International Risk Sharing and Portfolio Choice with Non-separable Preferences

June 2015

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Abstract

This paper aims to account for the Backus-Smith puzzle in a generally calibrated two-country DSGE model with endogenous portfolio choice in international bonds and equities. There are multiple shocks, including shocks to TFP, labour supply, investment, government spending and monetary policy. Hence, there are more risks than can be spanned by international trade in equities and bonds, i.e. markets are incomplete. The utility function in the benchmark model is non-separable in consumption and leisure and there is external habit formation in consumption. We compare the benchmark model with models that differ according to preference/habit specification and financial market structure. We find that the benchmark model with non-separable preferences across consumption and leisure, habit formation and incomplete financial markets implies almost zero correlation between relative consumption and the real exchange rate while generating bond and equity portfolios that are broadly in line with the data. What is more, the cross-country correlation of consumption is lower than the cross-country correlation of output in our benchmark model, which has proved to be a difficult fact to match in IRBC models. Non-separable preferences are found to be crucial to generating these results but financial market structure plays only a minor role.

JEL Classification: F31, F41

Keywords: Portfolio choice, international risk sharing, consumption-real exchange rate anomaly, Backus-Smith puzzle, non-separable preferences, incomplete markets.

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1 Introduction

In standard international real business cycle (IRBC) models, efficient risk sharing across countries requires that consumption is higher in the country where it is cheaper to consume, which implies a positive and high correlation between relative consumption and the real exchange rate.\(^1\) However, in the data the correlation between relative consumption and the real exchange rate is low, or even negative, which is generally interpreted as indicating a lack of consumption risk sharing (Backus and Smith, 2003, Kollmann, 2005). Standard open economy macro models often fail to account for this empirical fact, which came to be known as the ‘Backus-Smith puzzle’ or ‘consumption-real exchange rate anomaly.’

Notable explanations of the Backus-Smith puzzle include Benigno and Thoenissen (2008) and Corsetti, Dedola and Leduc (2008) (hereafter BT and CDL) who emphasize the role of asset market incompleteness alongside certain goods market frictions.\(^2\) These features give rise to large wealth transfers across countries in response to country-specific productivity shocks, as a result of which relative consumption and the real exchange rate move in opposite directions.

One drawback, however, is that in the BT and CDL models international asset trade is restricted to a single non-contingent bond, which is an extreme assumption given the recent trends in financial globalization. In recent work, Benigno and Küçük (2012) show that these models can no longer account for the Backus-Smith puzzle when this assumption is relaxed by introducing a second internationally traded bond. The presence of a second traded bond in the Benigno and Küçük model allows a greater degree of risk sharing than in the BT and CDL models and this appears to be important in reversing the consumption-real exchange rate results. This suggests that the Backus-Smith puzzle is likely to re-emerge in models which allow for international trade in a realistic set of assets, such as bonds and equities.

Thanks to recent developments in macroeconomic modelling that allow the characterization of country portfolios in open economy general equilibrium models with trade in multiple assets, there is now a large literature that explains the determinants and composition of international portfolios in various model settings.\(^3\) Although this literature provides extensive answers to what drives international portfolios, it is silent about whether these portfolios can be reconciled with the Backus-Smith puzzle. This paper aims to bridge this gap in the literature.

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\(^1\) The implied correlation is positive provided that the real exchange rate is defined as the price of the foreign consumption basket in units of the domestic consumption basket.

\(^2\) The combination of incomplete financial markets and goods markets frictions is an important aspect of BT and CDL. Chari, Kehoe and McGrattan (2002) show that it is very hard to generate a low correlation between relative consumption and the real exchange rate relying only on market incompleteness.

\(^3\) Heathcote and Perri (2013), Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2007, 2010), Coeurdacier and Gourinchas (2013), Devereux and Sutherland (2008) are among the papers that explain the motives behind international portfolio positions - home equity bias in particular. See Coeurdacier and Rey (2012) for an extensive survey on the recent portfolio choice literature.
Given the results in Benigno and Küçük (2012), one can go down three different routes to tackle the ambitious task of accounting for the Backus-Smith puzzle in a model with trade in bonds and equities. The first of these is to include many sources of risk in the model – many shocks – so that asset trade in bonds and equities cannot come close to ensuring high risk sharing. The idea is the following: A portfolio which is optimal when the only source of risk are productivity shocks might imply valuation effects that go the wrong way when there are other shocks such as risk premium shocks or government spending shocks.\(^4\)

The second possible route that might reconcile trade in multiple assets with the Backus-Smith puzzle is to break the tight link between the marginal utility of consumption and the level of consumption by moving away from separable utility and CRRA preferences. In a model where preferences are non-separable across consumption and leisure and across time (as in the case of habit formation in consumption), marginal utility of consumption depends not only on current consumption but also on past consumption as well as labour effort. In such a model, relative consumption and real exchange rate can go in different directions even if relative marginal utilities and real exchange rate move together. According to this approach, a negative correlation between relative consumption and real exchange rate does not necessarily imply a lack of risk sharing in the data as there might be high risk sharing in terms of the marginal utility of consumption if not in terms of its level.

A possible third route that might resolve the Backus-Smith puzzle in a model with multiple assets is to allow for financial market imperfections other than market incompleteness. The existing literature mostly relies on financial market incompleteness to understand the apparent lack of international risk sharing. However, financial imperfections such as borrowing frictions or restrictions in asset trade might also have important implications for risk sharing. In the current paper, we explore the first two routes described above and leave this third one for future work.

We aim to account for the Backus-Smith puzzle in a generally calibrated two-country DSGE model with endogenous portfolio choice in international bonds and equities. There is Calvo-style price and wage setting and monetary policy in each country is described by a Taylor rule. We allow for goods market frictions such as the presence of non-tradable goods as in BT and CDL. We specify many shocks in the spirit of Smets and Wouters (2003) and Devereux, Senay and Sutherland (2014). Hence, there are more risks than can be spanned by international trade in equities and bonds, i.e. markets are incomplete. The utility function in the benchmark model is non-separable across consumption and leisure and there is external habit formation in consumption (non-separability across time) which is supported by many estimated DSGE models in the literature.

\(^4\)An alternative approach by Arslan, Keleş and Kılıç (2012) relies on trend shocks and country-specific risk aversion coefficients to account for the lack of international risk sharing in a two-country two-good model with international trade in domestic and foreign bonds.
We compare the benchmark model with models that differ according to preference/habit specification. We find that the benchmark model with non-separable preferences across consumption and leisure and with habit formation implies a low – almost zero – correlation between relative consumption and the real exchange rate while generating bond and equity portfolios that are broadly in line with the data. What is more, the cross-country correlation of consumption is lower than the cross-country correlation of output in our benchmark model, which has proved to be a difficult fact to match in IRBC models known as the ‘quantity puzzle.’

When utility is separable across consumption and leisure, introducing habit formation helps reduce the correlation between relative consumption and the real exchange rate, but does not bring it down to levels comparable to the data. Besides, this setup does not generate equity home bias in our model. Likewise, when there are no habits, specifying non-separable preferences across consumption and leisure does not bring about a meaningful reduction in the consumption-real exchange rate correlation. Hence, it is the combination of these two features that allow us to reconcile the observed patterns in portfolio positions with a low correlation between consumption and the real exchange rate. These features are also essential to account for the quantity puzzle in our model.

One important result to note is that market incompleteness does not play a major role in driving our results. The correlation implied by the benchmark model is almost the same as the one obtained under the complete markets assumption. Hence, in our model, the low correlation between relative consumption and the real exchange rate does not necessarily imply a lack of international risk sharing across countries. In fact, there is high risk sharing in our benchmark model as relative marginal utilities of consumption are closely related to changes in real exchange rate through trade in bonds and equities. The low consumption-real exchange rate correlation in our model, instead, stems from the fact that the marginal utility of consumption depends not only on current consumption but also on leisure and past consumption due to non-separable preferences. In other words, non-separable utility drives a wedge between relative consumption levels and the real exchange rate even if the portfolio ensures high risk sharing in terms of the marginal utility of consumption.

We are not the first to look at the Backus-Smith puzzle in a model with non-separable preferences. Although they do not explicitly discuss their role, BT and CDL also specify non-separable preferences across consumption and leisure. Recently, Raffo (2010) and Karabarbounis (2012) set out different models with non-separable utility to account for the consumption-real exchange rate anomaly under complete markets. In Raffo (2010), variable capacity utilization, Greenwood, Hercowitz and Huffman (1988) (GHH) preferences and investment-specific shocks are key in generating the results, while in Karabarbounis (2013) it is the home-production sector and the associated labour wedge that matter. Dmitriev and Krznar (2012) and Stathopoulos (2012) point at the role
of habit formation in addressing the Backus-Smith puzzle with complete markets using simpler models.

There is a comprehensive literature on country portfolios in open economy macro models reviewed by Coeurdacier and Rey (2012). Most of the literature on county portfolios (with a few exceptions) does not explicitly touch on the role of non-separable preferences. Using different model setups, Jermann (2002) and Matsumoto (2012) show that non-separability across consumption and leisure can generate home equity bias. On the other hand, Stathopoulos (2012) finds that time non-separability in the form of external habits in consumption can explain home equity bias in an endowment model with home bias in consumption.

Insights of many of these papers also apply to our model. However, our paper is different from the existing literature in many respects. First, we use a generally calibrated DSGE model with many features that are shown to be important by the empirical DSGE literature such as sticky prices and wages, capital adjustment costs, and variable capacity utilization. Secondly, we allow for endogenous portfolio choice in bonds and equities. Third, we solve our model under incomplete financial markets. Our contribution is, thus, to offer a comprehensive open economy DSGE model that can be used to study international spillovers through asset trade without generating a counterfactual high correlation between relative consumption and real exchange rate.

