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Abstract

This paper estimates a normalized constant elasticity of substitution production function for Turkey using the data between 1991-2014. Employing a system approach, the elasticity of substitution, direction of the technical change and total factor productivity are determined. The results indicate that elasticity of substitution is around 0.8 and significantly below unity in Turkey. The dynamics of the technical progresses of inputs signal a slowing productivity growth in labour, and a falling productivity in capital. These findings imply that in Turkey, the average growth of the total factor productivity is very low, if not zero, and labour-augmenting technical progress is slightly dominant over time.

Keywords: CES function, elasticity of substitution, technical change, factor shares, Turkey
Jel classification: C22, E23, E25

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1 Introduction

Production function is a mathematical expression that describes the relation between inputs and output of an economy. It is a tool to analyse the medium to long term developments in real GDP and supply side factors, and thus is used to forecast potential output growth. Cobb-Douglas is a production function that has been extensively used in neoclassical growth theory. This function has been popular not only since it is easy to work with but also it was able to explain some stylized facts of long-term economic growth: capital and labour have stable shares of national income while capital to output ratio is constant, and capital intensity (capital per worker) and labour productivity grow at a sustained rate.\(^1\)

With unit elasticity of substitution between capital and labour Cobb-Douglas is a special case of the constant elasticity of substitution (CES) production function. Moreover, direction of the technical change cannot be differentiated in a Cobb-Douglas specification. However, if there is a bias in the technical progress the factor shares may not be constant or the capital to output ratio can be non-stationary (Klump et al., 2007). Therefore, if the technical progress becomes more labour or capital-augmenting, then the Cobb-Douglas is not an appropriate specification anymore. Trends in the factor shares or in the capital to output ratio calls for the CES function, which lets bias in the technical change.

Assuming that there are only two factors of production, and that the production function is homogenous of degree one, the CES is derived from the definition of elasticity of substitution. However, the efficiency and distribution parameters of the derived CES function are functions of arbitrary constants of integration. As explained in Klump et al. (2012), normalization is necessary for identifying the constants of integration in an economically meaningful way. Therefore, when normalized, the efficiency and distribution parameters have empirical correspondences. They are expressed in terms of fixed points and independent of the elasticity of substitution.

Using a normalized CES function and a system approach, Klump et al. (2004)

\(^1\)These are four of the six stylized facts about the growth of developed countries observed in the 20\(^{th}\) century highlighted in Kaldor (1961).
show that labour-augmenting technical progress is dominant compared to the capital-augmenting one, and the elasticity of substitution is smaller than unity for the US between 1953-1998. Klump et al. (2007) suggest a similar result for the Euro Area using quarterly data between 1970-2003. The results of León-Ledesma et al. (2009) imply that among various estimation equations and techniques, the normalized CES function estimated as a system together with the first order conditions of the profit maximization is the best combination as it is robust to sample size variation and alternative forms of technical progress.

According to the Word Development Indicators of the World Bank, with a GDP of around $0.8 trillion Turkey is the 18th largest economy in the world in 2014. Over the last 15 years, Turkey has been one of the fastest growing economies, on average, in Europe, and indeed the fastest one after the global financial crisis. In addition, the country’s integration with the world has been increasing as its trade to GDP ratio has doubled since 1991. Therefore, the growth in Turkey worths attention. From the supply side, growth depends on the production function, and following Klump et al. (2007) this paper aims to estimate a normalized CES production function for Turkey using the data between 1991-2014. Employing a system approach, the elasticity of substitution, direction of the technical change and total factor productivity are determined. To authors’s best knowledge, this is the first study that models Turkish real GDP as a normalized CES estimated with a supply-side system approach. Many studies that focus on the growth and productivity developments in Turkey characterizes the Turkish economy as Cobb-Douglas. Altuğ et al. (2008), Üngör (2012), and Üngör (2013b) are some good examples of such.

This paper shows that neither the factor shares nor the ratio of labour share to capital share are stationary in Turkey. This suggests that there is technical progress both in the labour and capital but their dynamics are different. Hence, the differing technical progresses in factors call for a CES production function. The findings of this paper show that the elasticity of substitution is around 0.8

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2 Source is the IMF World Economic Outlook, October 2015 database.
3 The data are from the World Bank. Trade is the sum of exports and imports measured at current prices. While the share of trade in GDP was 30 percent in 1991, it increased to 60 percent as of 2014 in Turkey.
and significantly below unity in Turkey. Level of labour-augmenting technical progress is log-linear while that of capital-augmenting one is hyperbolic. At the point of normalization the change in the technical progress of labour is found to be positive while that of capital is estimated to be negative. The dynamics of the technical progresses signal a slowing productivity growth in labour, and a falling productivity in capital. These findings imply that in Turkey the average growth of the total factor productivity (TFP) is very low, if not zero, and labour-augmenting technical progress outweighs the capital-augmenting one slightly over time. The estimation results bring out two interesting points. First, higher productivity of capital relative to labour is matched with a falling ratio of labour to capital share, and vice versa. Second, the composition of capital stock has an effect on the productivity of capital.

The rest of this paper is organized as follows: Section 2 presents the CES function for the scope this paper and briefly discusses the idea of normalization. Section 3 introduces the estimation equations, together with the model of technical progress and the approximation used to estimate TFP. Section 4 explains the data. Section 5 presents the results of the estimations while section 6 shows the robustness of the results. Finally, Section 7 concludes.

