Estimating NAIRU for the Turkish Economy Using Extended Kalman Filter Approach

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ABSTRACT

This paper estimates NAIRU (Non-Accelerating Inflation Rate of Unemployment) for the Turkish economy as an unobserved stochastic variable. In doing so, the study adopts an empirical framework that is based on a systems approach. More specifically, the framework combines an Okun-type relationship between output gap and unemployment gap with a Phillips curve equation, and also imposes stochastic laws of motion for NAIRU and potential output, while assuming the parameters to be time-varying. However, the requirement to simultaneously estimate parameters and to solve the state-space problem introduces nonlinearity, which can be handled by employing Extended Kalman Filter (EKF), i.e. the use of standard Kalman filter equations to the first-order Taylor approximation of the nonlinear model about the last estimate. Estimation results suggest that NAIRU moves in tandem with the actual unemployment, but it follows a more volatile path than the latter. Accordingly, the estimated NAIRU series reacts more sharply to the crises than the actual unemployment. This observation is in line with the prior studies reporting the relatively persistent nature of actual unemployment compared to the non-accelerating inflation rate of unemployment. All of the derived series are plausible and capture the significant turning points of the economy. As for coefficients, the time-varying parameters indicate a stable, yet quite a weak link between unemployment and inflation. Meanwhile, the coefficient of exchange rate in the Phillips curve equation suggests a weakening, but significant pass-through to inflation. Moreover, estimation results also point to the presence of considerable inertia in inflation. To sum up, findings of this study provide guidance for future research on NAIRU, which is an important tool for monetary policy. The findings also lay the basis for further work that may adopt EKF. But most importantly, this study emphasizes the need for a more flexible and comprehensive framework for the conduct of monetary policy.

Keywords: NAIRU, Systems approach, Phillips curve, Okun law, Time-varying parameter, Extended Kalman Filter, Inertia, Monetary policy.

JEL codes: C32, C63, E24, E31.

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“I have become convinced that the NAIRU is a useful analytic concept. It is useful as a theory to understand the causes of inflation. It is useful as an empirical basis for predicting changes in the inflation rate. And, it is useful as a general guideline for thinking about macroeconomic policy.”

(Stiglitz, 1997)

I. INTRODUCTION

Policymakers are confronting the increasing challenge to predict the future course of price dynamics in meeting their overriding goal of maintaining price stability. Another important challenge that the policymakers face in envisaging the general price outlook is to utilize all the available information. In doing so, NAIRU – the Non-Accelerating Inflation Rate of Unemployment stands out as a key variable in foreseeing the future course of price dynamics.

NAIRU is the rate of unemployment at which there is no upward or downward pressure on inflation rate.\(^1\) The concept arose in the wake of the popularity of the Phillips curve that summarized the observed negative correlation between unemployment and inflation (Phillips, 1958).\(^2\) This correlation, which was previously seen for the U.S. by

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\(^1\) An early form of NAIRU is found in the work of (Lerner, 1951), which referred to it as "low full employment" attained via the expansion of aggregate demand, in contrast with the "high full employment" which adds incomes policies (wage and price controls) to demand stimulation. NAIRU was later on introduced as the “non-inflationary rate of unemployment” in Modigliani and Papademos (1975) as an improvement over the “natural rate of unemployment” concept, which was initially put forward by Phelps (1967) and Friedman (1968) as the level of unemployment that is consistent with equilibrium in the structure of real wages. The analysis supporting the natural rate hypothesis was controversial and empirical evidence suggested that the natural rate varied over time in ways that could not easily be explained by changes in the labor market structure. As a result, the natural rate terminology was largely supplanted by that of the NAIRU. This corresponded to the rate of unemployment below which inflation would accelerate, but without making a commitment to any particular theoretical explanation or a prediction that the rate would be stable over time.

\(^2\) The inverse relationship between inflation (money wage changes) and unemployment was documented by Phillips (1958) and later extended by the significant contributions from Samuelson and Solow (1960), which postulated the inverse relation between unemployment and inflation using price inflation instead of nominal wage inflation. The inverse relationship between inflation and unemployment, which was described via the Phillips curve, implied that it was possible for governments to tolerate higher rate of inflation in return for lower unemployment and this trade-off became an essential part of the policymaking process. However, during the 1970s, Phillips curve faced serious attacks as many countries experienced simultaneously high levels of inflation and unemployment, also known as stagflation. On the theoretical side, Phelps (1967) and Friedman (1968) rejected the idea of a long-run trade-off and suggested that a trade-off between unemployment and inflation would only be possible in the short run by including expected inflation and the natural rate of unemployment to the Phillips curve equation. Accordingly, the trade-off arises due to the inability of agents to adjust their expectations to anticipated inflation in the short-run, whereas, in the long-run, agents adjust their expectations and actual unemployment returns to the natural rate of unemployment. However, Lucas (1972) showed that, assuming rational expectations, agents may adjust their expectations quickly, so Phillips curve is vertical even in the short run. This New Classical view was later attacked by the New Keynesian Approach, which argued that there might be an inflation-
Fisher (1926), persuaded some analysts that it was impossible for governments to simultaneously target low unemployment and price stability. Therefore, it was government's duty to settle on an unemployment and inflation combination, which is optimal in terms of social welfare.

