heteroscedasticity (FIGARCH) model by generalizing the IGARCH model to allow for persistence in the conditional variance. It is worthwhile to investigate whether long memory property of volatility in financial time series can affect the measurement of market risk.

Hence, the objective of this paper is to determine the best method for VaR competition by evaluating the performances of different VaR models. Using data from an emerging market, namely the Turkish futures market, we will provide new evidence showing how VaR is affected by model misspecification when variance follows a long memory process. It is worthwhile to investigate this issue since there exists only a few published papers that take long memory property of volatility in the estimation of VaR of market indices (see for example Wu and Shieh, 2007; Tang and Shieh, 2006). Moreover, to the authors' best knowledge, no such study has been done on the Turkish futures market. In contrast to the mature markets, investors in emerging markets may tend to react slowly and gradually to new information. Modeling the long memory in volatility has become an integral part of risk measurement and investment analysis in these markets. In this study, we use the FIGARCH (1, $d$ ,1) model to examine the long-run dependence in the Turkish stock index futures price series. We consider the models with innovations following three different distributions: the normal, the Student- $t$ , and the skewed Student- $t$ .

The rest of the paper is organized as follows: Section 2 provides brief information on the Turkish futures market. The methodology is presented in Section

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<sup>3</sup> Since the introduction of autoregressive conditional heteroscedasticity (ARCH) model by Engle (1982), many extensions of this model have been produced. Some of them are the generalized ARCH (GARCH) and exponential GARCH (EGARCH) models. One of the special cases of the GARCH model is the Integrated GARCH (IGARCH) model introduced by Engle and Bollerslev (1986) to capture infinite persistence.

3. Section 4 gives information about the data and reports the empirical results. Section 5 is devoted to conclusions.

## 2. The Turkish Futures Market

The Turkish Derivatives Exchange (TurkDEX) was established in 2003 and formal trading in futures contracts started on 4 February, 2005. The TurkDEX is the only entity authorized by the Capital Markets Board (CMB) to launch a derivatives exchange in Turkey and according to the CMB regulations, membership to the TurkDEX is restricted to financial intermediaries. It currently has 84 members (66 brokerage firms and 18 banks) and all members are direct clearing members. Clearing is handled by the Istanbul Stock Exchange (ISE) Settlement and the Custody Bank Inc. (Takasbank). There is considerable interest in the potential success of this new market because of its role in price discovery and risk management prospects for the Turkish capital markets.

The TurkDEX has a fully electronic exchange system with remote access. The trading session starts from 09:30 to 17:10 without break. The contracts which are listed in the TurkDEX include index futures (ISE-30 and ISE-100), currency futures (USD/TRY and EUR/TRY), interest rate futures (for 91-day T-bill, 365-day T-bill and T-benchmark), commodity futures (cotton and wheat), and precious metal futures (gold). Currently, about 87 percent of the total value of the TurkDEX is on the index futures. Trading in the futures market has grown remarkably in past three years. Total trading volume increased from 3 billion TRY in 2005 to 280 billion TRY in May, 2008.<sup>4</sup>

## 3. Methodology

The GARCH model proposed by Bollerslev (1986) requires joint estimation of the conditional mean and variance equations. It is assumed that the disturbance term,  $\varepsilon_t$ , of the conditional mean is normally distributed with zero mean and time-varying variance,  $\sigma_t^2$ . The GARCH( $p, q$ ) model is specified as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \quad (1)$$

All the roots of  $\alpha(L)$  and  $[1 - \alpha(L) - \beta(L)]$  are constrained to lie outside the unit circle to ensure the stationarity of the process. When the polynomial

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<sup>4</sup> In US dollar figures, the total trading volume in May, 2008 was about USD 225 billion.

$[1 - \alpha(L) - \beta(L)]$  contains a unit root, the GARCH process is said to be integrated in variance (IGARCH) process (Engle and Bollerslev, 1986) and expressed as follows:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t \quad (2)$$

Where  $\phi(L) \equiv [1 - \alpha(L) - \beta(L)](1-L)^{-1}$  and  $v_t = \varepsilon_t^2 - \sigma_t^2$ .