2 CES function and the idea of normalization

The construction of the CES function starts from the definition of the elasticity of substitution, $\sigma \in [0, \infty)$, between capital and labour. $\sigma$ is defined as the percentage change in the capital-labour ratio divided by the percentage change in the marginal rate of technical substitution ($MRTS$). $MRTS$ is the rate at which labour can be substituted for capital along an isoquant line. In other words, it is the marginal product of labour ($Y_L$) divided by the marginal product of capital ($Y_K$). Therefore, the elasticity of substitution shows how easy it is to shift between capital and labour (Miller, 2008). The larger the $\sigma$, the easier to substitute.\(^4\)

\(^4\)When $\sigma > 1$ ($< 1$), capital and labour are substitutes (complements).
\[
\sigma = \frac{\% \Delta K/L}{\% \Delta MRTS} = \frac{dK/L}{dY_L/Y_K} \frac{Y_L/Y_K}{K/L} = \frac{f'(k)[f(k) - kf''(k)]}{-kf''(k)f(k)}
\]  

(1)

The last term on the right hand side of equation (1) is obtained assuming that there are only two factors of production, capital and labor, and that the production function is homogenous of degree one. As expressed in León-Ledesma et al. (2009), a second order partial differential equation of the last term of (1) in \( k \) is:

\[
y_t = a[k_t^{\sigma-1} + b]^{\sigma - 1}.
\]  

(2)

Assuming the production function as \( Y_t = F(e^{g_K(t)}K_t, e^{g_L(t)}L_t) \), \( y_t = \frac{Y_t}{e^{g_K(t)}L_t} \) represents per capita output in productivity units. Likewise, \( k_t = \frac{K_t}{e^{g_L(t)}L_t} \) is the capital-labour ratio in productivity units. The terms \( e^{g_K(t)} \) and \( e^{g_L(t)} \) capture the capital and labour-augmenting technical progress, or the productivity level of capital and labour, respectively. \( a \) and \( b \) are arbitrary constants of integration.

After some simple algebra equation (2) can be expressed as:

\[
Y_t = C \left[ \pi \left( e^{g_K(t)}K_t \right)^{\sigma-1} + (1 - \pi) \left( e^{g_L(t)}L_t \right)^{\sigma-1} \right]^{\frac{\sigma}{\sigma - 1}}
\]  

(3)

where \( C = a(1+b) \frac{\sigma}{\sigma - 1} \) is called as the efficiency parameter, and \( \pi \in (0, 1) = \frac{1}{1+b} \) is the distribution parameter. Equation (3) is the factor augmenting CES production function.\(^5\) For a given \( \sigma \), if the values of \( a \) and \( b \) were known, \( C \) and \( \pi \) could have been determined. This is the point where normalization comes into play. Normalization means selection of a baseline point. It is necessary for identifying the constants of integration in an economically meaningful way and independent of the elasticity of substitution. A meaningful identification of \( a \) and \( b \) is given by the fact that elasticity is a point elasticity (Klump et al., 2012, León-Ledesma et al., 2009). This means that it is measured at a particular isoquant line, with a

\(^5\)Loosely speaking, if \( g_L(t) = 0 \) (\( > 0 \)) and \( g_K(t) > 0 \) (\( = 0 \)) over time, then the technical progress is capital-augmenting (labour-augmenting). If \( g_L(t) = g_K(t) > 0 \), than the technical progress is (Hicks-)neutral. If \( g_L(t) > 0 \neq g_K(t) > 0 \), then the technical progress is factor-augmenting.
particular distribution parameter and capital-labour values. That is to say, in a dynamic setting $\sigma$ is identified at a particular point in time, $t=t_0$. Then, equation (3) can be written as:

$$Y_t = Y_0 \left[ \pi_0 \left( e^{g_K(t)} \frac{K_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \pi_0) \left( e^{g_L(t)} \frac{L_t}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

(4)

where $\pi_0 = \frac{r_0 K_0}{r_0 K_0 + w_0 L_0}$ stands for capital share, and $1-\pi_0$ represents labour share in the total earnings of capital and labour. $r_0$ is real rental price of capital, while $w_0$ is the real wage rate of labour at the point of normalization. Equation (4) is the normalized CES function. Clearly, $a$ and $b$ matches with a combination of $Y_0$, $K_0$, $L_0$ and $\pi_0$, whose values can be approximated from the data. The efficiency and distribution parameters in equation (4) are $Y_0$ and $\pi_0$, respectively. Hence, both of the parameters have an empirical correspondence, and independent of $\sigma$. Therefore, normalization means fixing a baseline point for time, and thus for output, labour, capital and marginal rate of technical substitution. In the context of Klump and Preissler (2000), normalization helps to distinguish different CES families as they emerge with different values of $Y_0$, $K_0$, $L_0$ and $\pi_0$. In other words, members of a CES family have common benchmark values of production, capital, labour and the factor shares, but they differ in their elasticity of substitution. Thus, in a normalized CES once the fixed values are determined, a comparative-static analysis in elasticity of substitution is possible.

How are the fixed values determined? As explained by Klump et al. (2004), choosing a benchmark value can be considered as defining an appropriate value for a variable. An appropriate value can be detected from the data and should include

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6 Imperfect competition represents many markets better than perfect competition in reality. This paper assumes imperfect competition in the product market. This assumption is indeed consistent with Rodrik (1988) who argues that imperfect competition is in fact more pervasive in the industrial sectors of the developing countries than of the developed ones. Note that since imperfect competition is assumed in this paper, $\pi_0$ is not equal to share of capital in national income. The derivation of equation (4) can be found in Appendix 1.

7 In a non-normalized CES function (i.e. equation (3)), however, the distribution and efficiency parameters have no clear theoretic or empirical meaning. Instead, they are parameters that depend on the elasticity of substitution. Therefore, a non-normalized CES function is not suitable for comparative-static exercises in elasticity of substitution (Klump et al., 2012).
as much information as possible. Therefore, benchmark points can be calculated as sample averages. Specifically, \( K_0 = \bar{K} \), \( L_0 = \bar{L} \) and \( Y_0 = A \bar{Y} \) are calculated as geometric averages, while \( \pi_0 = \bar{\pi} \) and \( t_0 = \bar{t} \) are measured as simple averages. \( A \) is called the scaling parameter. Due to the non-normality of the CES, sample averages of the inputs, simple or geometric, need not exactly coincide with the sample average of the output. The possible emergence of this fact is captured by the scaling parameter \( A \) (Klump et al., 2007). However, since the difference between \( \bar{Y} \) measured as the sample average of \( Y_t \) and \( \bar{Y} \) calculated from the sample averages of the inputs is not expected to be too large, \( A \) is considered to be around 1. Hence, the final specification of the normalized CES function used in this paper is:

\[
Y_t = A \bar{Y} \left[ \frac{\bar{\pi}}{\pi} \left( e^{g_K(t) \frac{K_t}{K}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi}) \left( e^{g_L(t) \frac{L_t}{L}} \right)^{\frac{\sigma-1}{\sigma}} \right]. \tag{5}
\]

