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Neslihan KAYA
A COMPARISON OF OPTIMAL POLICY RULES FOR PRE AND POST INFLATION TARGETING ERAS: EMPIRICAL EVIDENCE FROM BANK OF CANADA*

Neslihan KAYA

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Abstract

In this paper, we derive policy rules of Bank of Canada for different preferences over the goal variables in their loss functions. The optimal rules are derived for the pre and post inflation-targeting eras. According to the results, the monetary policy rule of the Bank of Canada for the pre inflation-targeting era is best described with a loss function that attaches equal weight to inflation, interest rate smoothing incentive and the output gap in the loss function. In the post-inflation targeting era the optimal interest rate attaches the highest weight to inflation rate in the loss function; followed by the interest rate smoothing incentive and then the output gap. The inclusion of the exchange rate as another goal variable in the loss function does not significantly alter the results in approximating the actual policy rate of Bank of Canada. Next, simulations of demand (positive) and supply (negative) shocks are carried out for the post-IT period for two cases where the monetary policy rule is mimicked by (i) an ad-hoc Taylor rule and (ii) the derived optimal rule. The results indicate that the ad-hoc Taylor rule brings down inflation rates more quickly compared to the derived optimal rule, but only at the cost of higher contraction in output and more volatile interest rates.

Keywords: Inflation targeting, optimal monetary policy rule, linear-quadratic regulator problem.

JEL codes: E52, E58, C63

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1 Economist, Central Bank of the Republic of Turkey, Research and Monetary Policy Department, Ulus, 06100, Ankara, Turkey. E-mail: Neslihan.Kaya@tcmb.gov.tr. Phone: (90)3125075482, Fax: (90)3125075732.
1. Introduction

Pursuing a credible and effective monetary policy is the primary responsibility of the central banks. In this regard, inflation targeting (IT) serves as a monetary policy framework to attain low and stable inflation. It is mainly characterized by (i) public announcement of an official medium-term inflation target or target ranges, (ii) forward looking strategy, (iii) transparency in conducting the monetary policy, and (iv) accountability of the monetary authority in achieving the target. Since its first adoption in the beginning of 1990s, the IT regime became more and popular, mainly due to its alleged macroeconomic benefits, such as decreasing inflation and its volatility (Mishkin and Schmidt-Hebbel, 2007), decreasing output volatility (Petursson, 2005; Goncalves and Salles, 2008) or interest and exchange rates volatility (Batini and Laxton, 2007; Lin, 2010).

Adoption of IT seemed to help policy makers to anchor public expectations for inflation; however, there is mixed evidence as to whether the way the monetary policy conducted has changed following the adoption of IT. Cecchetti and Ehrmann (1999) assess the changes in central bank aversion to inflation. They find that both for inflation targeting and non-inflation targeting countries there is apparent aversion of the central banks to inflation, however, for the inflation targeting central banks this aversion has increased significantly. Corbo, Landerreteche and Schmidt-Hebbel (2001) analyze the 1980 –1999 period for different country groups and pose the question whether inflation targeting affected the central bankers’ behavior in setting their policy rates. Their results indicate that the reaction of interest rates to both inflation and output shocks has declined significantly among the inflation targeting group of countries throughout the 1990s suggesting that they could achieve their inflation targets with smaller changes in interest rates due to the credibility gain. Neumann and Von Hagen (2002) estimate Taylor rules and find that the reaction of the central banks to output and inflation has indeed changed following the adoption of IT.

Even though inflation targeting policy seems to have proven itself useful, there are several sources of uncertainties surrounding the policy makers, one of them related to the role of the exchange rate in monetary policy, that is whether the policy makers should in any form take into account the exchange rate in forming their policies. Ball (1999) extends the Svensson (1997) and Ball (1997) models (which find support of Taylor rules in which the interest rate responds solely to output and inflation) to an open economy model and finds that in open economies Taylor rules do not perform well as the monetary
policy affects the economy through exchange rate as well as the interest rate channels. However, the role assigned for exchange rate in the monetary policy rule is not always positive. Svensson (2000) builds a small, open economy model with micro foundations and his simulations reveal that the interest rate rule that reacts to the exchange rate reduces the standard deviation of inflation but increases the variance of output. Taylor (1999) considers seven developed countries and concludes that reacting to the exchange rate either has made the performance worse or it has only led to some minor improvements. Taylor (2001) explains the exchange rate implications of Ball (1999), Svensson (2000) and Taylor (1999) by discussing that directly reacting to the exchange rate is not necessary since the standard Taylor rule reacts to inflation and output which are already affected by the exchange rates.

