Risk Sharing and Real Exchange Rate: The Roles of Non-tradable Sector and Trend Shocks

September 2013

Hüseyin Çağrı AKKOYUN
Yavuz ARSLAN
Mustafa KILINÇ
Risk Sharing and Real Exchange Rate: 
The Roles of Non-tradable Sector and Trend Shocks

Hüseyin Çağrı Akkoyun    Yavuz Arslan    Mustafa Kılınç

September 2013

Abstract

In this paper, we show that tradable and non-tradable TFP processes of the US and Europe have unit roots and can be modeled by a vector error correction model (VECM). Then, we develop a standard two country and two good (tradable and non-tradable) DSGE model. Our model implies that using cointegrated TFP processes improves the real exchange rate (RER) volatility and risk sharing puzzles compared to the model with transitory TFP processes. Cointegrated TFP shocks, or trend shocks, generate significant income effects, and amplify the mechanisms that produce high RER volatility. Moreover, trend shocks break the tight link between relative consumption and RER for low and high values of trade elasticity parameters.

Key Words: Trends Shocks, Risk Sharing, Real Exchange Rates

Jel Classification: E32, F41, F44

---

1We thank Juan F. Rubio-Ramirez, Fatih Ekinci, Hande Küçük and seminar participants at the CEE 2012 Annual Conference at Bogazici University, Central Bank of the Republic of Turkey and Fall 2012 Midwest Macroeconomics Meetings at the University of Colorado Boulder for valuable comments and suggestions.

Hüseyin Çağrı Akkoyun, Yavuz Arslan and Mustafa Kılınç: Central Bank of the Republic of Turkey, İstiklal Cad. No:10, Ulus, Ankara, Turkey. Phone: +90-312-507 5400. e-mails: cagri.akkoyun@tcmb.gov.tr, yavuz.arslan@tcmb.gov.tr, mustafa.kilinc@tcmb.gov.tr. The views expressed in this paper do not necessarily represent those of Central Bank of the Republic of Turkey or its staff.
1 Introduction

The two well known puzzles in the international real business cycle literature are the risk sharing and the RER volatility puzzles. The standard international real business cycle (IRBC) models, introduced by Backus, Kehoe and Kydland (1992,1995), the so called BKK model, generates nearly perfect correlation between relative consumption and RER which implies a very high level of risk sharing. However, Backus and Smith (1993) and Kollmann (1995) document that the correlation between relative consumption and RER is negative in the data which means that a country consumes more when its consumption basket is relatively expensive. Another puzzling feature of the data that IRBC models fail to mimic is the relative volatility of RER with respect to output. Models generate very low values for RER volatility at odds with data which is referred as exchange rate volatility puzzle.

In this paper, we first show that tradable and non-tradable TFP processes of the US and Europe have unit roots and the relation between these processes can be well captured by VECM. Then, we develop an international macro model with two countries, two goods (tradable and non-tradable) and non-contingent single bond. Our model with cointegrated TFP processes improves the real exchange rate (RER) volatility and risk sharing puzzles compared to the model with transitory TFP processes. In our setting, the source of improvement is cointegrated TFP processes that act as persistent transitory shocks.

Cointegrated TFP shocks, or trend shocks, generate significant income effects, and amplify the mechanisms that produce high volatility in the RER. Moreover, the mechanism modeled in the paper breaks the tight link between relative consumption and RER for low and high values of trade elasticity parameters. For low values of trade elasticity, a positive trend shock to the TFP in one country increases the demand for tradable goods in that country due to income effect. Since tradable goods are not easily substitutable, the prices of tradable and non-tradable goods increase, and in turn RER appreciates. As RER appreciates together with a higher consumption in that country, the model generates a negative correlation between relative consumption and RER, implying low risk sharing as in the data. For high values of the trade elasticity parameter, a positive trend shock increases the demand for both sectors but the price change in tradable sector is limited since tradable goods are now substitutes. Although, tradable price does not vary much, non-tradable prices increase since tradable and non-tradable goods are complements. In turn, RER appreciates and this again breaks the risk sharing.

The stubborn nature of the risk sharing and RER volatility puzzles attracted widespread interest in the literature. Several papers analyze risk sharing within different frameworks. For example, Heathcote and Perri (2002) loosens the complete market assumption in BKK and introduces non-contingent single asset.
They conclude that trade related statistics improve in financial autarky case. Chari, Kehoe and Mcgrattan (2002) show that in a world with sticky prices and a non-contingent bond, risk sharing puzzle still exists while their model generates high RER volatility. Among others, two studies that improve on risk sharing puzzle are Corsetti, Dedola and Leduc (2008), here after CDL, and Benigno and Thoenissen (2008), here after BT. CDL set up an IRBC model with tradable and non-tradable sectors and solve mentioned two puzzles for low and high trade elasticity parameters. BT use a two-country two-sector, tradable and non-tradable, model with an explicit role of monetary policy. They obtain Backus-Smith relation and argue that Balassa-Samuelson effect produces negative cross-correlation between relative consumption and RER. However, the relative volatility of RER is small for low and high trade elasticities. A recent paper by Benigno and Küçük (2011) investigates the consumption-RER anomaly in an environment with different asset market structures. For the single bond case they exhaustively explain the movements in the relative consumption and RER for different values of the trade elasticity parameter. However, they find that if the asset markets consist of two bonds, results in single bond case disappear and risk sharing puzzle reemerges.

The studies discussed above assume that TFP processes are stationary. However, Aguiar and Gopinath (2007), in their seminal work, allow the trend component of the TFP process to follow a stochastic path. By using this innovation, their small open economy model can explain some characteristic features of the emerging economies. Rabanal, Rubio-Ramirez, and Tuesta (2011), here after RRT, embed the trend shocks into a two country framework and show that TFP processes for the US and the rest of the world are cointegrated. Their model generates higher RER volatility compared to the similar models in which TFP processes are assumed to be transitory. In a similar paper, Ireland (2011) compares the sources of growth in the US and Euro Area (EA) where technology, preference and investment-specific technology shocks are cointegrated in an estimated two country model.

Our model is closely related with the CDL and RRT. The main difference from the CDL is modelling TFPs as cointegrated processes instead of transitory processes. CDL obtains negative correlation between relative consumption and RER for high values of trade elasticity parameter only with persistent transitory shocks which is not supported by the data. However, our model can address the Backus-Smith puzzle


2 However, Arslan et al. (2012) shows that in the two bond-case, trend shocks breaks the risk sharing in the model.

3 Rest of the world consists of Euro area, Japan, Canada, United Kingdom, and Australia.
with cointegrated TFP processes. On the other hand, our main difference from RRT is the inclusion of the non-tradable sector in the model. This extra feature improves the results for Backus-Smith puzzle significantly whereas the improvement in RER volatility is limited.

Rest of the paper is as follows. Section 2 presents the model and calibration of parameters Section 3 shows our results and Section 4 implements robustness analysis. Finally, Section 5 concludes.

2 Model

We setup a two-country two-sector production economy model with a single tradable bond. Our model includes tradable and non-tradable sectors similar to the model in CDL and BT. However unlike them, we assume TFP processes are cointegrated as in RRT and Ireland (2011)\(^4\).

Our world economy consists of two countries, one representing home country (H) and the other representing foreign country (F). Sectors are indexed as \(i = HT, HN, FT, FN\) representing the home tradable and non-tradable sectors and foreign tradable and non-tradable sectors, respectively and time period is denoted with \(t = 0, 1, 2, \ldots\) subscripts. In each country, firms use capital, labor and sector specific labor technology to produce tradable inputs and non-tradable inputs. Production sharing takes place in intermediate goods, so countries use both home and foreign tradable inputs to produce their respective intermediate goods. Then, they combine this intermediate good with non-tradable input to produce their distinctive final goods, which later to be consumed or invested by the representative households of each countries.

2.1 Tradable and Non-tradable Goods-Producing Firms

The perfectly competitive tradable and non-tradable good producer firms combine capital and labor with their sector specific technology through a Cobb-Douglas production function to obtain the tradable goods \((Y^i_{HT}, Y^i_{FT})\) and the non-tradable goods \((Y^i_{HN}, Y^i_{FN})\). The production function is:

\[
Y^i_t = (K^i_t)^\alpha (Z^i_t L^i_t)^{1-\alpha}
\]

where \(i \in \{HT, HN, FT, FN\}\). The firm rents \(K^i_t\) units of capital and hires \(L^i_t\) units of labor in the production with technology \(Z^i_t\). Capital share in production is \(\alpha\), which lies between 0 and 1. Tradable and non-tradable firms in both countries maximize their profits (2) by taking all prices as given:

\[^4\]These studies have only tradable sector.
\[
\max_{K_t^i \geq 0, L_t^i \geq 0} P_t^i Y_t^i - Q_t^i K_t^i - W_t^i L_t^i
\]

where \( P_t^i \) is the price of tradable or non-tradable goods, \( Q_t^i \) is the rental price for capital, and \( W_t^i \) is the wage for \( i \in \{HT, HN, FT, FN\} \).

### 2.2 Intermediate Goods Producing Firms

There is production sharing between countries, i.e. both countries use each other’s tradable inputs to produce their respective tradable intermediate goods. Competitive intermediate goods producer firms of the home country uses \( A_t^H \) units of home tradable good and \( B_t^H \) units of foreign tradable good to produce \( Y_t^{H,\text{int}} \) units of intermediate good which is used by final good firms of home country. Home country intermediate good producers use a constant elasticity of substitution production technology as follows:

\[
Y_t^{H,\text{int}} = \left[ (1 - \omega_1)^{\frac{1}{\theta}} (A_t^H)^{\frac{\theta}{\theta-1}} + (\omega_1)^{\frac{1}{\theta}} (B_t^H)^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}}
\]

where \( \theta \) measures the elasticity of substitution between the home tradable input \( A_t^H \) and foreign tradable input \( B_t^H \), and \( \omega_1 \) is the share of foreign tradable input in home country’s intermediate good production. Firms maximize their profits (equation 4) by taking all prices as given:

\[
\max_{A_t^H \geq 0, B_t^H \geq 0} P_t^{H,\text{int}} Y_t^{H,\text{int}} - P_t^A A_t^H - P_t^B B_t^H
\]

where \( P_t^{H,\text{int}} \) is the price of home intermediate good.