2 Risk Sharing and Portfolio Choice with Non-Separable Preferences

Before describing all the details of our model, in this section we first provide analytical expressions which show how the optimal risk sharing condition and the portfolio solution change with non-separable preferences across consumption and leisure, and across time (modelled as habit formation in consumption).

2.1 Separable Preferences: Revisiting the Backus-Smith puzzle

Consider the following utility function, where preferences are separable in consumption and labour effort,

$$ U_t = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{C_{t+i}^{1-\rho}(z)}{1-\rho} - \lambda \frac{H_{t+i}^{1+\phi}(z)}{1+\phi} \right\} $$

(1)

where $\beta$ is the discount factor, $C_t(z)$ is the consumption of household $z$, $H_t(z)$ is labour supply, $\rho$ and $\phi$ denote the inverse of intertemporal elasticity of substitution in consumption and leisure, respectively. The utility function of foreign households is similar to (1). Foreign variables are marked with an asterisk.
Provided that markets are complete, marginal utility of consumption is equalized across countries when adjusted by the respective price levels given by the real exchange rate:

\[ U_{C,t} = \frac{U_{C,t}^*}{Q_t} \] (2)

where \( U_{C,t} \) and \( U_{C,t}^* \) denote home and foreign marginal utility of consumption, respectively.

\[ U_{C,t} = \frac{d}{dC_t} U(C_t, H_t) = C_t^{-\rho} \]

\[ U_{C,t}^* = \frac{d}{dC_t} U(C_t^*, H_t^*) = C_t^{*-\rho} \]

This condition can be stated as follows in linearized form

\[ \hat{U}_{C,t} - \hat{U}_{C,t}^* + \hat{Q}_t = 0 + O(\varepsilon^2), \] (3)

where a hat over a variable denotes log-deviations from the deterministic steady-state unless stated otherwise, i.e. for any variable \( x \), \( \hat{x} = \log(\frac{x}{x^*}) \). The term \( O(\varepsilon^n) \) denotes the residual of approximation of order \( n \) and higher. For separable preferences and CRRA utility, the efficiency condition implies a one-to-one relationship between relative consumption and real exchange rate in linearized form

\[ \hat{C}_t - \hat{C}_t^* = \frac{\hat{Q}_t}{\rho} + O(\varepsilon^2). \] (4)

This condition implies a perfect correlation between relative consumption and real exchange rate. However, this implication is not supported at all by the data, as shown by Backus and Smith (1993) and Kollmann (1995) – the so-called Backus-Smith puzzle. A natural response to this finding is to relax the complete market assumption by specifying incomplete markets.

Under incomplete markets, risk-sharing condition links expected changes in the relative marginal utility of consumption to expected changes in the real exchange rate:

\[ E_t \hat{U}_{C,t+1} - \hat{U}_{C,t} - \left( E_t \hat{U}_{C,t+1} - \hat{U}_{C,t}^* \right) = -E_t \Delta \hat{Q}_{t+1} + O(\varepsilon^2), \] (5)

Assuming separable utility and CRRA preferences, this can be written in terms of expected changes.

\[ E_t \delta_{C,t+1} - \delta_{C,t} - \left( E_t \delta_{C,t+1} - \delta_{C,t}^* \right) = -E_t \Delta \delta_{Q,t+1} + O(\varepsilon^2), \] (6)

The assumption here is that initial wealth levels are equivalent across countries.

\[ E_t \delta_{C,t+1} - \delta_{C,t} - \left( E_t \delta_{C,t+1} - \delta_{C,t}^* \right) = -E_t \Delta \delta_{Q,t+1} + O(\varepsilon^2), \] (6)

This is obtained by taking a first-order approximation of home and foreign agents’ Euler equations with respect to the internationally traded bond and combining the two equations.
in relative consumption

\[ E_t \Delta \hat{C}_{t+1} - E_t \Delta C^*_t = \frac{E_t \Delta \hat{Q}_{t+1}}{\rho} + O(\varepsilon^2), \]  

(6)

According to this condition, consumption growth is expected to be higher in the country where the real exchange rate is expected to depreciate. For this condition to hold there needs to be at least one internationally traded asset. If the number of internationally traded assets is large enough such that all risks can be spanned, the risk sharing condition is given by equation (4) instead.

### 2.2 Risk Sharing with Non-separable Preferences

To understand how non-separable preferences interact with portfolio choice and international risk sharing now assume that household \( z \) in the home country maximizes a utility function of the following form

\[ U_t = E_t \prod_{i=0}^{\infty} \beta^i \left\{ (C_{t+i}(z) - hC_{t+i-1})^{1-\rho} (1 - H_{t+i}(z))^{\phi} \right\}, \quad \rho > 0, \phi \leq 0, \quad 0 \leq h \leq 1. \]  

(7)

where \( hC_{t-1} \) is the stock of (external) habits in period \( t \) which depends on aggregate consumption in period \( t - 1 \) and \( h \) is the persistence of habit formation. This specification is also known as ‘catching up with the Joneses’ after Abel (1990) since an individual household’s marginal utility depends also on how much other households consumed in the previous period. Note that for \( \phi < 0 \) and \( 0 < h \leq 1 \) this utility function implies non-separability both across consumption and leisure and across time. In the case where there are no habits (i.e. \( h = 0 \)), the functional form for the non-separability across consumption and leisure is of the form in Stockman and Tesar (1995) and Benigno and Thoenissen (2008).

We now describe how the utility function in (7) changes the risk sharing condition given in the previous section. The marginal utility of consumption in home and foreign country, \( U_{C,t} \) and \( U^{*}_{C,t} \) respectively, are now given by

\[ U_{C,t} \equiv \frac{d}{dC_t} U(C_t, C_{t-1}, H_t) = C_{X,t}^{-\rho}(1-H_t)^{\phi}, \]  

(8)

\[ U^{*}_{C,t} \equiv \frac{d}{dC^*_t} U^*(C^*_t, C^*_{t-1}, H^*_t) = C_{X,t}^{* -\rho}(1-H^*_t)^{\phi}, \]  

(9)
where $C_{X,t}$ and $C^*_{X,t}$ denote habit-adjusted consumption levels in each country

$$C_{X,t} \equiv C_t - hC_{t-1},$$

$$C^*_{X,t} \equiv C^*_t - hC^*_{t-1}.$$

Consider a log-linear approximation to the marginal utility of consumption given in (8)

$$^\Upsilon C_{t} = \frac{\hat{C}_{X,t}}{\hat{L}_t} + O(\varepsilon^2)$$

$$= -\rho(\hat{C}_t - h\hat{C}_{t-1}) - \zeta \hat{L}_t + O(\varepsilon^2), \text{ where } \zeta \equiv \phi \frac{\hat{L}}{1-L} > 0. \quad (10)$$

According to (10), an increase in this period’s consumption lowers current marginal utility, but raises it in the next period because the consumer wants to consume more tomorrow if today’s consumption is high.\(^7\) Hence, consumption smoothing motive is stronger in the presence of habits.

Assuming financial markets are incomplete, we can rewrite the risk sharing condition in (5) by plugging in (10) and its foreign counterpart

$$\frac{1}{1-h}(E_t \Delta \hat{C}_{t+1} - E_t \Delta \hat{C}^*_{t+1}) - \frac{h}{1-h}(\Delta \hat{C}_t - \Delta \hat{C}^*_t),$$

$$+ \frac{\zeta}{\rho}(E_t \Delta \hat{L}_{t+1} - E_t \Delta \hat{L}^*_{t+1}) = \frac{E_t \Delta \hat{Q}_{t+1}}{\rho} + O(\varepsilon^2) \quad (11)$$

It is apparent from (11) that habits and non-separability across consumption and leisure introduce a wedge between expected changes in relative consumption and real exchange rate.\(^8\) To build intuition, first focus on the effect of habit formation by letting $\zeta = 0$. In this case, international risk sharing requires that expected changes in habit-adjusted consumption are equalized across countries when adjusted by the real exchange rate. Provided that consumption is persistent, i.e. has a high AR(1) coefficient, and $h$ is sufficiently large, a shock that raises expected growth of relative consumption will not cause a one-to-one change in expected real exchange rate since it will be partly offset by the rise in relative growth of habits given by $\frac{h}{1-h}(\Delta \hat{C}_t - \Delta \hat{C}^*_t)$ in (11). This in turn might help account for the Backus-Smith puzzle.

Now let’s turn to the effect of the non-separability across consumption and leisure by setting $h = 0$. Then (11) simplifies as

$$E_t \Delta \hat{C}_{t+1} - E_t \Delta \hat{C}^*_{t+1} + \frac{\zeta}{\rho}(E_t \Delta \hat{L}_{t+1} - E_t \Delta \hat{L}^*_{t+1}) = \frac{E_t \Delta \hat{Q}_{t+1}}{\rho} + O(\varepsilon^2). \quad (12)$$

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\(^7\)Note that all households are identical in equilibrium. Hence, ‘catching up with the Joneses’ can be interpreted as ‘catching up with past consumption’ on an aggregate level.

\(^8\)When utility is separable and there is no habit formation, i.e. $\zeta = 0$ and $h = 0$, this equation simplifies to (6).
Holding expected consumption growth constant, $\zeta < 0$ implies that labour effort is expected to be lower in the country where prices are expected to become cheaper (equation (12)). To put it differently, efficiency requires that agents in the more expensive country work more to equalize their marginal utility of consumption – its expected change – to that of the foreign agents. For example, higher growth in relative labour supply in response to a shock that raises the growth of relative consumption would imply that the real exchange rate would not depreciate as much as it would if preferences were separable. This potentially drives a wedge between relative consumption and the real exchange rate. Hence, depending on the degree of complementarity between consumption and leisure in the utility function, measured by $\zeta$, and the volatility of labour supply relative to the volatility of consumption, the correlation between relative consumption and real exchange rate can be lowered.

