Central Bank Review
Vol. 13 (May 2013), pp.79-102
ISSN 1303-0701 print / 1305-8800 online

https://www3.tcmb.gov.tr/cbr/

OPTIMAL RESERVE MANAGEMENT PATTERN FOR TURKISH BANKS

Anil Talasli and Suheyla Ozyildirim*

ABSTRACT In this paper we model a representative bank's optimal reserve management pattern by adopting institutional aspects of the Turkish required reserve regime. The cost minimization problem, in general, takes into account the expected opportunity cost of holding reserves and penalty costs for not fulfilling the liability of reserve requirement. We extend this problem for reserve carry-over provision and use of late liquidity window facility and compute the optimal reserve pattern by dynamic programming. We show that optimal reserve pattern remains constant during the first week of the maintenance period and has an upward trend especially during the last three days. The analysis of changes in reserve requirements, carry-in amounts, liquidity shocks, policy interest rates, and remuneration of reserves provide some important insight into the commercial banks' reserve holding behaviors.
JEL G21, E52, E58, C61
Keywords Optimal Reserve Management, Reserve Requirements, Maintenance Period, Liquidity Shock, Carry-in

ÖZ Bu çalışmada Türkiye'de faaliyet gösteren temsili bir bankanın optimal zorunlu karşılık (rezerv) yönetimi modellenmiştir. Temel olarak, bankanın maliyet minimizasyon problemi hem rezerv tutmanın fırsat maliyetini hem de zorunlu karşılık yükümlülüğünün yerine getirilmemesi sonucu oluşan cezaları içermektedir. Bu temel maliyetlere ek olarak çalışmadaki minimizasyon problemi, rezerv taşıma opsiyonu ve Geç Likidite Penceresi borçlanmasını da içerecek şekilde genişletilerek bankanın optimal rezerv yönetim kalıbı, dinamik programlama yöntemi ile hesaplanmıştır. Model sonucuna göre, temsili bankanın rezerv seviyesi tesis döneminin ilk haftasında yatay seyretmekte ve son üç gün artış eğilimine girmektedir. Tutulması gereken rezerv düzeyi, politika faiz oranı, taşınan rezerv seviyesi değişiklikleri ile rezervlere faiz ödenmesi ve farklı likidite şoku büyüklük ve dağılımları için yapılan model duyarlılık analizi sonuçları ticari bankaların rezerv tutma davranışlarına ilişkin önemli bilgiler sunmaktadır.
TÜRKİYE'DE FAALİYET GÖSTEREN BANKALAR İÇİN OPTİMAL REZERV YÖNETİMİ
JEL G21, E52, E58, C61
Anahtar Kelimeler Optimal Rezerv Yönetimi, Zorunlu Karşılıklar, Zorunlu Karşılık Tesis Dönemi, Likidite Şokları, Rezerv Taşıma Opsiyonu

* **TALASLI:** Central Bank of the Republic of Turkey, Markets Department, İstiklal Cad. 10, 06100-Ankara, Turkey, anil.talasli@tcmb.gov.tr ▪ **OZYILDIRIM:** Bilkent University, Faculty of Business Administration 06800 Bilkent, Ankara, Turkey, suheyla@bilkent.edu.tr ▪ The views expressed in this paper belong to the authors only and do not represent those of the Central Bank of the Republic of Turkey or its staff.

1. Introduction

The management of bank reserves is important for understanding not only operational banking behavior but also for understanding the channel through which central banks achieve their liquidity management strategies in the financial system. Reserves and the reserve management function are directly linked to the core tasks of a central bank, namely ensuring price and financial stability. By controlling the banking system's liquidity, a central bank can provide smooth liquidity conditions for both banks and their customers. In many countries, banking regulation and supervision is not one of the tasks of a central bank. This raises an interesting question: remuneration of reserve balances provides an incentive for banks to hold reserve balances, but may not be the appropriate way to motivate the holding of balances for prudential purposes, since the central bank may not be the supervisor (Gray, 2011). However, the central bank as overseer of the large value payment system has an interest in ensuring that members of the payment system have sufficient liquidity. The management of bank reserves and creation of a well-designed framework for monetary policy lead lesser market volatility, lower costs for banks, and lesser complication to the central bank's policy making efforts (Bartolini et al., 2001; Carpenter and Demiralp, 2011).

Understanding the pattern of bank reserves and factors influencing reserve holdings seems to be more important than ever for central bankers considering the recent experiences of commercial banks in the global financial crisis which started in 2007. During the financial crisis in 2008-2009, the provision of reserves to the banking system was an important central bank instrument. In several countries, this led, in practice, to change in systems for management of bank reserves (see for example, Norges Bank, 2012). Hauck and Neyer (2011) present evidence that average aggregate reserve holding patterns of European banks changed significantly after the crisis. They show that banks met their reserve requirements by front-loading of reserves and held significantly less reserves at the end. Prior to the financial crisis, aggregate required reserves were fulfilled smoothly over the maintenance period in the euro area, however, as uncertainty increased especially during the first phase of the recent crisis, banks feared for significant deposit withdrawals and front-loaded reserve holdings.

The general reserve management problem is a choice of a bank which relates to the costs incurred by the foregone interest when a bank meets its reserve requirement and the penalty costs for having reserves fall below the reserve requirement. The probability that the reserve account of a bank falls below its reserve requirement is given by a stochastic process for the flows to/from bank's accounts. By this stochastic process and two other relevant state variables the bank's optimal decision can be analyzed. The uncertainty stems from the fact that certain operations of withdrawals and deposits take place after the bank sets its daily reserve position target.

The literature on the optimization of reserve demand has started with Poole (1968) in which he develops a single period model for reserve demand. The subsequent research have extended Poole's optimization model by constructing multi-day period models with some structural settings like overdraft penalty and carry-over. Poole (1968) shows that much of the demand for reserve balances, is related to uncertainty. In other words, there exists a positive relation between a bank's reserve demand and uncertainty about the flow of payments. The open market desk needs to derive the pattern of reserve demand over the days of the maintenance period and it can perform this task either through the level of demand or indirectly through the level of money market interest rate. In the literature, there are several studies that examine interest rates to provide empirical evidence on the reserve demand. For example, Campbell (1987) investigates the federal funds behavior during 1980-1983 and presents evidence that there exist some predictable changes in the funds rate through the maintenance week. Under lagged reserve accounting, before the announcement of aggregate reserve level, banks know their reserve requirements but not the reserve level of other banks. It is shown that if all banks underestimate the aggregate reserve level, the announcement will raise the federal funds rate which causes evidence against the martingale hypothesis.[1] Similarly, Hamilton (1996) provides statistical evidence against the hypothesis that federal funds rate follows a martingale over the two week reserve maintenance period. With the help of a theoretical model of federal funds market, it is claimed that line limits and transaction costs cause banks in the U.S. not to regard reserves held on different days of the week to be perfect substitutes. Hamilton (1997), Furfine (1998), Spindt and Hoffmeister (1988), Griffith and Winters (1995), and Carpenter and Demiralp (2006a) look at the variability of federal funds rate across the maintenance period and find a pattern that reserves are lower at the end of the first week but higher on the last days of the two weeks maintenance period. Kopecky and Tucker (1993)

[1] According to martingale hypothesis, the interest rate on day t (i_t) should be equal to the rate banks' expect to hold the following day, $t+1$, $i_t=E_t i_{t+1}$, where $E_t i_{t+1}$ is the expectations formed on information available on day t about the value of i_{t+1}. The arbitrage opportunities tend to drive i_t and $E_t i_{t+1}$ in such a way that they become equal.

constructs a two-period model of a profit maximizing bank's behavior on settling/non-settling days to explain the observed differential in interest rate variances over the maintenance period.