In view of the fact that NAIRU is derived by exploiting the short-run tradeoff between unemployment and inflation, the gap between actual unemployment and NAIRU indicates whether there is a risk of inflationary build-up in the economy. In other words, there must be some level of unemployment, i.e. the NAIRU, which is consistent with a stable inflation (Ball and Mankiw, 2002). Therefore, if a contractionary monetary policy shock increases unemployment above NAIRU, inflation rate will decelerate, while the inflation rate will accelerate if an expansionary monetary shock decreases the unemployment rate below NAIRU.

NAIRU is often associated with the concept of natural or structural unemployment, which is the unemployment rate that is identified with the structural, institutional or behavioral characteristics of the economy (Fabiani and Mestre, 2000). However, the non-accelerating inflation rate of unemployment may not always coincide with the natural rate of unemployment in the short run. In other words, the unemployment rate consistent with a stable inflation may deviate from its long-run equilibrium value when shocks have hysteresis effects on the labor market dynamics.3

In addition, NAIRU may well vary over time or inflation may accelerate even if unemployment declines to rates that are compatible with stable inflation (Estrella and Mishkin, 2000). Besides, at times when there are large swings in oil or raw material prices, it is clear that unemployment would have to fluctuate sharply in order to stabilize inflation (Boone et al., 2002). Thus, measuring NAIRU is challenging both at the conceptual and the empirical level.

unemployment trade-off in the short run, even assuming rational expectations, due to the presence of nominal rigidities in prices and wages. This implies that prices and wages cannot instantaneously adjust to changes in economic conditions (Fuhrer and Moore, 1995; Gali and Gertler, 1999; Clarida et al., 1999; Blanchard and Gali, 2007). The Phillips curve is now seen as too simplistic, with the unemployment rate replaced by more accurate predictors of inflation.

3 The hysteresis effect was initially introduced by Blanchard and Summers (1987a). Other seminal works on the hysteresis effect are Blanchard and Summers (1987b, 1987c) and Ball (2009).
There are numerous techniques developed to measure NAIRU. In this regard, NAIRU can be modeled from an economic theory perspective based on a Cobb-Douglas production function setting (Layard et al., 1991; Nickell, 1997). Alternatively, NAIRU can be modeled as a deterministic function of time (Staiger et al., 1997a, 1997b; Cross et al., 1998) or as a function of demographics and labor market dynamics (Weiner, 1993; Staiger et al., 1997b). NAIRU can also be modeled as an unobserved stochastic process (King et al., 1995; Staiger et al., 1997a; Gordon, 1997).

Another method for measuring NAIRU is through detrending techniques that can be applied by adopting simple statistical approaches. The detrending method may be univariate, bivariate or multivariate. In this regard, a simple way for univariately estimating NAIRU is by regressing unemployment on a linear time trend or a quadratic time trend. Another way for univariately estimating NAIRU is to implement filtering techniques, which are the Hodrick-Prescott, Baxter-King and Christiano-Fitzgerald filter. These filters basically split unemployment series to its cyclical and trend component.

NAIRU can be estimated bivariately on the basis of a Phillips curve equation assuming that shifts of the Phillips curve share a common trend with the unemployment rate (Laubach, 2001). Finally, NAIRU can be estimated via a multivariate filter by using a model that specifies the relationship between inflation, output and unemployment through the Phillips curve equation and the Okun law (Laxton and Tetlow, 1992; Richardson et al., 2000; Benes and N'Diaye, 2004). In this setting, exogenous variables are also taken into consideration in order to account for supply-side factors that explain inflation.

Another methodology for estimating the non-accelerating inflation rate of unemployment is to treat NAIRU as an observed variable. In doing so, an unobserved components model is developed where Kalman filter features out as the appropriate estimation algorithm. Kalman Filter has been extensively used in the recent economics literature as a recursive estimation technique. It is a powerful algorithm that can be easily employed in linear state-space models, as noted in Harvey (1990). Kuttner (1994) is a

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4 For further details, see Hodrick and Prescott (1997), Baxter and King (1999) as well as Christiano and Fitzgerald (2003).
5 For further details, see Kalman and Bucy (1961), Kalman (1960) and Hamilton (1994a, 1994b).
A seminal paper that utilizes this approach for analyzing the U.S. economy, while Gerlach and Smets (1999) adopt this approach for the European economies.

Even though these methods perform well in industrialized economies with rare incidences of excessive boom-bust cycles, they fail in emerging market economies where extreme volatility is typical. In order to better capture the volatility and the structural change that seem to be likely in these economies, one might allow the parameters of an unobserved components model to vary over time. However, this requires the adoption of a non-linear state-space form.

In a non-linear state-space setting where the state variables and the time-varying parameters need to be estimated simultaneously, the EKF can be proposed as the only possible algorithm. Even though EKF is a powerful algorithm for solving non-linear state-space models, it has been employed in a rather limited number of studies (Grillenzoni, 1993; McKiernan, 1996; Bacchetta and Gerlach, 1997).

As for the Turkish economy, there are only a few studies that implement EKF (Özbek et al., 2003; Özbek and Özlace, 2005; Özbek and Efe, 2000, 2004; Kara et al., 2007; Kara et al., 2007). However, these previous attempts utilizing EKF do not estimate NAIRU. Meanwhile, there are several studies that estimate NAIRU in Turkey (Şıklar et al., 1999; Yavan, 1997; Kaya and Yavan, 2007; Bildirici, 1999; Yiğit and Gökçe, 2012; Temurlenk and Başar, 2012; Gianella et al., 2008). Yet, none of the studies adopt a non-linear framework.