To sum up, both sources of non-separability in preferences have a potential to break the tight link between relative consumption and real exchange rate. Whether they can account for the Backus-Smith puzzle is a quantitative question that requires a numerical solution of the model.

According to these explanations of the Backus-Smith anomaly, a low correlation between relative consumption and the real exchange rate do not indicate or a lack of risk sharing across countries. Non-separability of preferences introduce a wedge between relative consumption and the real exchange rate also under complete markets. This can be seen clearly by log-linearizing (4), and substituting (10) for marginal utility

$$\left(\hat{C}_t - h\hat{C}_{t-1}\right) - \left(\hat{C}^*_t - h\hat{C}^*_{t-1}\right) + \frac{\zeta}{\rho} \left(\hat{L}_t - \hat{L}_t^*\right) = \frac{\hat{Q}_t^*}{\rho} + O(\varepsilon^2).$$

### 2.3 Portfolio choice with Non-separable Preferences

As discussed above, this paper extends the literature on the Backus-Smith puzzle in two directions. The first is the introduction of non-separable preferences. The second is to analyse a case where asset trade is allowed in a realistic set of assets while the economic environment is subject to multiple shocks. Asset markets are therefore incomplete and it is necessary explicitly to consider equilibrium portfolio allocation across available assets. In the analysis below we characterize optimal portfolios using the approximation techniques proposed in Devereux and Sutherland (2011). Before we consider the details of the model it is useful first to consider how the solution technique can be applied to the case of non-separable preferences.

According to the Devereux-Sutherland method, a second-order approximation of optimal portfolio choice equations alongside a first-order approximation of non-portfolio equations of the model is sufficient to pin down steady-state portfolios. Optimal steady-state portfolios are then given by the conditional covariance between relative marginal utility of consumption and excess return on
the $i^{th}$ asset:

$$\text{Cov}_t \left[ (\hat{U}_{C,t+1} - \hat{U}_{C,t+1}^* + \hat{Q}_{t+1}), \hat{r}_{x,t+1} \right] = 0 + O(\varepsilon^3)$$

(14)

According to this condition, agents in each country choose a portfolio that minimizes deviations from the perfect risk sharing condition given by (3). In other words, the optimal portfolio is a portfolio that offers a high return when marginal utility of consumption is high relative to the rest of the world (when converted into the same unit). Provided that there are enough assets to span all sources of uncertainty that affect relative marginal utilities, we can find an optimal portfolio that replicates the complete market outcome. Since we specify a general model with many shocks, international trade in bonds and equities cannot replicate the complete market outcome.

Plugging in the first order approximation of the marginal utilities of consumption for separable preferences and CRRA utility the portfolio orthogonality condition is simply

$$\text{Cov}_t \left[ \hat{C}_{t+1} - \hat{C}_{t+1}^* - \frac{\hat{Q}_{t+1}}{\rho}, \hat{r}_{x,t+1} \right] = 0 + O(\varepsilon^3)$$

(15)

On the other hand, for the non-separable utility function given by (7) steady-state portfolios are determined by

$$\text{Cov}_t \left[ (\hat{C}_{t+1} - h\hat{C}_t) - (\hat{C}_{t+1}^* - h\hat{C}_t^*) + \frac{\zeta}{\rho} (\hat{L}_{t+1} - \hat{L}_{t+1}^*) - \frac{\hat{Q}_{t+1}}{\rho}, \hat{r}_{x,t+1} \right] = 0 + O(\varepsilon^3)$$

(16)

That is, agents will choose a portfolio that minimizes fluctuations in relative consumption levels adjusted by past consumption habits as well as relative labour effort. As we discuss later, these extra hedging motives are key in generating realistic portfolio positions alongside a low relative consumption-real exchange rate correlation.

3 A General Model with Non-Separable Preferences

We now describe a fully specified DSGE model that incorporates household preferences of the non-separable form given in (7). Apart from non-separability across consumption and leisure, the model shares many of the same basic features of the closed economy models developed by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Households consume a basket of non-traded final goods and home and foreign produced traded final goods. Final goods are produced by monopolistically competitive firms which use intermediate goods as their only input. Final goods prices are subject to Calvo-style contracts. Intermediate goods are produced by

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9We approximate our model around the symmetric steady state in which steady-state net foreign asset position and steady-state inflation rates are assumed to be zero.
perfectly competitive firms using labour and real capital as inputs. Intermediate goods prices are perfectly flexible. Capital stocks are subject to adjustment costs. Households supply homogeneous labour to monopolistically competitive labour unions. The labour unions supply differentiated labour to firms in the intermediate goods sector. The wages charged by labour unions are subject to Calvo-style contracts. All profits from firms in the intermediate and final goods sectors and surpluses from labour unions are paid to households.

We allow trade in equities and bonds. Home and foreign equities represent claims on aggregate firm profits of each country, and home and foreign nominal bonds are denominated in the currency of each country.

The following sections describe the home country in detail. The foreign country is identical. An asterisk indicates a foreign variable or a price in foreign currency. The model is closely related to Devereux, Senay and Sutherland (2014) but is replicated below for completeness.

3.1 Households

Household preferences are given by

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \left( C_{t+i}(z) - hC_{t+i-1} \right)^{1-\rho} (1 - H_{t+i}(z))^{\phi \Delta_{t+i}} \right\}, \rho > 0, \phi \leq 0, \ 0 \leq h \leq 1. \quad (17)$$

which is identical to (7) except that we have added a shock to labour supply preferences in the form of $\Delta_{t+i}$. We assume $\Delta_t = \hat{\Delta} \exp(\hat{\Delta}_t)$ where $\hat{\Delta}_t = \psi_{\Delta} \hat{\Delta}_{t-1} + \varepsilon_{\Delta,t}$, $0 \leq \psi_{\Delta} < 1$ and $\varepsilon_{\Delta,t}$ is a zero-mean normally distributed i.i.d. shock with $\text{Var} [\varepsilon_{\Delta}] = \sigma^2_{\Delta}$.

$C$ is aggregate consumption defined across traded and non-traded goods and is given by

$$C_t = \left[ \frac{1}{\eta} C_{N,t}^{\eta-1} + (1 - \eta) \frac{1}{\eta} C_{T,t}^{\eta-1} \right]^{\frac{\eta}{\eta-1}} \quad (18)$$

where $0 \leq \eta \leq 1$ and $\varkappa > 0$ is the elasticity of substitution between traded and non-traded goods. $C_N$ is aggregate consumption of non-traded goods, and $C_T$ is aggregate consumption of traded goods, given by

$$C_{T,t} = \left[ \gamma^\frac{\theta}{2} C_{H,t}^{\theta-1} + (1 - \gamma)^\frac{\theta}{2} C_{F,t}^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \quad (19)$$

where $C_H$ and $C_F$ are aggregators over individual home and foreign produced goods. The elasticity of substitution across individual goods within all sectors is $\lambda_t > 1$. The parameter $\theta$ in (19) is the elasticity of substitution between home and foreign traded goods. The parameter $\gamma$ measures the importance of consumption of the home good in preferences over traded goods. For $\gamma > 1/2$, we
have ‘home bias’ in consumption. The aggregate CPI for home households consistent with (18) is

\[ P_t = \left[ \eta P_{N,t}^{1-\kappa} + (1 - \eta) P_{T,t}^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \]  

(20)

where \( P_N \) and \( P_T \) are the price indices for traded and non-traded goods where

\[ P_{T,t} = \left[ \gamma P_{H,H,t}^{1-\theta} + (1 - \gamma) P_{F,H,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \]  

and where \( P_{H,H} \) is the price index of home traded goods for home consumers and \( P_{F,H} \) is the price index of foreign traded goods for home consumers. The corresponding prices for foreign consumers are \( P_{H,F} \) and \( P_{F,F} \).

The flow budget constraint of the home country household is given by

\[ P_t C_t + P_t F_t = w_t H_t + P_t \Pi_t + P_t \Theta_t - P_t T_{D,t} + P_t \sum_{k=1}^{N} \alpha_{k,t-1} r_{kt} \]  

(22)

where \( F_t \) denotes home country net external assets in terms of the home consumption basket, \( w_t \) is the nominal home nominal wage, \( \Pi_t \) is real profits of all home firms, \( \Theta_t \) is the surplus of labour unions and \( T_{D,t} \) is lump-sum taxes imposed on households. The final term represents the total return on the home country portfolio where \( \alpha_{k,t-1} \) represents the real external holdings of asset \( k \) (defined in terms of the home consumption basket), purchased at the end of period \( t - 1 \) and \( r_{k,t} \) represents the gross real return on asset \( k \). We allow for trade in \( N = 4 \) assets; home and foreign equity and home and foreign nominal bonds. Note that \( F_t = \sum_{k=1}^{N} \alpha_{k,t} \).

Home nominal bonds represent a claim on a unit of home currency in each period into the infinite future, i.e. nominal bonds are assumed to be perpetuities. The real price of the home bond is denoted \( Z_{B,t} \). The gross real rate of return on a home bond is thus \( r_{B,t+1} = (1/P_{t+1} + Z_{B,t+1})/Z_{B,t} \). For the foreign nominal bond, the real return on foreign bonds, in terms of home consumption, is \( r_{B',t+1} = (Q_{t+1}/Q_t)(1/P_{t+1} + Z_{B',t+1}^*)/Z_{B,t}^* \), where \( Q_t = S_t P_t^*/P_t \) is the real exchange rate (where \( S \) is the price of the foreign currency in terms of the home currency).