### 3 Normalized supply-side system estimation

Assuming imperfect competition, the profit maximization of firms yields equation (6) and (7). Additionally, equation (5) can be rearranged as equation (8). Hence, the three-equation supply-side system is arrived:

\[
\ln \left( \frac{W_t L_t}{P_t Y_t} \right) = \ln \left( \frac{1-\bar{\pi}}{1+\mu} \right) + \frac{1-\sigma}{\sigma} \left[ \ln \left( \frac{Y_t/\bar{Y}}{L_t/\bar{L}} \right) - \ln(A) - g_L(t) \right] \tag{6}
\]

\[
\ln \left( \frac{R_t K_t}{P_t Y_t} \right) = \ln \left( \frac{\bar{\pi}}{1+\mu} \right) + \frac{1-\sigma}{\sigma} \left[ \ln \left( \frac{Y_t/\bar{Y}}{K_t/\bar{K}} \right) - \ln(A) - g_K(t) \right] \tag{7}
\]

\[
\ln \left( \frac{Y_t}{L_t} \right) = \ln \left( \frac{A \bar{Y}}{\bar{L}} \right) + g_L(t) - \frac{\sigma}{1-\sigma} \ln \left[ \frac{1}{\bar{\pi}} e^{1-\sigma (g_L(t) - g_K(t))} \left( \frac{K_t/\bar{K}}{L_t/\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi}) \right] \tag{8}
\]
where $\mu$ shows the mark-up, determined by the price elasticity of demand.\footnote{8} However, since the aggregate demand function is unknown, $\mu$ is proxied as $\frac{P_t Y_t}{W_t L_t + R_t K_t} - 1$. $W_t$ and $R_t$ are nominal wages and rental price, respectively. Clearly, $\frac{W_t L_t}{P_t Y_t}$ represents the share of labour while $\frac{R_t K_t}{P_t Y_t}$ stands for the share of "non-profit" capital in national income, since due to imperfect competition $P_t Y_t = W_t L_t + R_t K_t + \text{Profit}_t$. Equation (6), (7), and (8) are estimated by the method of non-linear seemingly unrelated regression. $g_L(t)$ and $g_K(t)$ are replaced by equation (9).

3.1 Technical progress

Taking into account the fact that technical progress can be non-neutral and technical growth can change over time, following Klump et al. (2004) and Klump et al. (2007), the technical progresses of both inputs are modelled as a Box-Cox transformation, which is a flexible function introduced by Box and Cox (1964):\footnote{9}

$$g_i(t) = \begin{cases} \gamma_i t_0 \left( \frac{t}{t_0} \right)^{\lambda_i} - 1 & \text{for } \lambda_i \neq 0 \\ \gamma_i t_0 \ln \left( \frac{t}{t_0} \right) & \text{for } \lambda_i = 0 \end{cases}$$

where $i=K$ and $L$. Equation (9) is obtained under the assumption that at $t=t_0$, change in the technical progress of input $i$ over time should be constant $\gamma_i$. Equation (9) shows that the level of technical progress is a function of time, the point of normalization and the curvature parameter $\lambda_i$. When $\lambda_i=1$ ($=0$ $[<0]$, the technical progress has linear (log-linear) [hyperbolic] dynamics. $e^{g_i(t)}$ is considered as the productivity level of factor $i$. Therefore, the productivity growth is:

$$\frac{dg_i(t)}{dt} = \begin{cases} \gamma_i \left( \frac{t}{t_0} \right)^{\lambda_i-1} & \text{for } \lambda_i \neq 0 \\ \gamma_i \frac{t_0}{t} & \text{for } \lambda_i = 0 \end{cases}$$

\footnote{8}The derivations of equation (6), (7) and (8) can be found in Appendix 2. \footnote{9}For example, using a CES function, Raurich et al. (2011) assume that technical progress is labour-augmenting and time-varying.
Note that when $\lambda_i=1$, productivity growth is constant. Hence, as put by Klump et al. (2007) Box-Cox modelling of technical progress allows the data rather than the researcher to decide on the dynamics of technical progress.

### 3.2 Total factor productivity

Using the production function (5), what can be said about the total factor productivity? Clearly, to compute the TFP one needs to separate it out from the rest of the production function, and this is done via the Kmenta approximation. Similar to Kmenta (1967) and Klump et al. (2004), applying a second order Taylor series expansion to equation (5) for $\sigma$ around $\sigma=1$ yields:

$$
\ln \left( \frac{Y_t}{L_t} \right) \approx \pi g_K(t) + (1 - \pi)g_L(t) - \left( \frac{1 - \sigma}{\sigma} \right) \frac{\pi(1 - \pi)}{2} (g_L(t) - g_K(t))^2 + \\
\ln \left( \frac{\Delta Y}{L} \right) + \pi \ln \left( \frac{K_t/K}{L_t/L} \right) - \\
\left( \frac{1 - \sigma}{\sigma} \right) \frac{\pi(1 - \pi)}{2} \left( \ln \left( \frac{K_t/K}{L_t/L} \right) \right)^2.
$$

Therefore, consistent with the earlier notations of productivity, the TFP is:

$$
\text{TFP}_t = e^{\pi g_K(t) + (1 - \pi)g_L(t) - \left( \frac{1 - \sigma}{\sigma} \right) \frac{\pi(1 - \pi)}{2} (g_L(t) - g_K(t))^2}.
$$

Hence, the growth in TFP becomes:

$$
\frac{\text{TFP}_t}{\text{TFP}_t} = \pi g_K(t) + (1 - \pi)g_L(t) - \left( \frac{1 - \sigma}{\sigma} \right) \pi(1 - \pi) (g_L(t) - g_K(t)) (g_L(t) - g_K(t))
$$

where $\dot{x}$ shows the change in variable $x$ with respect to time.

