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LEARNING, MONETARY POLICY, AND HOUSING PRICES

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ABSTRACT This paper evaluates different types of simple monetary policy rules according to the determinacy and the learnability of rational expectations equilibrium criteria within a dynamic stochastic general equilibrium framework. Incorporating housing prices and collateralized borrowing into the standard model allows us to answer important policy questions. One objective is to investigate whether responding to housing prices affects determinacy and learnability of rational expectations equilibrium (REE). For this purpose, we work with a New Keynesian model in which housing plays an accelerator role in business cycles as a collateralized asset. The results show that for current data rule, responding to asset prices does not improve learnable outcomes but for a monetary policy with lagged data and forward-looking rules we see improved learnable outcome if current housing prices are available to monetary authority. Moreover, we examine the effects of interest rate inertia and price stickiness on E-stability of REE.
JEL E4, E5
Keywords Monetary policy rules, Determinacy, Learning, Housing prices

ÖZ Bu çalışma, farklı para politikası kurallarını rasyonel beklentiler dengesinin belirlilik (determinacy) ve öğrenilebilirlik (learnability) kiriterlerine göre değerlendirmektedir. Standart bir modele konut ve kredi piyasalarını da dahil etmek önemli para politikası sorularına cevap verme imkanı tanımaktadır. Çalışmanın bir amacı da, konut fiyatlarındaki hareketlere tepki veren para politikası kurallarının rasyonel bekleyişler dengesinin belirliliğine ve öğrenilebilirliğine etki edip etmediğini incelemektir. Bunun için, konut sektörünün iş çevrimleri üzerinde ivmelendirme etkisi oluşturduğu bir yeni keynesyen model kullanılmıştır. Sonuçlar, cari dönem verilerini dikkate alan para politikası kuralları için konut fiyatlarına verilen tepkiyle birlikte, öğrenilebilirliğin artmadığını ancak geçmiş dönem ve ileriye dönük verilere tepki veren para politikası kuralları için öğrenilebilirliğin daha geniş politika parametrelerinde mümkün olduğunu göstermektedir. Son olarak faiz oranı ataleti ve fiyat katılıklarının rasyonel bekleyişler dengesinin öğrenilebilirliğini nasıl etkilediği incelenmiştir.
ÖĞRENME, PARA POLITIKASI VE KONUT FIYATLARI
JEL E4, E5
Anahtar Kelimeler Para politikası kuralı, Belirlilik, Öğrenme, Konut fiyatları

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1. Introduction

Determinacy and learnability are two important criteria to evaluate Taylor-type monetary policy feedback rules where central bank adjusts the short-term nominal interest rate in linear response to deviations of inflation from some target inflation and to deviations of output from some target output.

A monetary policy rule that induces indeterminacy is viewed as undesirable since agents may not be able to coordinate on an equilibrium among the many that exist. If the equilibrium is determinate, it is assumed that coordination will exist on that equilibrium. However, it is not clear how and whether such coordination will exist. On the other hand, it is clear for the "learnability" criterion how agents reach the equilibrium. This is one reason why "learnability" criterion is taken into account in addition to "determinacy" criterion to evaluate monetary policy rules. A rational expectations equilibrium (REE) is learnable if agents can learn the fundamental equilibrium if they do not possess rational expectations but are able to create forecasts by using recursive learning algorithms, such as recursive least squares. Evans and Honkapohja (2001) generate the concept of expectationally stable (E-stable) REE that we can use interchangeably with learnable REE.

Several papers discuss indeterminacy problem of monetary policy rules, including Bernanke and Woodford (1997), Carlstrom and Fuerst (2000, 2001), Woodford (1999). Instability under learning problem with the monetary policy rules has first been raised by Howitt (1992) and suggested that the analysis of monetary policy rules should be supplemented by stability under learning. More recently, Bullard and Mitra (2002) study learnability of REE and evaluate alternative policy rules in this context.

The basic framework in the learning literature is the New Keynesian model. Dynamic Stochastic General Equilibrium Model (DSGE) with imperfect competition and sticky pricing allows a tractable mechanism for learning analysis. However, this basic framework does not consider the asset market. Recent research provide empirical evidence that housing market has significant effect on consumption (Leamer, 2007) and this effect is magnified when housing is used as collateral asset for the borrowing (Muellbauer, 2008). The lack of well-defined credit market in which housing can be used as a collateralized asset and lack of asset market in the equipped model limit the learning analysis for several policy issues

including the question of "should central bank respond to housing prices?" or "how should central bank respond to credit growth?"

The goal of this paper is to fill this gap in the learning literature. To our knowledge, this is the first paper in the learning literature incorporating a credit market with collateralized housing asset into a small DSGE model. This allows us to reply the questions above and to study E-stability of REEs for different monetary policy rules by considering housing market.

We employed a New Keynesian (DSGE) model in which housing and borrowing play key roles. In our model, housing serves not only as a shelter but also acts as a collateralized asset on which credit-constrained agents can borrow to finance their consumption. The eligibility of agents to borrow is related to the value of the asset. When the value of housing increases, agents' borrowing capacity expands, and this allows them to purchase more housing and consumption goods. Higher housing demand increases housing prices more and creates a housing boom. Increased aggregate demand, therefore, increases work hours and produces an economic expansion. This propagation mechanism is absent in a standard DSGE model.

The rest of the paper is organized as follows. Section 2 describes the economic environment. Section 3 provides the interest rate rules that we study. Section 4 lays out the indeterminacy of equilibrium issue. Section 5 gives insights on adaptive learning. Section 6 presents the results for our experiments. Section 7 investigates whether responding to housing prices is beneficial from learning perspective. Section 8 gives the results of estimation of Fed interest rate policy rule and the last section concludes.