Home equities represent a claim on aggregate profits of all firms in the home traded, non-traded, final and intermediate sectors. The real payoff to a unit of the home equity purchased in period \( t \) is defined to be \( \Pi_{t+1} + Z_{E,t+1} \), where \( Z_{E,t+1} \) is the real price of home equity and \( \Pi_{t+1} \) is real aggregate profits. Thus the gross real rate of return on the home equity is \( r_{E,t+1} = (\Pi_{t+1} + Z_{E,t+1})/Z_{E,t} \).

We let the foreign equity act as the \( N \)th asset, so that \( r_{N,t+1} = r_{E',t+1} \).

Optimal portfolio choices for the home and foreign countries respectively imply

\[ E_t U_{C,t+1}(r_{k,t+1} - r_{N,t+1}) = 0, \quad k = 1..N-1. \]  

(23)
\[ E_t U_{C,t+1}^k \frac{(r_{k,t+1} - r_{N,t+1})}{Q_{t+1}} = 0, \quad k = 1..N - 1. \] (24)

where \( U_{C,t+1} \) and \( U_{C,t+1}^* \) denote the marginal utility of consumption defined as in (8) and (9) – the only difference being the presence of labour supply shock, \( \Delta_{t+i} \).

### 3.2 Government

We assume that total government expenditure is exogenous and subject to stochastic shocks. In particular, \( G_t = \hat{G} \exp(\hat{G}_t) \) where \( \hat{G}_t = \psi_G \hat{G}_{t-1} + \varepsilon_{G,t} \), \( 0 \leq \psi_G < 1 \) and \( \varepsilon_{G,t} \) is a zero-mean normally distributed i.i.d. shock with \( \text{Var}[\varepsilon_G] = \sigma^2_{\hat{G}} \). The allocation of government expenditure across traded and non-traded goods and across individual goods is governed by aggregators similar to those of consumers.

All government spending is financed via lump sum taxes on households, \( T_D \), and firms, \( T_C \). The budget constraint is \( P_{G,t}G_t = P_{T_D,t} + P_{T_C,t} \) with the assumption that \( P_{T_D} = (1 - \tau)P_{G,t}G \) and \( P_{T_C} = \tau P_{G,t}G \) where \( \tau \) is a fixed parameter which determines the share of profit taxes in the overall tax take. \( P_{G,t} \) is the price index of government purchased goods and is given by \( P_{G,t} = \eta P_{N,t} + (1 - \eta)P_{H,H,t} \).

### 3.3 The Labour Market

There are labour unions that hire homogeneous labour from households in a perfectly competitive primary labour market at wage rate \( w_t \). They act as monopolistic competitors in a secondary labour market where they sell differentiated labour to intermediate goods firms. Labour union \( z \) charges \( w_t(z) \) in the secondary market and faces a downward sloping demand for its variety of labour as follows

\[ L_t(z) = L_t \left( \frac{w_t(z)}{W_t} \right)^{-\xi} \]

where \( L_t \) is aggregate demand for labour and \( W_t \) is the aggregate wage in the secondary labour market and \( \xi \) is the elasticity of substitution between labour varieties.

The choice of \( w_t(z) \) is subject to Calvo (1983)-style sticky-wage contracts with partial backward indexation. In each period \( w_t(z) \) can be optimally reset with probability \( 1 - \varsigma \) or partially indexed to past aggregate wage inflation with probability \( \varsigma \) where the degree of indexation is given by \( \varpi \) (where \( 0 \leq \varpi \leq 1 \)).

Labour union \( z \) maximizes

\[ E_t \sum_{i=0}^{\infty} \Omega_{t+i} \left[ L_{t+i}(z) \frac{w_t(z)}{P_{t+i}} - L_{t+i}(z) \frac{w_{t+i}}{P_{t+i}} \right] \]

when choosing \( w_t(z) \) where \( \Omega_t \) is the stochastic discount factor of home households. The aggregate
surplus of labour unions is given by $\Theta_t = L_t (W_t - w_t) / P_t$ and is paid to households.

### 3.4 Firms

Firms in traded and non-traded sectors in each country are divided between final and intermediate sectors. Intermediate goods firms use labour and fixed capital. Labour is fully mobile between sectors but capital is immobile. The structure of the intermediate sector is similar in the traded and non-traded sectors so the equations shown below apply to both sectors. Variables for the traded and non-traded sectors are indicated with subscripts $T$ and $N$.

There is a unit mass of firms in each of the non-traded and traded sectors at both the final and intermediate levels.

#### 3.4.1 Final goods

Each firm in the final goods sector of sector $j$ produces a single differentiated product. Sticky prices are modelled in the form of Calvo-style contracts with a probability of re-setting price given by $1 - \kappa$ and partial backward indexation with the degree of indexation given by $\omega$ (where $0 \leq \omega \leq 1$). We assume producer currency pricing (PCP) in the benchmark model and consider local currency pricing (LCP) as a model variant.

If firms use the discount factor $\Omega_t$ to evaluate future profits, then in the PCP case, firm $z$ chooses $p_{H,H,t}(z)$ and $p_{H,F,t}(z)$ in home currency to maximize

$$
E_t \sum_{i=0}^{\infty} \Omega_{t+i} \kappa^i \left\{ y_{H,H,t+i}(z) \frac{p_{H,H,t}(z) - q_{T,t+i}}{P_{t+i}} + y_{H,F,t+i}(z) \frac{p_{H,F,t}(z) - q_{T,t+i}}{P_{t+i}} \right\} 
$$

where $y_{H,H}(z)$ is the demand for home traded good $z$ from home buyers and $y_{H,F}(z)$ is the demand for home good $z$ from foreign buyers and $q_T$ is the price of the intermediate good in the traded goods sector.

In the non-traded sector firm $z$ chooses $p_{N,t}(z)$ to maximize

$$
E_t \sum_{i=0}^{\infty} \Omega_{t+i} \kappa^i y_{N,t+i}(z) \frac{p_{N,t}(z) - q_{N,t+i}}{P_{t+i}}
$$

where $y_N(z)$ is the demand for non-traded good $z$ and $q_N$ is the price of the intermediate good in the non-traded goods sector.

Monopoly power in the final goods sector implies that final goods prices are subject to a mark-up given by $v_t = \lambda_t / (\lambda_t - 1)$. The mark-up is assumed to be subject to stochastic shocks such that $v_t = \bar{v} \exp(\hat{v}_t)$ where $\hat{v}_t = \psi_v \hat{v}_{t-1} + \varepsilon_{v,t}$, $0 \leq \psi_v < 1$ and $\varepsilon_{v,t}$ is a zero-mean normally distributed i.i.d. shock with $\text{Var}[\varepsilon_v] = \sigma_v^2$. 

13
3.4.2 Intermediate goods

The representative firm in the intermediate goods sector \(j\) (where \(j = N, T\)) combines labour, \(L_j\), and capital, \(K_j\), to produce output \(Y_j\) using a standard Cobb-Douglas technology,

\[
Y_{j,t} = A_{j,t}(z_{j,t}K_{j,t-1})^{1-\mu}L_{j,t}^\mu,
\]

where \(0 \leq z_{j,t} \leq 1\) is capacity utilization, \(L\) is an index defined across all individual varieties of labour supplied by labour unions and \(A_{j,t} = \exp(\tilde{a}_{j,t})\) is a common stochastic productivity shock across all intermediate goods firms in sector \(j\). Productivity shocks follow a joint process of the form

\[
\begin{bmatrix}
\hat{a}_{T,t} \\
\hat{a}_{T,t}^2 \\
\hat{a}_{N,t} \\
\hat{a}_{N,t}^2
\end{bmatrix} = A
\begin{bmatrix}
\hat{a}_{T,t-1} \\
\hat{a}_{T,t-1}^2 \\
\hat{a}_{N,t-1} \\
\hat{a}_{N,t-1}^2
\end{bmatrix} + \varepsilon_{a,t}
\]

(27)

where \(\varepsilon_{a}\) is a vector of mean-zero normally distributed i.i.d. shocks with covariance matrix \(\Sigma_a\).

The capital accumulation equation in sector \(j\) is

\[
K_{j,t+1} = I_{j,t} + (1 - \varrho)K_{j,t}
\]

where \(0 \leq \varrho \leq 1\) is the rate of depreciation.

Capital is subject to adjustment costs given by \(\varphi(t_4I_{j,t})\) where we assume \(\varphi(\tilde{I}_j) = \varphi'(\tilde{I}_j) = 0\), \(\varphi''(\tilde{I}_j) > 0\) and \(t_4\) is a stochastic shock to investment costs which is common to both traded and non-traded sectors, where \(u_t = \exp(i_t)\) and \(i_t = \psi_{1,t-1} + \varepsilon_{i,t}, 0 \leq \psi_i < 1\) and \(\varepsilon_{i,t}\), is a zero-mean normally distributed i.i.d. shock with \(\text{Var}[\varepsilon_i] = \sigma_i^2\). Capital has the same composition as consumption (see equations (18) and (19)) so the price of investment goods is given by (20).

Firms are assumed to incur costs of unused capacity which are given by \(F(z_{j,t+i})\) where we assume \(F(1) = 0, F'(1) > 0\) and \(F''(1) > 0\).