Similarly, intermediate good firms of foreign country uses \( A_t^F \) units of home tradable good and \( B_t^F \) units of foreign tradable good to produce \( Y_t^{F,\text{int}} \) units of intermediate good which is used by final good-producer firms of foreign country. Foreign country intermediate good producers use a constant elasticity of substitution production technology as follows:

\[
Y_t^{F,\text{int}} = \left[ (\omega_1)^{\frac{1}{\theta}} (A_t^F)^{\frac{\theta}{\theta-1}} + (1 - \omega_1)^{\frac{1}{\theta}} (B_t^F)^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}}
\]

where \( A_t^F \) is the home tradable input, \( B_t^F \) is the foreign tradable input, and \( \omega_1 \) is the share of home tradable input in foreign country’s intermediate good production. Firms maximize their profits (equation 6) by taking all prices as given:

\[
\max_{A_t^F \geq 0, B_t^F \geq 0} P_t^{F,\text{int}} Y_t^{F,\text{int}} - P_t^A A_t^F - P_t^B B_t^F
\]
where \( P^F_{t, \text{int}} \) is the price of foreign intermediate good.

### 2.3 Final Goods-Producing Firms

Home final goods producing firms combine \( Y^{H, \text{int}}_t \) units of home intermediate good and \( Y^{HN}_t \) units of home non-tradable goods through a production function with constant elasticity of substitution in order to produce \( Y^H_t \) units of final home good with price of \( P^H_t \):

\[
Y^H_t = \left(1 - \omega_2\right) \frac{1}{\theta_2} \left(Y^{H, \text{int}}_t \right)^{\frac{\theta_2 - 1}{\theta_2}} + \left(\omega_2\right) \frac{1}{\theta_2} \left(Y^{HN}_t \right)^{\frac{\theta_2 - 1}{\theta_2}} \left(1 - \omega_2\right)^{\theta_2-1} \tag{7}
\]

where \( \theta_2 \) is the elasticity of substitution between intermediate good and non-tradable good, and \( \omega_2 \) is the share of non-tradable input in home country’s final good production.

The foreign final goods-producing firms use \( Y^{F, \text{int}}_t \) units of foreign intermediate good and \( Y^{FN}_t \) units of foreign non-tradable goods in order to produce \( Y^F_t \) units of final home good with price of \( P^F_t \), either to consume or invest. Foreign final goods-producing firm uses a constant elasticity of substitution production technology as follows:

\[
Y^F_t = \left(1 - \omega_2\right) \frac{1}{\theta_2} \left(Y^{F, \text{int}}_t \right)^{\frac{\theta_2 - 1}{\theta_2}} + \left(\omega_2\right) \frac{1}{\theta_2} \left(Y^{FN}_t \right)^{\frac{\theta_2 - 1}{\theta_2}} \left(1 - \omega_2\right)^{\theta_2-1} \tag{8}
\]

Both final good producers maximize their profits by taking the prices as given.

### 2.4 Representative Households

The expected life time utility of the representative home consumer is described by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\mu (1 - L^H_t)^{\theta_2} \left(1 - L^{HN}_t - L^H_t \right)^{(1 - \mu)} (1 - \gamma)}{1 - \gamma} \right\} \tag{9}
\]

where \( C^H_t \) is the consumption, \( \beta \) is the discount factor, \( \gamma \) is the risk aversion parameter, and \( \mu \) is the share parameter between consumption and leisure. Households provide labor services \( L^H_t \) to tradable and \( L^{HN}_t \) to non-tradable firms in their countries at the wage rates of \( W^H_t \) and \( W^{HN}_t \). They also own the capital stock in both sectors (\( K^H_t \) and \( K^{HN}_t \)) and rent it to the firms at rates \( Q^H_t \) and \( Q^{HN}_t \). Both labor and capital are mobile across sectors within the country but they are immobile across countries. Households can trade an international bond \( D^H_t \). In total, income of households are wage income from labor supply, rent income from capital supply and the interest income from the international bond. Households use their income to finance their consumption \( C^H_t \) and investment in tradable \( I^H_t \) and non-tradable sector.
$I_t^{HN}$, and to buy new bonds $D_{t+1}^H$. They also pay adjustment costs for capital and bond changes. Then the budget constraint for households in home country is:

$$ P_t^H C_t^H + P_t^H I_t^{HT} + P_t^H I_t^{HN} + \frac{P_t^H D_{t+1}^H}{R_t} + P_t^H \frac{\phi_d}{2} U_t^H \left( \frac{D_{t+1}^H}{U_t^H} \right)^2 \leq W_t^{HT} L_t^{HT} + W_t^{HN} L_t^{HN} + Q_t^{HT} K_t^{HT} + Q_t^{HN} K_t^{HN} + P_t^H D_t^H $$

(10)

where $1/R_t$ is the price of one unit bond at time $t$ that matures at period $t+1$. $\phi_d > 0$ is the adjustment cost parameter for bond holdings that closes small open economy models as discussed in Schmitt-Grohe and Uribe (2003). Bond adjustment cost is scaled by a factor $U_t^H = Z_t^{HT}$ as in Ireland (2011) to achieve consistency in the model and ensure a zero adjustment cost along the steady-state growth path. The law of motion for the both capital of tradable and non-tradable sectors are:

$$ K_{t+1}^{HT} \leq (1-\delta)K_t^{HT} + I_t^{HT} - \frac{\phi_k}{2} \left( \frac{I_t^{HT}}{K_t^{HT}} - \eta_t^{HT} \right)^2 K_t^{HT} $$

(11)

$$ K_{t+1}^{HN} \leq (1-\delta)K_t^{HN} + I_t^{HN} - \frac{\phi_k}{2} \left( \frac{I_t^{HN}}{K_t^{HN}} - \eta_t^{HN} \right)^2 K_t^{HN} $$

(12)

where $\phi_k > 0$ is the adjustment cost parameter for capital, $\delta$ is the depreciation rate ($1 > \delta > 0$), $\eta_t^{HT}$ and $\eta_t^{HN}$ are the parameters that provide zero cost along the steady-state growth path.

The representative home consumer maximizes the expected utility (9) subject to the constraints (10), (11) and (12) where the choice variables are $(C_t^H, L_t^{HT}, L_t^{HN}, I_t^{HT}, I_t^{HN}, K_{t+1}^{HT}, K_{t+1}^{HN}, D_{t+1}^H)$. The problem of the representative foreign consumer is symmetric, where $C_t^F$ is the foreign consumption, $L_t^{FT}$ and $L_t^{FN}$ are hours worked, $I_t^{FT}$ and $I_t^{FN}$ are investment, $K_{t+1}^{FT}$ and $K_{t+1}^{FN}$ are capital stocks in tradable and non-tradable sectors respectively, and $D_{t+1}^F$ is the bond holdings.

### 2.5 Trade Variables and Equilibrium Conditions

Home country’s net exports are expressed in the units of home final good:

$$ N_t^H = (P_t^{HT} A_t^F - P_t^{FT} B_t^H) / P_t^H $$

(13)

For both countries there are two more relevant prices, i.e. terms of trade and real exchange rates. We define the terms of trade, $ToT_t$, as the ratio of its import prices to its export prices, $ToT_t = P_t^{FT} / P_t^{HT}$. We define the real exchange rate, $ReR_t$, as the ratio of foreign final goods prices to home final goods.
prices, $ReR_t = P_t^F/P_t^H$. An increase in the $ToT$ means a depreciation of the terms of trade of the home country by making its export prices less expensive or import prices more expensive. An increase in the $ReR$ implies a depreciation of real exchange rate for the home country and an appreciation for the foreign country.

Market clearing conditions for tradable good sectors in home and foreign countries are given as:

$$ Y_t^{HT} = A_t^H + A_t^F $$

$$ Y_t^{FT} = B_t^H + B_t^F $$

Since there is no trade in final goods, final goods in the country is used for consumption, investment and adjustment costs and we have the following resource constraints:

$$ Y_t^H = C_t^H + \frac{\phi_d}{2} U_t^H \left( \frac{D_{t+1}^H}{U_t^H} \right)^2 + T_t^{HT} + T_t^{HN} $$

$$ Y_t^F = C_t^F + \frac{\phi_d}{2} U_t^F \left( \frac{D_{t+1}^F}{U_t^F} \right)^2 + T_t^{FT} + T_t^{FN} $$

Also there is only one bond in the international financial markets and the net supply is zero, giving us:

$$ D_t^H + D_t^F = 0 $$

### 3 Vector Error Correction Model (VECM)

This section documents the cointegration relation between the TFP levels of tradable and non-tradable sectors in both countries. First, we describe data and show that all four TFP processes include unit root. Second, all six pairs of TFP processes are cointegrated. Finally, we estimate the TFP system with three cointegrating vectors.

#### 3.1 Data and Cointegration

We calculate the TFP processes for tradable and non-tradable sectors by using the 60 industry database of Groningen Growth and Development Centre as in BT. The data spans a period between 1979 and 2003. We take the US as the home country and EU-15 as the foreign country. Fifty seven sectors in each country are grouped into two as tradable and non-tradable sectors\(^5\). The TFP processes for each group

\(^5\)Detailed list is available upon request.
is equal to the logarithms of the weighted sum of labor productivities. We denote TFP processes of home tradable, home non-tradable, foreign tradable and foreign non-tradable sectors with \( \ln(Z_{HT}^t), \ln(Z_{HN}^t), \ln(Z_{FT}^t), \ln(Z_{FN}^t) \), respectively.