2. Economic Environment

The model is based on that of Iacoviello (2005). But, this model differs from Iacoviello's model in several ways. In this model, we do not include physical capital and accumulation, we do not assign housing asset in the production function, and we do not have entrepreneurs. The economy is a discrete time, infinite horizon model populated by infinitely-lived agents. In the economy, there are patient household, impatient household, intermediate good producers, retailers, and a central bank. Households have an endowment of housing asset and labor time. Households consume, work, and demand housing asset. They consume final goods, they work for intermediate good producers and they have utility by holding housing asset. The patient household maximizes a lifetime utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta_1^t \left(\ln C_{1,t} + d_t \ln h_{1,t} - \frac{L_{1,t}^{\eta}}{\eta} \right)$$

subject to the budget constraint

$$C_{1,t} + q_t h_{1,t} + \frac{R_{t-1} b_{1,t-1}}{\pi_t} = b_{1,t} + q_t h_{1,t-1} + w_1 L_{1,t} + F_t \quad (1)$$

where E_0 is the expectation operator, β_1 is the discount factor, C_{1t} is consumption, h_{1t} is the housing asset, and L_{1t} is hours worked, b_t represents real holdings of one period loan, R_{t-1} is the nominal interest rate, q_t is real housing price, w_1 is real wage, π_t is the gross inflation, and F_t represents real profits received from the retailers in good markets. The variable d_t is a preference shock that shifts the marginal rate of substitution between housing and consumption/leisure. The subscript "1" is used to tag all variables of the patient household.

By putting housing in the utility function we implicitly assume that housing services are proportional to the housing stock. In this model, housing stock does not depreciate and there is no housing construction. Stock of housing is fixed. Moreover, there is no rental market for housing. For these reasons, if a household wants to increase housing (shelter service), she needs to buy housing asset. Patient and impatient households are distinct from one another by the assumption of different discount rates, where impatient household has a lower discount rate than the patient household. We allow borrowing and lending among households, and the distinction in discount rates designates impatient household as a borrower and patient household as a lender in the equilibrium.[1] Borrowing is in nominal terms, constrained by a collateral value and allowed only among households. There are two other differences between lender and borrower other than discount rate. First, lender own retailers and get dividend income F_t. On the other hand, borrower does not own any firm. Second, the borrower uses her housing asset as collateral to borrow from the lender. Borrower maximizes her lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta_2^t \left(\ln C_{2,t} + d_t \ln h_{2,t} - \frac{L_{2,t}^{\eta}}{\eta} \right)$$

subject to the budget constraint,

$$C_{2,t} + q_t h_{2,t} + \frac{R_{t-1} b_{2,t-1}}{\pi_t} = b_{2,t} + q_t h_{2,t-1} + w_2 L_{2,t} \quad (2)$$

and borrowing constraint,

$$b_{2,t} \leq m E_t \left(\frac{q_{t+1} h_{2,t} \pi_{t+1}}{R_t} \right) \quad (3)$$

The subscript "2" will be used to denote variables of the impatient household. A requirement of $\beta_2 < \beta_1$ ensures that this household is more

[1] In the text, we will use lender (borrower) and patient (impatient) household interchangeably.

impatient than the lender and will need to borrow from her. The amount that the debtor can borrow is bounded by $mE_t\left(\frac{q_{t+1}h_{2,t}\pi_{t+1}}{R_t}\right)$ where 0 < m < 1. In other words, a fraction 1-m of the housing value cannot be used as collateral. One can broadly think of 1-m as the down payment rate, or think of m as the loan-to-value ratio. As shown in Iacoviello (2005), this setup ensures that the borrowing constraint will always be binding in the steady state. In order to have a constant consumption path for the borrower we need a collateral constraint.

Intermediate good producers (wholesaler firms) hire from both types of households to produce homogenous intermediate goods. They produce homogenous good Y_t according to

$$Y_t = A_t L_{1,t}^{\alpha} L_{2,t}^{1-\alpha} \tag{4}$$

where $0 < \alpha < 1$, A_t represents the technology shock. When the intermediate goods are produced, retailers purchase them at the wholesale price P_t^w in a competitive market, and transform them into final goods and sell them at price P_t. We denote the markup of final over intermediate goods as $X_t = \frac{P_t}{P_t^w}$. The producers maximize their profit by

$$\frac{Y_t}{X_t} - w_{1,t}L_{1,t} - w_{2,t}L_{2,t}$$

subject to 4.

Retailers are monopolistically competitive firms set their prices every period with some probability, $1-\theta$. They are Calvo-type price setters. This sector is standard in the literature used to incorporate sticky pricing into the model. We buy this from the literature directly. The optimal pricing decision of retailer i

$$\max_{P^0} \sum_{t=0}^{\infty} \theta^k E_t \left\{ \beta_1^k \frac{C_{1,t}}{C_{1,t+k}} \left[\frac{P_t^o - P_{t+k}^{\omega}}{P_{t+k}} Y_{t+k}(i) \right] \right\}$$

subject to its individual demand curve

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t^f$$

Incorporating monopolistically competitive retailer sector to the model allows us to have the New Keynesian Philips curve

$$\hat{\pi}_t = \beta_1 E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t \tag{5}$$

where $\kappa = \frac{1-\theta}{\theta}(1-\beta_1\theta)$.

There is also a central bank that implements a Taylor-type interest rate rule targeting inflation and output for the benchmark. For the benchmark case we equipped with current data rule as

$$R_t = R\left(R_{t-1}/R\right)^{r_r}\left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi}\left(\frac{y_t}{y}\right)^{r_y}\right]^{1-r_r} e_{R,t}$$

where r_π and r_y are the responding parameters for inflation and output deviations, respectively. r_r allows for interest rate inertia. R, π, y are the steady state values for interest rate, inflation, and output.

We do not provide details of solution and linearization of the model. Nevertheless, Appendix provides calibration, steady state, and the linearized form of the model. We calibrate the model with the US data and some parameters are selected from the empirical evidence.

Our model is different from the standard New Keynesian model. Borrowers are credit constrained by a proportion of their discounted expected future housing value. When the house price expectations or house prices increases, their net worth increases and it expands their borrowing capacity. Thus they can consume more and demand more housing which increase output and increase house prices more. Housing and the credit market have real effects on the economy and they propagate the effect of any shock on the economy by the mechanism described above. For this reason, housing prices and credit market deserve a close attention from the policymakers. Our economic environment allows us to study the stability of REE by considering credit markets and housing.