Interest rate smoothing incentive is another crucial issue for the policymakers and is widely accepted as a characteristic of monetary policy in most industrial countries. It refers to the tendency for monetary authorities to adjust official interest rates mainly in sequences of small steps in the same direction: that is they adjust rates gradually, and with relatively few reversals of direction (Cobham, 2003). One of the most common explanations of why monetary authorities smooth interest rates lie in official dislike of the financial market volatility. In the literature, models addressing this incentive improve upon ad-hoc Taylor rules. Detailed evidence on the existence of interest rate smoothing was first provided for the United States by Rudebusch (1995). Subsequent studies such as Goodhart (1997) and Lowe and Ellis (1997) provide evidence for a wider range of countries. Cobham (2003) shows that policy has been smoothest in the period of inflation targeting with Bank of England control of interest rates since 1997.

Taking the findings above as given, we build a model for the Canadian economy to explore the changes in monetary policy rules following the adoption of IT. For this purpose, we use a loss function that includes not only inflation and output gap, but also exchange rate and interest rate smoothing incentive. The motivation behind using more flexible loss functions is to improve over ad-hoc Taylor rules to capture the changing preferences of policymakers between the two monetary policy regimes. Bank of Canada has officially adopted inflation-targeting regime in 1991, which makes it the second central bank in the world adopting this framework, following the Reserve Bank of New Zealand. Therefore, Canada serves as a good example, as it has a quite long history of inflation targeting. Moreover, there are several studies exploring the effects of IT on the Canadian economy. Studies such as Johnson (1997) and Honda (2000) find that IT failed to decrease the level and expectations of inflation in Canada, supported by Ball and
Sheridan (2005) which use a panel of countries including Canada. Miles (2008) estimates the impact of IT on inflation persistence and uncertainty. It finds that while IT appears to shorten the life of inflation shocks, it has actually increased uncertainty for any given level of inflation. As to our knowledge, there are not any studies investigating whether the policy making behavior of Bank of Canada has changed following the adoption of IT, within a framework where policy rules are derived separately for the two eras and then compared. Our results indicate the presence of a significant change in the policy making behavior. For the pre-inflation targeting era, the optimal rule attaches equal weight to inflation, interest rate smoothing incentive and the output gap in the loss function; whereas in the post-inflation targeting era the optimal interest rate attaches the highest weight to inflation rate in the loss function; followed by the interest rate smoothing incentive and then the output gap. The inclusion of the exchange rate in the loss function does not significantly alter the results.

Section 2 of the paper introduces an empirical model of the Canadian economy. Section 3 describes the optimal linear regulator problem, presents the state space representation of the model and the numerical algorithm to obtain the optimal instrument rules. Section 4 presents the main findings of the study. The derived optimal policy rules are interpreted, the actual policy rates and the derived policy rates are compared for the two monetary policy eras and the implications of simulations of a demand and supply shock are discussed. Finally, Section 5 concludes.

2. An Empirical Model of an Inflation Targeting, Open Economy: Canada

The model, which is a version of Ball (1999), is expressed as follows:

\[ \pi_{t+1} = \alpha_{\pi_1} \pi_t + \alpha_{\pi_2} \pi_{t-1} + \alpha_{\pi_3} \pi_{t-2} + \alpha_{\pi_4} \pi_{t-3} + \alpha_y y_t + \alpha_{rer} (rer_t - rer_{t-1}) + \epsilon_{\pi,t+1} \tag{2.1} \]

\[ y_{t+1} = \beta_{y_1} y_t + \beta_{y_2} y_{t-1} + \beta_r r_t + \beta_{rer} rer_t + \epsilon_{y,t+1} \tag{2.2} \]