To capture long-memory observed in the volatility of financial return series, Baillie *et al.* (1996) modified the IGARCH model by replacing the first difference operator  $(1-L)$  in Eq. (2) by the fractional differencing operator  $(1-L)^d$  with  $0 < d < 1$  and proposed the fractional integrated GARCH model (FIGARCH). The fractionally integrated extension of the GARCH model expands the variance equation by considering fractional differences. The FIGARCH( $p, d, q$ ) model is given by:

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]v_t \quad (3)$$

Where  $0 < d < 1$ , such that the model reduces to a GARCH model for  $d = 0$  and to an IGARCH model for  $d = 1$ . The impact of a shock on the conditional variance of FIGARCH( $p, d, q$ ) processes decrease at a hyperbolic rate when  $0 < d < 1$ . Thus, the long memory dynamics in volatility is taken into account by the fractional integration parameter  $d$ .

### **Value at Risk**

VaR is used to ensure that the financial institutions can still be in business during severe adverse market fluctuations. By computing VaR, the manager of a financial institution could have some idea on the minimum amount that is expected to lose with a probability  $\alpha$  over a given time horizon. Hence, VaR at level  $\alpha$  means that in a given time the potential maximum loss for a portfolio will not exceed VaR with a  $(1 - \alpha)$  confidence level. For instance, a  $\alpha = 1\%$  one-day VaR of USD 1 million indicates that 99 out of 100 days, we could expect to realize a loss of at most USD 1 million.

In this study, following Wu and Shieh, 2007 and Tang and Shieh, 2006, the values of VaR are calculated using the FIGARCH(1, $d$ ,1) model under three distributions including the normal, Student- $t$ , and skewed Student- $t$ . The one-step-ahead forecast of the conditional mean and conditional variance is computed at time  $t-1$ . It is also assumed that investors have both long and short trading position. The VaRs of  $\alpha$  quantile for long and short trading position are estimated as:

Under normal distribution,

$$VaR_{long} = \hat{\mu}_t - z_{\alpha} \hat{\sigma}_t \quad (4)$$

$$VaR_{short} = \hat{\mu}_t + z_{\alpha} \hat{\sigma}_t \quad (5)$$

where  $\hat{\mu}_t$  and  $\hat{\sigma}_t$  are conditional mean and conditional variance, respectively.  $z_{\alpha}$  is the left or right quantile at  $\alpha\%$  for the normal distribution.

Under Student- $t$  distribution,

$$VaR_{long} = \hat{\mu}_t - st_{\alpha,v} \hat{\sigma}_t \quad (6)$$

$$VaR_{short} = \hat{\mu}_t + st_{\alpha,v} \hat{\sigma}_t \quad (7)$$

where  $st_{\alpha,v}$  is the left or right quantile at  $\alpha\%$  for the Student- $t$  distribution.

Under skewed Student- $t$  distribution,

$$VaR_{long} = \hat{\mu}_t - skst_{\alpha,v,\gamma} \hat{\sigma}_t \quad (8)$$

$$VaR_{short} = \hat{\mu}_t + skst_{\alpha,v,\gamma} \hat{\sigma}_t \quad (9)$$

where  $skst_{\alpha,v,\gamma}$  is the left or right quantile at  $\alpha\%$  for the skewed Student- $t$  distribution with  $v$  degrees of freedom and asymmetry coefficient  $\gamma$ . If  $\gamma < 1$ , the VaR value for long trading position will be bigger than that of short trading position, and vice versa.

#### **Accuracy for VaR estimates**

To test the accuracy of the estimated VaR values, a likelihood-ratio test proposed by Kupiec (1995) is used. The test can be employed to test whether the sample point estimate is statistically consistent with the VaR model's prescribed confidence level. Thus, testing the accuracy of the model is equivalent to testing the null hypothesis that the probability of failure on each tail ( $f$ ) equals the model's specified probability ( $\alpha$ ).<sup>5</sup> To judge the performance of VaR model, we measure the difference between the pre-specified VaR level and the failure rate. If the difference is close to zero then we conclude that the VaR model is specified very well. The likelihood-ratio test statistic is expressed as follows:

$$LR = -2 \ln \left[ (1 - \alpha)^{N-x} (\alpha)^x \right] + 2 \ln \left[ (1 - \hat{f})^{N-x} (\hat{f})^x \right] \sim \chi_1^2 \quad (10)$$

<sup>5</sup> The failure rate is commonly used in testing the effectiveness of VaR models. The failure rate is that the proportion of the number of times the observations exceed the forecasted VaR to the number of all observations (see Tang and Shieh, 2006).

where  $\hat{f} = \frac{x}{N}$ .