The absence of previous studies on NAIRU that consider the highly volatile nature of the Turkish economy, leaves us with a gap for estimating the non-accelerating inflation rate of unemployment using time-varying parameters in a state-space form. This gap establishes the main motivation of this study. To our knowledge, this is the first formal attempt to estimate NAIRU for the Turkish economy in a non-linear setting.

We believe that this paper serves useful in several ways. First of all, findings of this study provide guidance for future research on NAIRU, which is an important tool for

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monetary policy. In addition, the study contributes to the existing literature by jointly estimating NAIRU and its time-varying relationship with inflation in a Phillips curve setting. The estimation of NAIRU in this multivariate framework also produces other significant variables like potential output, output gap and inflation. Moreover, the results shed light on the course of time-varying parameters that indicate inflation persistence and the contribution of demand and supply-side factors to inflation. Furthermore, the findings also lay the basis for further work that may adopt EKF. But most importantly, this study emphasizes the need for a more flexible and comprehensive framework in the conduct of monetary policy.7

It should be underlined that this paper does not intend to explain the evolution of unemployment dynamics in the Turkish economy, nor does it seek to determine the underlying structural forces driving the natural rate of unemployment. Also, the paper does not attempt to explain price developments, but it solely tries to exploit the information contained in the price dataset in order to extract the unobservable NAIRU.

It should also be kept in mind that the models tested in the paper are chosen according to their in-sample properties and the degree to which they are able to match the behavior of the original series. Thus, the model selection is based on the informal optimization of the plausibility of the resulting estimates, but without any regard to their forecasting properties.

The organization of the paper is as follows. The next section presents the baseline empirical model on NAIRU and its state-space representation as well as the alternative model specification with the corresponding state-space representation. The third section discusses the estimation results, which are followed by a further analysis imposing dynamic homogeneity under both specifications as well as the interpretation of time-

7 Obviously, a more flexible and comprehensive framework in the conduct of monetary policy can be provided by means of a more comprehensive set of policy instruments. This can further be strengthened by legal amendments to the central bank law and subsequent changes to the operational framework. As for Turkey, the CBRT has already increased flexibility and the scope of the monetary policy by conducting a new framework starting from late 2010. In this regard, the conventional inflation targeting regime was modified by adopting financial stability as a supplementary objective and a new monetary policy mix was implemented that included additional policy tools for pursuing multiple objectives (Başçı and Kara, 2011; Kara, 2013, Alper et al., 2012; Kılıçarslan and Özel, 2012). The new monetary policy framework, which observed financial stability as a supplementary objective, was already backed by the required legal basis. In other words, as stipulated in Central Bank Law No.1211, in addition to maintaining the price stability objective, the CBRT is liable to take precautions for enhancing the stability of the financial system. In doing so, the CBRT is empowered to use, determine and implement monetary policy instruments at its own discretion (Central Bank Law No.1211 is available at [http://www.tcmb.gov.tr/yeni/eng/](http://www.tcmb.gov.tr/yeni/eng/)).
varying parameters. Finally, the last section concludes this paper. The state-space representation of the EKF is provided in the Appendix.

II. SYSTEM SPECIFICATION

The set of models that have been used to estimate NAIRU is essentially based on a system of equations. The system of equations, which constructs our empirical framework, is in the spirit of Fabiani and Mestre (2004). Accordingly, the system is composed of a Phillips curve, which determines the relation between unobserved cyclical factors and inflation; an Okun-type relationship that links output gap to unemployment gap; and a set of equations defining the law of motion for potential output and NAIRU.

II.1. Baseline Model Specification

The baseline model specification is in the spirit of Fabiani and Mestre (2004), while also assuming time-varying parameters given the highly volatile nature of the Turkish economy. The variables used in the baseline model are as follows: \( \pi \) is the inflation rate (first difference of the log of price index); \( z \) is a vector of supply-side variables (normally taken to be changes in import prices; real exchange rate or the nominal exchange rate) that pose pressure on inflation; \( u \) is the unemployment rate; \( y \) is the (log of) output level; \( u^* \) and \( y^* \) represent the NAIRU and (the log of) potential output, respectively. Correspondingly, \( u_{gap} \) and \( y_{gap} \) are unemployment gap and the output gap.

The system is composed of the following seven equations:

Inflation is assumed to behave according to accelerationist-type Phillips curve equation such that:

\[
\pi_t = \alpha_{1,t}\pi_{t-1} + \alpha_{2,t}\pi_{t-2} + \alpha_{3,t}u_{gap,t} + \alpha_{4,t}z_t + \epsilon_t
\]

According to the above equation, inflation is assumed to be a function of built-in inflation, demand-pull inflation and cost-push inflation, which can be captured by the inclusion of lagged inflation terms, the unemployment gap and the change in nominal
exchange rate.\textsuperscript{8} The coefficients $\alpha_{1,t}$ and $\alpha_{2,t}$, which show the degree of inflation persistence, are expected to be greater than zero. The coefficient $\alpha_{3,t}$ denotes the degree of demand-pull inflation and it is expected to be less than zero. Finally, $\alpha_{4,t}$ is the degree of pass-through from exchange rate to inflation and it is expected to be greater than zero. Obviously, $\varepsilon_{i}^{\pi}$ is the disturbance term.