\[ rer_{t+1} = y r_t + \epsilon_{rer,t+1} \tag{2.3} \]

where, \( \pi_t \) is the CPI-based, annualized quarterly inflation; \( y_t \) is the output gap; \( rer_t \) is the real exchange rate index - where an increase in the index is to be interpreted as an appreciation of the domestic currency against the US dollar- and \( r_t \) is the real interest rate, defined as the nominal interest rate, \( i_t \), minus the inflation rate. All variables are demeaned; therefore no constants appear in the equations. The pre-IT period is defined as the period between 1970 and 1990 and the post-IT starts from 1991 and ends at the third quarter of 2013.
Equation 2.1 is an open economy Phillips curve where inflation rate is expressed as a function of four lags of itself, the output gap and the change in the real exchange rate. The coefficient of the output gap measures the response of inflation to fluctuations in aggregate demand, capturing one of the most important channels of the transmission of monetary policy to inflation and is expected to be positive. $\alpha_{rer}$ captures the pass-through of appreciation of domestic currency to prices and is expected to be negative. Equation 2.2 is an open economy IS curve where output gap is written as a function of its own lags, the real interest rate and the real exchange rate. $\beta_r$, the coefficient of real interest rate is expected to be negative, meaning that a tightening of the monetary policy (higher interest rates) leads to a decline in the economic activity. $\beta_{rer}$, together with the coefficient of output gap in Phillips curve - $\alpha_y$ - determines how a change in the monetary policy is transmitted to inflation. As for the real exchange rate, $\beta_{rer}$ is expected to be negative, as a real appreciation of the domestic currency is expected to lead to a decline in the total demand –both domestic and foreign– for domestic goods; thereby leading to a decline in output. Equation 2.3 is a simple representation of the Uncovered Interest Parity, which is of the type employed in Ball (1999). It links the real exchange rate and the real interest rate and aims at capturing the idea that an increase in the interest rate makes domestic assets more attractive and leads to capital inflows thereby leading to the appreciation of the domestic currency. Thus, $\gamma$ is expected to have a positive sign.

In this model, inflation is affected through two monetary channels. For instance, a contractionary monetary policy decreases the output gap, leading to a decline in inflation through the Phillips curve. At the same time, through the Uncovered Interest Rate Parity condition, it leads to the appreciation of the domestic currency, which, again through the Phillips curve, leads to a decline in the inflation rate.

The system represented above is estimated by the Seemingly Unrelated Regression method\(^2\) for the pre-IT and post-IT Canadian economy. The estimated set of parameters for the pre-IT and the post-IT can be found in Table 1 below:

<table>
<thead>
<tr>
<th>Table 1: Parameters of the model</th>
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<tbody>
<tr>
<td>Pre-IT</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$\alpha_{x1}$</td>
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<tr>
<td>$\alpha_{x2}$</td>
</tr>
<tr>
<td>$\alpha_{x3}$</td>
</tr>
<tr>
<td>$\alpha_{x4}$</td>
</tr>
<tr>
<td>$\alpha_y$</td>
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<td>$\alpha_{rer}$</td>
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<tr>
<td>$\beta_{y1}$</td>
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<tr>
<td>$\beta_{y2}$</td>
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</table>

\(^2\) Taking into account the cross-correlation of the error terms we carried out a SUR estimation instead of individually estimating the equations by OLS.
Notice that equations 2.1 - 2.3 do not include a monetary policy rule. To estimate the monetary policy rule we proceed as follows. We define a quadratic loss function for Bank of Canada that is minimized subject to a model of the Canadian economy, estimated above. The solution of linear quadratic optimization problem results in an optimal rule for Bank of Canada. In this exercise the weights of the goal variables in the loss function are assigned arbitrarily. Then, we carry out the exercise for several ad-hoc weights in the loss function. Having obtained several optimal rules this way, we are comparing them with the actual rate of the Bank in order to find out the loss function that Bank of Canada resembled the most. This is an extension of Rudebusch & Svensson (1998) where the same optimization problem is solved for the US economy.