The research on optimal reserve demand in various reserve regimes starts with Longworth (1989), in which he examines the optimal reserve pattern in a multi-day maintenance period for Canadian reserve system. Similarly, Clouse and Dow (1999) examine optimal reserve demand in a two period maintenance period with heterogeneous banks but without carry-over provision for U.S. and extend this study later with more realistic assumptions and solve the decision problem of a representative bank in a 14-day maintenance period with carry-over provisions (Clouse and Dow, 2002). Coelho and Pinto (2004) derive optimal reserve management pattern of a hypothetical bank in Brazil by taking account the systemic properties of the Brazilian regime.

In this study, we analyze the optimal behavior of a representative Turkish bank in managing its Turkish lira (TL) compulsory reserves in a 14-day maintenance period. Banks face with two types of costs in reserve management: daily opportunity costs and penalty cost for not accomplishing the requirement. And they try to minimize the overall cost of holding reserves in a maintenance period. The existence of carry-over of reserve surpluses/deficiencies to the next period will connect the two independent maintenance periods and causes the cost minimization problem to be more complicated. The decisions of a bank in current period will have impacts on reserve holdings in the next period, so the cost minimization problem has to cover not only the expected costs associated with current period, but also the discounted expected future costs. In other words, the existence of carry-over provision connects all subsequent maintenance periods which requires dynamic programming setup in solving the bank's cost minimization problem.

Using benchmark parameters for Turkish reserve system, we find that optimal reserve pattern remains constant during the first week of the maintenance period and has an upward trend in the second week especially during the last three days. However, this pattern of reserve demand seems to be different from the observed pattern in Turkey. Recently, Talaslı (2010, 2012) examines bank reserves econometrically during the period January 2005-August 2010 to extract the day specific demand for excess reserves and show that the banks in Turkey tend to hold high levels of excess reserves on the first day of the maintenance period. In other words, Turkish banks generally prefer to be locked in early in the maintenance period and accept the costs of holding non-interest earning excess reserves since the costs associated with failing reserve requirements or avoiding overnight overdraft are higher. It is argued that this pattern might be associated with the turbulent past of the Turkish economy which caused the banks that experienced high volatility episodes to act with a precautionary motive in their reserve management. Incorporating different skewness to the liquidity shocks, our model captures the observed bank reserve pattern highlighting the importance of understanding the overall liquidity conditions for central bank to stabilize the money markets effectively.

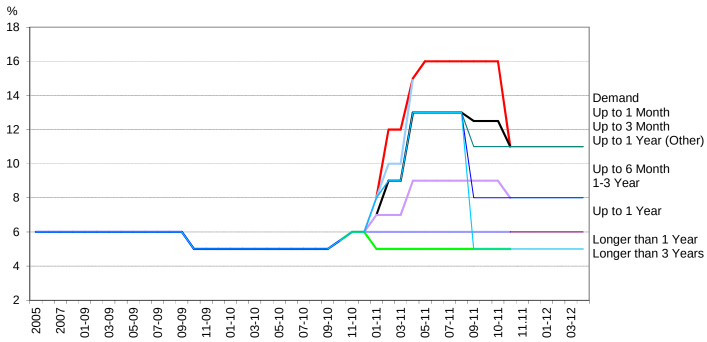
The remainder of this paper is organized as follows: Section 2 provides information about the characteristics of the reserve market structure in Turkey. This section also includes a detailed explanation of the carry-over provision. Section 3 presents the model constructed to solve bank's optimization problem in reserve management. Section 4 analyzes the results of the model under various scenarios. Section 5 provides conclusions and remarks.

2. Reserve Market Structure in Turkey

2.1. Reserve Computation and Maintenance Periods

The rules governing the required reserve system in Turkey are set by the Central Bank of the Republic of Turkey (CBRT).[2] Turkish banks are required to hold a pre-determined amount of TL reserves averaged over 14 day maintenance period beginning on Friday and ending on Thursday. The TL required reserve ratios were differentiated according to the maturity structure of liabilities according to the Communiqué dated 12 November 2010 as seen in Figure 1.

Figure 1. Evolution of TL Required Reserve Ratios



[2] The official document about the required reserve system which is Communiqué No. 2005/1 is available only in Turkish and can be accessed from the CBRT website (http://www.tcmb.gov.tr).

The amount of a bank's required reserves is calculated from its reservable liabilities in its balance sheet two weeks earlier. Since the reserve maintenance period begins after the reserve computation period, banks face no uncertainty during a given maintenance period about the level of their required reserves.[3] The amendment to Communiqué No. 2005/1 on December 15, 2009, permitted banks to carry forward a reserve amount that is excess or deficient of not more than 10 percent of their required reserves. It can only be carried for the next maintenance period. Otherwise there must be a penalty to exercise for the banks that violate these limits. Table 1 provides the rules for carry-over and the amount of reserves subject to penalty conditional on the sign of carry-in. Suppose $C_t<0$ and $R<RR$. Then, we expect C_{t+1} to be the maximum of the difference between required reserves and reserves held and the 10 percent of required reserves. Since $R<RR$, both the terms in the formula will be negative and the reserves to be carried-over would be limited by the maximum allowance. Under this case, the reserves subject to a penalty, P_t, would be equal to the sum of C_t, difference (D) and the maximum allowable carry-over if this sum is negative or zero.

Table 1. Rules for Carry-over and Amount of Reserves Subject to Penalty

		C_{t+1}	P_t
$C_t<0$	$R<RR$	max[(-0.1)xRR,D]	min[C_t+D+(0,1)xRR,0]
	$RR\leq R<RR-C_t$	0	min(C_t+D,0)
	$R>RR-C_t$	min[(0,1)xRR,D+C_t]	0
$C_t\geq 0$	$R<RR-C_t$	max[(-0,1)xRR,D+C_t]	min[C_t+D+(0,1)xRR,0]
	$RR-C_t<R\leq RR$	0	0
	$R>RR$	min[(0,1)xRR,D]	0

C_t:Carry-in from the previous period ; RR:Required reserves ; R:Reserves held ; D: Difference between required reserves and reserves held ; P_t: Reserve amount subject to penalty.