First, we provide evidence for presence of one unit root in the TFP processes. Table 1 presents the test results for augmented Dicky-Fuller test (ADF), Elliot, Rothenberg and Stock (1996) detrended residual test, and Phillips and Perron (1988) test (PP). The lag length is chosen according to Schwarz information criterion. We assume a constant and a trend in each specification. All tests do not reject the null hypothesis of having a unit root for levels at the 5% critical value. Also, tests reject the null hypothesis for first-difference series. So, TFP processes are nonstationary and include one unit root.

<table>
<thead>
<tr>
<th>( \ln(Z_{HT}^t) )</th>
<th>ADF</th>
<th>ERS</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>-2.13*</td>
<td>-2.42*</td>
<td>-2.38*</td>
</tr>
<tr>
<td>First-diff.</td>
<td>-4.57</td>
<td>-4.44</td>
<td>-4.58</td>
</tr>
<tr>
<td>( \ln(Z_{HN}^t) )</td>
<td>Level</td>
<td>-1.19*</td>
<td>-2.15*</td>
</tr>
<tr>
<td>First-diff.</td>
<td>-8.50</td>
<td>-8.50</td>
<td>-10.81</td>
</tr>
<tr>
<td>( \ln(Z_{FT}^t) )</td>
<td>Level</td>
<td>-2.56*</td>
<td>-2.79*</td>
</tr>
<tr>
<td>First-diff.</td>
<td>-4.68</td>
<td>-4.89</td>
<td>-11.85</td>
</tr>
<tr>
<td>( \ln(Z_{FN}^t) )</td>
<td>Level</td>
<td>-1.67*</td>
<td>-1.82*</td>
</tr>
<tr>
<td>First-diff.</td>
<td>-4.67</td>
<td>-4.81</td>
<td>-4.67</td>
</tr>
</tbody>
</table>

* denotes null hypothesis of unit root is not rejected at 5% level

Second, we investigate the cointegration relation between the six possible pairs of these four TFP processes. For this purpose, we use the Johansen (1991) trace and maximum eigenvalue cointegration tests. Table 2 reports the results for both tests under the "intercept without deterministic trend" specification and lag length is set to zero. The tests for all pairs indicate one cointegrating equation at 5% level.
Table 2: Pairwise Cointegration

<table>
<thead>
<tr>
<th>Vectors</th>
<th>Eigenvalue</th>
<th>Trace</th>
<th>p-value</th>
<th>Max-Eigen.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((Z_{t}^{HT}, Z_{t}^{HN}))</td>
<td>0</td>
<td>0.60</td>
<td>26.24</td>
<td>0.01</td>
<td>21.76</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.17</td>
<td>4.47</td>
<td>0.35</td>
<td>4.47</td>
</tr>
<tr>
<td>((Z_{t}^{HT}, Z_{t}^{FT}))</td>
<td>0</td>
<td>0.79</td>
<td>41.47</td>
<td>0</td>
<td>37.18</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.16</td>
<td>4.29</td>
<td>0.37</td>
<td>4.29</td>
</tr>
<tr>
<td>((Z_{t}^{HT}, Z_{t}^{FN}))</td>
<td>0</td>
<td>0.88</td>
<td>57.37</td>
<td>0</td>
<td>51.16</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.23</td>
<td>6.21</td>
<td>0.18</td>
<td>6.21</td>
</tr>
<tr>
<td>((Z_{t}^{HN}, Z_{t}^{FT}))</td>
<td>0</td>
<td>0.81</td>
<td>46.01</td>
<td>0</td>
<td>39.27</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.24</td>
<td>6.74</td>
<td>0.14</td>
<td>6.74</td>
</tr>
<tr>
<td>((Z_{t}^{HN}, Z_{t}^{FN}))</td>
<td>0</td>
<td>0.88</td>
<td>59.10</td>
<td>0</td>
<td>51.80</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.26</td>
<td>7.30</td>
<td>0.11</td>
<td>7.30</td>
</tr>
<tr>
<td>((Z_{t}^{FT}, Z_{t}^{FN}))</td>
<td>0</td>
<td>0.90</td>
<td>62.92</td>
<td>0</td>
<td>55.90</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.25</td>
<td>7.02</td>
<td>0.13</td>
<td>7.02</td>
</tr>
</tbody>
</table>

3.2 The VECM Model

The above analysis show that each pair of TFP processes are cointegrated. However, this is not adequate to ensure a balanced growth path. For balanced growth, \(\ln(Z_{i}^{t})\) and \(\ln(Z_{j}^{t})\) should be cointegrated with cointegrating vector \((1,-1)\). Formally, the two variable VECM system can be written for \(i, j \in \{HT, HN, FT, FN\}\) and \(i \neq j\):

\[
\Delta \log(Z_{i}^{t}) = c_{i} + \kappa_{i} (\log(Z_{t-1}^{i}) - \gamma \log(Z_{t-1}^{j}) - \log(\zeta)) + \varepsilon_{i,t} \tag{19}
\]

\[
\Delta \log(Z_{j}^{t}) = c_{j} + \kappa_{j} (\log(Z_{t-1}^{i}) - \gamma \log(Z_{t-1}^{j}) - \log(\zeta)) + \varepsilon_{j,t} \tag{20}
\]

where \(\Delta\) is first-difference operator, \((1,-\gamma)\) is cointegrating vector, \(\varepsilon_{t} \sim N(0,\sigma)\) and \(\varepsilon_{t}^{*} \sim N(0,\sigma^{*})\) are noise terms. As stated in RRT, if the hypothesis \(\gamma = 1\) cannot be rejected in this system, then balance growth cannot be rejected. Cointegrated vector \((1,-1)\) implies that any positive deviation in growth rate difference, \(\log(Z_{t-1}^{i}) - \log(Z_{t-1}^{j})\), decreases the growth of \(\Delta \log(Z_{i}^{t})\) and increases \(\Delta \log(Z_{j}^{t})\) for \(\kappa > 0\) and \(\kappa^{*} < 0\). This relation ensures a balanced growth path for the existence of steady state of the system.

Table 3 reports the likelihood ratio test statistics for \(\gamma = 1\) under the specification "intercept without
deterministic trend" and lag length is set to zero. Test statistics show that $\gamma = 1$ cannot be rejected for all pairs for significance level 1%. Thus, likelihood test indicates that balanced growth path cannot be rejected.

<table>
<thead>
<tr>
<th>Table 3: Likelihood Ratio Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{HT}, Z_t^{HN} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{HT}, Z_t^{FT} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{HT}, Z_t^{FN} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{HN}, Z_t^{FT} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{HN}, Z_t^{FN} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\left( Z_t^{FT}, Z_t^{FN} \right)$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Our purpose is to estimate a VECM model for the four variable $-\ln(Z_t^{HT}), \ln(Z_t^{HN}), \ln(Z_t^{FT}), \ln(Z_t^{FN})$-system that ensures balanced growth. We expand the two variable system in Ireland (2011) to four variable system:

\[
\ln(Z_t^{HT}/Z_{t-1}^{HT}) = (1 - \rho^{HT}) \ln(z^{HT}) + \rho^{HT} \ln(Z_{t-1}^{HT}/Z_{t-2}^{HT}) + \kappa^{HTHN} \ln(Z_t^{HT}/Z_{t-1}^{HN}) \\
+ \kappa^{HTFT} \ln(Z_t^{HT}/Z_{t-1}^{FT}) + \kappa^{HTFN} \ln(Z_t^{HT}/Z_{t-1}^{FN}) + \epsilon_t^{HT} \tag{21}
\]

\[
\ln(Z_t^{HN}/Z_{t-1}^{HN}) = (1 - \rho^{HN}) \ln(z^{HN}) + \rho^{HN} \ln(Z_{t-1}^{HN}/Z_{t-2}^{HN}) + \kappa^{HNHT} \ln(Z_t^{HN}/Z_{t-1}^{HT}) \\
+ \kappa^{HNTF} \ln(Z_t^{HN}/Z_{t-1}^{FT}) + \kappa^{HNFN} \ln(Z_t^{HN}/Z_{t-1}^{FN}) + \epsilon_t^{HN} \tag{22}
\]
\[
\ln(Z_{t}^{FT}/Z_{t-1}^{FT}) = (1 - \rho^{FT}) \ln(z^{FT}) + \rho^{FT} \ln(Z_{t-1}^{FT}/Z_{t-2}^{FT}) + \kappa^{FTHT} \ln(Z_{t-1}^{FT}/Z_{t-1}^{HT}) \\
+ \kappa^{FTHN} \ln(Z_{t-1}^{FT}/Z_{t-1}^{HN}) + \kappa^{FTFN} \ln(Z_{t-1}^{FT}/Z_{t-1}^{FN}) + \epsilon_{t}^{FT}
\]

(23)

\[
\ln(Z_{t}^{FN}/Z_{t-1}^{FN}) = (1 - \rho^{FN}) \ln(z^{FN}) + \rho^{FN} \ln(Z_{t-1}^{FN}/Z_{t-2}^{FN}) + \kappa^{FNHT} \ln(Z_{t-1}^{FN}/Z_{t-1}^{HT}) \\
+ \kappa^{FHNH} \ln(Z_{t-1}^{FN}/Z_{t-1}^{HN}) + \kappa^{FNTF} \ln(Z_{t-1}^{FN}/Z_{t-1}^{FT}) + \epsilon_{t}^{FN}
\]