3. The Optimal Linear Regulator Problem

Let the model be given by the equations 2.1 - 2.3 above, where \( \varepsilon_{\pi,t+1} \), \( \varepsilon_{y,t+1} \) and \( \varepsilon_{rer,t+1} \) are i.i.d. disturbances with zero mean and variances \( \delta_{\pi,t+1}^2 \), \( \delta_{y,t+1}^2 \) and \( \delta_{rer,t+1}^2 \), respectively. Following the definition of Rudebusch and Svensson (1998), inflation targeting in our analysis is interpreted as “having a loss function for the monetary policy where the deviations of inflation from an explicit inflation target will always be given some weight but it will not necessarily be given all the weight”.

For a discount factor \( \delta \), \( 0 < \delta < 1 \), the intertemporal loss function in quarter \( t \) is considered as:

\[
E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau},
\]

where, the period loss function is defined as:

\[
L_t = \lambda_\pi \pi_t^2 + \lambda_y y_t^2 + \lambda_{rer} rer_t^2 + \lambda_i (i_t - i_{t-1})^2 \quad \text{and} \quad \lambda_\pi, \lambda_y, \lambda_{rer}, \lambda_i \text{ are the weights on price stability, output stability, exchange rate stability and interest rate smoothing, respectively and } \pi_t, y_t, rer_t \text{ and } (i_t - i_{t-1}) \text{ are the goal variables. Referring to Rudebusch and Svensson (1998), we are employing a “flexible” version of inflation targeting where goal variables other than inflation are allowed to enter the loss function with a non-zero coefficient, that is, } \lambda_y, \lambda_{rer} \text{ and } \lambda_i \text{ are non-zero.}
It is shown in Rudebusch and Svensson (1998) that we can define the optimization problem for $\delta = 1$ and then interpret the intertemporal loss function as the unconditional mean of the period loss function as shown below:

$$E[L_t] = \lambda \pi \text{Var}[^t\pi] + \lambda \gamma \text{Var}[^t\gamma] + \lambda \text{Var}[^t\text{re}r] + \lambda \text{Var}[^t\text{re}r_{t-i_{t-1}}]$$

In this paper, we will be assuming $\delta = 1$ and hence 3.2 will be the standard loss function of policymakers, which turns out to be the weighted sum of the unconditional variances of the four goal variables.

### 3.1. State Space Representation of the Model

The state space representation of the model in equations 2.1 - 2.3 is as follows:

$$X_{t+1} = AX_t + Bi + \nu_{t+1}$$

where $X_t$ is a $9 \times 1$ vector of the state variables, $A$ is a $9 \times 9$ matrix, $B$ and $\nu$ are $9 \times 1$ column vectors shown as below:

$$X_t = \begin{bmatrix}
\pi_{t+1} \\
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
y_{t+1} \\
y_t \\
\text{re}r_{t+1} \\
\text{re}r_t \\
\gamma_t \\
l_t
\end{bmatrix}, \quad A = \begin{bmatrix}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_y & 0 & -\alpha & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\beta_r & 0 & 0 & \beta_1 & \beta_2 & \beta_{\text{re}r} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

The observation equation is also conveniently defined as below:

$$Y_t = C_X X_t + C_i i_t$$

, where $Y_t$ is a $4 \times 1$ vector of the goal variables, $C_X$ is a $4 \times 9$ matrix and $C_i$ is a $4 \times 1$ vector; which are expressed as follows:

$$Y_t = \begin{bmatrix}
\pi_t \\
y_t \\
\text{re}r_t \\
(l_t - l_{t-1})
\end{bmatrix}, \quad C_X = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}, \quad C_i = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}$$

With the vectors and matrices defined as above; the period loss function can be defined as follows:

$$L_t = Y'_t K Y_t$$
where $K$ is a $3 \times 3$ matrix with a diagonal $(\lambda_\pi, \lambda_y, \lambda_{rer})$ and all the other elements of the matrix are equal to zero.