The representative firm in sector \(j\) chooses \(L_{j,t}, I_{j,t}\) and \(K_{j,t}\) to maximize the real discounted value of dividends, given by

\[
E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Upsilon_{t+i} \left[ q_{j,t+i}Y_{j,t+i} - \frac{W_{t+i}}{P_{t+i}}L_{j,t+i} - I_{j,t+i} - \varphi(t_4I_{j,t+i}) - F(z_{j,t+i}) \right]
\]

subject to the production function and capital accumulation equations where \(q_j\) is the price of intermediate goods in sector \(j\). \(\Omega_t\) is the stochastic discount factor of shareholders of the firm. \(\Upsilon_t\) is a shock which affects the cost of funds to firms. Smets and Wouters (2003) refer to this as a shock to external finance premium. We assume that \(\Upsilon_t = \exp(\tilde{\Upsilon}_t)\) and \(\tilde{\Upsilon}_t = \psi_\Upsilon \tilde{\Upsilon}_{t-1} + \varepsilon_{\Upsilon,t}, 0 \leq \psi_\Upsilon < 1\)
and \( \varepsilon_{T,t} \) is a zero-mean normally distributed i.i.d. shock with \( \text{Var}[\varepsilon_T] = \sigma_T^2 \).

### 3.5 Aggregate output and employment

Total private sector expenditure is given by

\[
D_t = C_t + I_{N,t} + I_{T,t} + \varphi(t_t I_{N,t}) + \varphi(t_t I_{T,t}) + I \left( z_{N,t} \right) + I \left( z_{T,t} \right). \tag{28}
\]

Home purchases of home non-traded and home traded final goods are

\[
D_{N,t} = \eta \left( \frac{P_{N,t}}{P_t} \right)^{-\eta} D_t \tag{29}
\]

\[
D_{H,t} = \gamma (1 - \eta) \left( \frac{P_{T,t}}{P_t} \right)^{-\eta} \left( \frac{P_{H,H,t}}{P_{T,t}} \right)^{-\theta} D_t \tag{30}
\]

Market clearing for good \( z \) in the home country non-traded final goods sector implies

\[
y_{N,t}(z) = \left( \frac{p_{N,t}(z)}{P_{N,t}} \right)^{-\lambda} [D_{N,t} + \eta G_t]
\]

Equilibrium in the market for good \( z \) in the home country traded final goods sector implies \( y_{T,t}(z) = y_{H,H,t}(z) + y_{H,F,t}(z) \) where

\[
y_{H,H,t}(z) = \left( \frac{p_{H,H,t}(z)}{P_{H,H,t}} \right)^{-\lambda} [D_{H,t} + (1 - \gamma) G_t] \quad y_{H,F,t}(z) = \left( \frac{p_{H,F,t}(z)}{P_{H,F,t}} \right)^{-\lambda} D_{H,t}^* \tag{31}
\]

and \( D_{H,t}^* \) is the foreign demand for home traded goods (defined analogously to (30)).

Aggregate GDP for the home economy is given by

\[
Y_t = \frac{P_{N,t}}{P_{Y,t}} [D_{N,t} + \eta G_t] + \frac{P_{H,H,t}}{P_{Y,t}} [D_{H,t} + (1 - \eta) G_t] + \frac{S_t P_{H,F,t}^*}{P_{Y,t}} D_{H,t}^*
\]

where \( P_{Y,t} \) is the GDP deflator, which we define as follows

\[
P_{Y,t} = \eta P_{N,t} + (1 - \eta) (1 - g) (1 - \gamma) P_{H,H,t} + (1 - \eta) (1 - g) (1 - \gamma) S_t P_{H,F,t}^*
\]

where \( g \) is the steady-state share of government spending in GDP.

Total after-tax dividends aggregated across all intermediate and final goods firms in both traded
and non-traded sectors are given by

$$\Pi_t = \frac{P_{Y,t}}{P_t} Y_t - \frac{W_t}{P_t} L_t - I_{N,t} - I_{T,t} - \varphi(t) I_{N,t} - \varphi(t) I_{T,t} - F(z_{N,t}) - F(z_{T,t}) - T_{C,t}$$

Equilibrium in the primary labour market implies $L_{N,t} + L_{T,t} = L_t = H_t$.

### 3.6 Monetary Authorities

We assume that monetary authorities follow a Taylor rule to set the nominal rate of return on the nominal bonds of their respective currencies. For the home country, this is described by

$$i_t = \beta^{\frac{1}{1-\vartheta}} i_{t-1} \left( \frac{P_t}{P_{t-1}} \right)^{\chi} \left( \frac{Y_t}{\bar{Y}_t} \right)^{\delta} \exp(\varepsilon_{m,t}) \right)^{1-\vartheta}$$

where $0 \leq \vartheta < 1$, $\chi > 1$, and $\delta > 0$, and $\bar{Y}_t$ represents potential output of the home country. $\varepsilon_{m,t}$ is a random monetary policy disturbance which is zero-mean, i.i.d. and normally distributed with $\text{Var}[\varepsilon_{m}] = \sigma_m^2$.

In (32) the nominal interest rate is determined as a function of the historic CPI inflation rate, which is in line with the actual practice in countries that have been following inflation targeting policies. The CPI also has the advantage of being the most visible and relevant price index for guiding monetary policy.

We assume that potential output, $\bar{Y}_t$, is constant. As our purpose is to represent actual rather than optimal monetary policy, we ignore the impact of shocks on $\bar{Y}_t$, which can change the welfare relevant measure of $\bar{Y}_t$. In practice policy makers are not able directly to observe shocks affecting potential output and therefore tend to measure potential output using a moving average measure of actual output.

Rule (32) allows for a degree of partial adjustment in monetary policy, which is determined by the parameter $\vartheta$.

### 4 Quantitative Results

#### 4.1 Benchmark Parameter Values

The benchmark parameter values used in the numerical analysis are listed in Table 1. Most of the parameter values are chosen in the same way as in Devereux, Senay and Sutherland (2014).

The value chosen for the discount factor, $\beta$, yields a steady state rate of return of approximately 4%. The rate of depreciation of real capital, $\varphi$, is set at 0.025, and is consistent with an annual rate of depreciation of 10%. The capital adjustment cost function is parameterized to yield a variance
of total investment which is approximately 3 times the variance of GDP (which is consistent with
the data for most developed economies). The capacity utilization cost function is parameterized in
line with the estimates of Smets and Wouters (2003, 2005, 2007).

The elasticity between home and foreign traded goods, \( \theta \), is set to 1.5 as in the benchmark
parameterization of Backus, Kehoe and Kydland (1994). The share of non-traded goods in the
consumption basket, \( \eta \), the elasticity of substitution between traded and non-traded goods, \( \varepsilon \), and
the share of home traded goods in the traded consumption basket, \( \gamma \), are based on an approximate
average of values used in Benigno and Thoenissen (2008), Corsetti, Dedola and Leduc (2008) and
Stockman and Tesar (1995). In the case of \( \gamma \), the value is chosen to imply a steady state share of
external trade of approximately 20%.

The elasticity of substitution between individual final goods, \( \lambda \), and the Cobb-Douglas coefficient
on labour in the production function of intermediate goods, \( \mu \), are chosen to yield a steady state
monopoly mark-up of 11% and share of capital in output of 0.33. The implied steady state share
of dividends in GDP is approximately 0.15.

The Calvo parameters for price and wage setting, \( \kappa \) and \( \zeta \), are chosen to imply an average period
between price or wage changes of 4 quarters. The degree of backward indexation in price and wage
setting, \( \omega \) and \( \varpi \), and \( \rho \) (risk aversion) are consistent with the estimates of Smets and Wouters

In the benchmark model where the utility function is given by (17), the value of \( \phi \) (inverse
elasticity of labour) is set such that agents devote nearly 30 percent of their time to work at the
steady-state (see Table 1). When solving the alternative model where utility is separable across
consumption and leisure, we set \( 1/\phi = 0.67 \) as in Smets and Wouters (2003). In this alternative
calibration the value of \( \Delta \) is set such that steady-state labour effort is approximately the same as
in the case where utility is non-separable across consumption and leisure.

The values of the Taylor rule parameters \( \delta \) and \( \theta \) are broadly consistent with the estimates of,
for example, Clarida, Gali and Gertler (1998, 2000) and Smets and Wouters (2003, 2005, 2007).\(^{10}\)

The steady state share of government spending in GDP, \( g \), is set at 0.2 and the share of dividend
taxes in total taxes, \( \tau \), is set at 0.15 (which is approximately the same as the assumed steady state
share of dividends in total income).

The covariance matrix of innovations of productivity shocks, \( \Sigma_a \), and the degree of persistence
in productivity shocks, \( A \), are set in line with Benigno and Thoenissen (2008) who use annual data.
We adjust the parameters such that the quarterly TFP series generated using these parameters give
annual TFP series that are consistent with the parameters estimated by Benigno and Thoenissen
(2008).

\(^{10}\)Note that the value of \( \delta \) is adjusted to take account of the difference between annual and quarterly measures of
the nominal interest rate and rate of inflation.
The parameters of the other shock processes are approximately based on the estimates of Smets and Wouters (2003, 2005, 2007). The standard deviation of labour supply shocks, \( \sigma_\Delta \), was adjusted to take account of the effect of the non-separability across consumption and leisure. We set this parameter to 0.025 when we are solving the model variant where utility is separable across consumption and leisure. However, when solving the benchmark model we set this parameter such that the standard deviation of labour is approximately equivalent to that obtained when preferences are separable across consumption and leisure.

4.2 Portfolio Holdings and Business Cycle Moments

Tables 2a and 2b report data on international bond and equity portfolios and Hodrick-Prescott (HP) filtered business cycle moments alongside model counterparts. The benchmark model with non-separable utility across consumption and leisure and across time does a good job in matching the volatilities of key macro variables. The volatility of labour is slightly higher in the model than in the data. The volatility of the real exchange rate is very low compared to the data, which is a failure common to a large class of international business cycle models.