3. Interest Rate Rules

We evaluate several simple interest rate rules according to the determinacy and learnability of rational expectations equilibrium criteria. The linearized version of the benchmark interest rate rule incorporated in our model when $r_r = 0$ is

$$\hat{R}_t = r_\pi \hat{\pi}_t + r_y \hat{y}_t + \hat{e}_t \qquad \text{(Rule 1a)}$$

where variables with a hat correspond to percentage deviations from the steady state equilibrium. This interest rate rule is the current data rule where central bank reacts to deviations of inflation and output from steady state values. By using this rule in the model we assume that central bank has information on the current economic data. A central bank may want to react to housing prices and credit volume because of the reasons described in the previous section. If this is the case then the interest rate rule takes the form with $r_r = 0$:

$$\hat{R}_t = r_\pi \hat{\pi}_t + r_y \hat{y}_t + r_q \hat{q}_t + r_b \hat{b}_{2,t} + \hat{e}_t \qquad \text{(Rule 1b)}$$

$$\hat{R}_t = r_\pi \hat{\pi}_t + r_y \hat{y}_t + r_q \hat{q}_t + \hat{e}_t \quad \text{(Rule 1c)}$$

$$\hat{R}_t = r_\pi \hat{\pi}_t + r_y \hat{y}_t + r_b \hat{b}_{2,t} + \hat{e}_t \quad \text{(Rule 1d)}$$

Central bank may not have current data of the aggregate variables. It may react to the lagged variables or expectations of those variables. Thus, as in Bullard and Mitra (2002) we will also consider lagged data rule and forward-looking rule as

$$\hat{R}_t = r_\pi \hat{\pi}_{t-1} + r_y \hat{y}_{t-1} + \hat{e}_t \quad \text{(Rule 2)}$$

$$\hat{R}_t = r_\pi \pi^e_{t+1} + r_y \hat{y}^e_{t+1} + \hat{e}_t \quad \text{(Rule 3)}$$

where expectations are not necessarily rational. Agents form the expectations by using information available at time t-1.

If we investigate an answer for the two important policy questions "should central bank react to housing prices?" and "should central bank react to credit volume?" with lagged data and forward-looking rules, rules 2 and 3 need to be modified as in rule 1b, 1c and 1d. The modified version of rules 2 and 3 include housing and credit volume reaction components, as in current data case. Evaluating these innovative monetary policies under determinacy and learning context provides a rigorous examination of two important policy questions in a different perspective. In this paper, we will focus only on the first question.

4. Determinacy

In general equilibrium theory, the basic concepts evolve from the study of economic equilibrium. Basically, an equilibrium for a market based economy consists of an allocation and price levels at which the commodities of the economy are traded. The relationship between the prices at which commodities of the economy are traded and allocated with the properties of the economy that determine those prices and allocation is key to the study of equilibrium theory. While using the theory to study macroeconomics, we may encounter the shortcoming of not having a determinate equilibrium. The relationship between prices and fundamentals may not be unique. In the case of determinacy, given the preferences, technology and endowments, with exogenous shocks that shift any fundamental of the economy, we know how the economy evolves, since there is only one path for prices and allocation evolvement. If there is indeterminacy this relationship between prices and allocation with fundamentals breaks up and we will be unable to predict how the economy will behave. Non-fundamental shocks will affect the outcomes.

For the purpose of our study, determinacy is the case when we have unique relationship that gives a unique path of evolvement of the variables. In other words, determinacy is the case when we have one unique

convergence path of the endogenous variables to the unique steady state, given the exogenous and state variables. If the unique convergence path does not exist, then; there are multiple paths to the steady state where endogenous variables such as inflation and output respond to random events not related to economic fundamentals, such as preferences, technology or any factor that affects the structural relations determining inflation and output. Moreover, these equilibrium paths also include the equilibria in which fundamental disturbances cause fluctuations in inflation and output that are arbitrarily greater than the degree that actual structure of the economy has been affected. In these cases, macroeconomic stability can be ruined by self-fulfilling expectations. The general view accepts these situations as undesirable outcomes.

We do not show technical details on how indeterminacy is detected. This is pretty standard in the literature. Following Benhabib and Farmer (1998), we consider the models that allow us to have a system of equations which is the linearized form of the model around its steady state. [2] For our experiments, we follow the same method to detect the determinacy that the correct number of eigenvalues inside the unit circle should be satisfied given the number of exogenous and predetermined variables. We do not provide the details of the methodology since it is standard in the literature.

We plot the combination of policy coefficients (r_π , r_y) that gives determinacy and indeterminacy. We find that, even for the Taylor-type rules (not a pure-inflation-targeting rule) the Taylor principle, r_π>1, still suffices to have determinate outcome for a large range of r_y for every monetary policy rule we consider. We give the results of determinacy and learnability within the same graphs in the next section. In section 7, we also examine whether responding to housing prices causes indeterminacy. We find that for small values of housing coefficient, r_q, the determinacy region of policy coefficients does not change and for high values, i.e. r_q>0.5, it changes but not substantially.

Carlstrom and Fuerst (2007) examine the question "should monetary policy respond to asset prices?" from the vantage point of equilibrium determinacy. In their model, a rise in inflation tends to reduce firm profits which causes a decline in asset prices. Thus, if central bank also responds to the asset prices this will be a negative force to the central bank's overall response to inflation and the Taylor principle may not be satisfied. The issue of indeterminacy arises because of the negative relationship between inflation and asset prices. However in our model asset (housing) prices and

[2] We give the linearized form of the model in the Appendix.

inflation most likely move in the same direction and the same source of indeterminacy does not exist in our model.