(24)

where \(z^{HT}, z^{HN}, z^{FT}, z^{FN}\) are long-run average steady-state growth rates of \(Z_{t}^{HT}, Z_{t}^{HN}, Z_{t}^{FT}, Z_{t}^{FN}\); \(\rho^{HT}, \rho^{HN}, \rho^{FT}, \rho^{FN}\) are persistency parameters for growth rates, \(\kappa^{ij}\)'s are negative correction parameters for shocks that determine the convergence speed across sectors, where \(i, j \in \{HT, HN, FN, FT\}\) and \(i \neq j\); and \(\epsilon_{t}^{HT}, \epsilon_{t}^{HN}, \epsilon_{t}^{FT}, \epsilon_{t}^{FN}\) are Gaussian random processes with zero mean and standard deviations \(\sigma^{HT}, \sigma^{HN}, \sigma^{FT}, \sigma^{FN}\), respectively. Equation (21) implies that growth rate of \(Z_{t}^{HT}\) depends on the its own growth in the previous period through persistency parameter. It also depends on the relative growth rates \(Z_{t-1}^{HT}/Z_{t-1}^{HN}, Z_{t-1}^{HT}/Z_{t-1}^{FT}, Z_{t-1}^{HT}/Z_{t-1}^{FN}\). If the level of technology in home tradable sector is higher than remaining three sector at time \(t-1\), the growth rate of \(Z_{t}^{HT}\) decrease while the growth rate of remaining sector increase through equations (22)-(24) at time \(t\) to ensure balanced growth rates in the long run.

Actually, the model defined with equations (21)-(24) is a VECM model with three cointegrating vectors under the restriction \(\kappa^{ij} = \kappa\) for all \(i, j \in \{HT, HN, FN, FT\}\) and \(i \neq j\). Otherwise, system cannot be represented by 3 cointegrating vectors. The cointegrating vectors for the system are:

\[
u_{t}^{HT} = 3 \ln(Z_{t}^{HT}) - \ln(Z_{t}^{HN}) - \ln(Z_{t}^{FT}) - \ln(Z_{t}^{FN})
\]

(25)

\[
u_{t}^{HN} = 3 \ln(Z_{t}^{HN}) - \ln(Z_{t}^{HT}) - \ln(Z_{t}^{FT}) - \ln(Z_{t}^{FN})
\]

(26)

\[
u_{t}^{FT} = 3 \ln(Z_{t}^{FT}) - \ln(Z_{t}^{HT}) - \ln(Z_{t}^{HN}) - \ln(Z_{t}^{FN})
\]

(27)

So, we can rewrite the system in equations (21)-(24) by using the cointegrating vectors:

\[
\Delta Z_{t}^{HT} = (1 - \rho^{HT}) \ln(z^{HT}) + \rho^{HT} \Delta Z_{t-1}^{HT} + \kappa \nu_{t-1}^{HT} + \epsilon_{t}^{HT}
\]

(28)

\[
\Delta Z_{t}^{HN} = (1 - \rho^{HN}) \ln(z^{HN}) + \rho^{HN} \Delta Z_{t-1}^{HN} + \kappa \nu_{t-1}^{HN} + \epsilon_{t}^{HN}
\]

(29)
\[
\Delta Z_t^{FT} = (1 - \rho^{FT}) \ln(z^{FT}) + \rho^{FT} \Delta Z_{t-1}^{FT} + \kappa v_{t-1}^{FT} + \epsilon_t^{FT}
\]

(30)

\[
\Delta Z_t^{FN} = (1 - \rho^{FN}) \ln(z^{FN}) + \rho^{FN} \Delta Z_{t-1}^{FN} - \kappa(v_{t-1}^{HT} + v_{t-1}^{HN} + v_{t-1}^{FT}) + \epsilon_t^{FN}
\]

(31)

### 3.3 Calibration

Most of the parameter values are standard and taken from the literature, as presented in Table 4. Since we use annual data in our analysis we set the discount factor \( \beta \) to 0.96, and depreciation rate \( \delta \) to 0.1. We follow CDL and set consumption share in household’s utility \( \mu \) to 0.34. We calibrate the capital share in production \( \alpha \) to 0.36 as in Backus et al. (1995). We use \( \gamma = 2 \) as the risk aversion parameter.

There are a number of estimates of the elasticity between tradable inputs (\( \theta \)) in the literature. CDL discusses that the estimates of \( \theta \) range from 0.1 to 2 and they show that trade elasticity values of around 0.5 and 4 solve the Backus-Smith puzzle.\(^6\) RRT simulate their model for \( \theta = 0.62 \) and \( \theta = 0.85 \). Ireland (2011) estimates the elasticity parameter and finds \( \theta = 1.47 \) for nonstationary model with additional shocks to investment and preferences. Thus, given the large range of parameters used in the literature, we find it instructive to study with different values of elasticity parameter, \( \theta = 0.5, 0.62, 0.85, 1.47, 2, 4 \). The share of non-tradable goods in the final good production is \( \omega_2 = 0.5 \), following Stockman and Teaser (1995). The import share in intermediate good production is calibrated as \( \omega_1 = 0.2 \) by combining the value of share parameter in Ireland (2011) with the share of non-tradable goods in final good production. The elasticity between intermediate goods and non-tradable goods is set to be \( \theta_2 = 0.5 \) implying that they are complements.

The steady state growth rate of TFP shocks, \( z^{HT}, z^{HN}, z^{FT}, z^{FN} \) are estimated by using the annual data series, 1979:2004, and set to 1.004, the average growth rate of tradable and non-tradable sectors in the US and Europe by using 60 industry database of Groningen Growth and Development Centre. The growth rates of TFP shocks are set to be equal to achieve a balanced growth path. The parameters \( \eta^{HT}, \eta^{HN}, \eta^{FT}, \eta^{FN} \) are set to \( \eta^i = z^i - (1 - \delta), i \in \{HT, HN, FN, FT\} \), that provide no capital adjustment costs along the balanced growth path as discussed earlier. Also, \( \phi_d = 0.001 \) is chosen as in Ireland (2011) to have a unique balanced growth path. The capital adjustment cost parameter \( \phi_k = 0.001 \) is calibrated to achieve a plausible standard deviation ratio between investment and output.

\(^6\)CDL includes a distribution sector in their model. So, their trade elasticity parameters correspond to \( 0.85/2 \approx 0.43 \) and \( 8/2 = 4 \).
The persistency parameters, correction parameter and standard deviations of shocks are estimated separately by assuming symmetry between home and foreign countries as in CDL by using the VECM. Table 5 presents the results. The maximum likelihood estimates of persistency parameters for tradable sector $\rho^{HT} = \rho^{FT} = 0.46$ and non-tradable sector $\rho^{HN} = \rho^{FN} = 0.2$, are in line with the previous findings in the literature.\(^7\) The estimate of the correction parameter is $\kappa = -0.015$. Also, we simulate our model for a set of persistency parameters in the robustness section.

<table>
<thead>
<tr>
<th>Table 4: Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
</tr>
<tr>
<td>Discount factor</td>
</tr>
<tr>
<td>Depreciation rate</td>
</tr>
<tr>
<td>Risk aversion</td>
</tr>
<tr>
<td>Consumption share in utility</td>
</tr>
<tr>
<td>Capital share in production</td>
</tr>
<tr>
<td>Elasticity between home and foreign tradable goods</td>
</tr>
<tr>
<td>Share of imports in intermediate goods</td>
</tr>
<tr>
<td>Elasticity between intermediate goods and non-tradable goods</td>
</tr>
<tr>
<td>Share of non-tradables in final goods</td>
</tr>
<tr>
<td>Steady state growth rates of TFP shocks</td>
</tr>
<tr>
<td>Bond adjustment cost</td>
</tr>
<tr>
<td>Capital adjustment cost</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5: Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
</tr>
<tr>
<td>Correction parameter</td>
</tr>
<tr>
<td>Persistency parameter for tradable sector</td>
</tr>
<tr>
<td>Persistency parameter for non-tradable sector</td>
</tr>
<tr>
<td>Standard deviation of tradable shocks</td>
</tr>
<tr>
<td>Standard deviation of non-tradable shocks</td>
</tr>
</tbody>
</table>

\(^7\)Aguiar and Gopinath (2007) utilize GMM to estimate persistency parameter of trend growth for small open economy model with one sector. Their estimation for Canada ranges between 0.03 and 0.29 for the period 1981Q1 to 2003Q2. Ireland (2011) uses Bayesian methods for a two country model without non-tradable sector and estimates US and EU persistency parameters as 0.1519 and 0.3845 respectively for the period 1970Q1 to 2007Q4.
4 Results

4.1 Nonstationary Model

We stationarize our model by dividing all variables with their growth rates along the steady state growth path. The resulting model that includes stationarized variables is linearized around the steady state. We simulate the model 100 times for 1250 periods and reconstruct nonstationary variables by utilizing growth rates. Before calculating the statistics we remove the first 1000 periods. The HP-filtered statistics for the nonstationary model are given in Table 6. Table 6 also includes the data statistics, which are taken from BT. We display the statistics for the values of trade elasticity parameter $\theta = 0.5, 0.62, 0.85, 1.47, 2, 4.$

<table>
<thead>
<tr>
<th>Data</th>
<th>Nonstationary Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>$\theta = 0.5$</td>
</tr>
<tr>
<td>1.57</td>
<td>1.34</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>0.76</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(Y)$</td>
<td>4.33</td>
</tr>
<tr>
<td>$\sigma(N)/\sigma(Y)$</td>
<td>0.31</td>
</tr>
<tr>
<td>$\sigma(ReR)/\sigma(Y)$</td>
<td>6.16</td>
</tr>
<tr>
<td>$\sigma(ToT)/\sigma(Y)$</td>
<td>2.12</td>
</tr>
<tr>
<td>$\sigma(C,C^*)$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\sigma(C,C^*,ReR)$</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma(C,C^*,ReR)$</td>
<td>-0.45</td>
</tr>
<tr>
<td>$\sigma(Y,ReR)$</td>
<td>-0.09</td>
</tr>
<tr>
<td>$\rho(ReR)$</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The first main result of our model is the success in breaking risk sharing for low ($\theta = 0.5$) and high ($\theta = 4$) trade elasticities. The main mechanism generating the Backus-Smith relation for $\theta = 0.5$ is as follows: A trend shock to technology increases the demand for home tradable goods due to strong wealth

---

8 Appendix D includes the normalization process and gives full system of equations.
9 We use the same data source with BT for estimation of TFP processes.
10 CDL states that if U.S. import prices is replaced with the trade-weighted export deflators, this value becomes 3.02.
effect coming from the future expected growth. Since tradable goods are not substitutes across countries and there is home-bias in production, wealth effect dominates the substitution effect and in turn the price of home tradables increases substantially. This results in an appreciation of RER following the appreciation in TOT. On the other hand, the foreign country experiences negative wealth effect due to increase in home tradable prices. Thus, home country with relatively more expensive consumption basket consumes more and we observe negative correlation, -0.11, between relative consumption and RER.