### 3.2. The Optimal Rule

With 3.1.1 and 3.1.3 above, the problem is written as a standard stochastic linear regulator problem, where 3.1 is minimized each quarter subject to 3.1.1. The standard problem is solved in Oudiz and Sachs (1985); applied in Rudebusch & Svensson (1998) and their solution is used here. The solution gives a linear feedback rule for the instrument which is of the form $i_t = fX_t$, where $f$ is a $1 \times 9$ row vector. As we assume that the discount factor, $\delta$, is equal to 1; the optimal rule converges to the rule that minimizes 3.2.

The optimal policy rule is of the following form, where the explanatory variables on the right hand side are the state variables represented in $X_t$:

$$
[i_t] = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} rer_t \\ rer_{t-1} \end{bmatrix} + [d_1][i_{t-1}]  \quad \text{(3.2.1)}
$$

Equation 3.2.1 can be interpreted as a monetary policy rule where the coefficients are derived from a structural setup in which the Central Bank is assumed to minimize a hypothetical loss function. In this rule, the parameters $a_1, a_2, a_3$ and $a_4$ measure the reaction of the interest rate to the current and lagged values of inflation rate. The long-run reaction of the interest rate to the inflation rate - which would be measured by the sum of these four parameters - is expected to be positive. The response of the interest rate to the output gap is measured with the parameters $b_1$ and $b_2$. As an increase in the output gap is a sign of a demand pressure in the economy, this would normally lead to a need for contractionary monetary policy, thereby leading to an increase in the policy rate. The optimal rule reacts to the current and lagged values of the real exchange rate. As appreciation has a contractionary effect on aggregate demand, a negative long-run reaction - measured by the sum of $c_1$ and $c_2$ - of the policy rate to exchange rate is expected. Finally, the parameter $d_1$ would capture the interest smoothing incentive of the policy-making authorities.

### 4. Results

The optimal policy rules are derived for several illustrative cases of weights on inflation and output variability, interest rate smoothing and exchange rate variability in the loss function. First, the resulting optimal policy rules are interpreted and then compared to the
actual policy rates of both pre and post inflation-targeting era. Next, we discuss the implications of two types of shocks; namely the demand and the supply shock for the post inflation targeting era.

4.1. Interpreting the Optimal Policy Rules

The weights for the goal variables in the loss functions are given in Table 2 and 3. For the loss functions that include solely the inflation rate, the output gap and the interest rate smoothing; the weights on the first two are selected as to resemble the weights that Rudebusch and Svensson (1998) has adopted. For the interest rate smoothing incentive, the weight is selected to be either 0.5 or 1. For the loss functions that also include the real exchange rate, the weight on the real exchange rate is selected to be 0.5. The derived optimal rules for the pre-IT period are presented in Table 2 and for the post-IT period in Table 3. The responses of the policy rates to the variables of the model vary significantly depending on the monetary policy era and are sensitive to the weights on the goal variables in the loss function.

Regardless of the differences in the weights of the loss functions, the responses of the policy rates to current and all lagged rates of inflation is much less in the post inflation-targeting era compared to the pre inflation targeting era; which is in line with our expectations as the Phillips curve in the model is backward looking. Since the idea of the inflation targeting is built on the logic of policy rates reacting to future rates of inflation, we would expect the policy rate in the post inflation targeting era to react much less to the lagged values of inflation compared to the pre inflation targeting era. As for the output gap, for all types of loss functions considered, the policy rules react positively in the current period and negatively in the lagged period - the cumulative reaction in the two periods being positive in each case. Comparing the pre-IT results with the post-IT results; the policy rate tends to react more aggressively to output gap in pre-IT period, however this cannot be generalized to all rules. It is also observed that interest rate smoothing is always much stronger in the post-IT period compared to the pre-IT period. This is in line with the argument that monetary authorities abstain from creating too much variability in interest rates, as the incentive not to disrupt the financial markets increases with the IT regime. As for the exchange rate, for all types of loss functions, the interest rate reacts negatively in the current period, which is offset by a positive reaction in the lagged period. For all cases, in line with our expectations, the cumulative reaction is found to be negative, though negligible.