The issue of indeterminacy has been raised in the monetary policy literature. Some of this research includes studies by Bernanke and Woodford (1997), Christiano and Gust (1999), Clarida et al. (2000) and Carlstom and Fuerst (2000, 2001). The common method of analyzing monetary policy rules in this context is to find a rational expectations equilibrium and if a monetary policy rule induces indeterminacy, to view that monetary policy rule as undesirable. If there is indeterminacy, then; there may exist many equilibria and the system will be highly volatile. Bernanke and Woodford (1997) study the behavior of the economy when central bank targets the forecast of inflation by setting interest rate to minimize the deviations of inflation forecasts from some specific target. They show that forecast-based policy rules in many cases lead to the non-uniqueness of the rational expectations equilibrium. Carlstom and Fuerst (2001) study money-in-the-utility-function models and show that the timing of the trade changes the conditions for a model to have determinacy. They conclude that in a sticky price model with cash-in-advance timing, a central bank should follow a backward-looking rule. Christiano and Gust (1999) study a limited participation model in which monetary non-neutrality is not coming from price stickiness but rather from credit market friction. They evaluate the different Taylor-type monetary policy rules in terms of their success in protecting the economy from bad outcomes like the ones seen in 1970s. High inflation faced during the 70s is viewed as the outcome of monetary policies that permitted self-fulfilling inflation expectations. Their results suggest that these self-fulfilling expectations can be ruled out by implementing a Taylor-type rule that responds aggressively to inflation and weakly to the output. Clarida et al. (2000) estimate a forward-looking monetary policy reaction function using the postwar US macroeconomic data for the periods before and after 1979. They argue that one of the reasons behind the unstable macroeconomic data before 1979 was the weak response of monetary policy to inflation, which allowed self-fulfilling inflation expectations. They embed the estimated monetary policy rules into a simple macroeconomic model to determine their implications for equilibrium properties of inflation and output. They find that the estimated rule for the period before 1979 is in the indeterminacy region of the calibrated macro model. The policy parameters, which are located in the region denoting indeterminacy, are accepted as being harmful. We work with our own model, shown in the Appendix, and determine the parameter space of monetary rules which induce determinacy and detect the harmful parameters of the different policy rules.

5. Learning

A monetary policy framework that induces indeterminacy is viewed as undesirable since agents may not be able to coordinate on an equilibrium. If an equilibrium is determinate, it is assumed that coordination will exist on that equilibrium. However, as Bullard and Mitra (2002) point out, it is not clear how and whether such coordination will exist. One additional criterion they suggest to evaluate a monetary policy is the potential of agents to learn the equilibrium. They argue that the central bank should choose a monetary policy rule that is associated with learnable rational expectations equilibria. The basic question is whether agents can learn the fundamental equilibrium if they do not possess rational expectations but are eligible to create forecasts by using recursive learning algorithms. We consider recursive least squares in our calculations and follow the methodology of Bullard and Mitra (2002) and Evans and Honkapohja (1999, 2001) to evaluate monetary policy rules whether they can learn fundamental equilibrium. We use the criterion of expectational stability to decide whether rational expectations equilibria are stable under real time recursive learning dynamics. Evans and Honkapohja (1999, 2001) show that for a wide variety of macroeconomic models, expectational stability of rational expectations equilibrium governs the local convergence of real time recursive learning algorithms.

We follow Evans and Honkapohja (2001) and Bullard and Mitra (2002) to define expectational stability (E-stability). The system, the linearized version of which is given in the Appendix, can be written in the form

$$y_t = \alpha + \beta E_t y_{t+1} + \delta y_{t-1} + \rho w_t \tag{6}$$
$$w_t = \varphi w_{t-1} + e_t$$

For learning analysis the basic required concept is mapping from the perceived law of motion (PLM) to the actual law of motion (ALM). We assume agents have PLM of the form

$$y_t = a + b y_{t-1} + c w_t$$

and forecasts

$$E_t y_{t+1} = a + b y_t + c \varphi w_t$$

If we put this to Equation 6 we get the ALM and then we can get mapping T(a,b,c) from PLM to ALM. E-stability conditions are determined by the differential equation:

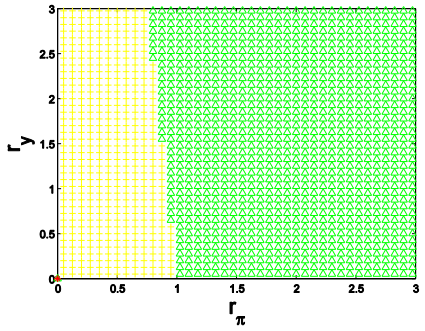
$$\frac{d}{dt}(a,b,c) = T(a,b,c) - (a,b,c) \tag{7}$$

A "Minimum State Variable" solution is E-stable if the differential equation is locally stable at that point. Evans and Honkapohja (2001) have shown that E-stability conditions obtained from the mapping from PLM to ALM provide conditions for the asymptotic stability of a rational expectations equilibrium (REE) under least squares learning.

6. Results

In this section, we give the results for determinacy and E-stability for the policy rules mentioned above. For the current data rule what we find is consistent with the literature; the determinacy condition is the same with the learnability condition. In other words, we can say every determinate equilibrium is also E-stable. We plot the combination of policy rule coefficients of output and inflation deviations that lead to a determinate and learnable equilibrium. Figure 1 shows that the region for $r_\pi > 1$ is both determinate and E-stable, which corresponds to the Taylor principle. The intuition is simple. Starting at an REE equilibrium, if inflation expectations rise by 1%, the interest rate needs to be increased by more than 1% to raise the real interest rate, to cool down the economy and to push the inflation back to its RE value. When the interest rate policy rule is also responding to output deviation in addition to inflation, like the one we use in our calculations, then $r_\pi > 1$ is still required but not that strictly, as r_y is also playing a role in the value of the interest rate. In Figure 1 we give the results based on the parameters with the calibration given in the Appendix. The condition $r_\pi > 1$ does not change much with the changes in parameters. However, we want to expose one parameter that makes a substantial improvement in the determinacy region. The indeterminacy (left) region changes substantially with the parameter κ of the New Keynesian Philips curve (Equation 5). The requirement of $r_\pi > 1$ for determinacy is not very strict for low values of κ.

Figure 1. E-stability of Current Data Rule



Parameter space that allows for determinacy and E-stability for the current data rule (rule 1), $r_r = 0$, $r_q = 0$ and $\theta = 0.75$. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria. Determinacy: $r_\pi > 1$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ satisfies E-stability.