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10Following Klump et al. (2004), without $\sigma$ deviating too much from the unity, the term $\left( \frac{1 - \sigma}{\sigma} \right) \pi(1 - \pi)(g_L(t) - g_K(t)) \ln \left( \frac{K_t/K}{L_t/L} \right)$ has been dropped from the approximation, since it has been considered to have second order importance. Therefore, it should be addressed that the closer the elasticity of substitution to unity, the more precise the approximation is.
4 Data

The data are annual. The real GDP ($Y_t$), nominal GDP ($P_tY_t$), and employment ($L_t$) series are for the overall economy and obtained from the Turkstat. Employment corresponds to the total number of employees and the self-employed.\textsuperscript{11} According to the definition of the OECD, compensation of employee has two main components: wages-salaries and social insurance contributions payable by employers.\textsuperscript{12} The average compensation, $W_t$, in Turkey is calculated in line with this definition. The Social Security and General Health Insurance Law (Law No: 5510) in Turkey classifies insurants into three: employee under service contract (EUSC), self-employed and civil-servant. The compensation of the EUSC and self-employed are computed with the data obtained from the Social Security Institution (SSI) assuming that the contribution of the employer is 20 percent of gross wage.\textsuperscript{13} The compensation of the civil servants is calculated using the data from the Ministry of Finance.\textsuperscript{14} Hence, the average compensation, $W_t$, is obtained as a weighted average of the compensation of EUSC, self-employed and civil-servants.\textsuperscript{15} Weights are the ratio of number of insurants in each category to the total number of insurants. Though the average compensation computed using the data from the SSI and Ministry of Finance is believed to be accurate, it is short. It dates back up to 2003. Therefore, using these data labour share of income can be computed from 2003 onwards. From the AMECO database of the European Commission, using nominal compensation per employee and nominal GDP per employed, which is a sum of employees and self-employed, labour income share for Turkey can be computed. Hence, in the construction of labour income share for the years preceding 2003, AMECO database is employed.

For capital stock ($K_t$), the capital services series of Demiroğlu (2012) is used.\textsuperscript{16} 

\textsuperscript{11} Therefore, factors that affect the productivity of labour, such as education and health status, are considered to be captured in $g_L(t)$.

\textsuperscript{12} See https://stats.oecd.org/glossary/detail.asp?id=396.

\textsuperscript{13} This assumption seems plausible taking into account the fact that the contribution of government to workers employed in the public sector is 20 percent on average between 2003-2014.

\textsuperscript{14} No assumption for the contribution of employer for the civil-servants is necessary as it is available in the general budget expenditures of the Ministry of Finance.

\textsuperscript{15} In Gollin (2002), Klump et al. (2004), and Trapp (2015) average compensation of a self-employed is assumed to be equal to that of an employee.

\textsuperscript{16} As well explained in OECD (2015), when the purpose is to determine the role of capital in
This series is composed of two types of capital: machinery-equipment and building stock. It weighs each type of capital according to their marginal product. The real user cost of capital ($r_t$) is also taken from Demiroğlu (2012).

The data span is mainly constrained by the capital stock series, which are available from 1987 onwards. Thus, taking into account the period after financial liberalization in Turkey, the estimations cover 1991-2014.\textsuperscript{17} The fact that Turkey has experienced crises in 1994, 2001 and 2008 increases the volatility of the series. Therefore, to smooth out fluctuations and obtain trends, five year moving averages of the series are used in the estimations.

Put forth by Kaldor (1961), some stylized facts of long-term economic growth in developed countries were: capital and labour have stable shares of national income while capital to output ratio is constant, and capital intensity and labour productivity grow at a sustained rate. These facts were able to explained by the simple Cobb-Douglas production function, which let it shoot to fame. However, Blanchard (1997) documents that the capital share of income increases between 1980-1995 in the Continental Europe upon the adverse labour demand shifts. Bentolila and Gilles (2003) argue that there are large cross-country differences in the behaviour of the labour share even though the countries are relatively similar from a technological point of view. While the the labour share has an upward trend in Japan, it is downward trending in France between 1965-1995. Likewise, Harrison (2002) suggests that labour shares are not constant over time and across countries. Over 1960-1997 period, labour shares in poor countries fell, while shares in rich countries rose. These changes in labour shares are found to be driven by the changes in the factor endowments, government spending, and the globalization. Jones (2015) shows that there is a marked decline in the labour share of income and a corresponding rise in capital share between 2000-2010 in the US. Similarly, Klump et al. (2007) find that neither the factor shares, the aggregate mark-up nor production, the correct measure of $K_t$ is not the "capital stock" but the "capital services". In papers on growth, this fact is seldomly stated explicitly. Moreover, $K_t$ is quite oftenly called capital stock though it is measured as capital services indeed. This paper is not an exception in this respect.\textsuperscript{17}

\textsuperscript{17}The relatively short sample may be considered as a drawback of this study, though the sample used is the longest possible one. Hence, the findings can improve as more data accumulate over time.
the capital to output ratio are stationary in 1970-2003 period in the Euro-area. Therefore, relatively recent empirical findings deviate from what was suggested 50 years ago, casting serious doubt on the validity of the Cobb-Douglas production function today.\footnote{Jones and Romer (2010) suggest six new facts of growth; increase in the extent of the market, acceleration in growth rates, variation in growth rates, large TFP differences, increase in human capital and rise in the return to human capital. They discuss that the first two of these facts can be captured by ideas, the next two can be explained by institutions and the last two can be covered by the human capital. Hence, they argue that a formal model of growth should include the accumulation and interaction of ideas, institutions and human capital.}

Graph 1 plots the share of labour and capital in national income in Turkey. The share of labour is 0.4 on average between 1991-2014. Hence, the total share of capital \( \left( \frac{R_t K_t + \text{Profit}_t}{P Y_t} \right) \) in national income is 0.6. Half of the 0.6 belongs to the non-profit capital \( \left( \frac{R_t K_t}{P Y_t} \right) \) while the rest pertains to the profit. Typically, the share of labour in national income is taken in the vicinity of 0.65 for developed countries.\footnote{See for example Musso and Westermann (2005) and Husabø (2013).} However, as shown in Senhadji (2000) and Abu-Qarn and Abu-Bader (2007) the share of labour in developing countries appears to be smaller than the one in developed countries. Table 1 shows the shares of factors computed in various studies on Turkey. Using the Fully Modified estimator, Senhadji (2000) estimates the share of capital as 0.62 for Turkey between 1960-1994. Likewise, İsmihan and Metin-Özcan (2006) employ two different estimation methodologies and find the share of capital as 0.58 and 0.65 in 1960-2004 period. Using a cointegration framework, Abu-Qarn and Abu-Bader (2007) report a considerably high share for capital, 0.72, in Turkey. Finally, Ünveren and Sunal (2015) estimate a dynamic general equilibrium model with imperfect competition for 1983-2010. Their results imply that the share of labour is 0.38. Hence, the existing literature on the factor shares in Turkey suggests that once truly estimated, the share of labour in Turkey is not only smaller than the one used for the developed countries, but also smaller than the share of capital. The factor shares computed in this paper are no exception.