Now we have a series of optimal monetary reaction functions, each of which corresponds to a set of parameter weights used in the loss function. To detect which of these functions
can best mimic the actual policy rule of Bank of Canada, first we regress the actual policy rates of Bank of Canada in the pre and post-IT period, on each of the derived policy rules, as below:

\[ i_{t,act} = \alpha_0 \cdot i_{t-1,act} + \alpha_1 \cdot i_{t, act} + \varepsilon_t \]

where, \( i_{t,act} \) is the actual policy rate, and \( i_{t, opt} \) is the derived optimal policy rate. Then, we calculate the minimum of the squared residuals between the actual and the derived policy rates. The results point out that the optimal rule that best mimics the actual rate corresponds to different loss functions for the pre and post inflation targeting eras, implying a significant change in the policymaking process.

For the pre inflation-targeting era, the best-fitted rule corresponds to the one that is derived from the loss function with weights of 1, 1, 1 and 0 on inflation, output, interest rate smoothing and the real exchange rate, respectively (Rule K in Table 2). However, for the post inflation targeting era, it corresponds to the one that is derived from the loss function with weights of 1, 0.2, 0.5 and 0 (Rule G in Table 3). Thus, both for the pre and post inflation targeting the optimal rules behave similarly in terms of the weights on the real exchange rate. The major difference between the two rules stems from the weights on inflation, output gap and interest rate-smoothing incentive of the monetary authorities. For the post inflation-targeting era, the optimal interest rate attaches the highest weight in the loss function to inflation; followed by the interest rate smoothing incentive and then the output gap. However, for the pre inflation-targeting era the optimal policy rate attaches the same weight to inflation, interest rate smoothing incentive and the output gap. These two optimal rules, that best capture the actual policy rate for the two eras are formulated as follows:

**Pre-IT optimal policy rule:**

\[
[i_t] = [0.92 \ 0.08 \ 0.12 \ 0.05] \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3}
\end{bmatrix} + [0.64 \ -0.13 \ Y_t \ -0.16 \ 0.12] \begin{bmatrix}
rer_t \\
rer_{t-1} \\
rer_{t-2} \\
rer_{t-3}
\end{bmatrix}
\]

+ [0.17][i_{t-1}]

**Post-IT optimal policy rule:**

\[
[i_t] = [0.30 \ 0.02 \ 0.04 \ 0.03] \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3}
\end{bmatrix} + [0.24 \ -0.10 \ Y_t \ -0.04 \ 0.03] \begin{bmatrix}
rer_t \\
rer_{t-1} \\
rer_{t-2} \\
rer_{t-3}
\end{bmatrix}
\]

+ [0.70][i_{t-1}]

Analyzing these policy rules in detail, we see that they support the views expressed above. For example, in the pre-IT period; a 1% increase in the output gap increases the policy rate at about 0.64 percentage point in the current period which is partially offset by
a 0.13 percentage points decline in the next period, meaning a long-run reaction of 0.51 percentage point increase in the policy rate. For the post-IT period, the long-run reaction of the policy rate to a 1% increase in the output gap is found to be 0.14 percentage point; implying that in the pre-IT period, the reaction of the optimal policy rate to output gap is almost four times the reaction in the post-IT era. The same exercise can be repeated for the real exchange rate in the policy rules. In the pre-IT period, a 1% appreciation in the real exchange rate leads to a 0.16 percentage point decline in the policy rate which is partially offset by a 0.12 percentage point increase: implying a long-run reaction of 0.04 percentage point decline in the policy rate. For the post-IT period, a 1% appreciation would yield a long-run reaction of 0.01 percentage point decline in the policy rate. The reaction in the pre-IT period is four times the reaction in the post-IT period though the reactions in both periods can be considered to be negligible. Thus, Canadian monetary authorities have behaved in a way consistent with Taylor (2001)’s finding that the inclusion of the real exchange rate in the policy rules doesn’t improve the performance of the economy at all or it only makes a minor improvement.