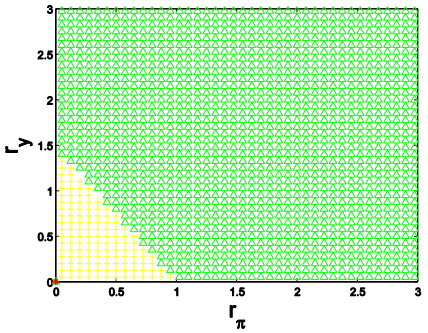
In Figure 2, we give the results for the same model and parameters with a lower value of κ. We see that the indeterminacy region has decreased substantially. Smaller values of κ mean, output deviations tend to affect

inflation less. This is a situation where monetary policy raises interest rates with positive output deviations which do not have high inflation pressures. This weakened link between inflation and output allows one to achieve determinacy with $r_\pi < 1$ and for some positive values of r_y.

The current data rule is criticized as being unrealistic; opponents of the current data rule argue that policymakers cannot observe the contemporaneous data when they decide on the interest rate. One suggestion in order not to have this simultaneity problem is to replace the contemporaneous data rule with the lagged data rule. Monetary policy responds to lagged data of inflation and output deviations:

$$\hat{R}_t = r_\pi \hat{\pi}_{t-1} + r_y \hat{y}_{t-1} + \hat{e}_t$$

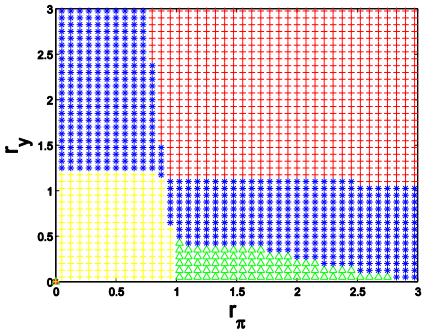
Figure 2. E-stability of Current Data Rule with High Price Stickiness



Parameter space that allows for determinacy and E-stability for the current data rule (rule1), r_r=0, r_q=0 and θ = 0.90. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable outcomes. Determinacy: $r_\pi > 1$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ satisfies E-stability.

In Figure 3, we give the determinate and learnable regions for the lagged data rule. The yellow region is indeterminate and E-unstable, where $r_\pi < 1$ and $r_y < 1.2$, the green region is determinate and E-stable, where $r_\pi > 1$ with low values of r_y, the red region is explosive, where $r_\pi > 1$ and high values of r_y ($r_y > 1.2$) and the rest of the regions, which are blue, are the determinate but E-unstable regions.

Figure 3. E-stability of Lagged Data Rule



Parameter space that allows for determinacy and E-stability for the lagged data rule (rule 2), r_r=0 and r_q=0. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria. Blue region with stars represents determinate and E-unstable outcomes. Red with squares represents explosive outcomes. Determinacy: $r_\pi > 1$ with $r_y < 1.2$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ with low values of r_y satisfies E-stability.

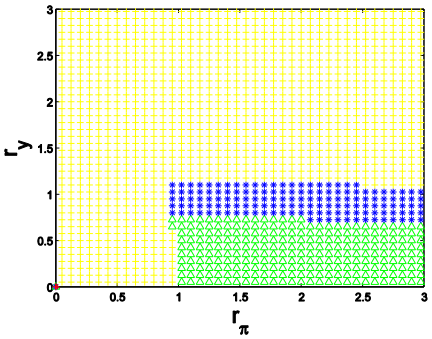
The other rule for monetary authority to consider is the forward-looking rule. Monetary policy responds to expectations of inflation and output deviations:

$$\hat{R}_t = r_\pi \pi^e_{t+1} + r_y \hat{y}^e_{t+1}$$

These are the expectations of agents using available data at time t-1 where π_t and y_t are not in the information set of agents. We adopt this assumption to avoid simultaneity problem between π_t and π^e_{t+1} ($E^*_t \pi_{t+1}$) and between y_t and y^e_{t+1} ($E^*_t y_{t+1}$). That means all the expectations of all the endogenous variables $E_t y_{t+1}$ (not only the output) at Equation 6 are linear functions of (1, y_{t-1}, w_t).

In Figure 4, we give the determinate and learnable regions for the forward-looking rule. The determinate and learnable region is the green region, where $r_\pi > 1$ and $r_y < 0.7$. There is small region of determinate and E-unstable region, with a blue color, roughly $r_\pi > 1$ and $0.7 < r_y < 1.1$.

Figure 4. E-stability of Forward-looking Rule



Parameter space that allows for determinacy and E-stability for the forward-looking rule (rule3) r_r=0 and r_q=0. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria. Blue region with stars represents determinate and E-unstable outcomes. Determinacy: $r_\pi > 1$ with $r_y < 1.1$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ with $r_y < 0.7$ satisfies E-stability.

From these exercises, we found that the determinacy requirements are not enough to bring about the learnability of an REE equilibrium. The E-stability criterion is also linked to the Taylor principle. What we see from the exercises is that $r_\pi > 1$ is necessary but not sufficient for learnability. Only for the current data rule for a wide scope of parameter values, $r_\pi < 1$ with high values of r_y leads to learnable outcomes but again for r_y=0, $r_\pi > 1$ is needed for learnability. With the same model, we examine the different monetary policy rules and find the coefficients that minimize the loss function of the central bank. We assume that central bank's loss function consist of the volatilities of the two variables – output and inflation in which central bank's goal is to pick policy parameters r_π, r_y and possibly r_q and r_b that minimize the loss function. We allow policy rule parameter values to be $0 \leq r_y \leq 2$, $1 < r_\pi \leq 3$. When we look at each coefficient that minimizes the loss function, we see that not all of them satisfy the determinacy and the learnability criteria. We show the possibility that even some policy coefficients are the best ones that minimize the deviations of output and inflation, the RE equilibrium with which they are associated may not be the one that is determinate and/or learnable. In addition to the Taylor principle, one also needs to check the learnability criterion to suggest policy parameters.