Factor income shares and the mark-up in Turkey are found to be non-stationary. The trends in the share of labour and mark-up are opposite, while the trend in capital is relatively smooth and declining (Graph 1 and Graph 2). Indeed, since
mark-up in Graph 2 is proxied as \( \frac{P_t Y_t}{W_t L_t + R_t K_t} - 1 \), higher volatility in total earnings of labour renders mark-up as a mirror image of \( W_t L_t \). Graph 3 plots the ratio of labour share to capital share, which slopes downward until 2006 and upward later on. Consistent with Graph 3, Boratav (2006) argues that the period from early 1990s to 2005 has been marked by the weakening of labour against capital. The fall in unionization, rise in informal labour and privatization have been cited as the reasons for the decrease in the share of labour compared to capital. Together with a rapid growth in nominal GDP, these facts- which possibly caused a subdued growth in labour and a rise in unemployment- might have led to a rise in mark-up to a level as high as 60 percent in mid 2000s. On the other hand, the active labour market and employment incentive policies, and higher minimum wage increases after the global financial crisis might have brought on an increase in the share of labour relative to capital, hence a reversal in the trend in mark-up. If there is a change in the direction of the technical progress, the factor shares may not be constant or the capital to output ratio can be non-stationary (Klump et al. (2007)). Therefore, in the case of Turkey, the fact that factor shares and their ratio lack stationarity suggests that there is technical progress both in the labour and capital but their dynamics are different. Hence, the differing technical progresses in factors call for a CES production function for Turkey.

5 Results

Table 2 presents the estimation results of equation (6), (7) and (8) for Turkey using annual data 1991-2014. The determinants computed from the residuals of the equations are also shown. Additionally, the Augmented Dickey Fuller (ADF) test results for stationarity of the residuals of each equation can be found in Table 2. TFP growth is calculated as an average using equation (13).

Column (1) of Table 2 shows that all the estimated parameters are significant at 1 percent level. The estimated elasticity of substitution is around 0.8 in Turkey.\(^{20}\)

\(^{20}\)Klump et al. (2007) estimate the elasticity of substitution as 0.65 and 0.6 for the US and Euro area, respectively. Therefore, \( \sigma \) estimated for Turkey is higher than the elasticity of substitution estimated for the US and Euro area. This is consistent with Duffy and Papageorgiou (2000), which suggest that capital and labour are more substitutable in rich countries compared
It is found to be significantly below 1. This suggests that labour and capital are complements and capital deepening, *ceteris paribus*, decreases the share of capital in Turkey. The mark-up is estimated as 37 percent.\(^{21}\) It is consistent with the average of the proxied mark-up shown in Graph 2. However, it should be noted that \(\mu\) is sensitive to measurement errors in \(W_tL_t\) and \(R_tK_t\). The scaling parameter, \(A\), is found to be very close to 1 as expected. The share of capital in total income of labour and capital, \(\pi\), is estimated as 0.44, which is in line with the data. The technical progress in labour is modelled best when the curvature parameter is assumed to be 0. Hence, the technical progress in labour is found to be log-linear with \(\gamma_L\) estimated as 1.5 percent. In other words, labour productivity increases at a decreasing rate.\(^{22}\) On average, labour productivity growth is 2.3 percent in 1991-2014 period.\(^{23}\) \(\lambda_K\) is estimated as -0.5. \(\gamma_K\) is found to be negative with 1.1 percent.\(^{24}\) These parameters suggest that the technical progress in capital is hyperbolic, and the capital productivity decreases at a decreasing rate.

\(^{21}\)Klump et al. (2007) estimate the size of mark-up as 5 percent for the US. Using a different estimation scheme Raurich et al. (2011) find it as 31 percent for Spain. Badinger (2007) shows that the mark-up lies between 30-40 percent in 10 EU countries. Using 1991-1997 period, Ceritoğlu (2002) calculates the mark-up in subsectors of manufacturing industry for Turkey and shows that it is around 45 percent in the sectors with the largest value added. Kivilcim et al. (2002) outlay that the mark-up in manufacturing had a rising trend from 33.5 percent in 1989 to 47 percent in 1994. They also argue that sectoral mark-ups are heterogeneous. In a recent study, Ünveren and Sunal (2015) compute the mark-up as 70 percent for Turkey in 1983-2010 period. However, if the market concentration has not soared, this value seems high taking into account the increased openness of the country.

\(^{22}\)Üngör (2013a) argues that the service sector has lower labour productivity growth between 2002-2007 compared to agriculture and industry in Turkey. However, over the same time period, employment share of services increases. Therefore, Üngör (2013a) suggests that the service sector represents a drag in the aggregate labour productivity growth.

\(^{23}\)According to the Conference Board Total Economy Database, labour productivity growth per person employed is 2.2 percent in Turkey between 1991-2014. Similarly, from the OECD Statistics growth in real GDP per hour worked is calculated as 2.3 percent between 1995-2014. Employing a longer time horizon, 1968-2005, Yılmaz (2015) finds the growth in labour productivity as 2.7 percent for Turkey. In a shift-share analysis framework, he shows that the main determinant of labour productivity improvements in Turkey is the within productivity gains (i.e gains due to capital deepening, technological progress and reduction in the misallocation across plants within the sectors).