Based upon payment flows, each bank has a daily forecast of its expected reserve position. Banks adjust their reserve holdings by participating to open market operations, borrowing or lending in money markets and in the interbank money market established within the CBRT. A bank's forecast about its reserve position may differ from the realized levels due to liquidity shocks. For any realized deficiency, a bank will incur deficiency penalty costs at the end of the maintenance period and for excess reserves banks will have an opportunity cost.

[3] Maintenance period consists of the days for which a bank's end-of-day reserve balances are used to determine whether it fulfills its reserve requirement. A multi-day maintenance period with a reserve-averaging provision can benefit the central bank in managing the liquidity in the banking system and also the banks' management of their own liquidity.

2.2. Remuneration and Penalties

Although the remuneration of TL required reserves has changed over time, the TL reserve balances are not remunerated since 23 September 2010. The reserve system does not allow banks to have negative end-of-day balances (overnight overdrafts). Hence, they can avoid overdrafts by using the CBRT's standing facilities which in general have higher costs compared to borrowing from money markets. If a bank fails to meet reserve requirements on time or with insufficient amounts, it is required to hold interest-free deposits in blocked accounts over the next 14-day maintenance period. The amount of reserves subject to a penalty is double the deficient portion of required level. If the institution fails to hold the deficient amount, the bank is obliged to make an interest payment on the average deficient amount at a rate equal to 150 percent of the CBRT's announced maximum overnight lending rate which is the rate in the Late Overnight Liquidity Window (LON) facility.[4]

2.3. The Dynamics of Carry-over

Since bank's reserve management involves uncertainties especially related to liquidity needs, it is a difficult task for a bank to have its end-of-day reserve balance equal to the targeted amount. Therefore, central banks introduce some flexibility for banks in meeting their reserve requirements. The most common type of amendment in reserve system that facilitates flexibility is to have carry-over allowances from one maintenance period to the next.

Figure 2, 3, and 4 depict the carry-over and penalty rules for a representative bank with zero, positive and negative carry-in levels. The bank is assumed to have a required reserve level of 100 and consequently the institution will have a maximum carry-over allowance of $\pm$ 10 (10 percent of 100).

For the solid line in Figure 2, the institution has a zero carry-in from the previous period (C_t=0). Maintained reserve balances exceeding the required level of, for instance, 100 can be carried-over into the next period up to the maximum amount of 10. Similarly, deficiencies up to a maximum of 10 are eligible for carry-over. The difference between required reserves (which is equal to 100 in our example) and any reserve holdings below 90 would be treated as a reserve requirement deficiency and be subject to a penalty.

[4] The CBRT provides LON Facility such that banks can borrow from the Central Bank against collateral, and lend to the Central Bank within the transaction hours. The banks can exercise this facility between 16.00-17.00 hours and on the last working day of the maintenance period between 16.00-17.15.

For the solid line in Figure 3, the institution has a positive carry-in from the previous period (C_t=5). Maintained reserve balances exceeding the required level of 100 can be carried-over into the next period up to the maximum amount of 10. The deficiency for the current period that may be carried to the next is measured relative to bank's "adjusted" required reserve balance which is simply the required reserve level minus carry-in of 5. The difference between required reserves and any reserve holdings below 85 would exceed the maximum possible value of carry over allowance and would be subject to a penalty for reserve requirement deficiency. The condition that a bank cannot repeatedly carry forward the same excess is shown by the region between 95 and 100.

Figure 2. Carry-out and Penalty Functions with no Carry-in

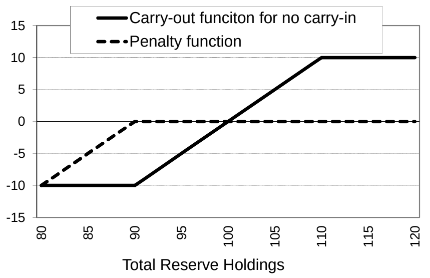
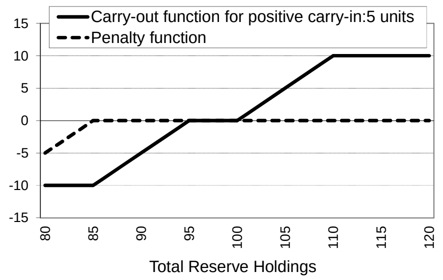
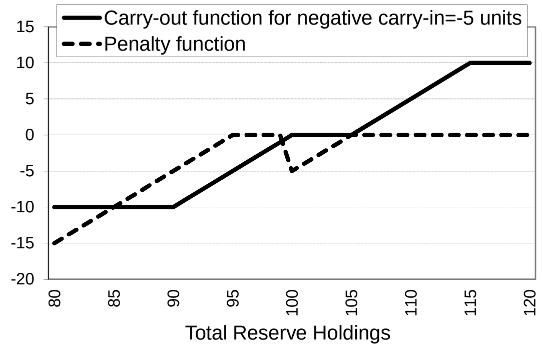


Figure 3. Carry-out and Penalty Functions with +5 Carry-in



For the institution that has a negative carry-in from the previous period (C_t=-5), the carry-out function is shifted to the right. In this case, the bank cannot repeatedly carry forward the same deficiency to the next period and it is shown by the region between 100 and 105. Maintained reserve balances exceeding the "adjusted" required level of 105 is eligible to be carried over into the next period up to the maximum amount of 10. The deficiency for the current period may be carried to the next period for reserve holdings below 100. The difference between required reserves and any reserve holdings below 95 would exceed the maximum allowable carry over allowance and would be subject to a penalty for reserve requirement deficiency. The reserve levels on the region between 100 and 105 are also subject to a penalty.

Figure 4. Carry-out and Penalty Functions with -5 Carry-in



3. Modeling of Banks' Reserve Management

In this section, we provide the details of the model which is constructed in line with the Turkish reserve market structure. The model is formed for a representative bank in a 14-day maintenance period (10 business day) with penalties for reserve deficiencies, costs related to Late Overnight Window borrowing, accounting for weekends, and the existing carry-over rules.

The optimization problem is solved for a bank that chooses a sequence of reserves over 10 business days of the maintenance period, given a pre-determined value of carry-in (C_t). The bank minimizes the expected current period cost of reserve management together with the discounted expected costs in all future maintenance periods. The bank is assumed to have exposure to three different kinds of costs in reserve management. These costs include the opportunity cost of holding non-interest bearing reserves, potential borrowing from CBRT LON facility, and finally the reserve requirement deficiencies. Since the reserve account of a bank in Turkey cannot fall below zero, there exists no overdraft penalty as in the U.S. reserve market. However, the representative bank in our model is assumed to experience a negative liquidity shock after the money markets are closed and forced to cover its short position by borrowing from the CBRT. This cost is included to the model similar to the overdraft penalty that we explained in the previous section.