Alternative models yield standard deviations that are close to the benchmark model. One apparent difference is in the relative volatility of consumption. Consumption is less volatile when utility is separable across consumption and leisure. Comparing columns (3) and (4) reveal that habit formation reduces the volatility of consumption even further. This is because when there is habit formation, households have a preference towards smoother consumption. This is not so apparent when we compare the relative volatility of consumption under non-separable utility given by columns (1) and (2) in Table 2a. However it is still true in terms of the absolute level of consumption volatility given by \( \text{Std}(C) \).

Comovement measures show that consumption, investment and employment are positively correlated with output for all model variants. The correlation of consumption with output is low compared to the data in the case of non-separability across consumption and leisure regardless of habits, but other correlation measures are roughly in line with the data counterparts.

**Portfolio Holdings**

The data on international portfolios point at the following observations which generally hold for most advanced economies: (i) positive FX exposure (long position in foreign bonds), (ii) home bias in equity.\(^{11}\) As reported in Table 2b column (1), the benchmark model with non-separability across consumption and leisure and external habits can generate a positive FX exposure \( \bar{\alpha}_B / \beta \bar{Y} > 0 \) alongside home equity bias \( \bar{x}_E > 0.5 \). Although model variants with different utility specifications also imply a long position in foreign bonds, none of them can generate home equity bias given the

\(^{11}\)See Lane and Shambaugh (2010) and Coeurdacier et al. (2007) for details on the portfolio data.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>Elasticity of substitution between individual goods</td>
<td>$\lambda = 10$</td>
</tr>
<tr>
<td>Inverse of the elasticity of labour supply</td>
<td>$\phi = -3.4$</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$h = 0.75$</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\rho = 1.5$</td>
</tr>
<tr>
<td>Share of home goods in consumption basket</td>
<td>$\gamma = 0.58$</td>
</tr>
<tr>
<td>Elasticity of substitution between home and foreign goods</td>
<td>$\theta = 1.5$</td>
</tr>
<tr>
<td>Share of labour in production of intermediate goods</td>
<td>$\mu = 0.67$</td>
</tr>
<tr>
<td>Taylor rule: coefficient on inflation</td>
<td>$\chi = 1.5$</td>
</tr>
<tr>
<td>Taylor rule: coefficient on output</td>
<td>$\delta = 0.1$</td>
</tr>
<tr>
<td>Taylor rule: interest rate smoothing</td>
<td>$\vartheta = 0.85$</td>
</tr>
<tr>
<td>Share of non-traded goods in consumption basket</td>
<td>$\eta = 0.45$</td>
</tr>
<tr>
<td>Elasticity of substitution between traded and non-traded</td>
<td>$\kappa = 0.45$</td>
</tr>
<tr>
<td>Share of government spending in output</td>
<td>$g = 0.2$</td>
</tr>
<tr>
<td>Share of profit taxes in total taxes</td>
<td>$\tau = 0.15$</td>
</tr>
<tr>
<td>Elasticity of substitution between individual labour varieties</td>
<td>$\xi = 10$</td>
</tr>
<tr>
<td>Calvo wage setting and indexation parameters</td>
<td>$\zeta = 0.75$, $\varpi = 0.5$</td>
</tr>
<tr>
<td>Calvo price setting and indexation parameters</td>
<td>$\kappa = 0.75$, $\omega = 0.75$</td>
</tr>
<tr>
<td>Capital adjustment costs</td>
<td>$\varphi^n(\tilde{I})\tilde{I} = 0.25$</td>
</tr>
<tr>
<td>Depreciation of real capital</td>
<td>$\varrho = 0.025$</td>
</tr>
<tr>
<td>Capacity utilization costs</td>
<td>$\frac{f^n(1)}{f'(1)} = 0.2$</td>
</tr>
<tr>
<td>Labour supply shocks</td>
<td>$\psi_\Delta = 0.9$, $\sigma_\Delta = 0.008$</td>
</tr>
<tr>
<td>Mark-up shocks</td>
<td>$\psi_v = 0.0$, $\sigma_v = 0.0015$</td>
</tr>
<tr>
<td>Investment cost shocks</td>
<td>$\psi_i = 0.9$, $\sigma_i = 0.001$</td>
</tr>
<tr>
<td>Government spending shocks</td>
<td>$\psi_G = 0.9$, $\sigma_G = 0.003$</td>
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<td>Risk premium shocks</td>
<td>$\psi_T = 0.0$, $\sigma_T = 0.006$</td>
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<tr>
<td>Monetary shocks</td>
<td>$\sigma_m = 0.0012$</td>
</tr>
<tr>
<td>Productivity shocks</td>
<td></td>
</tr>
<tr>
<td>$A = \begin{bmatrix} 0.95 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0.95 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0.59 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0.59 \end{bmatrix}$</td>
<td>$\Sigma_a = \begin{bmatrix} 0.0134^2 &amp; 0.000081 &amp; 0 &amp; 0 \ 0.000081 &amp; 0.0134^2 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0.0079^2 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0.0079^2 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Table 2a: Business Cycle Moments (Volatility)

<table>
<thead>
<tr>
<th></th>
<th>Non-separability across cons. and leisure</th>
<th>Separability across cons. and leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Habits (1)</td>
<td>No Habits (2)</td>
</tr>
<tr>
<td>Std(Y)</td>
<td>1.58</td>
<td>1.23</td>
</tr>
<tr>
<td>Std(N)</td>
<td>1.19</td>
<td>1.40</td>
</tr>
<tr>
<td>Std(C)/Std(Y)</td>
<td>0.76</td>
<td>0.83</td>
</tr>
<tr>
<td>Std(I)/Std(Y)</td>
<td>4.55</td>
<td>3.31</td>
</tr>
<tr>
<td>Std(N)/Std(Y)</td>
<td>0.75</td>
<td>1.14</td>
</tr>
<tr>
<td>Std(Q)/Std(Y)</td>
<td>3.06</td>
<td>0.45</td>
</tr>
<tr>
<td>Corr(Y,C)</td>
<td>0.84</td>
<td>0.23</td>
</tr>
<tr>
<td>Corr(Y,I)</td>
<td>0.91</td>
<td>0.84</td>
</tr>
<tr>
<td>Corr(Y,N)</td>
<td>0.87</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Notes: Data is from Mandelman et al. (2011) calculated for the U.S. and an aggregate of 15 countries for the period between 1973:1 to 2006:4.

The Backus-Smith Puzzle

The benchmark model is successful in bringing down the correlation between relative consumption and the real exchange rate to almost zero as in the data. On the other hand, model variants with different preferences imply much higher correlations. First consider the model where preferences are separable across consumption and leisure and there is no habit formation – column (4) in Table 2b. In this case, the correlation between relative consumption and real exchange rate is almost perfect even though markets are incomplete – there are four internationally traded assets and twenty different shocks. Introducing consumption habits reduces the correlation to 0.73 – column (3) in Table 2b – which is still very high compared to the data. Hence, introducing habit formation in a model with otherwise separable preferences is not enough to address the Backus-Smith puzzle. Introducing non-separability across consumption and leisure does not account for the puzzle on its own either. Column (2) in Table 2b shows that without habits, a model with non-separability across consumption and leisure reduces Corr(C-C*,Q) to only 0.78.

The Quantity Puzzle

In the data, the correlation of output across countries is higher than that of consumption. The lower panel of Table 2b shows that this ordering can only be matched by the benchmark model. In other model variants, cross-country consumption correlations are much higher than cross-country output correlations.

These results suggest that non-separability across consumption and leisure and consumption habits are crucial for a generally calibrated Smets and Wouters type open economy model to match
Table 2b: Portfolio Holdings and Risk Sharing

<table>
<thead>
<tr>
<th></th>
<th>Non-separability across cons. and leisure</th>
<th>Separability across cons. and leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data (1)</td>
<td>No Habits (2)</td>
</tr>
<tr>
<td>$\alpha_{B^*/\beta Y}$</td>
<td>0.53</td>
<td>1.01</td>
</tr>
<tr>
<td>$\bar{x}_E$</td>
<td>0.62</td>
<td>0.89</td>
</tr>
<tr>
<td>Corr(C-C$,Q$)</td>
<td>-0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Std(U$<em>C^<em>-U^</em></em>{C^*}+Q)/Std(Y)$</td>
<td>-</td>
<td>0.08</td>
</tr>
<tr>
<td>Corr(Y,Y$^*$)</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Corr(C,C$^*$)</td>
<td>0.36</td>
<td>0.41</td>
</tr>
<tr>
<td>Corr(I,I$^*$)</td>
<td>0.28</td>
<td>0.56</td>
</tr>
<tr>
<td>Corr(N,N$^*$)</td>
<td>0.40</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Notes: Portfolio data are for the U.S and are taken from Coeurdacier et al. (2007). $\alpha_{B^*/\beta Y}$ denotes foreign bond portfolio of steady-state output. $\bar{x}_E$ denotes the share of home equity held by home agents. Data for cross-country correlations is from Mandelman et al. (2011) calculated for the U.S and an aggregate of 15 countries between 1973:1 to 2006:4.

the basic facts on international portfolios and account for the Backus-Smith and quantity puzzles at the same time.