Role of Policy Inertia on Equilibrium

Policy inertia is a well-documented behavior of central banks in industrialized countries. For the US, empirical findings show that the Fed changes the short-term interest rate gradually. Rudebusch (1995) gives a statistical analysis that shows the target funds rate is changed in limited

amounts and is rarely reversed and the target level gradually increases or gradually decreases. Sack (1998) estimates a reaction function using the US data in the following form:

$$R_t^* = r^* + r_\pi \hat{\pi}_t + r_y \hat{y}_t$$

where $R_t{}^*$ is the funds target rate, π_t is the deviation of inflation from the target inflation, and y_t is the deviation of output from its potential level. The other parameters are coefficients. To incorporate the partial adjustment of the funds rate, it also estimates the following specification:

$$R_t = \theta R_{t-1} + (1-\theta) R_t^*$$

The estimated value of θ by OLS is 0.63, with a standard error of 0.08, which is a large and significant number. There are other estimates that are not far from this number. Monetary policy inertia is subject to the criticism that policymakers are not moving fast enough to respond effectively to new information. The explanations for the question of why central bankers are unwilling to respond quickly to incoming information about the economy may vary. Bullard and Mitra (2007) provide a new explanation to the existing literature, which hasn't been addressed. They study Woodford's (2003) model, which can be written as a system of three equations with the endogenous variables, inflation and output, and an exogenous variable, interest rate. They provide an analytical explanation and intuition for this simple model of how policy inertia enhances the possibility of the determinacy and learnability of an REE. They see this contribution of policy inertia to determinacy and learnability as a complementary result to the research, which deals with the question of why we observe policy inertia as a practice of central banks.

We study our benchmark model to see the contribution of policy inertia to the determinacy and learnability of the equilibrium. Kanik and Xiao (2010) use reaction functions without policy inertia for the same model that we have used in this paper, to measure the sum of volatilities with the news shocks. When we add inertia to the reaction function and do the same exercises with news shocks, we see higher volatility of output and inflation. An intuitive reason for this result might be that monetary policy is not responding quickly and large enough to the incoming information about the economy. Therefore, including inertia in the monetary policy increases the loss of the central bank. Our results show that including inertia does not increase welfare, but rather it contributes to the determinacy and learnability of the REE. With this exercise, we provide another evidence for the main findings of Bullard and Mitra (2007).

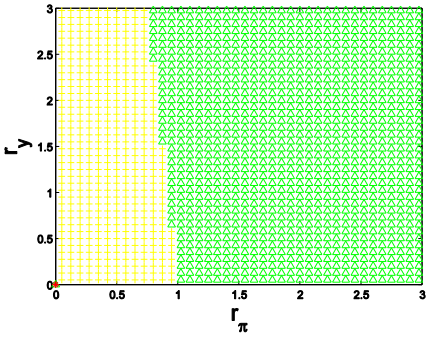
In the Figures 5, 6 and 7, as in the figures above, we give the combination of output and inflation coefficients for the determinacy and learnability of

the benchmark model. Now, we use the following specification for the interest rate reaction function:

$$\hat{R}_t = (1-r_r)r_\pi \hat{\pi}_t + r_y \hat{y}_t + r_r \hat{R}_{t-1} + \hat{e}_t$$

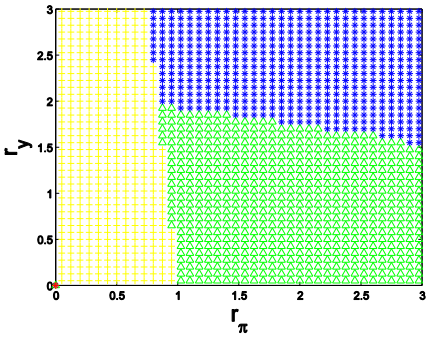
What is new in this form is only the degree of inertia, r_r. For the experiments, we set r_r to 0.65, which is not far from empirical estimates. We show the results for the contemporaneous data rule, the lagged data rule and the forward-looking rule in Figures 5, 6 and 7, respectively. The results are consistent with those of Bullard and Mitra (2007) who do not analyze the contemporaneous data rule, as the Taylor principle is enough to achieve both learnable and determinate outcome and there is not a large problem for monetary policy to be cautious about. However, we also get the results for the current data rule and confirm that inertia has no effect on determinacy and learnability under the current data rule. For the lagged data rule and forward-looking rule, in Figures 6 and 7, we see a highly extended region for learnable and determinate equilibria when compared to non-inertial policy which can be seen in Figures 3 and 4.

Figure 5. E-stability of Current Data Rule with High Interest Rate Inertia



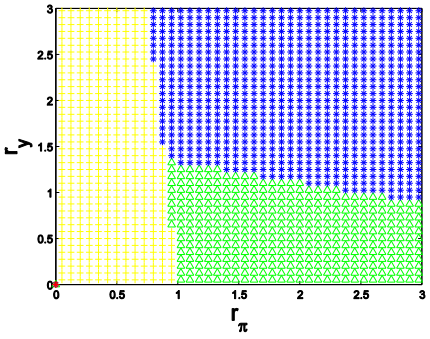
Parameter space that allows for determinacy and E-stability for the current data rule, $r_r = 0.65$, $r_q = 0$. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria.

Figure 6. E-stability of Lagged Data Rule with High Interest Rate Inertia



Parameter space that allows for determinacy and E-stability for the lagged data rule, $r_r = 0.65$, $r_q = 0$. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria. Blue region with stars represents determinate and E-unstable outcomes.

Figure 7. E-stability of Forward-looking Rule with High Interest Rate Inertia



Parameter space that allows for determinacy and E-stability for the forward-looking rule, $r_r = 0.65$, $r_q = 0$. Yellow region with squares represents the parameter space for indeterminacy and non-learnability. Green region with triangles represents determinate and E-stable equilibria. Blue region with stars represents determinate and E-unstable outcomes.

7. Should Central Bank Respond to Housing Prices?

The policy rules we have tried for determinacy and learnability do not consider cases of responding to asset prices. In the policy rule, we add housing price that makes the central bank reacting to housing price developments as in rule 1b and rule 1c. We worked with, again, three different rules; contemporaneous data, lagged data and forward-looking rule.

If central bank implements contemporaneous data rule, including housing prices into the interest rate policy rule does not make any change on the learnability region of coefficients.