$x$  is the number of observations exceeding (in absolute value) the forecasted VaR in the sample and  $N$  is the sample size. Under the null hypothesis the  $LR$  test statistic is distributed as chi-squared with 1 degree of freedom.

#### 4. Data and Empirical Results

##### *Data*

Trading of index futures in Turkey began on February 4, 2005. In this study, the futures on ISE-30 is studied.<sup>6</sup> The ISE-30 index futures contract uses the ISE-30 index, which composed of 30 most liquid Turkish stocks that are traded in the continuous market as the underlying index. Daily closing futures prices index is used for the period beginning on 4 February 2005 and ending on 4 April 2008. The sample data were obtained from the Turkish Derivatives Exchange. The basic statistical characteristics of the return series are summarized in Table 1.<sup>7</sup> The distribution of returns over the sample period is negatively skewed and is characterized by statistically significant kurtosis, suggesting that the underlying series are leptokurtic, that is, the series have a fatter tail and higher peak as compared with a normal distribution. Likewise, the Jarque-Bera ( $JB$ ) test results indicate that the ISE-30 futures returns are not normally distributed. Table 1 also reports the Ljung-Box statistics. They are estimated both for the returns and squared returns. From the test statistics, we can reject the null of white noise and assert that the return series are autocorrelated, suggesting the existence of volatility clustering of an ARCH process in the series.

The plots of the daily closing futures prices index and the respective return series are also presented in Figure 1a and 1b.

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<sup>6</sup> The ISE-30 index futures contracts are the most traded futures contracts on The Turkish Derivatives Exchange (TurkDEX).

<sup>7</sup> Daily returns are defined as  $r_t = \ln(p_t) - \ln(p_{t-1})$ .

**Table 1**  
**Summary Statistics for the ISE-30 Index Futures Returns**

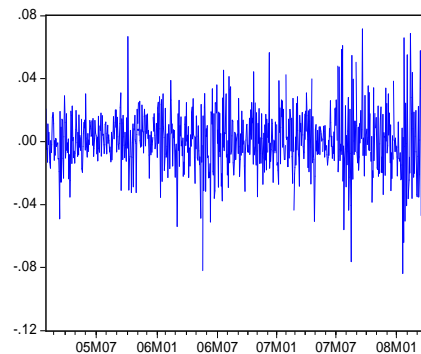
	ISE-30
No. of observation	802
Mean	0.048
Standard deviation	1.914
Skewness	-0.097
Kurtosis	5.153
Minimum	-8.382
Maximum	7.149
J-B	156.213*
$Q(10)$	11.373**
$Q(20)$	31.733*
$Q(40)$	54.356**
$Q_s(10)$	93.425*
$Q_s(20)$	161.300*
$Q_s(40)$	224.841*

Notes: J-B denotes Jarque-Bera normality test statistic. \* and \*\* denote significance levels at 1% and 5%, respectively.  $Q(\cdot)$  and  $Q_s(\cdot)$  are the Ljung-Box statistic for returns and squared returns up to 10, 20, and 40 lags, respectively.

**Figure 1a. Daily Price Series for Futures Index**



**Figure 1b. Daily Return Series for Futures Index**



### **Empirical Results**

Before examining the volatility and estimating VaR, we test for stochastic trends in the autoregressive representation of the return series using unit root tests. The ADF and KPSS tests are used to check whether or not the series is stationary,  $I(0)$ . Table 2 reports the results of both tests. The results indicate that the ADF statistic significantly rejects the null hypothesis of unit roots for daily return series. As for the KPSS test, the test statistics indicate that return series is insignificant to reject



the null hypothesis of stationarity, suggesting that they are stationary processes  $I(0)$ .<sup>8</sup> Hence, the futures daily returns are stationary and suitable for our empirical analysis.