Okun law runs from output gap to unemployment gap such that:

$$y_{gap_{i}} = \beta_{1,t}y_{gap_{i-1}} + \varepsilon_{i}^{y_{gap}}$$

(2.2)

where $\beta_{1,t}$ is expected to be negative and $\varepsilon_{i}^{y_{gap}}$ is the error term.

The unemployment gap is modeled as an autoregressive process:

$$u_{gap_{i}} = \delta_{1,t}u_{gap_{i-1}} + \varepsilon_{i}^{u_{gap}}$$

(2.3)

where $\delta_{1,t}$ is expected to be positive and less than unity, while $\varepsilon_{i}^{u_{gap}}$ is the error term.

The Beveridge-Nelson (1981) decomposition of output and unemployment is as follows:

$$y_{i} = y_{i}^{*} + y_{gap_{i}}$$

(2.4)

$$u_{i} = u_{i}^{*} + u_{gap_{i}}$$

(2.5)

Potential output and NAIRU are assumed to follow a local linear trend model:

$$y_{i}^{*} = y_{i-1}^{*} + \gamma_{i-1} + \varepsilon_{i}^{y_{i}^{*}}$$

(2.6)

$$u_{i}^{*} = u_{i-1}^{*} + \eta_{i-1} + \varepsilon_{i}^{u_{i}^{*}}$$

(2.7)

\textsuperscript{8} Based on the triangle model of Gordon (1991), inflation is viewed to have three root causes: built-in inflation (inflation results from past events and persists in the present), demand-pull inflation (falling unemployment rates or rising real gross domestic product feeds into inflation), and cost-push inflation (increases in the cost of goods and services raise inflation).
where the two stochastic trends $\gamma$ and $\eta$ are defined as:

$$
\gamma_t = \gamma_{t-1} + \varepsilon^\gamma_t
$$

(2.8)

$$
\eta_t = \eta_{t-1} + \varepsilon^\eta_t
$$

(2.9)

Obviously, $\varepsilon^\gamma_t$, $\varepsilon^\mu_t$, $\varepsilon^\tau_t$ and $\varepsilon^\eta_t$ are the disturbance terms.

The model can be represented in state-space as follows:

$$
x(t) = Fx(t-1) + Gu(t) + e_1(t)
$$

(2.10)

$$
y(t) = Hx(t) + e_2(t)
$$

(2.11)

where $x(t)$ is the state vector, $y(t)$ is the observation vector, $F$ is the transition matrix, $H$ is the observation matrix and $G$ is a known matrix. $e_1(t)$ and $e_2(t)$ denote vectors of normally distributed i.i.d. shocks, which are assumed to be uncorrelated with each other and have covariance matrices $R_1$ and $R_2$, respectively. Furthermore, $u(t)$ is the vector of exogenous variables.

The measurement equation, which shows the evolution of the observed variables (inflation, output and unemployment), can be described as a function of the observed state variables as follows:

$$
\begin{bmatrix}
\pi_t \\
y_{t-1} \\
ugap_t \\
y_g^* \\
ugap^* \\
y_t \\
\eta_t
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_{t-1} \\
ugap_t \\
y_g^* \\
ugap^* \\
y_t \\
\eta_t
\end{bmatrix}
$$

(2.12)

The transition equation can be expressed as:
II.2. Alternative Model Specification

In order to analyze the robustness of our results to different model specifications, we investigated the possibility that the output gap might better capture the inflation dynamics than the unemployment gap. In this respect, instead of the unemployment gap, output gap is used as an indicator for measuring the stance of aggregate demand in the Phillips curve. Correspondingly, Okun law is reversed from unemployment gap to output gap. Therefore, the baseline model is modified and equations (2.1) and (2.2) are replaced with the following equations:

\[ \pi_t = \alpha_{1,t} \pi_{t-1} + \alpha_{2,t} \pi_{t-2} + \alpha_{3,t} ygap_{t-1} + \alpha_{4,t} \zeta_t + \varepsilon_t^\pi \]  \hspace{1cm} (2.1')

\[ ugap_t = \beta_{1,t} ygap_{t-1} + \varepsilon_t^{ugap} \]  \hspace{1cm} (2.2')

where \( \alpha_{3,t} > 0 \) and \( \beta_{1,t} < 0 \). Furthermore, the output gap is modeled as an autoregressive process and hence equation (2.3) is modified as:

\[ ygap_t = \delta_{1,t} ygap_{t-1} + \varepsilon_t^{ygap} \]  \hspace{1cm} (2.3')

Clearly \( \delta_{1,t} > 0 \) and assumed to be less than unity for stationarity. These modifications imply a re-definition of the transition equation as below:
Meanwhile, the observation equation remains unchanged.

### III. ESTIMATION AND EMPIRICAL RESULTS

This section reports and discusses the estimation results of the models described above. The model utilizes quarterly data on inflation, output, unemployment and nominal exchange rate for the Turkish economy between 2000Q1 and 2013Q3.\(^9\)\(^{10}\) Inflation is measured as the logarithmic difference of the seasonally adjusted consumer price index. Output is seasonally adjusted Gross National Product series in logs and unemployment is seasonally adjusted series in percentages. Exchange rate is the logarithmic difference of the USD/TRY spot rate. Seasonal adjustment is handled via TRAMO/SEATS (Gómez and Maravall, 1996). The initial values are set according to (Chan and Hsiao, 2011).\(^{11}\) All estimations are conducted using Matlab.