The representative bank is assumed to minimize the total cost of reserve management over an infinite horizon. The optimization problem has two dynamic elements; the first one relates to the cost minimization of the current maintenance period, the second one is the inter-maintenance period optimization. In other words, the optimization problem has both finite and infinite horizons. The intra-maintenance period problem includes the bank's choices of daily reserve levels which provide sufficient protection against LON borrowing and also produce enough level of average reserve position that satisfies the reserve requirement. The inter-maintenance period problem arises because of the carry-over provision by which the decisions made during the current period have important impact on all decisions made at the next period.

The two elements of the dynamic problem can be written using Bellman equation. $V(C_t)$ is the associated value function for the amount of carry-in (C_t).

$$V(C_t) = \min \mathrm{E}[\,\psi_t + \beta V(C_{t+1})\,] \tag{1}$$

where ψ_t is the cost of reserve management during maintenance period t, β is the discount factor and E denotes expectations conditional on the information available at time t.

The bank chooses a targeted level of reserve balances (R^*_{it}) at i-th business day of the maintenance period t. Considering shock to its liquidity position for that day (Z_{it}), the bank's end-of-day reserve balance is determined by:

$$R_{it}=R^*_{it}+Z_{it} \qquad ; Z_{it} \sim N\left(0, \sigma_z^2\right) \tag{2}$$

It should be noted that there are two state variables in the dynamic problem, the first is the level of carry-in C_t and the second one is the cumulative average reserve position denoted by A_{it} . The level of carry-in is constant during a given maintenance period but evolves from one maintenance period to the next as described in Table 1. As shown below, the rules governing the carry-over can be denoted by a function $g(\,)$ that gives the equation of carry-over as a function of current period's carry-in and the average reserve position of the current period. The second state variable A_{it} , gives the cumulative average level of reserves in each day of the current maintenance period. Hence,

$$C_{t+1} = g\left(C_t, \left(\sum_{i=1}^{10} \alpha_i R_{it}\right) / 14\right) \tag{3}$$

$$A_{it} = \left(1 / \sum_{k=1}^{i-1} \alpha_k\right)\left(\sum_{k=1}^{i-1} \alpha_k (R^*_{kt} + Z_{kt})\right) \tag{4}$$

α_i and α_k = 3, if Friday (i=1,6); 1, otherwise

The equation of motion gives the relation of cumulative average reserve position between two consecutive days in the maintenance period.

$$A_{(i+1)t} = \gamma_i (R^*_{it} + Z_{it}) + (1-\gamma_i) A_{it} \tag{5}$$

where

$$\gamma_i = \alpha_i / (\sum_{k=1}^{i} \alpha_i)$$

We start solving the problem from the last day of the maintenance period *t*, given the carry-in state and the cumulative average reserve position of the previous 13 days, and choose R^*_{10} to minimize the expected costs. The cost function for a given day *i* is denoted by $W_{it}(C_t, A_i)$, so the function for the last day is given by

$$W_{10}(C_t, A_{10}) = \min_{R^*_{10}} E\left[i_{10} R_{10} + o(R_{10}) + d(C_t, A_{10}, R_{10}) + \beta V(C_{t+1})\right] \tag{6}$$

Late overnight borrowing occurs each day at a rate i^{LON} when the bank's reserve balance falls below zero because of a negative liquidity shock, incurring a total cost of $o(R_{it})$. In addition, $d(C_t, A_{10}, R_{10})$ denotes the cost of reserve requirement deficiencies.

$$o(R_{it}) = -i^{LON}(R_{it}) \text{ if } R_{it}<0; \text{ 0 } otherwise$$

For the middle days of the maintenance period the cost function changes since there is no deficit cost on the accomplishment of the reserve requirement.

$$W_i(C_t, A_{it}) = \min_{R^*_{it}} E\left[i_i R_{it} + o(R_{it}) + W_{i+1}(C_t, A_{(i+1)t})\right] \tag{7}$$

Since the optimization problem is solved by backward induction, the missing values of the one-day-ahead cost functions are calculated with a cubic spline interpolation. The cost equation for the first day is different because there is no initial value of cumulative average level of reserves. In addition, the cost function on the first day of the maintenance period coincides with the value function given by Eq. 1.

$$W_1(C_t) = V(C_t) = \min_{R^*_{it}} E\left[i_1 R_{1t} + o(R_{1t}) + W_2(C_t, A_{2t})\right] \tag{8}$$

It is necessary to derive the value function which forms the inter-maintenance period element of the optimization problem. In this study, we use both linear and log-linear functional forms and report only the findings with log-linear form since we haven't observed any significant difference between the two functional forms.

The successive approximation of the value function, $V(C_t) = a.b^{Ct}$, starts with an initial guess which is 1. Given V^1, the optimal choices of R^*_{it} can be calculated for all points on a grid of values (C_t,A_{it}). In our model, C_t runs from -5 to +5 by 1 so we have 11 cost functions for the first day which will create the new value functions V^2. The coefficients a and b of the log-linear function are forecasted by least squares method between the log of V and C_t. We continue this iterative process until the deviation of the sequence of coefficient estimates is below 1 percent, in other words the value function converges.

3.1. The Model Parameter Values

This section provides the values of all relevant parameters which we need to obtain numerical solutions to the bank's optimization problem. Firstly, we assume the representative bank has an average required reserve of 50 monetary units. As an approximation, the opportunity cost is assumed to be equal to the CBRT policy rate 5.75 percent. The discount factor β is roughly two weeks of interest at the policy rate and we assume no discounting within the maintenance period.

The overnight borrowing interest rate for the LON facility is taken as 12 percent and additionally the reserve requirement deficiencies are calculated over the policy rate. Besides these pecuniary costs, the bank is assumed to expose to some non-pecuniary costs for using LON facility and reserve requirement deficiencies. In order to approximate actual bank behavior, the non-pecuniary costs are assumed to be equal to the pecuniary charges. In this way, all costs are doubled in the numerical analysis. These non-pecuniary costs are also mentioned by Clouse and Dow (2002) as hidden costs with reputation concerns and additional office hours.

We use discrete intervals for parameters R_{it} , Z_{it} , A_{it} and C_t to solve the problem as follows: $R_{it} \in [0:1:100], Z_{it} \in [-50:1:50], A_{it} \in [0:1:150]$, $C_t \in [-5:1:5]$.[5] As base case scenario, standard deviation of shocks (σ_Z) is 25. Hence, the probability of each value of Z_{it} is calculated by first determining the probability of each point given a normal density function with zero mean and standard deviation of 25 and then scaling the probabilities proportionately to sum up to 1.

[5] The first and the last terms in the parenthesis indicate interval range and the term in the middle indicates the incremental change.