\(^{24}\)Though as small as 0.2-0.1 percent, Klump et al. (2007) find \(\gamma_K\) positive for the US and the Euro area. The fact that they use private non-residential capital stock for capital stock series can be a reason for the positive \(\gamma_K\) they find. In this paper, the public investments and all of the private construction investments are included in the capital stock series.
rate (Graph 4 and 5). The average growth in capital productivity is -2.6 percent, which is consistent with the rising capital to output ratio in Turkey (Graph 6). The TFP growth is calculated as low as 0.2 percent on average. Hence, the labour-augmenting technical progress is found to be slightly dominant.

Two interesting points emerge from the estimation results. First, when capital productivity is higher than labour productivity the relative share of labour to capital falls, and vice versa (Graph 3 and 4). One possible explanation is at the times when capital has higher productivity, the price of capital relative to wage is lower than its historical average. This may encourage capital accumulation and therefore lead to an increase in its share. The rise in the relative share of labour to capital after 2006 might be owing to the increase in the price of building stock compared to wages, limiting the rise in capital accumulation (Graph 7). Second, the composition of capital stock has an effect on the productivity of capital. Ceteris paribus, during 1991-2014, if Turkey had invested less in construction but more in machinery-equipment, the growth in the productivity of capital could have been relatively higher, though still negative (Graph 8). Column (2) of Table 2 presents the results of this case. Holding total investment constant, if the machinery-equipment investment had been more by an amount equaling to 30 percent of the construction investment, the estimate of $\gamma_K$ would have been increased to -0.6 percent from -1.1 percent. This implies that the average productivity growth in capital would have risen to -1.7 percent from -2.6 percent. Similar to the finding of negative capital productivity growth of this paper, OECD (2015) shows that capital productivity has fallen in most of the OECD countries over the 1995-2013 period.

6 Robustness check

Normalization introduces benchmark values, i.e. $Y_0$, $K_0$, $L_0$ and $t_0$, into a CES function. These benchmark values are calculated using the full sample average.

\footnote{Note that investing more in construction and less in machinery-equipment would have led to a greater fall in the productivity of capital. The estimation results of this exercise is available upon request.}

\footnote{OECD (2015) suggests that declining costs of capital relative to labour and the resulting fall in the use of labour per unit of capital have led to a fall in the capital productivity.}
However, how sensitive are the results shown in column (1) of Table 2 to the selection of baseline points? In other words, do the results change if the averages rest on a different time period? To check the robustness of the findings, two different time periods are used when computing the averages, namely 1991-2002 and 2003-2014. Table 3 presents that the parameter estimations are quite robust in general against the benchmark values. In particular, the estimated mark-up, elasticity of substitution and curvature parameter of the capital stay the same while the share of capital changes slightly. The most noticable changes occur in the estimations of $\gamma_L$ and $\gamma_K$. In both of the robustness checks, while the former rises to 3 percent, the latter falls to -3 percent. Hence, if the baseline points are computed using 1991-2002, the TFP displays 0.2 percent growth on average as before. Alternatively, if the benchmark values are averaged out using 2003-2014 period, the TFP growth is calculated as 0.3 percent. Therefore, the findings presented in Table 2 are robust to changes in baseline points.

7 Conclusion

Following Klump et al. (2007) this paper estimates a normalized CES production function for Turkey. Employing a system approach, the elasticity of substitution, direction of the technical change and total factor productivity are determined. As in Klump et al. (2004) and Klump et al. (2007), time-varying technical progresses of inputs are modelled via the Box-Cox transformation, while the TFP is calculated by the Kmenta approximation. The estimations cover 1991-2014.

The findings of this paper show that the elasticity of substitution is around 0.8, hence factor inputs are complements, and capital deepening decreases the share of capital in Turkey. The dynamics of the technical progresses signal a slowing productivity growth in labour, and a falling productivity in capital. These findings imply that in Turkey the average annual growth of the TFP is as low as 0.2 percent, and labour-augmenting technical progress surpasses the capital-augmenting one marginally. The last 25 years have witnessed 4 percent growth in real GDP on average. However, the findings show that only 5 percent of this growth has been due to the growth in TFP. If not reversed, the weak growth in TFP can hamper
the convergence of Turkey to developed economies, and prolong her graduation from the middle income.

The estimation results bring out two interesting points. First, when capital productivity is higher (lower) than labour productivity the relative share of labour to capital falls (rises) in Turkey. A possible explanation lies in the relation between factor prices and factor accumulation, i.e. relatively lower (higher) price of capital encourages (discourages) capital accumulation. Second, the composition of capital stock has an effect on the productivity of capital. Holding total investment constant, during 1991-2014, if Turkey had invested less in construction but more in machinery-equipment, the growth in the productivity of capital could have been relatively higher, though still negative. The latter point suggests that an increase in the relative contribution of machinery-equipment stock to growing capital deepening can improve the productivity of capital, and labour, hence the TFP. Overall, in addition to the size of growth, the composition of it will be a determinant on the convergence of Turkey in coming years.

The future research can calculate the potential growth of Turkey using the aggregate production function estimated in this paper. Different potential growth projections taking into account the fact that the composition of investments affects the productivity of factors, hence the overall productivity, would be valuable in terms of policy-making. Finally, inclusion of imported intermediate goods and CES functions estimated at different levels regarding the different types of capital, labour and sectors would be an extension to this paper.

References


Raurich, X., H. Sala, and V. Sorolla (2011). Factor shares, the price markup, and the elasticity of substitution between capital and labour. IZA Discussion


Appendix 1

This section shows the derivation of the normalized CES, i.e. equation (4). At \( t = t_0 \) equation (3) is:

\[
Y_0 = C \left[ \pi \left( e^{gK(0)} K_0 \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \pi) \left( e^{gL(0)} L_0 \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \tag{A1.1}
\]

Then:

\[
C = \frac{Y_0}{\left[ \pi \left( e^{gK(0)} K_0 \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \pi) \left( e^{gL(0)} L_0 \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}} \tag{A1.2}
\]

Taking into account the fact that \( e^{gL(0)} \) and \( e^{gK(0)} \) are 1 from equation (9):

\[
C = \frac{Y_0}{\left[ \pi \left( K_0 \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \pi) \left( L_0 \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}} \tag{A1.3}
\]