\(^9\) Our publicly available data come from the electronic data dissemination system of the CBRT at [http://tcmbf40.tcmb.gov.tr/cbt.html](http://tcmbf40.tcmb.gov.tr/cbt.html).

\(^{10}\) The time period covered in the analysis is crucial for the Turkish economy as it witnessed major crises and regime changes. To be more specific, between 2000Q1 and 2001Q1, the CBRT conducted an IMF-backed exchange-rate-based stabilization (ERBS) program, which eventually collapsed with the outbreak of the financial crisis in February 2001. Between 2001Q1 and the end-2001 is the period of crisis management and transition to implicit inflation targeting. As of 2002, the CBRT has conducted implicit inflation targeting followed by the implementation of inflation targeting regime starting from 2006. In the last quarter of 2008, the CBRT had to face challenges originating from the occurrence of the global crisis in September 2008. Consequently, the CBRT adopted anti-crisis measures, which were later withdrawn starting from April 2010. Finally, as of end-2010, the CBRT has conducted a new monetary policy framework, which departed from strict inflation targeting by the inclusion of financial stability as a supplementary objective to its primary goal of maintaining price stability (Ertuğrul and Seçlik, 2001; Kibrițçioğlu et al., 2002; Başçı and Kara, 2011; Başçı et al., 2007; Kara, 2008, 2013).

\(^{11}\) The state-space representation for EKF is given in the Appendix to the paper. Accordingly, it should be noted that the EKF needs predefined \(P_0, Q, R, \hat{x}_0\) for initialization. It is acceptable to assign \(P_0\) arbitrarily, however, the values must be large enough to allow good tracking of the parameters. If the states are measured, \(\hat{x}_0\) can be specified by taking the average of the first few data points.
III.I. Baseline Model Results

The baseline model results indicate that NAIRU follows a similar yet more volatile path than unemployment. This is an expected outcome given the reported evidence on the higher sensitivity of NAIRU to economic fluctuations than the actual unemployment as well as the relatively higher persistence of actual unemployment than NAIRU (Clark and Laxton, 1997). Accordingly, the hovering of unemployment rate during crisis periods in 2001 and 2008 is accompanied by a sharper increase in NAIRU. In return, unemployment gap falls below zero, thereby causing the inflation rate to increase during the same period.

Meanwhile, during periods when the NAIRU follows a stable path and moves close to actual unemployment, the inflation is also stable, i.e. non-accelerating as expected. This coincides with the implementation of implicit inflation targeting from 2002 to 2005 and the launch of inflation targeting regime as of 2006. However, the NAIRU starts to climb during the global crisis period. After reaching a peak as the adverse effects of the global crisis manifest on the Turkish economy, the NAIRU starts to fall down gradually as anti-crisis measures are implemented. Finally, the NAIRU follows a relatively steady path after the adoption of the new monetary policy mix.

As for potential output, the baseline model produces a series that is sensitive to the crisis episodes in the economy. In other words, estimation results suggest that the potential output declines during crisis periods in 2001 and 2008, whereas it continues to trend upwards otherwise. The model is also successful in capturing the dynamics of inflation over the period of analysis. Inflation settles on a downward course with the adoption of the ERBS program, which is then halted with the outbreak of the financial crisis in 2001 when the inflation surges significantly. Inflation re-settles on a declining track during the implicit inflation targeting regime. Afterwards, inflation follows a volatile, yet stable path (Figure 1).
III.2. Alternative Model Results

The NAIRU and unemployment gap estimations produced by the alternative model specification are very similar to the baseline model results. However, the alternative model yields a more volatile potential output and output gap than the original model. This is basically due to the fact that unlike the baseline model where unemployment gap is included in the Phillips curve equation, the output gap enters the Phillips curve equation in the alternative model specification, and so NAIRU and
unemployment gap are indirectly derived from potential output and the output gap. This obviously causes the output gap to be estimated with a higher precision. Meanwhile, inflation estimations are quite similar in both specifications (Figure 2).

**Figure 2. Alternative Model Results**

- **NAIRU (Baseline Model)**
- **NAIRU (Alternative Model)**
- **Unemployment Gap (Baseline Model)**
- **Unemployment Gap (Alternative Model)**
- **Potential Output (Baseline Model)**
- **Potential Output (Alternative Model)**
III.3. Sensitivity Analysis Imposing Dynamic Homogeneity

Dynamic homogeneity is the condition that permanent changes in inflation should not affect output in the long run. Imposing dynamic homogeneity to the above system of equations implies that the existence of a trade-off between inflation and output is only limited to short run. Hence, the restriction of dynamic homogeneity enables to derive a more meaningful NAIRU (Greenslade et al., 2003; Batini and Greenslade, 2006). The restriction also implies that prices are determined by nominal factors such as wages and imported costs (Gómez and Julio, 2000).

One way of imposing dynamic homogeneity to a system of equations is to restrict the sum of explanatory inflation terms in the Phillips curve equation to be equal to unity. Another way to impose the same condition is to use differenced inflation terms in the Phillips curve equation, and so the unity restriction is automatically satisfied (Fabiani and Mestre, 2004).
Since our original Phillips curve equation does not contain any differenced inflation terms, this paper sticks to the former approach in imposing dynamic homogeneity. Thus, the sum of lagged inflation terms in the Phillips curve equation is restricted to be equal to unity.