3.2. The Daily Optimal Pattern of Bank's Reserve Management

Figure 5 plots the average demand for reserves on each day of the maintenance period. We test the optimal pattern whether it comes from a well-behaved function or not by computing the average values of reserves for all C_t values. Figure 6 plots the average reserve levels after this simulation. Theoretically, the optimal pattern under base case scenario should not deviate much for different carry-in levels. As shown by Figure 6, we can conclude that our numerical model estimates are generated by a consistent, well-behaved function.

Figure 5. The Optimal Reserve Demand

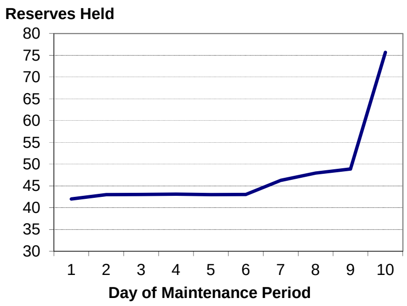
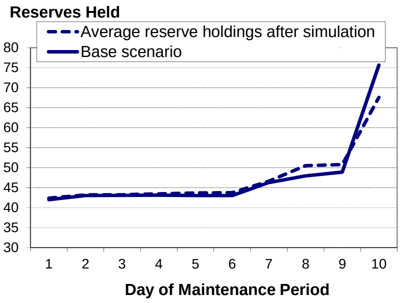


Figure 6. The Optimal Reserve Demand After Simulation



The characteristic features of the optimal intra-maintenance period reserve management can be summarized as follows:

(i) The bank enters the maintenance period with slightly lower reserve balance than required and keeps at this level throughout the first week,

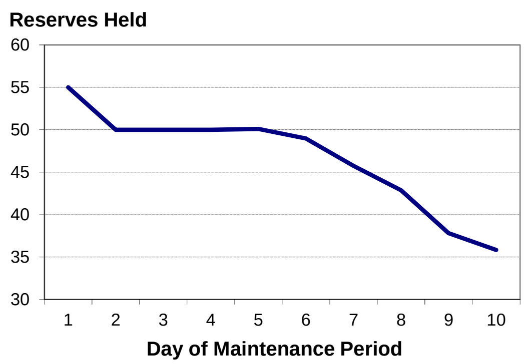
(ii) The bank starts to increase its demand for reserves gradually in the second week,

(iii) There is a significant upward spike in the demand for reserves on the last day of the maintenance period.

The upward spike on the last day and the tendency of holding more reserves on the last few days of the maintenance period is similar to the findings of previous works in the literature (see Bartolini et al., 2001, Clouse and Dow, 2002). These findings can be associated with the uncertainty over reserve needs causing bank's desire to avoid being locked-in with an excess reserve position. Moreover, Bartolini et al. (2001) explain this tendency with the existence of costs of trading. The authors claim that the banks have the most precise information on their reserve needs on the last day of the maintenance period and also the transaction costs induce them to delay trading. These two facts cause both money market interest rates and bank reserves to rise at the end of each maintenance period. The representative bank in our model does not want to be "long" on reserves late in the period which may cause ending the period with costly excess reserve position. Therefore, the bank hedges this risk by running short on reserves through much of the maintenance period and make the necessary adjustment on the last day.

Since the optimal pattern with the benchmark parameters is found to be different than the pattern derived from the recent empirical studies by Talaslı (2010, 2012), we change several of our assumptions and parameters and try to replicate the "actual" reserve management of banks in Turkey. As mentioned before, Talaslı (2010, 2012) shows that banks meet the bulk of their reserve requirement at the beginning of the reserve maintenance period implying that they hold significantly less reserves at the end. Similar pattern is observed among commercial banks in the euro area with the beginning of the financial crisis in summer 2007 (see Hauck and Neyer, 2011). Increasing uncertainties led European banks to frontload especially in the first phase of the financial crisis. Our alternative scenario is constructed by releasing the normality assumption of liquidity shocks. Under left-skewed normal distribution of shocks, where the probability of negative shocks is greater that the positive ones, the actual reserve management pattern seems to be the optimal pattern of the banks in Turkey (Figure 7). Thus, we can understand that during the time period that Talaslı (2010, 2012) examines bank reserve management patterns of Turkish banks, banks expect that negative liquidity shocks are more probable than positive shocks.

Figure 7. The Optimal Reserve Demand Under Alternative Scenario



4. Sensitivity Analysis of the Model

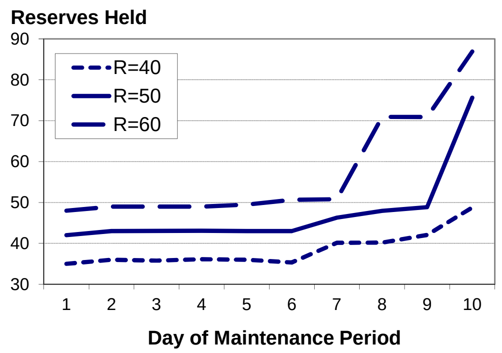
This section examines how the intra-period pattern changes in response to changes in our parameter settings first under normal then under left-skewed normal distribution of liquidity shocks.

4.1. Under Normal Distribution of Liquidity Shocks

4.1.1. Changes in the Level of Required Reserves

The effect of changes in the level of required reserves is shown in Figure 8. By changing the required reserve levels, the optimal pattern should shift parallel to the base case scenario. As expected, the optimal daily pattern shifts up/down with the increase/decrease in average required reserves. It should be noted that the representative bank adjusts its reserve balance significantly before the last day of the maintenance period when the average required reserve is 60. The increase in required reserve level causes the bank to make the appropriate adjustment before the last day.

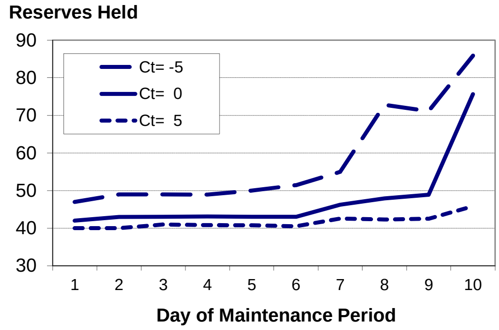
Figure 8. Effect of Required Reserves on Reserve Demand



4.1.2. Changes in the Level of Carry-In

Similar to the change in required reserves, different carry-in levels are expected to shift the optimal path parallel to the base case scenario. As depicted in Figure 9, when a bank enters the maintenance period with positive/negative carry-in, it makes a corresponding reduction/increase in its reserve holdings. Positive carry-in causes a smoother optimal pattern compared to the base case scenario. On the other hand, negative carry-in forces the bank to start increasing its reserve holding significantly before the last day.