If central bank implements forward-looking rule, including expected housing prices into the interest rule does not make any gain, does cause loss in the learnability region. In this case, agents do not have any current data information and form expectations based on the data in t-1.

If central bank implements a lagged data rule, including lagged values of housing prices into the interest rate rule does not make improvement on the learnability region of coefficients.

We see a substantial change for the lagged data rule when we include lagged house prices in the policy rule. We increase the policy parameter for housing step by step, and what we see is that the learnability region in Figure 3 disappears if r_q >0.3. For the other rules, there is loss in the learnability region but only for high values of the housing coefficient in the policy rule.

What is interesting about the analysis done for forward-looking and lagged data rules is the increase in the learnability region of policy coefficients when it is assumed that the central bank have the information on current housing prices and include it in its interest rate policy rule.

We present the results in Figure 8 for the lagged data rule and Figure 9 for the forward-looking rule. What we see is an improvement in the learnable region when we compare them with Figure 3 and Figure 4. Central bank has some gain by obtaining and using current housing data in the interest rate rule under adaptive learning.

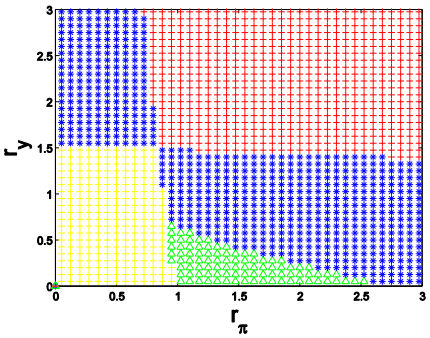
The results are not surprising for the lagged data rule, since what we see in Figure 1 is that, with current data rule, learnability is satisfied when the inflation parameter is bigger than one. These results are consistent with the idea that when the central bank does not respond to the current but the lagged endogenous variables, then it is not able to react to current shocks including sunspots. Then, what we expect is the volatility of variables will be higher and learnability of equilibrium will be much more difficult to achieve. Since the data that agents use to estimate the parameters of the economy and to make forecasts are much more volatile, that is why it is much harder to learn REE. Adding current housing prices to the interest rate rule mitigates the negative effect of not responding to the current output and inflation. When inflation and housing prices co-move, then a model with lagged inflation and current housing prices feedback interest rate rule actually reacts to the current inflation. That is why we expect less volatile past data for an economy with this feature and much easier learning of REE.

We do not provide the details but the learnable region for a policy rule with a feedback to the current inflation and lagged output gap is much higher than the one with lagged variables only, which supports our argument above.

The results show that if monetary authority does not have information of current data on inflation and output and the policy rule only responds to the lagged inflation and output gap or expected inflation and expected output gap then adding a current housing component to the policy rule provides learnable REE for a larger parameter space. Thus, responding to asset prices has a benefit in this case. However what is critical is the availability of accurate housing prices to the monetary policy.

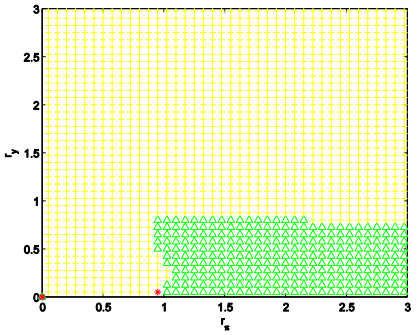
We did the same analysis for the interest rate policy rules where central bank reacts to the credit volume. What we see is loss of learnability regions of policy coefficients by including credit volume.

Figure 8. E-stability of Lagged Data Rule when Responding Current Housing Prices



Parameter space that allows for determinacy and E-stability for the lagged data rule, r_r=0 and r_q=0.5. Yellow region with squares represents parameter space for indeterminacy and non-learnability. Green region with triangle represents determinate and E-stable equilibria. Blue region with stars represents determinate and E-unstable outcomes. Red with squares represents explosive outcomes. Determinacy: $r_\pi > 1$ with $r_y < 1.2$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ with low values of r_y satisfies E-stability with extended region.

Figure 9. E-stability of Forward-looking Rule when Responding Current Housing Prices



Parameter space that allows for determinacy and E-stability for the forward-looking rule, r_r=0 and r_q=0.5. Yellow region with squares represents parameter space for indeterminacy and non-learnability. Green region with triangle represents determinate and E-stable equilibria. Determinacy: $r_\pi > 1$ with $r_y < 1.2$ satisfies determinate equilibrium. Learning: $r_\pi > 1$ with low values of r_y satisfies E-stability with extended region.

8. Taylor Type Policy Rule Estimation for the Fed

In this section we estimate a Taylor type rule for the Fed. We aim to reveal how the Fed has responded to inflation, output and in particular to house prices. When we get the estimation results, we can evaluate the Fed policy whether it has followed a learnable policy rule under our modeling framework. Standard form of Taylor type rule can be written by

$$R_t = r^* - (r_\pi - 1)\pi^* + r_\pi \pi_t + r_y y_t$$

where we use R is the federal funds rate, r^* is the equilibrium real federal funds rate, π^* target inflation, π_t is the inflation and y_t is the output gap. We use quarterly average of effective federal funds rate for R, GDP deflator over the contemporaneous and prior three quarters for π_t, HP-filtered data for y_t. This form of Taylor rule does not include interest rate inertia. For the estimation purposes, we add policy inertia and take differences to eliminate high serial correlation of errors. Our specification with policy inertia and without housing prices is:

$$\Delta R_t = (1 - r_r)(r_\pi \Delta \pi_t + r_y \Delta y_t) + r_r \Delta R_{t-1}$$

and the specification with inertia and housing prices is

$$\Delta R_t = (1 - r_r)(r_\pi \Delta \pi_t + r_y \Delta y_t + r_q \Delta q_t) + r_r \Delta R_{t-1}$$

We estimate these two interest rate rules for different periods to see whether policy reaction coefficients of the Fed to any variable has changed and whether after these changes coefficients are still in the parameter space that allows learnable REE in our model framework.