Figure-1 shows the actual policy rates versus the derived optimal rates that capture the actual rates the best: Rule K for the pre-IT period and Rule G for the post-IT period. In this practice we are not trying to capture the exact weights to find the actual policy rate but we are only choosing among a set of weights -commonly used in the literature- to find the best approximation of the actual policy rate of Bank of Canada, as our objective is to capture only the significant changes in the policy making process. For the pre-IT period, our results indicate that, in the first half of the period the derived policy rate stays above the actual one, but then this pattern is reversed in the second half of the period. Nevertheless, it would not be misleading to state that for the whole pre-IT period, the derived rule replicates the fluctuations in the actual policy rate. Hence, as a result, on average, we conclude that the derived rule (therefore, the associated weights on the goal variables) gives us a good picture of the reality. As for the post-IT period, the optimal rate and the actual rate display almost a perfect match. This result is further important as we use post-IT period to run simulations in the next section.

In the post-IT period, the most recent global financial crisis deserves special attention. With the onset of the 2008-2009 global financial crisis, all developed and developing country central banks have reacted to the devastating effects of the crisis by exerting -more than ever- large cuts in their policy rates. In line with this global trend of loose

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3 All other alternative rules are available upon request.
monetary policy, Bank of Canada has decreased its policy rates by 400 basis points, from 4.5 percentage points at the end of 2007 to 0.5 by the end of 2009. The optimal policy rate implied by the model successfully captures this downward movement of policy rates, as it responds to the considerable decline in economic activity and the parallel favorable conditions on the inflation front by declining the rates throughout the same period. Starting from the beginning of 2010 economic activity starts to pick up and therefore inflation starts to rise, which initiates the beginning of a normalization of policy rates, though gradually. However, starting from the second half of 2010, the actual and the derived policy rates diverge, in the sense that, the optimal rate offers to continue the tightening of the monetary policy whereas the Bank of Canada preferred to keep the rates steady at 1.25 percentage points. This is exactly the period when the Eurozone debt crisis intensified and posed great uncertainty and downside risks on the global economic activity. Thus, the optimal rule, in response to closing output gap and onset of moderate inflationary risks, implied tightening of the policy; whereas the Bank of Canada abstained from increasing the rates, taking into account the uncertainty associated with the global economic outlook.

4.2. Simulation Results
In this section the implications of the stochastic simulations; carried out for the post-IT period; are evaluated. Two types of basic shocks are run; namely a (positive) demand and a (negative) supply shock, introduced as unitary shocks to the system through the residuals of the IS curve and the Phillips curve, respectively. The simulations are carried out for a system, consisting of four equations; 2.1, 2.2 and 2.3 as above and a policy reaction function. As for the policy reaction function, two different policy rules are employed: Rule G, as it captures the actual policy rule in the post-IT period the best, and an ad-hoc Taylor rule that attaches a weight of 1.5 on inflation and 0.5 on output gap in the loss function. Both rules imply no reaction to exchange rate but, unlike the Taylor rule, Rule G has interest rate smoothing incentive. Figure 2 reveals impulse responses to the positive demand shock, and Figure 3 reveals impulse responses to the negative supply shock. Both types of shocks exert an upward pressure on inflation.