III.3.i. Estimated Series under Dynamic Homogeneity

The sensitivity analysis demonstrates that NAIRU follows a smoother path under the dynamic homogeneity restriction in the baseline model, while it follows a more volatile path in other specifications. Accordingly, the unemployment gap series estimated under the dynamic homogeneity restriction in the baseline model moves quite opposite to the unemployment gap series in other models. The unemployment gap series derived in this specification displays a marked divergence from unemployment gap series estimated under other specifications, especially prior to 2001 and after 2008. In other words, unemployment gap estimated under the dynamic homogeneity restriction in the baseline model points to a higher inflation, while unemployment gap series estimated under other model specifications implies a lower inflation during these episodes (Figure 3).

Dynamic homogeneity constraint does not notably affect potential output and output gap estimations in the baseline model. However, imposing the dynamic homogeneity restriction causes potential output and output gap series to react more sharply to the crisis episodes in the alternative model specification. As for inflation, the constraint for dynamic homogeneity leads to virtually no change under both specifications (Figure 3).

**Figure 3. Comparison of Series**

![Figure 3. Comparison of Series](image-url)
As a final step in the search for an improvement in our model specification and to better capture the dynamics of inflation, estimations are augmented with a smoothing procedure. Accordingly, the inflation series is smoothed using the Rauch-Tung-Striebel (RTS) smoothing algorithm. The RTS is a two-pass smoother, which consists of a forward pass EKF with a backward recursion smoother. The RTS smoother is successful in enabling the model-based inflation figures to better mimic the actual inflation (Figure 4).

### III.3.ii. Time-Varying Parameters

The analysis of time-varying parameters reveals that the impact of the past period’s inflation on current inflation is the weakest under the baseline model, whereas, inflation is subject to the highest persistence under alternative model specification with the dynamic homogeneity constraint. The results show that the coefficient of $\pi_{t-1}$ declines over time and reaches a lower plateau in all specifications, except for the baseline model yielding a stable coefficient throughout the period of analysis (Figure 5).

The impact of $\pi_{t-2}$ on current inflation is initially the strongest in the baseline model specification under dynamic homogeneity constraint, while the weakest impact is produced under the alternative model specification. The coefficient of $\pi_{t-2}$ declines in both specifications with the dynamic homogeneity constraint. On the other hand,

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12 For further details on the RTS smoother, see Rauch et al. (1965).
13 Smoothing is applied to all series including the time-varying parameters. However, the results are not reported here as the algorithm produces too smooth estimates.
coefficient of $\pi_{t-2}$ is stable in the alternative model, whereas it increases over time in the baseline model (Figure 5).

**Figure 5. Inflation Persistence**

![Coefficient for $\pi_{t-1}$ vs. Coefficient for $\pi_{t-2}$](image)

The analysis shows that demand-pull inflation is weak, yet stable. This can be proven by the steady course of coefficients for $\text{ugap}_{t-1}$ and $\text{ygap}_{t-1}$ over the period of analysis. Imposing dynamic homogeneity leads to higher values for both coefficients implying that demand-side factors are more effective in inflation dynamics if dynamic homogeneity is imposed (Figure 6).

**Figure 6. Degree of Demand-Pull Inflation**

![Coefficient for $\text{ugap}_{t-1}$ vs. Coefficient for $\text{ygap}_{t-1}$](image)

Finally, the analysis results indicate that cost-push inflation as measured by the degree of pass-through from exchange rate to inflation has declined considerably over the period of analysis. The pass-through coefficient is slightly higher under alternative specification than the baseline model with or without the dynamic homogeneity constraint (Figure 7).
V. CONCLUSION

Estimating the relationship between nominal and real variables in the Turkish economy is challenging given the instability that the Turkish economy has suffered in its recent past. Crisis episodes as well as regime shifts introduce structural breaks to the economy, which makes it even harder to estimate a plausible relation. In this regard, the disinflation period that the economy has undergone throughout the analysis period features out as another major challenge.

NAIRU is a vital concept in explaining the extent to which aggregate demand may expand without accelerating inflation. NAIRU, by definition, is the unemployment rate when inflation tends to be stable. Yet, attempting to model NAIRU in a period where inflation has not been stable is obviously a problem. This problem is even more intriguing for the Turkish economy, which is characterized by highly volatile output and rapidly changing macroeconomic dynamics. Apparently, conventional methods for estimating NAIRU may fail to capture this volatility, and produce too smooth trends.

This study takes the above discussion as its starting point and attempts to estimate NAIRU for the Turkish economy. The system of equations, which constructs our empirical framework, is in the spirit of Fabiani and Mestre (2004). Accordingly, the system is composed of a Phillips curve, which determines the relation between unobserved cyclical factors and inflation; an Okun-type relationship that links output gap to unemployment gap; and a set of equations defining the law of motion for potential output and NAIRU.
In view of the highly volatile nature of the Turkish economy, this study improves over the methodology in Fabiani and Mestre (2004) by introducing time-varying parameters into the model. Since time-varying parameters and state variables are estimated simultaneously, the model presents non-linearity, which can be handled via EKF- the use of standard Kalman filter equations to the first-order Taylor approximation of the nonlinear model about the last estimate.

EKF is a useful algorithm that can successfully control the issue of nonlinearity introduced by the requirement to simultaneously estimate time-varying parameters and solve the state problem. Moreover, the use of EKF avoids the problem of finding a ‘too smooth trend’ without having to resort to the strong restrictions that are imposed on the parameters in previous studies.