Profit maximization in factor markets requires marginal revenue product (MRP) of a factor equals to its price. MRP equals marginal revenue times marginal product. Hence, for labour:

\[
MRP_{lt} = W_t \tag{A1.4}
\]

\[
\frac{dTR_t}{dY_t} \frac{dY_t}{dL_t} = W_t \tag{A1.5}
\]

\[
\frac{d(P_tY_t)}{dY_t} \frac{dY_t}{dL_t} = W_t \tag{A1.6}
\]

\[
\left( \frac{dP_t}{dY_t} + P_t \right) \frac{dY_t}{dL_t} = W_t \tag{A1.7}
\]

\[
\left( \frac{1 + \varepsilon}{\varepsilon} \right) P_t \frac{dY_t}{dL_t} = W_t \tag{A1.8}
\]

where \( \varepsilon \equiv \frac{dY_t/Y_t}{dP_t/P_t} \) is the price elasticity of demand. Let \( 1 + \mu = \frac{\varepsilon}{1 + \varepsilon} \), \( w_t = \frac{W_t}{P_t} \) and \( r_t = \frac{R_t}{P_t} \). Equations (A1.4)-(A1.8) hold for capital, as well. Then, from (3):

\[
w_t(1 + \mu) = \frac{dY_t}{dL_t} = C \frac{\sigma}{\sigma - 1} \left[ \frac{\sigma}{\sigma - 1} \left( 1 - \pi \right) \left( e^{gL(0)} L_t \right)^{\frac{\sigma - 1}{\sigma}} L_t^{-1} \right]^{\frac{\sigma}{\sigma - 1}} \tag{A1.9}
\]
\[ r_t (1 + \mu) = \frac{dY_t}{dK_t} = C \frac{\sigma}{\sigma - 1} \lbrack \pi - \frac{1}{\sigma} \rbrack (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}} K_t^{-1} \]  
\text{(A1.10)}

where \( \lbrack \cdot \rbrack \) refers to \( \pi (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) (e^{gL(t)} L_t)^{\frac{\sigma-1}{\sigma}} \). Hence:

\[ \frac{w_t}{r_t} = \frac{(1 - \pi)}{\pi} \left( \frac{e^{gL(t)} L_t}{e^{gK(t)} K_t} \right)^{\frac{\sigma-1}{\sigma}} \]  
\text{(A1.11)}

\[ \frac{\pi}{1 - \pi} = \frac{r_t K_t}{w_t L_t} \left( \frac{e^{gK(t)} K_t}{e^{gL(t)} L_t} \right)^{\frac{1 - \sigma}{\sigma}} \]  
\text{(A1.12)}

Clearly, at \( t=t_0 \) equation (A1.12) is:

\[ \frac{\pi}{1 - \pi} = \frac{r_0 K_0}{w_0 L_0} \left( \frac{1}{\sigma} \right) \]  
\text{(A1.13)}

Plug equation (A1.3) and (A1.13) into (3):

\[ Y_t = Y_0 \left[ \frac{\pi (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}}}{\pi (K_0)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) (L_0)^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\sigma}{\sigma - 1}} \left[ \pi (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) (e^{gL(t)} L_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \]  
\text{(A1.14)}

\[ Y_t = Y_0 \left[ \frac{\pi (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}}}{\pi (K_0)^{\frac{1 - \pi}{\sigma}} (L_0)^{\frac{\sigma - 1}{\sigma}} + (1 - \pi) (L_0)^{\frac{\sigma - 1}{\sigma}} + \frac{(1 - \pi) (e^{gL(t)} L_t)^{\frac{\sigma-1}{\sigma}}}{\pi (K_0)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) (L_0)^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\sigma}{\sigma - 1}} \]  
\text{(A1.15)}

\[ Y_t = Y_0 \left[ \frac{\pi (e^{gK(t)} K_t)^{\frac{\sigma-1}{\sigma}}}{\pi (K_0)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) \left( L_0 \frac{e^{gL(t)} L_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \pi) (L_0)^{\frac{\sigma-1}{\sigma}} \left( \frac{1}{\pi} \frac{K_0}{L_0} \right)^{\frac{\sigma-1}{\sigma}} + 1} \right]^{\frac{\sigma}{\sigma - 1}} \]  
\text{(A1.16)}
\[ Y_t = Y_0 \left[ \left( \frac{e^{g_K(t) K_t}}{K_0} \right)^{\frac{\pi_0}{\sigma}} \left[ 1 + \frac{w_0 L_0}{r_0 K_0} \right] + \left( \frac{e^{g_L(t) L_t}}{L_0} \right)^{\frac{\pi_0}{\sigma}} \frac{r_0 K_0}{w_0 L_0 + 1} \right]^{\frac{1}{\sigma-1}} \]  

(A1.17)

Let \( \pi_0 \equiv \frac{r_0 K_0}{r_0 K_0 + \frac{w_0 L_0}{r_0 K_0}} \), then we arrive at the normalized CES function:

\[ Y_t = Y_0 \left[ \pi_0 \left( \frac{e^{g_K(t) K_t}}{K_0} \right)^{\frac{\pi_0}{\sigma}} + (1 - \pi_0) \left( \frac{e^{g_L(t) L_t}}{L_0} \right)^{\frac{\pi_0}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \]  

(4)

**Appendix 2**

This section shows the derivation of equation (6) and (7). Using equation (A1.8) and (5):

\[ \frac{w_t (1 + \mu)}{dL_t} = A \bar{Y} \left( \frac{\pi}{\sigma - 1} \right) \bar{Y}^{-1} \left( 1 - \frac{\pi}{\sigma} \right) \frac{1}{\sigma} \left( \frac{e^{g_L(t) L_t}}{L_t} \right)^{\frac{\sigma}{\sigma-1}} L_t^{-1} \]  

(A2.1)

where \( \bar{Y} \) refers to \( \left[ \pi \left( \frac{e^{g_K(t) K_t}}{K_0} \right)^{\frac{\pi_0}{\sigma}} + (1 - \pi) \left( \frac{e^{g_L(t) L_t}}{L_0} \right)^{\frac{\pi_0}{\sigma}} \right] \). Rearranging equation (A2.1):

\[ \frac{W_t L_t}{P_t Y_t} = \frac{(1 - \pi)}{(1 + \mu)} \left( \frac{Y_t}{\bar{Y}} \frac{1}{A e^{g_L(t) L_t}} \right)^{\frac{1}{\sigma}} \]  