4.3 Comparing Alternative Asset Market Structures

In this subsection, we compare our benchmark model with i) the model solved under the assumption of international trade in a single international bond, and ii) the model solved under the assumption of complete financial markets. Table 3a and 3b show that business cycle moments obtained by simulating the benchmark model are almost equal to those obtained by simulating the same model under different asset market assumptions. The only difference is in the volatility of the relative marginal utility of consumption adjusted by the real exchange rate, $\text{Std}(U^*_C-U^*_{C^*}+Q)/\text{Std}(Y)$, which measures deviations from perfect risk sharing given in equation (3). The relative volatility of "uninsured marginal utility" is about 5 times larger in single bond economy (the NC economy in Tables 3a and 3b) compared to our benchmark case (the NBE economy in Tables 3a and 3b). However, this difference in the degree of risk sharing does not imply a major difference in the Backus-Smith correlations which changes only slightly across different asset market assumptions. In other words, in our model the Backus-Smith correlation is quite low even under complete markets, indicating that a low or negative correlation between relative consumption and the real exchange rate does not have to be a consequence or an indicator of a lack of risk sharing across countries.12

12The result that the asset market structure does not matter much for the business cycle moments and the Backus-Smith correlation also holds for model variants with different preferences given in Tables 2a and 2b.
### Table 3a: Business Cycle Moments (Volatility)

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>NC</th>
<th>NBE</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Std(Y)</td>
<td>1.58</td>
<td>1.22</td>
<td>1.23</td>
</tr>
<tr>
<td>Std(N)</td>
<td>1.19</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>Std(C)/Std(Y)</td>
<td>0.76</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>Std(I)/Std(Y)</td>
<td>4.55</td>
<td>3.32</td>
<td>3.31</td>
</tr>
<tr>
<td>Std(N)/Std(Y)</td>
<td>0.75</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td>Std(Q)/Std(Y)</td>
<td>3.06</td>
<td>0.44</td>
<td>0.45</td>
</tr>
<tr>
<td>Corr(Y,C)</td>
<td>0.84</td>
<td>0.21</td>
<td>0.23</td>
</tr>
<tr>
<td>Corr(Y,I)</td>
<td>0.91</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>Corr(Y,N)</td>
<td>0.87</td>
<td>0.62</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Notes: See notes under Table 2a. NC denotes the model with a single internationally traded bond. NBE denotes benchmark model with trade in bonds and equities. CM denotes complete market solution.

### Table 3b: Portfolio Holdings and Risk Sharing

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>NC</th>
<th>NBE</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$\alpha_{B^*}/\beta Y$</td>
<td>0.53</td>
<td>NA</td>
<td>1.01</td>
</tr>
<tr>
<td>$\hat{x}_E$</td>
<td>0.62</td>
<td>1</td>
<td>0.89</td>
</tr>
<tr>
<td>Corr(C-C*,Q)</td>
<td>-0.04</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>Std($U_{C^<em>}-U_{C^</em>}+Q$)/Std(Y)</td>
<td>-</td>
<td>0.37</td>
<td>0.08</td>
</tr>
<tr>
<td>Corr(Y,Y*)</td>
<td>0.44</td>
<td>0.45</td>
<td>0.44</td>
</tr>
<tr>
<td>Corr(C,C*)</td>
<td>0.36</td>
<td>0.45</td>
<td>0.41</td>
</tr>
<tr>
<td>Corr(I,I*)</td>
<td>0.28</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>Corr(N,N*)</td>
<td>0.40</td>
<td>0.31</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Notes: See notes under Table 2a. NC denotes the model with a single internationally traded bond. NBE denotes benchmark model with trade in bonds and equities. CM denotes complete market solution.
4.4 Sensitivity Analysis

In this section, we check the sensitivity of our quantitative results with respect to the calibration of shocks and some key parameters regarding preferences and price setting behaviour. The purpose of this exercise is twofold. First, we want to see whether our main results are robust to certain parameters whose values vary across different studies, such as parameters governing sector-specific TFP shocks, elasticity of substitution between domestic and foreign goods etc. Second, by changing some parameters, such as the degree of indexation or the currency of export pricing, we want to see what features of the model are important for our model’s implications for portfolio positions and consumption-real exchange rate correlations.

4.4.1 Sensitivity with respect to TFP Shock Calibration

In our benchmark calibration, we set the parameters that determine sector-specific TFP shocks in line with BT, who use annual data for tradable and non-tradable sector TFP for the US as the home country and EU-15 plus Japan as the foreign country between 1979 and 2002. An alternative calibration is provided by CDL, who use annual data for the US (home) and an aggregate of Canada, Japan and EU15 (foreign) between 1970 and 2001. While the former uses Groningen Growth and Development Centre, 60-Industry database, the latter uses BLS data for the US and OECD STAN database for the foreign aggregate.

As in BT, we do the necessary adjustments in the persistence of shocks and the variance-covariance matrix originally given by CDL to obtain values compatible with our model which is calibrated to a quarterly frequency. In the CDL calibration, tradable sector TFP shocks are less persistent than non-tradable sector TFP shocks, but the relative size of tradable TFP shocks to non-tradable TFP shocks is larger compared to the BT calibration (Table 1 and Table 4, panel a). Besides, according to CDL, the covariance between different shocks are set to higher values.

Table 5a reports the business cycle moments obtained with the CDL calibration. Output and employment become more volatile compared to the benchmark calibration, which is re-stated in Table 5a for ease of comparison. The relative volatility in the real exchange rate goes up by about 25 percent compared to the benchmark. Consumption becomes more correlated with output, but this correlation remains low compared to the empirical counterpart. Other business cycle moments are very similar across the BT and CDL TFP shock calibrations. Table 5b compares the implications of these alternative shock calibrations for the main variables of interest. The cross-correlation between relative consumption and real exchange rate remains close to zero as in the benchmark calibration while portfolio positions change more drastically. The long position in foreign bonds is almost halved, while the equity portfolio comes close to full diversification under the CDL calibration. Besides, the correlation between cross-country consumption levels become higher compared to BT.
Table 4: Alternative TFP Shock Calibrations

a. CDL: CDL’s original calibration for annual TFP adjusted for quarterly frequency

\[
A = \begin{bmatrix}
0.93 & 0 & 0 & 0 \\
0 & 0.93 & 0 & 0 \\
0 & 0 & 0.98 & 0 \\
0 & 0 & 0 & 0.98
\end{bmatrix}
\quad \Sigma_a = \begin{bmatrix}
0.0147^2 & 0.000106 & 0.000038 & 0.000012 \\
0.000106 & 0.0147^2 & 0.000012 & 0.000038 \\
0.000038 & 0.000012 & 0.006^2 & -0.000005 \\
0.000012 & 0.000038 & -0.000005 & 0.006^2
\end{bmatrix}
\]

b. RT: Original asymmetric shock estimation of Rabanal and Tuesta (2013)

\[
A = \begin{bmatrix}
0.93 & 0 & 0 & 0 \\
0 & 0.94 & 0 & 0 \\
0 & 0 & 0.93 & 0 \\
0 & 0 & 0 & 0.97
\end{bmatrix}
\quad \Sigma_a = \begin{bmatrix}
0.0172^2 & 0 & 0 & 0 \\
0 & 0.0138^2 & 0 & 0 \\
0 & 0 & 0.0102^2 & 0 \\
0 & 0 & 0 & 0.0181^2
\end{bmatrix}
\]

Table 5a: Business Cycle Moments (Volatility) under Alternative Shock Calibrations

<table>
<thead>
<tr>
<th></th>
<th>BT</th>
<th>CDL</th>
<th>RT</th>
<th>RT (asym)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Std(Y)</td>
<td>1.58</td>
<td>1.23</td>
<td>1.39</td>
<td>1.43</td>
</tr>
<tr>
<td>Std(N)</td>
<td>1.19</td>
<td>1.40</td>
<td>1.55</td>
<td>1.65</td>
</tr>
<tr>
<td>Std(C)/Std(Y)</td>
<td>0.76</td>
<td>0.83</td>
<td>0.82</td>
<td>0.81</td>
</tr>
<tr>
<td>Std(I)/Std(Y)</td>
<td>4.55</td>
<td>3.31</td>
<td>3.29</td>
<td>3.15</td>
</tr>
<tr>
<td>Std(N)/Std(Y)</td>
<td>0.75</td>
<td>1.14</td>
<td>1.12</td>
<td>1.15</td>
</tr>
<tr>
<td>Std(Q)/Std(Y)</td>
<td>3.06</td>
<td>0.45</td>
<td>0.56</td>
<td>0.66</td>
</tr>
<tr>
<td>Corr(Y,C)</td>
<td>0.84</td>
<td>0.23</td>
<td>0.30</td>
<td>0.37</td>
</tr>
<tr>
<td>Corr(Y,I)</td>
<td>0.91</td>
<td>0.84</td>
<td>0.85</td>
<td>0.82</td>
</tr>
<tr>
<td>Corr(Y,N)</td>
<td>0.87</td>
<td>0.63</td>
<td>0.57</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Notes: See notes under Table 2a.
Table 5b: Portfolio Holdings and Risk Sharing under Alternative Shock Calibrations

<table>
<thead>
<tr>
<th></th>
<th>( \alpha_{B^*/\beta Y} )</th>
<th>Data</th>
<th>BT</th>
<th>CDL</th>
<th>RT</th>
<th>RT(\text{ asym} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x}_E )</td>
<td>0.53</td>
<td>1.01</td>
<td>0.52</td>
<td>0.49</td>
<td>0.49</td>
<td>0.50</td>
</tr>
<tr>
<td>Corr(C-C(C^*),Q)</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Std(U(U_{C^*}+Q))/Std(Y)</td>
<td>-</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Corr(Y,Y(^*))</td>
<td>0.44</td>
<td>0.44</td>
<td>0.45</td>
<td>0.21</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Corr(C,C(^*))</td>
<td>0.36</td>
<td>0.41</td>
<td>0.45</td>
<td>0.22</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Corr(I,I(^*))</td>
<td>0.28</td>
<td>0.56</td>
<td>0.60</td>
<td>0.52</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Corr(N,N(^*))</td>
<td>0.40</td>
<td>0.30</td>
<td>0.36</td>
<td>0.07</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

Notes: See notes under Table 2a.