**Table 2**  
**Unit Root Tests**

		ISE30 futures
ADF	$\eta_{\mu}$	-29.362(0)*
	$\eta_{\tau}$	-29.369(0)*
KPSS	$\eta_{\mu}$	0.144(1)
	$\eta_{\tau}$	0.067(0)

Note:  $\eta_{\tau}$  and  $\eta_{\mu}$  refer to the test statistics with and without trend, respectively.

\* denotes significance level at 1%.

The GARCH(1,1) and FIGARCH(1, $d$ ,1) models are estimated under normal, Student- $t$ , and skewed Student- $t$  distributions. The best fitting specifications are reported in Table 3.<sup>9</sup> As seen in the Table 3, the estimated GARCH parameters  $\alpha_1$  and  $\beta_1$  are all positive and statistically significant under three distributions. Moreover, the sum of the estimates of  $\alpha_1$  and  $\beta_1$  is very close to one, indicating that the volatility process is highly persistent. The results also indicate that the index futures daily returns exhibit significant fat tails as the estimated degrees of freedom parameter  $\nu$  is statistically significant under the skewed Student- $t$  distribution. As for the FIGARCH model, the estimated degree of integration  $d$ 's are found to be significantly different from zero, indicating that the volatility exhibits a long memory process in the Turkish futures market. That is, the volatility of index futures daily returns can be characterized by slowly mean-reverting fractionally integrated process. Hence, the result shows the importance of modeling long memory in volatility and suggests that future volatility depends on its past realizations and therefore, is predictable. As in the GARCH model, the skewed Student- $t$  distribution performs better than the normal distribution since the tail parameter  $\nu$  and asymmetric parameter  $\ln(\gamma)$  are statistically significant at 1%

<sup>8</sup> The KPSS is more powerful than Augmented Dickey-Fuller (ADF) test. The null hypothesis of the ADF test is that a time series contains a unit root, while the KPSS test has the null hypothesis of stationarity. Since the null hypothesis in ADF test is that a time series contains a unit root, this hypothesis is accepted unless there is strong evidence against it. However, this approach may have low power against stationary near unit root processes.

<sup>9</sup> The models with different orders are estimated for both GARCH and FIGARCH under three different distributions. The model selection is based on Akaike's information criterion (AIC) and Ljung-Box  $Q$ -statistics. The model which has the lowest AIC and passes  $Q$ -test simultaneously is used.

level. Table 3 also provides some diagnostics such as the  $Q$  statistics up to 10 lags for the squared standard residuals for the sampled return series. The  $Q$  statistics fails to reject the null hypothesis of independently and identically distributed squared standardized residuals. The ARCH LM test statistics for residuals also supports that there is no ARCH effect in the residuals.

**Table 3**  
**Estimation Results of ARMA-(FI)GARCH Models for the ISE-30 Index Futures Returns**

	GARCH(1,1)			FIGARCH(1,d,1)		
	Normal	Student- $t$	Skewed Student- $t$	Normal	Student- $t$	Skewed Student- $t$
$\mu$	0.149** (0.064)	0.106*** (0.055)	0.146** (0.062)	0.142** (0.063)	0.103*** (0.055)	0.139** (0.062)
$\psi_1$	0.009 (0.038)	0.002 (0.036)	-0.680* (0.218)	-0.580* (0.211)	0.006 (0.035)	-0.672* (0.207)
$\psi_2$	0.068*** (0.037)	-	0.065 (0.041)	0.081** (0.039)	-	0.071*** (0.040)
$\theta_1$	-	-	0.706* (0.215)	0.601* (0.210)	-	0.703* (0.204)
$\theta_2$	-	-	-	-	-	-
$\omega$	0.098** (0.043)	0.090*** (0.052)	0.088*** (0.052)	0.273** (0.116)	0.272*** (0.154)	0.271*** (0.152)
$\alpha_1$	0.089* (0.021)	0.093* (0.028)	0.1025* (0.031)	0.048 (0.111)	0.046 (0.152)	0.046 (0.150)
$\beta_1$	0.889* (0.025)	0.889* (0.034)	0.884* (0.035)	0.396* (0.136)	0.383** (0.188)	0.396** (0.191)
$d$				0.398* (0.091)	0.404* (0.108)	0.429* (0.114)
$\nu$	-		5.909* (1.225)	-	-	6.214* (1.263)
$\ln(\gamma)$	-		0.101*** (0.053)	-	-	0.095*** (0.052)
$\ln(L)$	-	-	-	-1605.33	-	-1583.085
AIC	1608.303	1587.790	1584.183	4.023	1586.691	3.973
ARCH(4)	4.026	3.974	3.973	4.023	3.974	3.973
$Q^2(10)$	0.474	0.452	0.387	0.155	0.138	0.179
	6.354	6.693	6.584	5.184	5.345	5.469