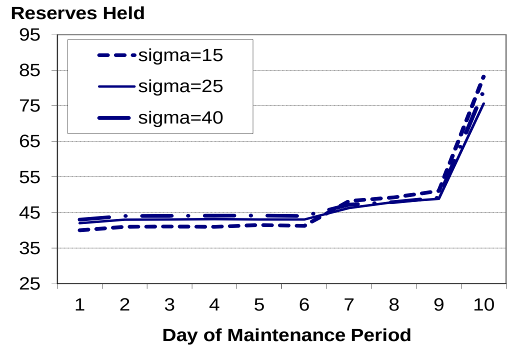
Figure 9. Effect of Carry-in



4.1.3. Changes in the Volatility of Liquidity Shocks

Figure 10 shows the demand for reserves in cases in which the standard deviation of liquidity shocks is equal to 15 and 40. The bank is expected to hold higher reserve balances due to increased uncertainty. Since the location of the optimal pattern does not shift significantly with the changes in uncertainty, with the increase in standard deviation of shocks, the bank ends the maintenance period with slightly higher cumulative average reserve position compared to the base case scenario.

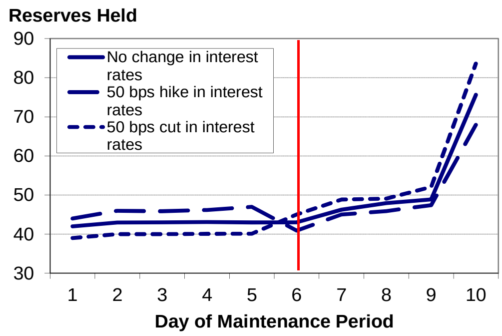
Figure 10. Effect of Uncertainty on Reserve Demand



4.1.4. Expected Interest Rate Decisions

Similar to Carpenter and Demiralp (2006b), we simulate maintenance periods in which the representative bank correctly anticipate a 50 basis points (bps) hike or cut in the policy interest rate on the sixth day of the period the rate decision. Figure 11 plots the results. In the event of an expected hike, the bank tends to minimize the opportunity cost and holds high level of reserves on the days prior to the rate decision and has lower balances at the second week of the maintenance period. On the other hand, the expected 50 bps cut will have the opposite tendency of the bank to hold lower reserves at the first week and compensate for the mean by having higher reserves during the second week.

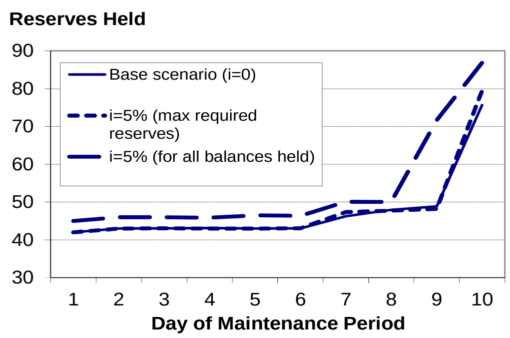
Figure 11. Effect of Expected Interest Rate Decisions



4.1.5. Remuneration of Reserves

Our base case scenario assumes no remuneration of reserve holdings. We perform simulations with interest paid only to average reserve requirements and to all reserve holdings including the excess reserves. Both simulations include interest payment equal to the CBRT overnight borrowing rate, which is 5 percent. Paying interest only to the reserve requirement has a very small effect on the optimal pattern. However, under this scenario the bank holds slightly higher reserve balances on average, as expected. On the other hand, paying interest to all reserve balances shifts the optimal path upwards and the bank's reserve adjustment starts from the 8th day. The parallel shift of the optimal pattern results from a reduction of the bank's opportunity cost.

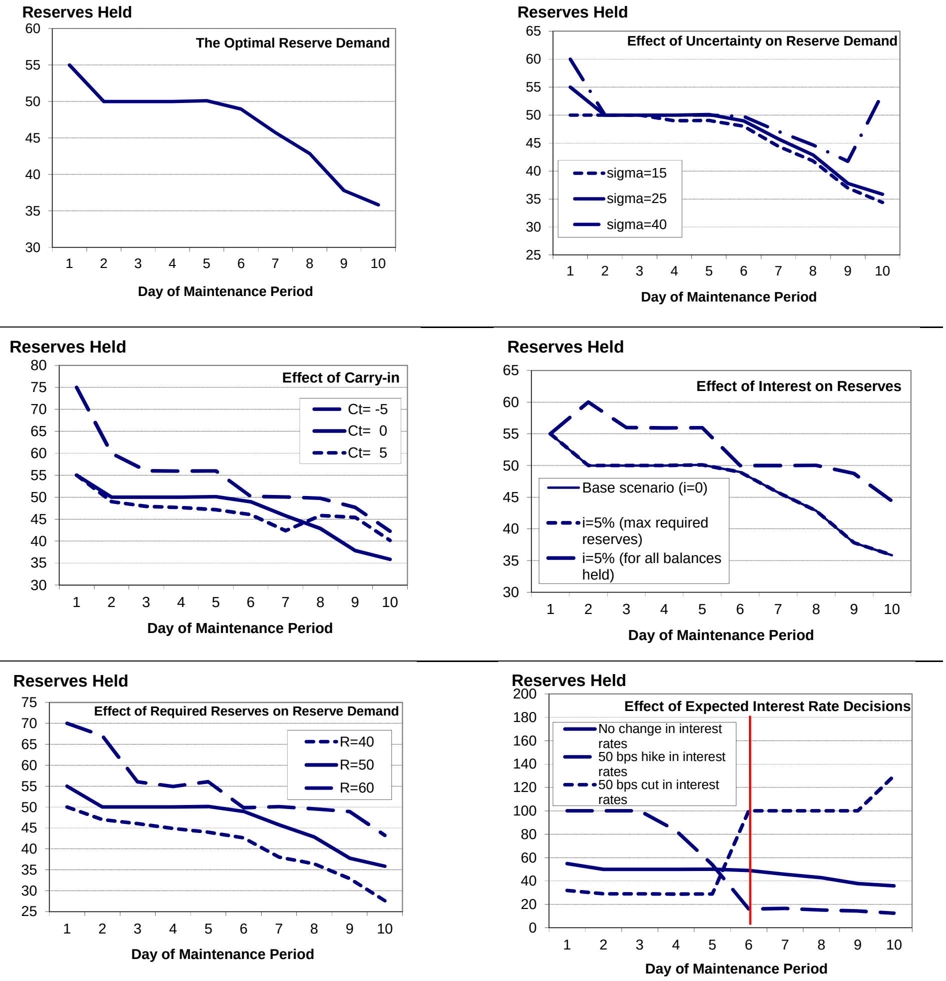
Figure 12. Effect of Interest on Reserves