The dynamic impulse responses of the model under the two rules basically share similar features, though they diverge in several important ways, such that the Taylor rule exhibits a large and quick policy rate increase in response to both of the shocks in order to get inflation on target, whereas the optimal rule gives smoother policy responses in line with the aim of not disrupting the financial markets by creating too much volatility in the interest rates. In response to a positive demand shock, in order to bring the rising inflation
back on track, Taylor rule offers a large policy rate hike that reaches its peak in the first quarter. In response to this rate hike, output starts to narrow down and the domestic currency appreciates, both of which put downward pressure on inflation. In turn, the Taylor rule implies a steady decline in the interest rate. Rule G yields a similar policy response, however, in line with the interest rate smoothing incentive, it offers a smoother rate hike which is smaller in magnitude but lasts longer than what Taylor rule yields. Also it does not reach its peak in the first quarter but rather it gradually increases and then declines smoothly. Therefore, the policy rate under Rule G initially stays below the Taylor rule, but starting from the third period it stays considerably above. In result, as the model assumes that interest rates affect inflation with two periods of lags, under both rules inflation increases until the third period following the demand shock, after which it starts to decline. The rates of decline in inflation are similar under both rules but the decline under Rule G lags the decline under Taylor rule. As a result, at any point in time between period 3 and 10, inflation is around 1 to 3.5 percent higher under Rule G, though this gap narrows down very fast. In the following periods after period 10, on the other hand, Rule G does a better job than the Taylor rule in mitigating inflation. The main difference between the impulse responses under both rules is that Taylor rule corrects the deviation of real exchange rate between periods 3 and 6, whereas Rule G corrects these deviations in longer term. However, the deviations in exchange rate under Rule G are not as high as the deviations in exchange rate under the Taylor rule, due to the gradual response of policy rate in the former one. The responses are similar in case of a supply shock. In both cases, Taylor rule and Rule G, yields a policy rate hike. The hike under the Taylor rule is large enough to increase the real interest rate, therefore the output decreases and the currency appreciates, both of which lowers inflation. However, Rule G, yields a smoother rate hike, which only after two periods leads to an increase in the real interest rates that brings down inflation. In sum, in both cases, Taylor rule guarantees to bring down inflation rates more quickly compared to Rule G but only at the cost of higher contraction in output and more volatile interest rates.

5. Conclusion
This paper derives optimal monetary policy rules of the Bank of Canada for the pre and post inflation targeting eras, in order to capture the changing preferences of the monetary policy makers between the two eras. We then compare impulse responses of model variables to demand and supply shocks for the post-IT period under two cases, where the monetary policy rule is mimicked by (i) an ad-hoc Taylor rule that reacts to solely inflation and output gap, (ii) the derived optimal rule for the post-IT period, Rule G.
There are important conclusions to be drawn from our study. First, it is observed that in the post inflation targeting era, the optimal policy rate that mimics the actual policy rate the best is derived from a loss function that attaches the highest weight to inflation, followed by interest smoothing incentive and the output gap. However, for the pre inflation targeting period the best fitted rule attaches equal weights on the goal variables in the loss function. This reflection of the preferences of the policymakers is in line with the spirit of inflation targeting as the primary objective of IT is to provide and maintain price stability and concerns regarding output stability can exist only to an extent where it does not hamper price stability. Second, the results are in line with Taylor (2001) in the sense that the inclusion of the exchange rate in the loss function does not significantly alter the results. Third, simulating a demand and a supply shock, we see that the impulse responses of the model under the two policy rules diverge in several ways. In response to both shocks, the Taylor rule exhibits a large and quick policy rate increase in order to get inflation on target. On the other hand, as the optimal rule –that best mimics actual policy rate of the Bank of Canada– possesses an interest rate smoothing behavior, it gives smoother policy responses in line with the aim of not disrupting the financial markets by creating volatile interest rates. In sum, in both cases of shocks, Taylor rule, which lacks interest rate smoothing behavior, guarantees to bring down inflation rates earlier than Rule G but only at the cost of higher contraction in output and more volatile interest rates.
REFERENCES


Table 2: Optimal Policy Rules for Pre IT

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Table 3: Optimal Policy Rules for Post IT

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Figure 1: Actual vs. Optimal Derived Policy Rates

Pre-IT period (Rule K)
- Actual policy rate
- Derived policy rate

Post-IT period (Rule G)
- Actual Policy Rate
- Derived Policy Rate

Figure 2: Impulse Responses to a Demand Shock in the Post-IT period

- Policy Rate_Rule G
- Policy Rate_Taylor rule
- Inflation rate_Rule G
- Inflation rate_Taylor rule
- Real exchange rate_Rule G
- Real exchange rate_Taylor rule
- Output Gap_Rule G
- Output Gap_Taylor rule
Figure 3: Impulse Responses to a Supply Shock in the Post-IT period

- Policy Rate: Rule G vs. Taylor rule
- Inflation rate: Rule G vs. Taylor rule
- Real exchange rate: Rule G vs. Taylor rule
- Output Gap: Rule G vs. Taylor rule
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