The mechanism for high trade elasticity parameter $\theta = 4$ works through the non-tradable sector. A positive trend shock increases the demand for both tradable and non-tradable goods. Since tradable goods are substitutes in this case, income effect cannot dominate substitution effect for tradables. However, we observe an increase in non-tradable prices because tradable and non-tradable goods in final good production are complements. The increase in non-tradable prices leads to an appreciation in RER and model improves the consumption-RER anomaly.

The discussed mechanisms in previous two paragraphs are closely related with CDL. However, CDL needs high persistency in transitory shocks to solve Backus-Smith puzzle which cannot be supported by data. On the other hand, we assume TFP processes as cointegrated processes instead of transitory processes and our model is able to address the puzzle for high trade elasticity without need of extra persistency.

Our model also generates high RER volatility for low trade elasticities ($\theta = 0.5$ and $\theta = 0.62$) stemming from the volatility in TOT. As the elasticity parameter increases TOT becomes less volatile since substitution effect becomes relatively more dominant with respect to income effect generated by trend shocks. However, after a certain degree of trade elasticity, variation in non-tradable prices improves the RER volatility, though not much, through the aforementioned mechanism for $\theta = 4$.

Although our model performs well in terms of Backus-Smith puzzle and RER puzzles, investment volatility of RER is low compared to the other RBC models. This issue is present also in RRT and the main reason behind this is the low persistency of the trend shocks together with the adjustment costs. Since the persistency in trend shocks is low, consumers do not vary their investment decision much for the future. As the persistency increases, the volatility of investment increases.\footnote{See robustness section for results with higher persistency parameters.} We can say that there is a trade-off between investment volatility and RER volatility.
4.2 Separate Effects of Trend Shocks and Non-tradables

In order to see the effects of trend shocks and non-tradables separately, we close each channel of our model one by one and repeat our simulation exercise. First, we assume that there are transitory TFP shocks instead of the trend shocks. We name this specification as stationary model. Stationary model is very close to the model of CDL except that we do not have a distribution sector. In the second exercise, we remove the non-tradable sector from our basic model, but we still assume that TFP shocks are cointegrated and name the model as nonstationary model with tradable sector. This version of the model is exactly the same as the model in RRT. However, our calibration is differs from the CDL and RRT since we use data set by Groningen Growth and Development Centre. Afterwards, we calculate the statistics by the same methodology in Table 6 and report the results in Table 7 and Table 8.

Table 7 illustrates that using trend shocks instead of transitory shocks to TFP significantly improves the performance of the model in terms of the Backus-Smith and the RER volatility puzzles. As discussed before, trend shocks acts like persistent transitory shocks. If home country receives positive shock to its trend, it generates a strong wealth effect in the country and domestic household increases its consumption more compared to case in which home country receives transitory shocks. Moreover, tradable good price movement is higher in nonstationary model relative to stationary model if the tradable goods are complements, where elasticity parameter is low $\theta = 0.50$. Thus, Backus-Smith correlation is negative, -0.11, in stationary model whereas the same correlation is highly positive, 0.76, for stationary model.

The improvement resulting from the trend shock diminishes as the substitution between tradable goods increase. However, for high values of trade elasticity parameter, the effect of trend shocks becomes evident again. For the trade elasticity parameter $\theta = 4$, fluctuations in tradable good prices are limited and most of the variation in RER comes from non-tradable good prices. Moreover, the non-stationary model generates almost zero correlation between relative consumption and RER whereas stationary model generates positive correlation 0.57.

$^{12}$The details for the calibration of stationary model and nonstationary model with only tradable sector are in Appendix A and B.
Table 7: The Effect of Trend Shocks

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(\text{ReR})/\sigma(Y)$</th>
<th>$\text{cor}(C/C^*, \text{ReR})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>6.16</td>
<td>-0.45</td>
</tr>
<tr>
<td><strong>Stationary Model</strong></td>
<td><strong>0.50</strong></td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.50$</td>
<td>1.25</td>
<td>0.76</td>
</tr>
<tr>
<td>$\theta = 0.62$</td>
<td>0.74</td>
<td>0.92</td>
</tr>
<tr>
<td>$\theta = 0.85$</td>
<td>0.48</td>
<td>0.96</td>
</tr>
<tr>
<td>$\theta = 1.47$</td>
<td>0.32</td>
<td>0.92</td>
</tr>
<tr>
<td>$\theta = 2$</td>
<td>0.28</td>
<td>0.84</td>
</tr>
<tr>
<td>$\theta = 4$</td>
<td>0.24</td>
<td>0.57</td>
</tr>
<tr>
<td><strong>Nonstationary Model</strong></td>
<td><strong>0.50</strong></td>
<td></td>
</tr>
<tr>
<td>$\theta = 0.50$</td>
<td>5.05</td>
<td>-0.11</td>
</tr>
<tr>
<td>$\theta = 0.62$</td>
<td>2.56</td>
<td>0.78</td>
</tr>
<tr>
<td>$\theta = 0.85$</td>
<td>1.19</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta = 1.47$</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td>$\theta = 2$</td>
<td>0.67</td>
<td>0.39</td>
</tr>
<tr>
<td>$\theta = 4$</td>
<td>0.84</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 8 reports the results for our basic model and nonstationary model with tradable sector. For the given values of trade elasticity parameter, our basic model, nonstationary model, exhibits a higher RER volatility with respect to nonstationary model with tradable sector because of the fluctuations in non-tradable good prices. For the Backus-Smith puzzle, the existence of non-tradable sector provides limited improvement but for high values of trade elasticity parameter the gap between Backus-Smith correlation of two models widens. Thus, we can conclude that addition of non-tradable sector to RRT shows its effect for higher trade elasticity parameters where the fluctuations in the non-tradable good prices dominate the movements in RER.
Table 8: The Effect of Non-Tradable Sector

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma(\text{RER})/\sigma(Y)$</th>
<th>$\text{cor}(C/C^*, \text{RER})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>6.16</td>
<td>-0.45</td>
</tr>
<tr>
<td><em>Nonstationary Model with tradable sector</em> ($\theta = 0.62$)</td>
<td>2.12</td>
<td>0.93</td>
</tr>
<tr>
<td><em>Nonstationary Model</em> ($\theta = 0.62$)</td>
<td>2.56</td>
<td>0.78</td>
</tr>
<tr>
<td><em>Nonstationary Model with tradable sector</em> ($\theta = 0.85$)</td>
<td>1.05</td>
<td>0.99</td>
</tr>
<tr>
<td><em>Nonstationary Model</em> ($\theta = 0.85$)</td>
<td>1.19</td>
<td>0.95</td>
</tr>
<tr>
<td><em>Nonstationary Model with tradable sector</em> ($\theta = 0.147$)</td>
<td>0.54</td>
<td>0.94</td>
</tr>
<tr>
<td><em>Nonstationary Model</em> ($\theta = 0.147$)</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td><em>Nonstationary Model with tradable sector</em> ($\theta = 2$)</td>
<td>0.43</td>
<td>0.85</td>
</tr>
<tr>
<td><em>Nonstationary Model</em> ($\theta = 2$)</td>
<td>0.67</td>
<td>0.39</td>
</tr>
<tr>
<td><em>Nonstationary Model with tradable sector</em> ($\theta = 4$)</td>
<td>0.32</td>
<td>0.58</td>
</tr>
<tr>
<td><em>Nonstationary Model</em> ($\theta = 4$)</td>
<td>0.84</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 1 displays the Backus-Smith performance of four models and summarizes the above tables. Three of the four models are the models presented above: nonstationary model, stationary model and nonstationary model with tradable sector. The last model is stationary model with tradable sector where we remove non-tradable sector and assume transitory shocks for TFP.\textsuperscript{13} Figure 1 shows that using trend shocks and non-tradable sector significantly improves the performance in terms of Backus-Smith puzzle. The performance of our basic model becomes more evident in the corner regions for the trade elasticity parameter. However, transitory models generates high correlation between relative consumption and risk sharing for all elasticities because of goods trade discussed by Cole and Obstfeld (1991).\textsuperscript{14}

\textsuperscript{13}The details for the transitory model with tradable sector are in Appendix C.