4.2. Under Left-Skewed Normal Distribution of Liquidity Shocks

As mentioned briefly in Section 3.2, the change in the assumption of the distribution of liquidity shock, we may have different pattern for the optimal reserve held by the banks. In case of left-skewed normal distribution of liquidity shocks, banks expect negative liquidity shocks more probable than positive shocks and hence, frontload reserves and decumulate over the maintenance period. Compared to our base scenario, banks start 40 percent more reserves and finish maintenance period with less reserves. In this subsection, we perform all of our sensitivity exercises for the left-skewed normal distribution of liquidity shocks and present **optimal patterns** in Figure 13. As expected, there are some different and interesting patterns when banks expect liquidity withdrawals more probable **during the maintenance period**. For example, increasing uncertainty on reserve demand (σ=40) seems to lead banks to start accumulating more reserves initially and decumulate until 9^{th} day and make an upward spike on the last day of the maintenance period. Similar to normal distribution cases, we again observe that optimal patterns have different slopes at the last day of the maintenance period in the non-normal distribution cases. However, increasing concerns of banks on negative liquidity shocks yield an interesting pattern. More specifically, it appears that the slope of the optimal pattern changes from negative to positive at the last day suggesting banks' concerns on liquidity risk seem to be transmitted to the next maintenance period. In the experiment of different levels of expected interest rate, we observe similar upward spike in the last day when banks expect interest rates to decline on the 6^{th} day of the maintenance period. Furthermore, these banks seem not to frontload and start accumulating reserves after the rate cut decision. In contrary, banks that are expecting interest rates to increase, frontload significantly higher reserves and decumulate swiftly following the rate hike decision.

Figure 13. The Sensitivity Analysis Under Alternative Scenario



5. Conclusion

This paper uses numerical optimization methods to solve optimal reserve management pattern by adopting institutional aspects of the Turkish required reserve regime. More specifically, a representative Turkish bank chooses a sequence of reserves over 10 business days of the maintenance period, given a pre-determined value of carry-in in order to minimize the expected current period cost of reserve management together with the discounted expected costs in all future maintenance periods. The problem has two dynamic elements; the first one relates to the cost minimization of the current maintenance period, the second one is the inter-maintenance period dynamic problem. The intra-maintenance period problem includes the bank's choices of daily reserve levels which provide sufficient protection against late overnight window borrowing and also produce enough level of average reserve position that satisfies the reserve requirement. The inter-maintenance period problem arises because of the carryover provision by which the decisions made during the current period have important impact on all decisions made at the next period.

According to our findings, the representative bank enters the maintenance period with slightly lower reserve balance than required and keeps this level throughout the first week, then starts to increase its demand for reserves gradually in the second week. This pattern is similar to the findings among U.S. banks although the optimum level of reserves is significantly lower on Fridays due to the penalty structure. We also show that there is a significant upward spike in the demand for reserves on the last day of the maintenance period suggesting less concern on stochastic withdrawals and a penalty for having insufficient reserves to meet withdrawals earlier in the maintenance period. The sensitivity analysis on changes in the level of required reserves, carry-in levels, volatility of liquidity shocks, policy interest rates and remuneration of reserve holdings yields slightly different pattern for optimal reserve holdings. In case of left-skewed normal distribution of liquidity shocks, banks expecting negative liquidity shocks to be more probable than positive shocks, frontload their reserve holdings and decumulate over the maintenance period.

Under normal distribution of liquidity shocks, we still observe fulfillment of the reserve requirements at the last days of maintenance period suggesting the desire to avoid being locked-in to a positive excess reserve position. The bank hedges against the possibility of being locked-in by running short on reserves through much of the maintenance period and make the necessary adjustment in its reserve accounts on the last day. The upward spike in reserve demand on the last day can be observed where efficient interbank money market trading takes place with relatively low trading costs which encourage banks to hold reserves below the required amount till the last days of the maintenance period. This phenomenon can also be seen in a system where full deposit insurance exists which diminishes the adverse effects of negative liquidity shocks and the magnitude of liquidity shocks are ignorable relative to the required reserve level.

Since our model is quite flexible in analyzing the effect of potential changes in required reserve regime, further topics on research may include the effect of interest rate uncertainty on reserve demand. By assuming a stochastic interest rate variable, the model can reveal interesting results on the responsiveness of optimal reserve demand to change in interest rates.

After introducing a stochastic process, the model can provide a useful tool in analyzing the banks' reserve demand especially for central banks following interest rate corridor approach.

References

Bartolini, L., G. Bertola, A. Prati, 2001, "Banks' Reserve Management, Transaction Costs, and the Timing of Federal Reserve Intervention" *Journal of Banking and Finance* 25 (7): pp. 1287–1317.

Campbell, John Y., 1987, "Money Announcements, the Demand for Bank Reserves, and the Behavior of the Federal Funds Rate within the Statement Week" *Journal of Money, Credit, and Banking* 19, 56-67.

Carpenter, S. and S. Demiralp, 2006a, "The Liquidity Effect in the Federal Funds Market: Evidence from Daily Open Market Operations" *Journal of Money, Credit, and Banking*, Vol. 38, June 2006, 901-920

Carpenter, S. and Demiralp, S., 2006b, "Anticipation of Monetary Policy and Open Market Operations" *International Journal of Central Banking*, June 2006, 25-63.

Carpenter, S., and Demiralp, S., 2011,"Volatility, Money Market Rates, and the Transmission of Monetary Policy" *Finance and Economics Discussion Series* No: 2011-22, Federal Reserve Board.

Clouse, J., Dow, J., 1999, "Fixed Costs and the Behavior of the Federal Funds Rate" *Journal of Banking and Finance* 23, 1015-1029.

Clouse, J., and J. Dow, 2002, "A Computational Model of Banks' Optimal Reserve Management Policy" *Journal of Economic Dynamics and Control* 26 (11): pp 1787-1814.

Coelho, C. A. and Pinto, J. C. C., 2004, "Modelling of Banks' Optimal Behavior in the Brasilian Funds Market" *Revista EconomiA*, Selecta, Brasília (DF) V, 3,155-178.

Hamilton, James D., 1996, "The Daily Market for Federal Funds" *Journal of Political Economy* 104, 26-56.

Hamilton, James D., 1997, "Measuring the Liquidity Effect" *American Economic Review* 87 (1): pp. 80-97

Hauck A. and Neyer, U., 2011, "Reserve Holding Incentives of Commercial Banks in the Euro Area During the Financial Crisis" *Heinrich-Heine-University*, manuscript.

Furfine, C., 1998, "Interbank Payments and the Daily Federal Funds Rate" Board of Governors of the Federal Reserve System, *Finance and Economics Discussion Series*, 1998-31.

Gray, S., 2011, "Central Bank Balances and Reserve Requirements" *IMF Working Paper* 11/36.

Griffith, M. and Winters, D., 1995, "Day-of-the-week Effects in Federal Funds Rates: Further Empirical Findings" *Journal of Banking and Finance* 19, 1265-1284.

Kopecky, Kenneth J., and Alan L. Tucker, 1993, "Interest Rate Smoothness and the Non-settling-day Behavior of Banks" *Journal of Economics and Business* 45, 297-314

Longworth, D., 1989, "Optimal Behavior of Direct Clearers in a World with Zero Reserve Requirements" *Bank of Canada* Working Paper.