Notes:  $\ln(\gamma)$  denotes asymmetry parameter.  $\nu$  is the tail parameter.

***In-sample VaR Analysis***

The in-sample VaR results computed under the three distributions: the normal, Student-*t*, and skewed Student-*t*. Since the estimated degree of integration parameters (*d*) are found to be significantly different from zero, the VaR values are calculated using only the FIGARCH models. The results of the FIGARCH(1,*d*,1) models are reported in Table 4. If comparing the three different distributions among the FIGARCH models, it is observed that the normal models have poor performance for both long and short trading positions. As  $\alpha$  ranges from 0.0025 to 0.0050 for long position, the failure rates significantly exceed the prescribed quantiles in the normal FIGARCH (1,*d*,1) model. The null hypothesis that failure rates equal to prescribed quantiles in the normal FIGARCH for long positions is rejected by the Kupiec LR test for  $\alpha$  values of 0.0025 and 0.0050, and is rejected for  $\alpha$  values of 0.9975 for short position. However, the Student-*t* FIGARCH models improve significantly on the in-sample VaR performance for long and short positions. All the Student-*t* models do not reject the null hypothesis that failure rates equal to the prescribed quantiles.

**Table 4**  
**In-Sample VaR Calculated by FIGARCH for ISE-30 Futures**

Short position				Long position			
$\alpha$	Failure rate	Kupiec	P-value	$\alpha$	Failure rate	Kupiec	P-value
Quantile				Quantile			
Gaussian distribution							
0.9500	0.9576	1.0276	0.3107	0.0500	0.0436	0.7121	0.3988
0.9750	0.9751	0.0001	0.9910	0.0250	0.0249	0.0001	0.9910
0.9900	0.9875	0.4579	0.4986	0.0100	0.0125	0.4579	0.4986
0.9950	0.9913	1.8309	0.1760	0.0050	0.0100	3.0904***	0.0788
0.9975	0.9938	3.1592***	0.0755	0.0025	0.0087	7.5450***	0.0755
Student- <i>t</i> distribution							
0.9500	0.9539	0.2587	0.6110	0.0500	0.0511	0.0211	0.8845
0.9750	0.9738	0.0455	0.8311	0.0250	0.0237	0.0574	0.8107
0.9900	0.9913	0.1369	0.7114	0.0100	0.0100	0.0001	0.9943
0.9950	0.9950	0.00003	0.9960	0.0050	0.0062	0.2277	0.6332
0.9975	0.9963	0.4291	0.5125	0.0025	0.0050	1.5402	0.2146
Skewed Student- <i>t</i> distribution							
0.9500	0.9576	1.0276	0.3107	0.0500	0.0586	1.1872	0.2759
0.9750	0.9788	0.5013	0.4789	0.0250	0.0237	0.0574	0.8107
0.9900	0.9925	0.5630	0.4531	0.0100	0.0112	0.1164	0.7330
0.9950	0.9963	0.2802	0.5966	0.0050	0.0075	0.8606	0.3536
0.9975	0.9963	0.4291	0.5125	0.0025	0.0050	1.5402	0.2146

Note: \*\*\* denotes significance at 10% level.