\textsuperscript{14}CDL is able to produce negative correlation for low $\theta$. However, the correlation that we obtained from transitory model is close to unity. The high persistency of TFP shocks and presence of distribution sector are the main sources of difference.
5 Robustness

In this section, we present the statistics of the nonstationary model for different values of persistency and correction parameters in Table 9. Regarding the robustness of persistency parameter, all sectoral persistency parameters of trend shocks set equal to each other for simplicity ($\rho = \rho^HT = \rho^FT = \rho^HN = \rho^FN$) except the baseline calibration which is taken from Table 5. The results show that the correlation between relative consumptions and RER weakens as persistency increases. The increase in persistency increases the investment of households at presence of positive technology shocks and weakens the instantaneous demand.

Second part of Table 9 presents robustness analysis with respect to the correction parameter ($\kappa$), which determines the spillover of shocks across the sectors and countries. In our benchmark case we have $\kappa = -0.015$, which implies a low spillover. We also simulate our model with a lower correction parameter $\kappa = -0.008$, implying much lower spillover. A technology shock to home country results in an increase in consumption due to expectation of high growth period. However, consumption in foreign country does not respond to this shock because of low spillover and as a result level of risk sharing is worsened, implied by lower value of correlation between RER and relative consumptions. As expected, if the level of spillover increase, benefit of foreign country from a positive shock in home country increases and risk sharing also increases as shown for parameter $\kappa = -0.03$. With a very strong spillover implied by $\kappa = -0.1$, positive risk sharing comes back.
6 Conclusion

This paper focuses on the Backus-Smith and the real exchange rate volatility puzzles in international macroeconomics. First, we show that TFP processes of tradable and non-tradable production in the US and Europe can be modelled by VECM. Then, we construct an international real business cycle model in which the TFP shocks are cointegrated. Cointegrated TFP shocks, or trend shocks, behave like highly persistent transitory shocks and strengthen the wealth effect. The strong wealth effect helps model to generate the Backus-Smith relation and the real exchange rate volatility for low and high values of elasticity parameters between tradable goods.

Our nonstationary model generates low investment volatility compared to data. Thus, one possible extension can be adding investment-specific shocks to our model. Other possible extension is enlarging the data sample for TFP processes and estimating more robust coefficients for VECM.

References


7 Appendices

7.1 Appendix A: Stationary Model

We also run our model by assuming stationary TFP processes to illustrate the importance of introduction of trend shocks. For this purpose, the production function of tradable and non-tradable sectors are modified as:

\[ Y_i^t = e^{\zeta_t}(K_i^t)^{\alpha}(Z_i^tL_i^t)^{1-\alpha}, i \in \{HT, HN, FN, FT\} \]  \hspace{1cm} (32)

where \( \zeta_t \) is transitory technology shock and it is assumed that \( Z_i^t = 1 \) \( \forall t = 0, 1, ... \) to obtain the stationary model. Then the shock processes are assumed to follow AR(1) processes:

\[ \zeta_t^i = \tilde{\rho}^i \zeta_{t-1}^i + \varepsilon_t^i \text{ for } i \in \{HT, HN, FN, FT\} \]  \hspace{1cm} (33)

where \( \tilde{\rho}^{HT}, \tilde{\rho}^{HN}, \tilde{\rho}^{FT}, \tilde{\rho}^{FN} \) are persistency parameters, and \( \varepsilon_t^HT, \varepsilon_t^HN, \varepsilon_t^FT, \varepsilon_t^FN \) are Gaussian random processes with zero mean and standard deviations \( \tilde{\sigma}^{HT}, \tilde{\sigma}^{HN}, \tilde{\sigma}^{FT}, \tilde{\sigma}^{FN} \).

We use cycles obtained by HP-filter (\( \lambda = 400 \)) in maximum likelihood estimation. Persistency parameters \( (\tilde{\rho}^{HT} = \tilde{\rho}^{FT} = 0.38, \tilde{\rho}^{HN} = \tilde{\rho}^{FN} = 0.2) \) and standard deviations \( (\tilde{\sigma}^{HT} = \tilde{\sigma}^{FT} = 0.017, \tilde{\sigma}^{HN} = \tilde{\sigma}^{FN} = 0.009) \) for the stationary version of the model are estimated by allowing a symmetry between tradable and non-tradable sectors using the same data. The persistency parameters are small when compared with the literature. However, note that BT also obtains small persistency parameters for non-tradable sector. Also, the correlation matrix of the \( \varepsilon_t^HT, \varepsilon_t^HN, \varepsilon_t^FT, \varepsilon_t^FN \):
7.2 Appendix B: Nonstationary Model with Tradable Sector

We obtain TFP series for both countries by combining the TFP processes for tradable and non-tradable sector in the 60 industry database of Groningen Growth and Development Centre. For this purpose, the production function of tradable and non-tradable sectors are modified as:

\[
Y_i^t = (K_i^t)^{\alpha}(Z_i^tL_i^t)^{1-\alpha}, i \in \{H, F\}
\]

where \(H\) represents home country and \(F\) represents foreign country. Then we estimate the following processes

\[
\ln\left(\frac{Z_H^t}{Z_H^{t-1}}\right) = (1 - \rho^H) \ln(z_H^t) + \rho^H \ln\left(\frac{Z_H^t}{Z_H^{t-1}}\right) + \kappa \ln\left(\frac{Z_H^{t-1}}{Z_H^{t-2}}\right) + \varepsilon_t^H
\]

\[
\ln\left(\frac{Z_F^t}{Z_F^{t-1}}\right) = (1 - \rho^F) \ln(z_F^t) + \rho^F \ln\left(\frac{Z_F^t}{Z_F^{t-1}}\right) + \kappa \ln\left(\frac{Z_F^{t-1}}{Z_F^{t-2}}\right) + \varepsilon_t^F
\]

where \(Z_H^t, Z_F^t\) are the TFP processes for home and foreign country, \(\rho^H, \rho^F\) are persistency parameters, and \(\varepsilon_t^H, \varepsilon_t^F\) are Gaussian random processes with zero mean and standard deviations \(\sigma^H, \sigma^F\).

Maximum likelihood estimates of persistency parameters are \(\rho^H = \rho^F = 0.46\), correction parameter is \(\kappa = -0.021\) and standard deviations are \(\sigma^H = 0.018\) and \(\sigma^F = 0.008\) by assuming a symmetry between persistency parameters of home and foreign country.

7.3 Appendix C: Stationary Model with Tradable Sector

Similar to Appendix A we modify the production function \(Y_i^t = e^{\xi_i^t}(K_i^t)^{\alpha}(Z_i^tL_i^t)^{1-\alpha}\) for \(i \in \{H, F\}\) where \(H\) represents home country and \(F\) represents foreign country. We estimate the following TFP processes by using the HP-filtered (\(\lambda = 400\)) log TFP data used in nonstationary model with one sector:

\[
\zeta_t^H = \tilde{\rho}^H \zeta_{t-1}^H + \epsilon_t^H
\]
\[
\zeta_t^F = \tilde{\rho}^F \zeta_{t-1}^F + \varepsilon_t^F
\]

where \(\zeta_{t}^H, \zeta_{t}^F\) are the TFP processes for home and foreign country, \(\tilde{\rho}_{t}^H, \tilde{\rho}_{t}^F\) are persistence parameters, and \(\varepsilon_t^H, \varepsilon_t^F\) are Gaussian random processes with zero mean and standard deviations \(\delta_{t}^H, \delta_{t}^F\). The maximum likelihood estimation results are AR terms \(\tilde{\rho}_{t}^H = 0.2, \tilde{\rho}_{t}^F = 0.2\) and standard deviations \(\delta_{t}^H = 0.011, \delta_{t}^F = 0.006\). The correlation matrix of shocks is very close to identity matrix.

### 7.4 Appendix D: System of Equations

Most of the variables listed above demonstrate nonstationary characteristics. Nonstationary variables are transformed into stationary versions by removing nonstationary terms. Stationary variables constitutes the stationary system that can be linearized around its steady state. The scaled variables are defined below.

**Home variables (24):**

\[
\begin{align*}
\xi_{t}^H &= C_t^H / U_{t-1}^H, \\
\delta_t^H &= I_t^H / U_{t-1}^H, \\
\xi_t^F &= I_t^F / U_{t-1}^F, \\
\delta_t^F &= L_t^F / U_{t-1}^F, \\
\beta_t^F &= \rho_0 / U_{t-1}^F, \\
\beta_t^H &= \rho_1 / U_{t-1}^F, \\
\eta_t^F &= \delta_t^F / L_t^F, \\
\zeta_t^F &= \delta_t^H / \beta_t^F, \\
\zeta_t^H &= \rho_1 / \beta_t^F, \\
\eta_t^H &= \zeta_t^F / \gamma_t^H, \\
\eta_t^F &= \zeta_t^H / \gamma_t^H, \\
\gamma_t^H &= \zeta_t^H / \eta_t^F, \\
\gamma_t^F &= \zeta_t^H / \eta_t^H.
\end{align*}
\]

**Foreign variables (24):**

\[
\begin{align*}
\xi_t^F &= C_t^F / U_{t-1}^F, \\
\delta_t^F &= I_t^F / U_{t-1}^F, \\
\xi_t^H &= I_t^F / U_{t-1}^H, \\
\delta_t^H &= L_t^H / U_{t-1}^H, \\
\beta_t^H &= \rho_0 / U_{t-1}^F, \\
\beta_t^F &= \rho_1 / U_{t-1}^F, \\
\eta_t^H &= \delta_t^H / \beta_t^F, \\
\zeta_t^H &= \delta_t^F / \beta_t^F, \\
\zeta_t^F &= \delta_t^F / \beta_t^F, \\
\eta_t^F &= \zeta_t^H / \gamma_t^H, \\
\eta_t^H &= \zeta_t^H / \gamma_t^F, \\
\gamma_t^H &= \zeta_t^H / \eta_t^H, \\
\gamma_t^F &= \zeta_t^H / \eta_t^F.
\end{align*}
\]

**Other variables (11):**

\[
\begin{align*}
\eta_t^H &= \beta_t^H, \\
\zeta_t^H &= \beta_t^F, \\
\zeta_t^F &= \beta_t^F, \\
\gamma_t^H &= \beta_t^H, \\
\gamma_t^F &= \beta_t^H.
\end{align*}
\]

**Shock ratios (12):**

\[
\begin{align*}
\eta_t^H &= \beta_t^H, \\
\zeta_t^H &= \beta_t^F, \\
\gamma_t^H &= \beta_t^H, \\
\gamma_t^F &= \beta_t^F.
\end{align*}
\]

The stationary system in terms of stationary variables are written below.