Table 1. Estimation Results

	$r_{\cdot}$	r_{π}	r_{y}	r_{q}	R^2
1987-2001	0.59	1.73	0.69		0.60
	(0.10)***	(0.73)**	(0.15)***		
1987-2001	0.57	1.16	0.44	0.39	0.61
	(0.10)***	(0.80)	(0.19)***	(0.20)*	
1987-2006	0.59	1.22	0.63		0.60
	(0.08)***	(0.58)**	(0.13)***		
1987-2006	0.57	0.94	0.51	0.14	0.61
	(0.08)***	(0.61)	(0.15)***	(0.13)	

Standard errors in parenthesis. *** p<0.01, ** p<0.05, * p<0.1

The results are given in Table 1. We estimate two equations for two different periods. 1987-2001 and 1987-2006. The reason is for the period 2002-2006 Fed has been criticized of ignoring the credit expansion and the rise of asset prices, in particular housing prices. This distinction of periods allows us to comment on those critics.

Coefficient for house prices is significant only at 10% level and only for the period 1987-2001. However, adding housing prices to the specification do not improve the fit of the standard model. These results show that Fed responded aggressively to inflation and output but does not consider housing prices, especially for the period 2002-2006. Nevertheless, the results for the specifications without housing show that Fed policy coefficients are in range of learnable REE under our model framework. However, it is not the case for the period 2002-2006 if we include housing prices in the monetary policy rule.

9. Conclusion

We evaluated different types of simple policy rules according to the determinacy and learnability of rational expectations equilibrium criteria and examined whether including housing prices in policy rule extends the range of parameter space that induces determinacy and learnability. We worked with three different rules; contemporaneous data, lagged data and forward-looking rule. Our results show that including asset prices in contemporaneous data interest rate rules in addition to the output gap and inflation has no positive effect for the learnability of equilibrium.

If monetary policy rule responds to the lagged inflation and output gap or expected inflation and expected output gap then adding the current housing component to the policy rule provides learnable REE for a larger parameter space.

We also examine the effects of interest rate inertia and price stickiness on E-stability of REE.

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Appendix

The Model

$$\frac{C_1}{Y}\hat{c}_{1,t}+\frac{C_2}{Y}\hat{c}_{2,t}=\hat{Y}_t$$

$$(\eta-1)\hat{L}_{1,t}-\hat{w}_{1,t}=-\hat{c}_{1,t}$$

$$(\eta-1)\hat{L}_{2,t}-\hat{w}_{2,t}=-\hat{c}_{2,t}$$

$$\hat{A}_t+\alpha\hat{L}_{1,t}+(1-\alpha)\hat{L}_{2,t}=-\hat{Y}_t$$

$$\hat{L}_{1,t}+\hat{w}_{1,t}=\hat{Y}_t-\hat{X}_t$$

$$\hat{L}_{2,t}+\hat{w}_{2,t}=\hat{Y}_t-\hat{X}_t$$

$$\frac{b_1}{Y}\hat{b}_{1,t}=\frac{b_2}{Y}\hat{b}_{2,t}$$

$$\hat{h}_{1,t}\frac{-qh_1}{Y}=\hat{h}_{2,t}\frac{qh_2}{Y}$$

$$\hat{c}_{1,t}=\hat{c}_{1,t+1}+E_t\hat{\pi}_{t+1}-\hat{R}_t$$

$$\hat{q}_t=(1-\beta_1)(\hat{d}_t-\hat{h}_{1,t})+\beta_1(\hat{q}_{t+1}-\hat{c}_{1,t+1})+\hat{c}_{1,t}$$

$$\frac{b_2}{Y}\hat{b}_{2,t}=\frac{C_2}{Y}\hat{c}_{2,t}+\frac{qh_2}{Y}\hat{h}_{2,t}+\frac{Rb_2}{Y}(\hat{b}_{2,t-1}+\hat{R}_{t-1}-\hat{\pi}_t)-\frac{1-\alpha}{X}(\hat{Y}_t-\hat{X}_t)$$

$$\hat{q}_t=\gamma_h E_t\hat{q}_{t+1}+(1-\gamma_h)(\hat{d}_t-\hat{h}_{2,t})+(1-m\beta_1)(\hat{c}_{2,t}-\omega E_t\hat{c}_{2,t+1})-m\beta_1 rr_t$$

$$\hat{b}_{2,t}=\hat{q}_{t+1}+\hat{h}_{2,t}+E_t\hat{\pi}_{t+1}-\hat{R}_t$$

$$\hat{\pi}_t=\beta_1+E_t\hat{\pi}_{t+1}-\kappa\hat{X}_t$$

$$\hat{R}_t=(1-r_r)r_\pi\hat{\pi}_t+r_y\hat{y}_t+r_r\hat{R}_{t-1}+\hat{e}_t$$

Capital letters represent steady state values for those variables and "^" are deviations from steady states.

C_1: Patient household consumption

C_2: Impatient household consumption

L_1: Patient household labor

L_2: Patient household labor

w_1: Patient household wage

w_2: Patient household housing asset

h_1: Patient household housing asset

h_2: Impatient household housing asset

Y: Output

π: Inflation

b_1: Patient household borrowing

b_2: Impatient household borrowing

X: Mark-up

q: Housing price

π: Inflation

R: Nominal interest rate

rr: Real interest rate

Steady State and Calibration

$$\beta_1 = 0.99,\ \beta_2 = 0.98,\ d = 0.08,\ \ \eta_1 = \eta_2 = 1.01,\ X = 1.05,$$

$$m = 0.90,\ \ \theta = 0.75,\ \ \pi = 1,$$

$$R = 1/\beta_1$$

$$\frac{C_2}{Y} = \frac{(1-\alpha)/X}{1+(R-1)\beta_1 md/(1-\gamma_e)}$$

$$\frac{qh_2}{Y} = \frac{C_2}{Y}\frac{d}{1-\gamma_e}$$

$$\frac{b_2}{Y} = \beta_1 m\frac{qh_2}{Y}$$

$$\frac{C_1}{Y} = 1-\frac{C_2}{Y}$$

$$\frac{qh_1}{Y} = \frac{C_1}{Y}\frac{d}{h_1}$$