The results reveal that the estimated parameters are fairly reasonable. NAIRU moves in tandem with the actual unemployment, but it follows a more volatile path than the latter. Consequently, the estimated NAIRU series is more responsive to the crisis periods than the actual unemployment, which is characterized by a relatively more persistent nature.

The same observation is true for potential output, which follows a more volatile path than the actual output. Hence, unlike earlier work on NAIRU, this study succeeds in deriving NAIRU and potential output series that do not seem to have an overly smooth trend. Accordingly, the two major crises in 2001 and 2008 are successfully captured by the estimated NAIRU series.

The time-varying parameters indicate a stable, but weak demand-pull inflation as evident by the considerably low and steady parameter for the unemployment gap. This result is in line with the previous studies in the literature citing the poor link between unemployment and inflation in the Turkish economy. This can be attributed to the fact that unemployment and inflation have very different dynamics and determinants.\(^\text{14}\) In

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\(^\text{14}\) Economic theory and previous empirical studies have identified a number of policy and institutional determinants of unemployment, which include unemployment benefits, taxes, trade union bargaining power and the structure of collective bargaining, employment protection legislation, anti-competitive product market regulation, active labor market policies, minimum wages and housing policies (Bassanini and Duval, 2007). Bildirici et al. (2012) report that rapid population growth, internal and external migration, technological advances, interregional differences in development, productivity and wages, educational policy, political and economic instability, inadequacy of investment in public and private sectors, labor
fact, an earlier work by Karahan et al. (2012) shows that inflation and unemployment trade-off only exists in the short run. Another work by Kuştepeli (2005) investigates the existence of a Phillips curve in Turkey and the results indicate no evidence of a meaningful relationship between inflation and unemployment for the Turkish economy. Finally, Uysal and Erdoğan (2003) report evidence of a statistically significant, yet quite a weak link between unemployment and inflation in Turkey.

The same conclusion holds true when output gap is used in the Phillips curve equation. This result is in line with the prior findings by Özbek and Özlale (2005), which contradicts with the more general view that demand-side dynamics are the main determinants of inflation in Turkey.

Meanwhile, estimation results suggest a weakening, but significant pass-through from nominal exchange rate to inflation. This result confirms earlier observations which indicate that pass-through of exchange rate to inflation has gradually weakened in the post-2001 period (Kara and Öğünç, 2005, 2008; Kara et al., 2007; Karasoy et al., 2005).

The estimation findings also point to the presence of considerable inertia in inflation. This can be proven by size as well as the relative stability of coefficients for past inflation. These results reinforces previous findings by Şahinöz and Saraçoğlu (2008), Özciçek (2011) and Tunay (2009) pointing to significant inflationary inertia even after the disinflation period in Turkey.

It should be noted that the estimations are based on aggregate data on unemployment, output and inflation. Obviously, the structure of unemployment may vary by sectors. Similarly, aggregate demand and inflation may have different dynamics by sub-categories. This may affect Phillips curve assumptions. More specifically, the degree of persistence and pass-through as well as the sensitivity of inflation to unemployment gap and output gap in the Phillips curve equation may vary depending on whether quality, low capacity utilization ratios, the inadequacy of training, credit and organization facilities for entrepreneurs are the main determinants of unemployment in Turkey. On the other hand, the main determinants of inflation in Turkey are viewed to be credit and exchange rates as the former plays an important role on the demand channel, while the latter is the main determinant of the cost channel (Kara, 2013).
aggregate data is used on inflation, output and unemployment. This obviously affects the derivation of NAIRU series as well.\textsuperscript{15}

In conclusion, this paper serves useful for future research on the Turkish economy and it provides guidance for further work on NAIRU. In addition, the study contributes to the existing literature by jointly estimating NAIRU and its time-varying relationship with inflation in a Phillips curve setting. The estimation of NAIRU in this multivariate framework also produces other significant variables like potential output, output gap and inflation. Moreover, the results shed light on the course of time-varying parameters that indicate inflation persistence and the contribution of demand and supply-side factors to inflation. Furthermore, the findings lay the basis for future work that may adopt EKF. But most importantly, this study emphasizes the need for a more flexible and comprehensive framework for the conduct of monetary policy.

\textsuperscript{15} For further details on the sensitivity of estimation results to using aggregate data, see (Altissimo et al., 2009; Tillmann, 2012; Ceritoğlu et al., 2012; Öğünç and Sarkin, 2011; Alp et al., 2012; Başer et al., 2012; Atuk et al., 2013; Erdoğan-Coşar et al., 2012; Özçöçek, 2011).
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APPENDIX

Non-Linear State-Space Models and the EKF

A non-linear state-space model can be represented as:

\[ x_{k+1} = f_k(x_k) + H_k(x_k)\xi_k \]  

(1)

\[ y_k = g_k(x_k) + \eta_k \]  

(2)

The \( f_k \) and \( g_k \) are vector-valued functions, while \( \xi_k \) and \( \eta_k \) represent white noise processes with the covariance matrices, \( Q_k \) and \( R_k \), respectively. The starting values for the EKF algorithm are:

\[ P_0 = \text{cov}(x_0) \]  

(3)

\[ \hat{x}_0 = E(x_0) \]  

(4)

As mentioned in Chui and Chen (1991), the updating equations can be written as:

\[ P_{k|k-1} = \left[ \frac{\partial f_{k-1}}{\partial x_{k-1}} (\hat{x}_{k-1}) \right] P_{k-1} \left[ \frac{\partial f_{k-1}}{\partial x_{k-1}} (\hat{x}_{k-1}) \right]' + H_{k-1} (\hat{x}_{k-1}) Q_{k-1} H_{k-1}' (\hat{x}_{k-1}) \]  