Tradable and non-tradable sectors:
\[ y_t^{HT} = (k_t^{HT})^\alpha (z_t^{HT} l_t^{HT})^{(1-\alpha)} \quad (39) \]
\[ \alpha p_t^{HT} q_t^{HT} = q_t^{HT} k_t^{HT} \quad (40) \]
\[ (1 - \alpha)p_t^{HT} q_t^{HT} = w_t^{HT} l_t^{HT} \quad (41) \]
\[ y_t^{FT} = (k_t^{FT})^\alpha (z_t^{FT} l_t^{FT})^{(1-\alpha)} \quad (42) \]
\[ \alpha p_t^{FT} q_t^{FT} = p_t^{FT} q_t^{FT} k_t^{FT} \quad (43) \]
\[ (1 - \alpha)p_t^{FT} q_t^{FT} = p_t^{FT} w_t^{FT} l_t^{FT} \quad (44) \]
\[ y_t^{HN} = (k_t^{HN})^\alpha (z_t^{HN} l_t^{HN})^{(1-\alpha)} \quad (45) \]
\[ \alpha p_t^{HN} q_t^{HN} = q_t^{HN} k_t^{HN} \quad (46) \]
\[ (1 - \alpha)p_t^{HN} q_t^{HN} = w_t^{HN} l_t^{HN} \quad (47) \]
\[ y_t^{FN} = (k_t^{FN})^\alpha (z_t^{FN} l_t^{FN})^{(1-\alpha)} \quad (48) \]
\[ \alpha p_t^{FN} q_t^{FN} = p_t^{FN} q_t^{FN} k_t^{FN} \quad (49) \]
\[ (1 - \alpha)p_t^{FN} q_t^{FN} = p_t^{FN} w_t^{FN} l_t^{FN} \quad (50) \]

Intermediate goods producers and final goods producers:

\[ y_t^{H,int} = \left[ (1 - \omega_1)^{\frac{1}{\pi}} (a_t^{H})^{\frac{\omega-1}{\pi}} + (\omega_1)^{\frac{1}{\pi}} (b_t^{H})^{\frac{\omega-1}{\pi}} \right]^{\frac{\pi}{\omega-1}} \quad (51) \]
\[ p_{t}^{HT} = p_{t}^{H,int} (y_{t}^{H,int})^{(1/\theta)} (1 - \omega_{1})^{(1/\theta)} (a_{t}^{H})^{(-1/\theta)} \] (52)

\[ p_{t}^{FT} = p_{t}^{H,int} (y_{t}^{H,int})^{(1/\theta)} \omega_{1}^{(1/\theta)} (b_{t}^{H})^{(-1/\theta)} \] (53)

\[ y_{t}^{F,int} = \left[ (\omega_{1})^{\frac{1}{\theta}} (a_{t}^{F})^{\frac{\theta_{2}-1}{\theta}} + (1 - \omega_{1})^{\frac{1}{\theta}} (b_{t}^{F})^{\frac{\theta_{2}-1}{\theta}} \right]^{\frac{\theta}{\theta_{2}}} \] (54)

\[ p_{t}^{HT} = p_{t}^{F,int} (y_{t}^{F,int})^{(1/\theta)} (\omega_{1})^{(1/\theta)} (a_{t}^{F})^{(-1/\theta)} \] (55)

\[ p_{t}^{FT} = p_{t}^{F,int} (y_{t}^{F,int})^{(1/\theta)} (1 - \omega_{1})^{(1/\theta)} (b_{t}^{F})^{(-1/\theta)} \] (56)

\[ y_{t}^{H} = \left[ (1 - \omega_{2})^{\frac{1}{\theta_{2}}} (y_{t}^{H,int})^{\frac{\theta_{2}-1}{\theta_{2}}} + (\omega_{2})^{\frac{1}{\theta_{2}}} (y_{t}^{HN})^{\frac{\theta_{2}-1}{\theta_{2}}} \right]^{\frac{\theta_{2}}{\theta_{2}-1}} \] (57)

\[ p_{t}^{H,int} = (y_{t}^{H})^{(1/\theta_{2})} (1 - \omega_{2})^{(1/\theta_{2})} (y_{t}^{H,int})^{(-1/\theta_{2})} \] (58)

\[ p_{t}^{HN} = (y_{t}^{H})^{(1/\theta_{2})} \omega_{2}^{(1/\theta_{2})} (y_{t}^{HN})^{(-1/\theta_{2})} \] (59)

\[ y_{t}^{F} = \left[ (1 - \omega_{2})^{\frac{1}{\theta_{2}}} (y_{t}^{F,int})^{\frac{\theta_{2}-1}{\theta_{2}}} + (\omega_{2})^{\frac{1}{\theta_{2}}} (y_{t}^{FN})^{\frac{\theta_{2}-1}{\theta_{2}}} \right]^{\frac{\theta_{2}}{\theta_{2}-1}} \] (60)

\[ p_{t}^{F,int} = p_{t}^{F} (y_{t}^{F})^{(1/\theta_{2})} (1 - \omega_{2})^{(1/\theta_{2})} (y_{t}^{F,int})^{(-1/\theta_{2})} \] (61)

\[ p_{t}^{FN} = p_{t}^{F} (y_{t}^{F})^{(1/\theta_{2})} \omega_{2}^{(1/\theta_{2})} (y_{t}^{FN})^{(-1/\theta_{2})} \] (62)

Representative households:

\[ w_{t}^{HT} k_{t}^{HT} + q_{t}^{HT} k_{t}^{HT} + w_{t}^{HN} k_{t}^{HN} + q_{t}^{HN} k_{t}^{HN} + d_{t}^{H} = c_{t}^{H} + \frac{\phi_{H}}{2} z_{t}^{HT} (d_{t+1}^{H})^{2} + i_{t}^{HT} + i_{t}^{HN} \] (63)

\[ (1 - \delta) k_{t}^{HT} + i_{t}^{HT} - \frac{\phi_{H}}{2} \left( \frac{i_{t}^{HT}}{k_{t}^{HT}} - \eta^{HT} \right)^{2} k_{t}^{HT} = z_{t}^{HT} k_{t+1}^{HT} \] (64)
(1 - \delta)k_{t+1}^H + i_{t+1}^H - \frac{\phi_k}{2} \left( \frac{i_{t+1}^H}{k_{t+1}^H} - \eta^H \right)^2 k_{t+1}^H = z_{t+1}^H k_{t+1}^H \quad (65)

\mu((c_t^H)\mu(1 - t_t^H - t_t^H)^{(1 - \mu)})^{1 - \gamma} = \lambda_t^H c_t^H \quad (66)

(1 - \mu)((c_t^H)\mu(1 - t_t^H - t_t^H)^{(1 - \mu)})^{1 - \gamma} = \lambda_t^H w_t^H (1 - t_t^H - t_t^H) \quad (67)

(1 - \mu)((c_t^H)\mu(1 - t_t^H - t_t^H)^{(1 - \mu)})^{1 - \gamma} = \lambda_t^H w_t^H (1 - t_t^H - t_t^H) \quad (68)

\lambda_t^H = \xi_t^H (1 - \phi_k \left( \frac{i_{t+1}^H}{k_{t+1}^H} - \eta^H \right)) \quad (69)

\lambda_t^H = \xi_t^H (1 - \phi_k \left( \frac{i_{t+1}^H}{k_{t+1}^H} - \eta^H \right)) \quad (70)

(z_t^H)^{(1 - \mu(1 - \gamma))} \xi_t^H = \beta \lambda_t^H q_{t+1}^H + \beta \xi_t^H \left[ 1 - \delta + \phi_k \left( \frac{i_{t+1}^H}{k_{t+1}^H} - \eta^H \right) \right]^2 k_{t+1}^H \quad (71)

(z_t^H)^{(1 - \mu(1 - \gamma))} \xi_t^H = \beta \lambda_t^H q_{t+1}^H + \beta \xi_t^H \left[ 1 - \delta + \phi_k \left( \frac{i_{t+1}^H}{k_{t+1}^H} - \eta^H \right) \right]^2 k_{t+1}^H \quad (72)

(z_t^H)^{(1 - \mu(1 - \gamma))} \lambda_t^H \left( \frac{1}{t_t^H} + \phi_d \phi_t^H \right) = \beta \lambda_t^H \quad (73)

(1 - \delta)k_t^F + i_t^F - \frac{\phi_k}{2} \left( \frac{i_t^F}{k_t^F} - \eta^F \right)^2 k_t^F = z_t^F k_{t+1}^F \quad (74)

(1 - \delta)k_t^F + i_t^F - \frac{\phi_k}{2} \left( \frac{i_t^F}{k_t^F} - \eta^F \right)^2 k_t^F = z_t^F k_{t+1}^F \quad (75)

\mu((c_t^F)\mu(1 - t_t^F - t_t^F)^{(1 - \mu)})^{1 - \gamma} = \lambda_t^F c_t^F \quad (76)