Norges Bank ,2012, "The Management of Bank Reserves: The System in Norway" http://www.norges-bank.no/en/price-stability/liquidity-management/the-liquidity-management - system/the-management-of-bank-reserves-the-system-in-norway/

Poole, W., 1968, “Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy” *Journal of Finance* 23 (5): pp. 769-791.

Spindt, P., Hoffmeister, R., 1988, “The Micromechanics of the Federal Funds Market: Implications for Day-of-the-week Effects in Funds Rate Variability” *Journal of Financial and Quantitative Analysis* 23: 401-416.

Talaslı, A., 2010, “The Demand for Excess Reserves in Turkey” *Journal of Banks and Bank Systems* 5 (4): 47-53.

Talaslı, A., 2012, “Türkiye’de Bankacılık Sistemi Zorunlu Karşılık Yönetimi” TCMB Uzmanlık Tezi.

TCMB Zorunlu Karşılık Uygulama Talimatı, 2011, http://www.tcmb.gov.tr.

TCMB Zorunlu Karşılıklar Hakkında 2005/1 Sayılı Tebliğ (2011). http://www.tcmb.gov.tr.

Appendix: The Optimization Problem Solved by Dynamic Programming

The optimization problem is to minimize the daily cost function

$W_j = f(i_o, i^{LON}, i_k, i_r, R_j, C_{t+1})$ by choosing a level of reserves (R_j) at each day of the maintenance period. The minimization procedure runs for 10 days and the optimal reserve levels for each day of the maintenance period are computed.

The parameters used in optimization and carry-over rules:

W_j**:** Daily cost function, i_o**:** Opportunity cost, i^{LON}**:** CBRT LON borrowing rate, i_k**:** Interest rate for reserve deficiency, i_r**:** Interest rate of remuneration, C_t**:** Carry-in from the previous period, C_{t+1}**:**Carry-over to the next period, ***Z*****:** Vector of liquidity shocks, $mean_Z$**:** Mean of liquidity shocks, $sigma_Z$**:** Standard deviation of liquidity shocks, ***A*****:** Vector of cumulative average reserve position, R_j**:** Possible choices of reserve holdings, ***RR*****:** Required reserve level, α**:** Coefficient of the value function (a), β**:** Coefficient of the value function (b), ***P*****:** Probability vector, ***R*****:** Reserves held; ***D*****:** Difference between required reserves and reserves held; P_t**:** Reserve amount subject to penalty.

Base case scenario: Normal distribution of liquidity shocks

P* = normal probability distribution function (Z, $mean_Z$, $sigma_Z$);

P = P*/sum(P*); scaling the probabilities proportionately to sum up to 1 ;

Alternative scenario: Left-skewed normal distribution of liquidity shocks

P* = skewed normal probability distribution function (Z, $mean_Z$, $sigma_Z$, shape);

P = P*/sum(P*); scaling the probabilities proportionately to sum up to 1;

The rules for carry-over (C_{t+1}) and penalty (P_t)

C_t		C_{t+1}
$C_t<0$	$R<RR$	max(-0.1xRR,D)
	$RR \le R < RR-C_t$	0
	$R>RR-C_t$	min(0,1xRR,$D+C_t$)
$C_t \ge 0$	$R<RR-C_t$	max(-0,1xRR,$D+C_t$)
	$RR-C_t<R \le RR$	0
	$R>RR$	min(0,1xRR,D)
		P_t
$C_t<0$	$R<RR$	min(C_t+D+0,1xRR,0)
	$RR \le R < RR-C_t$	min(C_t+D,0)
	$R>RR-C_t$	0
$C_t \ge 0$	$R<RR-C_t$	min(C_t+D+0,1xRR,0)
	$RR-C_t<R \le RR$	0
	$R>RR$	0

The daily cost functions to be minimized

$$W_{10} = (i_0 \max(0, R_{10} + Z) + 2i_d \max(0, -R_{10} - Z) + 0.998\,.\,4\; i_k P_t - i_r \left(A \frac{13}{14} + \frac{\max(0, R_{10} + Z)}{14}\right) + 0.998 \alpha \beta^{C_{t+1}})P$$

$$W_9 = (i_0 \max(0, R_9 + Z) + 2i_d \max(0, -R_9 - Z) + spline(A, W_{10}, \left(A \frac{12}{13} + \frac{\max(0, R_9 + Z)}{13}\right))P$$

$$W_8 = (i_0 \max(0, R_8 + Z) + 2i_d \max(0, -R_8 - Z) + spline(A, W_9, \left(A \frac{11}{12} + \frac{\max(0, R_8 + Z)}{12}\right))P$$

$$W_7 = (i_0 \max(0, R_7 + Z) + 2i_d \max(0, -R_7 - Z) + spline(A, W_8, \left(A \frac{10}{11} + \frac{\max(0, R_7 + Z)}{11}\right))P$$

$$W_6 = (i_0 \max(0, R_6 + Z) + 2i_d \max(0, -R_6 - Z) + spline(A, W_7, \left(A \frac{7}{10} + \frac{\max(0, R_6 + Z)}{10} \frac{3}{10}\right))P$$

$$W_5 = (i_0 \max(0, R_5 + Z) + 2i_d \max(0, -R_5 - Z) + spline(A, W_6, \left(A \frac{6}{7} + \frac{\max(0, R_5 + Z)}{7}\right))P$$

$$W_4 = (i_0 \max(0, R_4 + Z) + 2i_d \max(0, -R_4 - Z) + spline(A, W_5, \left(A \frac{5}{6} + \frac{\max(0, R_4 + Z)}{6}\right))P$$

$$W_3 = (i_0 \max(0, R_3 + Z) + 2i_d \max(0, -R_3 - Z) + spline(A, W_4, \left(A \frac{4}{5} + \frac{\max(0, R_3 + Z)}{5}\right))P$$

$$W_2 = (i0 \quad \max(0, R_2 + Z) + 2i_d \max(0, -R_2 - Z) + spline(A, W_3, \left(A \frac{3}{4} + \frac{\max(0, R_2 + Z)}{4}\right))P$$

$$W_1 = (i_0 \max(0, R_1 + Z) + 2i_d \max(0, -R_1 - Z) + spline(A, W_2, \max(0, R_1 + Z))P$$

The iteration process in estimating the coefficients of the value function

The value function is assumed to be log-linear: $V(C_t) = ab^{C_t}$. When we take the log of both sides we get; $\log V(C_t) = \log a + Ct.\log b$. In order to estimate the coefficients a and b, we use linear regression where $\log V(C_t)$ is dependent and C_t is the explanatory variable. Since the possible choice vector of C_t ranges between -5 to 5, we have 11 carry-in and consequently 11 value functions.