(A2.2)

Hence:

\[ \ln \left( \frac{W_t L_t}{P_t Y_t} \right) = \ln \left( \frac{1 - \pi}{1 + \mu} \right) + \frac{1 - \frac{\pi_0}{\sigma}}{\sigma} \left[ \ln \left( \frac{Y_t}{\bar{Y}} \right) - \ln(A) - g_L(t) \right] \]  

(6)

Similarly for capital:

\[ \ln \left( \frac{R_t K_t}{P_t Y_t} \right) = \ln \left( \frac{\pi}{1 + \mu} \right) + \frac{1 - \frac{\pi_0}{\sigma}}{\sigma} \left[ \ln \left( \frac{Y_t}{K_t} \bar{K} \right) - \ln(A) - g_K(t) \right] \]  

(7)
Table 1: Estimated factor income shares for Turkey.

<table>
<thead>
<tr>
<th>Study</th>
<th>Period</th>
<th>Capital Share</th>
<th>Labour Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senhadji (2000)</td>
<td>1960-1994</td>
<td>0.62</td>
<td>0.38</td>
</tr>
<tr>
<td>İsmihan and Metin-Özcan (2006)</td>
<td>1960-2004</td>
<td>0.76</td>
<td>0.24</td>
</tr>
<tr>
<td>Abu-Qarn and Abu-Bader (2007)</td>
<td>1960-1998</td>
<td>0.58, 0.65</td>
<td>0.42, 0.35</td>
</tr>
<tr>
<td>Ünveren and Sunal (2015)*</td>
<td>1983-2010</td>
<td>0.62</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Notes: The studies listed above assume Cobb-Douglas production function with constant returns to scale, and estimate the share of capital.
*Labour share is calculated from the estimated parameters in the paper. Capital share corresponds to the shares of non-profit capital plus the profits.
Table 2: The supply-side system estimation for Turkey, 1991-2014.

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.445***</td>
<td>0.453***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.374***</td>
<td>0.357***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.799***</td>
<td>0.707***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>$A$</td>
<td>0.998***</td>
<td>1.000***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0.015***</td>
<td>0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>-0.011***</td>
<td>-0.006***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\lambda_K$</td>
<td>-0.490***</td>
<td>-0.666***</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.112)</td>
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<td>TFP growth</td>
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<td>0.002</td>
</tr>
<tr>
<td>Determinant of residual covariance</td>
<td>6.01e-10</td>
<td>7.29e-10</td>
</tr>
</tbody>
</table>

**ADF test results for stationarity**

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF (6)</td>
<td>-3.161***</td>
<td>-3.074***</td>
</tr>
<tr>
<td>ADF (7)</td>
<td>-1.998**</td>
<td>-1.806*</td>
</tr>
<tr>
<td>ADF (8)</td>
<td>-1.855*</td>
<td>-1.770*</td>
</tr>
</tbody>
</table>

**p-values of restrictions**

<table>
<thead>
<tr>
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<th>Column (1)</th>
<th>Column (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 1$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Notes:* Standard errors in parenthesis. */**/*** show the 10, 5 and 1 percent significance level. TFP growth is calculated as an average using equation (13). ADF (i) shows the ADF test result of the residuals of equation (i).
Table 3: Robustness check of the supply-side system estimation for Turkey, 1991-2014.

<table>
<thead>
<tr>
<th>Benchmark values are averaged out using 1991-2002</th>
<th>Benchmark values are averaged out using 2003-2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\pi}$</td>
<td>$\bar{\pi}$</td>
</tr>
<tr>
<td>0.443***</td>
<td>0.419***</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>0.374***</td>
<td>0.374***</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>0.799***</td>
<td>0.799***</td>
</tr>
<tr>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$A$</td>
<td>$A$</td>
</tr>
<tr>
<td>1.050***</td>
<td>0.906***</td>
</tr>
<tr>
<td>(0.013)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>$\gamma_L$</td>
</tr>
<tr>
<td>0.029***</td>
<td>0.029***</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>$\gamma_K$</td>
</tr>
<tr>
<td>-0.030***</td>
<td>-0.030***</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\lambda_K$</td>
<td>$\lambda_K$</td>
</tr>
<tr>
<td>-0.490***</td>
<td>-0.490***</td>
</tr>
<tr>
<td>(0.087)</td>
<td>(0.087)</td>
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<tr>
<td>TFP growth</td>
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<tr>
<td>0.002</td>
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</tr>
<tr>
<td>Determinant of residual covariance</td>
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</tr>
<tr>
<td>6.01e-10</td>
<td>6.01e-10</td>
</tr>
</tbody>
</table>

**ADF test results for stationarity**

<table>
<thead>
<tr>
<th>$ADF_{(6)}$</th>
<th>$ADF_{(7)}$</th>
<th>$ADF_{(8)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.161***</td>
<td>-1.998**</td>
<td>-1.855*</td>
</tr>
</tbody>
</table>

**p-values of restrictions**

| $\sigma = 1$ | 0.00 | 0.00 |

*Notes:* Standard errors in parenthesis. */**/*** show the 10, 5 and 1 percent significance level. TFP growth is calculated as an average using equation (13). $ADF_{(i)}$ shows the ADF test result of the residuals of equation $(i)$. 
Graph 1: Share of labour and capital in national income

Graph 2: Mark-up

Graph 3: The ratio of labour share to capital share
Graph 4: Capital productivity, labour productivity and TFP in levels

Graph 5: Capital productivity growth, labour productivity growth and TFP growth

Graph 6: Capital to output ratio

Notes: Both the capital stock and the output are at constant prices.
Graph 7: Relative price movements

Notes: Price deflators and wage are measured relative to their 1991-2014 average. The dashed line at 1 shows the equality of deflator over wage to its historical mean.

Graph 8: Capital productivity growth

Notes: \( \frac{d g_k}{dt} (1) \) \([(2)] \) shows the change in the technical progress of capital of column 1 \([2]\) in Table 1.
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