**Out-of-Sample VaR Analysis**

In the previous subsection, we used the best model to calculate the VaR values. By comparing the VaR values using different models, we only know the past performance of the VaR models. However, the contribution of VaR calculations is its forecasting ability that provides information to financial institutions about the biggest loss they will incur (see Tang and Shieh, 2006). Hence, it is important to evaluate the forecasting ability of the VaR models. The one-step-ahead out-of-sample VaR values are calculated conditional on the available information on the  $t^{\text{th}}$  day.

The out-of-sample VaR results also calculated under the three distributions: the normal, Student- $t$ , and skewed Student- $t$ . The results of the FIGARCH(1, $d$ ,1) models are reported in Table 5. The models are re-estimated every 200 observations in the out-of-sample period. As in the in-sample VaR calculations, these out-of-sample VaR values are calculated with observed returns and results are recorded for evaluation of Kupiec LR test. The results indicate that for the short position, the Student- $t$  and skewed Student- $t$  perform better than the normal distribution. As for the long position, the performances of three distributions are very similar.

**Table 5**  
**Out-of-Sample VaR Calculated by FIGARCH for ISE-30 Futures**

Short position				Long position			
$\alpha$	Failure rate	Kupiec	P-value	$\alpha$	Failure rate	Kupiec	P-value
Quantile				Quantile			
Gaussian distribution							
0.9500	0.9150	4.3025**	0.0381	0.0500	0.0600	0.3968	0.5287
0.9750	0.9500	3.9923**	0.0457	0.0250	0.0250	0.0000	1.0000
0.9900	0.9750	3.2086***	0.0733	0.0100	0.0150	0.4379	0.5082
0.9950	0.9850	2.6118	0.1061	0.0050	0.0100	0.7776	0.3779
0.9975	0.9850	5.7820**	0.0162	0.0025	0.0100	2.5565	0.1098
Student-t distribution							
0.9500	0.9200	3.2316***	0.0722	0.0500	0.0600	0.3968	0.5287
0.9750	0.9550	2.6628	0.1027	0.0250	0.0350	0.7312	0.3925
0.9900	0.9850	0.4379	0.5082	0.0100	0.0100	0.0000	1.0000
0.9950	0.9850	2.6118	0.1061	0.0050	0.0100	0.7776	0.3779
0.9975	0.9950	0.3876	0.5336	0.0025	0.0050	0.3876	0.5336
Skewed Student-t distribution							
0.9500	0.9150	4.3025**	0.0381	0.0500	0.0800	3.2316***	0.0722
0.9750	0.9600	1.5665	0.2107	0.0250	0.0250	0.0000	1.0000
0.9900	0.9850	0.4379	0.5082	0.0100	0.0100	0.0000	1.0000
0.9950	0.9900	0.7776	0.3779	0.0050	0.0100	0.7776	0.3779
0.9975	0.9950	0.3876	0.5336	0.0025	0.0050	0.3876	0.5336

Note: \*\* and \*\*\* denote significance at 5% and 10%, respectively.

## 5. Conclusion

This paper has examined long memory property of the Turkish futures market. For modeling the volatility, the GARCH and FIGARCH models have been employed. The estimation results provide evidence supporting the FIGARCH models, in the sense that the FIGARCH models fit the data series better than the GARCH models. The results of the FIGARCH model show that estimates of the long memory parameters are significantly different from zero, suggesting that volatility series are long memory processes in the Turkish futures market. The estimation results also indicate that the skewed Student- $t$  distribution outperforms the normal distribution. The VaR values have also been estimated using the FIGARCH(1, $d$ ,1) model with three distributions. Comparing the estimated in-sample and out-of-sample VaR values based on Kupiec LR test, the skewed Student- $t$  model performs better than the normal distribution in describing the return series in the Turkish futures market.

In summary, since long memory model outperforms the traditional short-memory model risk analyzing methods requiring variance series, such as VaR, provide more efficient results when variance series of the ISE-30 index futures returns is filtered by the long memory model, rather than short memory model. Therefore, these findings would be helpful to the financial managers, investors and regulators dealing with the Turkish futures market.

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