(5)

\[ \hat{x}_{k|k-1} = f_{k-1}(\hat{x}_{k-1}) \]  

(6)

\[ K_k = P_{k|k-1} \left[ \frac{\partial g_k}{\partial x_k} (\hat{x}_{k|k-1}) \right] \left[ P_{k|k-1} \left[ \frac{\partial g_k}{\partial x_k} (\hat{x}_{k|k-1}) \right]' \right]^{-1} \left[ \frac{\partial g_k}{\partial x_k} (\hat{x}_{k|k-1}) \right]' + R_k \]  

(7)

\[ P_k = \left[ I - K_k \left[ \frac{\partial g_k}{\partial x_k} (\hat{x}_{k|k-1}) \right] \right] P_{k|k-1} \]  

(8)

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left[ y_k - g_k(\hat{x}_{k|k-1}) \right] \]  

(9)
where equations 5-9 denote the optimal Kalman gain, the updated estimate covariance, the updated state estimate, the predicted estimate covariance and the predicted state, respectively.

In order to apply EKF, the matrices in the state-space representation above should be written in terms of functions, which depend on the unknown parameter vector \( \theta \). In other words, let the matrices be represented by \( \Phi_k(\theta), G_k(\theta) \) and \( H_k(\theta) \). Furthermore, let \( \theta \) be a random walk process. In this case, the above equations can be rewritten as:

\[
x_{k+1} = \Phi_k(\theta_k)x_k + G_k(\theta_k)w_k \\
y_k = H_k(\theta_k)x_k + v_k
\]

and the parameter vector is:

\[
\theta_{k+1} = \theta_k + \zeta_k
\]

The state-space representation using functional form is:

\[
\begin{bmatrix}
x_{k+1} \\
\theta_{k+1}
\end{bmatrix} = \begin{bmatrix}
\Phi_k(\theta_k) & x_k \\
\theta_k & \zeta_k
\end{bmatrix} + \begin{bmatrix}
G_k(\theta_k)w_k
\end{bmatrix}
\]

\[
y_k = \begin{bmatrix}
H_k(\theta_k) & 0
\end{bmatrix} \begin{bmatrix}
x_k \\
\theta_k
\end{bmatrix} + v_k
\]

where equations 13 and 14 denote the state-space representation for the state and the observation vector, respectively. The above model is non-linear for which EKF can be readily applied. \( \zeta_k \) shows the white noise process for which the covariance matrix is assumed to be \( \text{cov}(\zeta_k) = S_k = S > 0 \). In the particular case where \( S = 0 \), the parameter vector is assumed to be time-invariant, where EKF cannot be operative. If EKF algorithm is applied to equations (13)-(14), depending on the following starting values:

\[
\begin{bmatrix}
\hat{x}_0 \\
\hat{\theta}_0
\end{bmatrix} = \begin{bmatrix}
E(x_0) \\
E(\theta_0)
\end{bmatrix}
\]
and \( P_0 = \begin{bmatrix} \text{cov}(x_0) & 0 \\ 0 & S_0 \end{bmatrix} \) \hfill (16)

We obtain

\[
\begin{bmatrix}
\hat{x}_{k|k-1} \\
\hat{\theta}_{k|k-1}
\end{bmatrix}
= \begin{bmatrix}
\Phi_{k|k-1}(\hat{\theta}_{k-1})\hat{x}_{k-1} \\
\hat{\theta}_{k-1}
\end{bmatrix} \hfill (17)
\]

\[
P_{k|k-1} = \begin{bmatrix}
\Phi_{k-1}(\theta_{k-1}) & \frac{\partial}{\partial \theta}(\Phi_{k-1}(\theta_{k-1}))\hat{x}_{k-1} \\
0 & I
\end{bmatrix} P_{k-1} \begin{bmatrix}
\Phi_{k-1}(\theta_{k-1}) & \frac{\partial}{\partial \theta}(\Phi_{k-1}(\theta_{k-1}))\hat{x}_{k-1} \\
0 & I
\end{bmatrix}^T + \begin{bmatrix}
G_{k-1}(\hat{\theta}_{k-1})\hat{\theta}_{k-1} & G'_{k-1}(\hat{\theta}_{k-1}) \\
0 & S_{k-1}
\end{bmatrix} \hfill (18)
\]

\[
K_k = P_{k|k-1} \begin{bmatrix} H_k(\hat{\theta}_{k-1}) & 0 \end{bmatrix} P_{k|k-1}^{-1} \begin{bmatrix} H_k(\hat{\theta}_{k-1}) & 0 \end{bmatrix}^T + R_k \hfill (19)
\]

\[
P_k = \left[ I - K_k \begin{bmatrix} H_k(\hat{\theta}_{k-1}) & 0 \end{bmatrix} \right] P_{k|k-1} \hfill (20)
\]

\[
\begin{bmatrix}
\hat{x}_k \\
\hat{\theta}_k
\end{bmatrix} = \begin{bmatrix}
\hat{x}_{k|k-1} \\
\hat{\theta}_{k|k-1}
\end{bmatrix} + K_k \left[ y_k - H_k(\hat{\theta}_{k-1})\hat{x}_{k|k-1} \right] \hfill (21)
\]
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