(1 - \mu)((c_t^F)\mu(1 - t_t^F - t_t^F)^{(1 - \mu)})^{1 - \gamma} = \lambda_t^F w_t^F (1 - t_t^F - t_t^F) \quad (77)
(1 - \mu)((\sigma^F_t)^\mu (1 - t_{FT}^F - t_{FT}^F)^{(1 - \mu)})^{1 - \gamma} = \lambda^F_t w_{t}^F (1 - t_{FT}^F - t_{FT}^F) \tag{78}

\lambda^F_t = \xi^F_t (1 - \phi_k \frac{i_{FF}^t}{k_{FF}^t} - \eta^F_t) \tag{79}

\lambda^F_t = \xi^F_t (1 - \phi_k \frac{i_{FN}^t}{k_{FN}^t} - \eta^F_t) \tag{80}

(z_{FT}^t)^{(1 - \mu(1 - \gamma))} \xi^F_t = \beta \lambda_{t+1}^F q_{t+1}^F + \beta \xi_{t+1}^F \left[ 1 - \delta + \phi_k \frac{i_{FF}^t}{k_{FF}^t} - \eta^F_t \right] - \frac{\phi_k}{2} \frac{i_{FF}^t}{k_{FF}^t} - \eta^F_t^2 \tag{81}

(z_{FT}^t)^{(1 - \mu(1 - \gamma))} \xi^F_t = \beta \lambda_{t+1}^F q_{t+1}^F + \beta \xi_{t+1}^F \left[ 1 - \delta + \phi_k \frac{i_{FN}^t}{k_{FN}^t} - \eta^F_t \right] - \frac{\phi_k}{2} \frac{i_{FN}^t}{k_{FN}^t} - \eta^F_t^2 \tag{82}

(z_{FT}^t)^{(1 - \mu(1 - \gamma))} \lambda^F_t \left( \frac{1}{r^F_t p_t^F} + \phi_d d_t^F \right) = \frac{\beta \lambda_{t+1}^F}{p_{t+1}^F} \tag{83}

\text{Trade variables and equilibrium conditions}

n_t^H = (z_{FT}^t)^{-1} p_t^H a_t^F - p_t^F b_t^H \tag{84}

p_t^F n_t^F = z_{t-1}^H p_t^F b_t^H - p_t^F a_t^F \tag{85}

\text{rer}_t = p_t^F \tag{86}

\text{tot}_t = p_t^F / p_t^H \tag{87}

y_t^H = a_t^H + a_t^F \left( z_{FT}^t \right)^{-1} \tag{88}

y_t^F = b_t^H z_{t-1}^H + b_t^F \tag{89}

d_t^H z_{t-1}^H + d_t^F = 0 \tag{90}
\[ y_t^H = c_t^H + \frac{\Phi_d}{2} z_t^{HT} (d_{t+1}^H)^2 + i_t^{HT} + i_t^{HN} \quad (91) \]

\[ y_t^F = c_t^F + \frac{\Phi_d}{2} z_t^{FT} (d_{t+1}^F)^2 + i_t^{FT} + i_t^{FN} \quad (92) \]

\[ p_t^H = 1 \quad (93) \]

**Exogenous shock process:**

\[ \ln(z_t^{HT}) = (1 - \rho^{HT}) \ln(z_{t-1}^{HT}) + \rho^{HT} \ln(z_t^{HT}) + \kappa^{HTHN} \ln(z_{t-1}^{HTHN}) \]

\[ + \kappa^{HTFT} \ln(z_t^{HTFT}) + \kappa^{HTFN} \ln(z_{t-1}^{HTFN}) + \epsilon_t^{HT} \quad (94) \]

\[ \ln(z_t^{HN}) = (1 - \rho^{HN}) \ln(z_{t-1}^{HN}) + \rho^{HN} \ln(z_t^{HN}) + \kappa^{HNHT} \ln(z_{t-1}^{HNHT}) \]

\[ + \kappa^{HNFT} \ln(z_t^{HNFT}) + \kappa^{HNFN} \ln(z_{t-1}^{HNFN}) + \epsilon_t^{HN} \quad (95) \]

\[ \ln(z_t^{FT}) = (1 - \rho^{FT}) \ln(z_{t-1}^{FT}) + \rho^{FT} \ln(z_t^{FT}) + \kappa^{FTHT} \ln(z_{t-1}^{FTHT}) \]

\[ + \kappa^{FTHN} \ln(z_t^{FTHN}) + \kappa^{FTFN} \ln(z_{t-1}^{FTFN}) + \epsilon_t^{FT} \quad (96) \]

\[ \ln(z_t^{FN}) = (1 - \rho^{FN}) \ln(z_{t-1}^{FN}) + \rho^{FN} \ln(z_t^{FN}) + \kappa^{FNHT} \ln(z_{t-1}^{FNHT}) \]

\[ + \kappa^{FNHN} \ln(z_t^{FNHN}) + \kappa^{FNFT} \ln(z_{t-1}^{FNFT}) + \epsilon_t^{FN} \quad (97) \]

**Shock ratios:**

\[ z_t^{HTHN} = \frac{z_t^{HTN}}{z_t^{HTN} z_{t-1}^{HTN}} \quad (98) \]

\[ z_t^{HTFT} = \frac{z_t^{HTF}}{z_t^{HTF} z_{t-1}^{HTF}} \quad (99) \]

\[ z_t^{HTFN} = \frac{z_t^{HTN}}{z_t^{HTN} z_{t-1}^{HTN}} \quad (100) \]
\[ z_{t}^{HNHT} = \frac{z_{t}^{HN}}{z_{t}^{HT}} z_{t-1}^{HNHT} \]  
(101)

\[ z_{t}^{HNFT} = \frac{z_{t}^{HN}}{z_{t}^{FT}} z_{t-1}^{HNFT} \]  
(102)

\[ z_{t}^{HNFN} = \frac{z_{t}^{HN}}{z_{t}^{FN}} z_{t-1}^{HNFN} \]  
(103)

\[ z_{t}^{FTHT} = \frac{z_{t}^{FT}}{z_{t}^{HT}} z_{t-1}^{FTHT} \]  
(104)

\[ z_{t}^{FTHN} = \frac{z_{t}^{FT}}{z_{t}^{HN}} z_{t-1}^{FTHN} \]  
(105)

\[ z_{t}^{FTFN} = \frac{z_{t}^{FT}}{z_{t}^{FN}} z_{t-1}^{FTFN} \]  
(106)

\[ z_{t}^{FNHT} = \frac{z_{t}^{FN}}{z_{t}^{HT}} z_{t-1}^{FNHT} \]  
(107)

\[ z_{t}^{FNNH} = \frac{z_{t}^{FN}}{z_{t}^{HN}} z_{t-1}^{FNNH} \]  
(108)

\[ z_{t}^{FNFT} = \frac{z_{t}^{FN}}{z_{t}^{FT}} z_{t-1}^{FNFT} \]  
(109)
Central Bank of the Republic of Turkey
Recent Working Papers
The complete list of Working Paper series can be found at Bank’s website (http://www.tcmb.gov.tr).

Unemployment and Vacancies in Turkey: The Beveridge Curve and Matching Function
(Birol Kanik, Enes Sunel, Temel Taşkın No. 13/35, September 2013)

Distributional and Welfare Consequences of Disinflation in Emerging Economies
(Enes Sunel Working Paper No. 13/34, August 2013)

Do We Really Need Filters In Estimating Output Gap?: Evidence From Turkey
(Evren Erdoğan Coşar, Sevim Kösem, Çağrı Sarıkaya Working Paper No. 13/33, August 2013)

The Role of Extensive Margin in Exports of Turkey: A Comparative Analysis

Alternative Tools to Manage Capital Flow Volatility
(Koray Alper, Hakan Kara, Mehmet Yörükoğlu Working Paper No. 13/31, July 2013)

How do Banks’ Stock Returns Respond to Monetary Policy Committee Announcements in Turkey?
Evidence from Traditional versus New Monetary Policy Episodes
(Güney Küçükçakaç argues, Deren Ünalms, Ibrahim Ünalms Working Paper No. 13/30, July 2013)

Some Observations on the Convergence Experience of Turkey
(Murat Ungör Working Paper No. 13/29, July 2013)

Reserve Option Mechanism as a Stabilizing Policy Tool: Evidence from Exchange Rate Expectations

GDP Growth and Credit Data

Yield Curve Estimation for Corporate Bonds in Turkey
(Burak Kanlı, Doruk Küçüksaraç, Özgür Özel Working Paper No. 13/26, July 2013)

Inattentive Consumers and Exchange Rate Volatility
(Mehmet Fatih Ekinci Working Paper No. 13/25, July 2013)

Non-core Liabilities and Credit Growth

Does Unemployment Insurance Crowd Out Home Production?

Home Ownership and Job Satisfaction
(Semih Tümen, Tuğba Zeydanlı, Çalışma Tebliği No. 13/22, May 2013)

Türkiye İşgücü Piyasasında Mesleklerin Önemi: Hizmetler Sektörü İstihdamı, İşgücü ve Ücret Kutuplaşması
(Semih Akçomak, H. Burcu Gürcihan Çalışma Tebliği No. 13/21, Nisan 2013)

Gecelik Kur Takası Faizleri ve İMKB Gecelik Repo Faizleri
(Doruk Küçüksaraç, Özgür Özel Çalışma Tebliği No. 13/20, Nisan 2013)

Undocumented Workers’ Employment Across US Business Cycles
(David Brown, Serife Genç, Julie Hothckiss, Myriam Quispe-Agnoli Working Paper No. 13/19, March 2013)

Systemic Risk Contribution of Individual Banks
(Hüseyin Çağrı Akköyün, Ramazan Karaşahin, Gürsu Keleş Working Paper No. 13/18, March 2013)