As an another alternative TFP shock calibration, we use the estimates provided by Rabanal and Tuesta (2013) – RT. RT use quarterly data for the US and the euro area between 1985:02 and 2004:04 to estimate the parameters governing sectoral TFP shocks. They assume that shocks are not correlated across countries or across sectors. The RT shock calibration is given in panel b of Table 4. We also consider a symmetric version of the RT calibration by using home country (US) estimates also for the foreign country. Columns (3) and (4) in Table 4a show that output and employment are more volatile compared to both BT and CDL. The relative volatility of the real exchange rate is somewhat closer to the data in both RT calibrations, more so in the case of asymmetric RT shocks.

The correlation between relative consumption and the real exchange rate is slightly higher under the asymmetric RT calibration. The size of the foreign bond portfolio becomes smaller compared to the benchmark, while the equity portfolio shows a slight bias for foreign equity. The cross-correlation between home and foreign variables is considerably lower since country-specific shocks are assumed to be uncorrelated.

In sum, the low correlation between relative consumption and real exchange rate implied by the benchmark model with non-separability across consumption and leisure and across time is robust to alternative sectoral TFP shock specifications. The cross-country correlation of consumption remains close to the cross-country correlation of output for different shock calibrations. As for portfolio implications, the long position in foreign bonds is a robust result, while home bias in equity is sensitive to alternative shock calibrations.
4.4.2 Sensitivity with respect to Parameter Choice

Table 6a reports sensitivity with respect to key parameters regarding household preferences. As illustrated in Table 2b, habit formation is important for reducing the cross-correlation between relative consumption and the real exchange rate. In the benchmark calibration, we set habit persistence to 0.75 which is well within the range of estimates reported by existing papers in the literature. In Table 6a we report results for lower and higher values of habit persistence. Accordingly, the cross-correlation between relative consumption and real exchange rate goes up to 0.23 for a lower degree of habit persistence \( (h = 0.65) \) and goes down to negative numbers for a higher degree of habit persistence \( (h = 0.85) \). A higher degree of habit persistence is associated with a larger long position in foreign bonds and an extreme home bias in equity. The relative volatility of the deviation from perfect risk sharing is higher for a higher degree of habit persistence and the cross-country correlation between consumption levels becomes lower and more in line with the empirical counterpart.

On the other hand, results are not that sensitive to the parameter that governs the labour supply elasticity. A higher labour elasticity brings model implied portfolios and consumption-real exchange rate correlation more in line with the data. It also helps bring down the cross-country correlation between consumption levels.

Setting a higher value for the coefficient of relative risk aversion works similarly to increasing habit persistence. Both imply larger portfolio positions and a lower correlation between relative consumption and the real exchange rate. Increasing the risk aversion parameter considerably lowers risk sharing as measured by the relative standard deviation of the relative marginal utility adjusted by the real exchange rate, denoted by \( \frac{\text{Std}(\mathcal{R}_k)}{\text{Std}(Y)} \) in the tables.

Lowering the trade elasticity to \( \theta = 0.85 \) increases the relative volatility of the real exchange rate and decreases the foreign bond position considerably, while it does not change the equity portfolio. Furthermore, the Backus-Smith correlation goes down to \(-0.20\). Though this calibration increases the cross-country correlation between output and consumption, the model implied ranking between the two remains consistent with the data. On the other hand, changing the elasticity of substitution between tradables and non-tradables does not make an important difference for our results.

Making the consumption basket more biased towards home produced goods – either by increasing \( \gamma \) or \( \eta \) parameters – makes the Backus-Smith correlation more negative, while it does not change portfolio positions in an important way.

Next, we turn to investigate the role of the parameters that are related to price setting behaviour and monetary policy. The benchmark calibration assumes that export prices are set in producer currency. When we change this assumption and let export prices be set in the currency of the importer (local currency pricing), the portfolio positions are not affected much, while the Backus-Smith correlation goes up to 0.18. The cross-country correlation of both output and consumption
Table 6a: Sensitivity with respect to Preference Parameters

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Risk Sharing</th>
<th>Correlation between</th>
<th>RER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}_{\beta^<em>}/\beta^</em>$</td>
<td>$\hat{\bar{X}}_E$</td>
<td>$\text{Std}(U^R)$</td>
</tr>
<tr>
<td>Data</td>
<td>0.53</td>
<td>0.62</td>
<td>-</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>1.01</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower Habit Persistence ($h = 0.65$)</td>
<td>0.81</td>
<td>0.75</td>
<td>0.07</td>
</tr>
<tr>
<td>Higher Habit Persistence ($h = 0.85$)</td>
<td>1.39</td>
<td>1.14</td>
<td>0.09</td>
</tr>
<tr>
<td>Higher Labour Supply Elas. ($\phi = -2.4$)</td>
<td>1.06</td>
<td>0.92</td>
<td>0.08</td>
</tr>
<tr>
<td>Lower Labour Supply Elas.($\phi = -4.4$)</td>
<td>0.99</td>
<td>0.86</td>
<td>0.08</td>
</tr>
<tr>
<td>Higher Risk aversion ($\rho = 2$)</td>
<td>1.40</td>
<td>1.16</td>
<td>0.12</td>
</tr>
<tr>
<td>Lower Trade Elasticity ($\theta = 0.85$)</td>
<td>0.21</td>
<td>0.86</td>
<td>0.06</td>
</tr>
<tr>
<td>Lower Elas. btw. T&amp;NT ($\kappa = 0.3$)</td>
<td>0.99</td>
<td>0.86</td>
<td>0.07</td>
</tr>
<tr>
<td>Higher Home Bias in Cons. ($\gamma = 0.8$)</td>
<td>0.76</td>
<td>0.98</td>
<td>0.06</td>
</tr>
<tr>
<td>Higher NT Cons. in Total ($\eta = 0.6$)</td>
<td>0.83</td>
<td>0.75</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6b: Sensitivity with respect to Price Setting/Monetary Policy Parameters

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Risk Sharing</th>
<th>Correlation between</th>
<th>RER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}_{\beta^<em>}/\beta^</em>$</td>
<td>$\hat{\bar{X}}_E$</td>
<td>$\text{Std}(U^R)$</td>
</tr>
<tr>
<td>Data</td>
<td>0.53</td>
<td>0.62</td>
<td>-</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>1.01</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local Currency Pricing</td>
<td>1.03</td>
<td>0.88</td>
<td>0.07</td>
</tr>
<tr>
<td>Lower Price Rigidity ($\kappa = 0.66$)</td>
<td>0.93</td>
<td>0.81</td>
<td>0.08</td>
</tr>
<tr>
<td>Lower Wage Rigidity ($\zeta = 0.2$)</td>
<td>1.08</td>
<td>0.93</td>
<td>0.07</td>
</tr>
<tr>
<td>No Indexation in prices ($\omega = 0$)</td>
<td>0.79</td>
<td>0.86</td>
<td>0.09</td>
</tr>
<tr>
<td>No Indexation in wages ($\varpi = 0$)</td>
<td>1.12</td>
<td>0.94</td>
<td>0.10</td>
</tr>
<tr>
<td>Taylor Rule:Inflation response ($\chi = 3$)</td>
<td>1.92</td>
<td>0.54</td>
<td>0.11</td>
</tr>
</tbody>
</table>

increase but the latter exceeds the former as in the data. Also, the relative volatility of the real exchange rate is considerably closer to the data compared to the benchmark.

Lowering Calvo parameters in price and wage setting, shutting off price and wage indexation and setting a larger coefficient on inflation in the Taylor rule all lower the Backus-Smith correlation to negative values. Portfolios seem to be relatively robust to parameters regarding price and wage setting.

5 Conclusion

This paper analyses the Backus-Smith puzzle in a calibrated DSGE model where preferences are non-separable across time (i.e. there is habit persistence) and non-separable across consumption...
and leisure. International financial trade is confined to nominal bonds and equities. Given the wide range of exogenous shocks incorporated into our model, financial markets are incomplete and risk sharing is less than perfect. Our benchmark model displays a low correlation between relative consumption and the real exchange rate. Portfolio allocations exhibit equity home bias and the cross-country correlation of consumption is lower than the cross-country correlation of output. The benchmark model therefore matches three important stylized facts.

A comparison between the benchmark model and model variants without habit persistence, with separable preferences across consumption and leisure, and with complete financial markets show that our results depend crucially on the presence of non-separability across consumption and leisure and habit persistence but financial market structure plays little role.

In view of the existing literature, our findings point at the importance of non-separable preferences, both across consumption and leisure and also across time, in building a model of the world economy that is consistent with the recent trends in financial globalization and the lack of a significant correlation between relative consumption levels and the real exchange rate. Our findings also contribute to the literature on country portfolios, by showing how non-separable preferences can change optimal portfolios in a general model with multiple shocks and also by investigating the risk sharing properties of endogenous asset trade.

The implications of our model open up an empirical debate regarding the role of non-separable preferences in international risk sharing. Earlier work by Lewis (1996) suggests that non-separable preferences alongside capital market restrictions can account for the low comovement of consumption across countries. It would be interesting to pursue this question further using more recent data.

In light of our findings, we think that it is important to investigate the role of financial market imperfections such as borrowing constraints both within-and across-countries rather than concentrating on asset market incompleteness per se. There are some recent papers that go along this direction. For example, Kollmann (2012) points at the role of within-country heterogeneity in accessing international financial markets in addressing Backus-Smith puzzle. Devereux and Yetman (2010) and Dedola and Lombardo (2012) build models of international financial integration and study the role of borrowing constraints for international propagation of shocks. We leave the investigation of the role of credit market frictions for international portfolio choice and risk sharing for future research.

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