TÜRKİYE CUMHURİYET MERKEZ BANKASI

Bargaining on Supply Chain Networks with Heterogeneous Valuations

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Abstract: In this study, a bargaining between buyers and sellers on a stationary two-sided supply chain network is modelled. Any further restrictions on the network structure is not imposed. Both buyers and sellers are allowed to make offers in the bargaining game. Furthermore, valuations of buyers for the good are heterogeneous. The results reveal that the equilibrium payoffs in the bargaining game that we study depend on the valuations of the buyers and the network positions of all players. As such, these two factors turn out to be the main sources of bargaining power.

Keywords: bargaining, heterogeneity, networks, supply chain

JEL Codes: C78, L14

Özet: Bu çalışmada, iki taraflı durağan bir tedarik zinciri üzerinde alıcılar ve satıcılar arasındaki pazarlık modellenmiştir. Ağ yapısı üzerinde başka herhangi bir kısıtlama tanımlanmamıştır. Pazarlık oyununda hem alıcıların hem de satıcıların teklif yapmasına izin verilmiştir. Ayrıca, alıcıların ürün için değerlemeleri heterojendir. Sonuçlar, üzerinde çalıştığımız pazarlık oyunundaki denge kazancının, alıcıların değerlemelerine ve tüm oyuncuların ağ üzerindeki pozisyonlarına bağlı olduğunu göstermektedir. Dolayısıyla, bu iki faktör pazarlık gücünün ana kaynakları olarak ortaya çıkmaktadır.

Anahtar Kelimeler: pazarlık, heterojenlik, ağ, tedarik zinciri

JEL Codes: C78, L14

Non-technical Summary

Bargaining between buyers and sellers plays a key role in determining the terms of trade in supply chains. Hence, bargaining is a well-studied topic in the supply chain literature. We identified three limitations which are very common in the literature. First, most of the existing supply chain literature adopt the Stackelberg modelling approach in which only one of the agents always makes an offer and the other one can just accept or reject the offer (i.e., one party is gifted with a significant bargaining power). Second, many papers restrict attention to the supply chain networks with one buyer-one seller, one buyer-multiple sellers, one seller-multiple buyers or two-sellers and two-buyers. Finally, the size of the pie subject to bargaining is the same for all bargaining pairs. As a result of these limitations, the effects of bargaining power, network structure, and the pie size on equilibrium outcomes could not be investigated in full generality. In reality, the bargaining power is more evenly distributed in supply chains, there are multiple sellers and buyers, and the size of the pie subject to bargaining varies across different buyer-seller pairs. In this paper, we develop a theoretical model of a supply chain network where (i) both sellers and buyers can make an offer in the bargaining, (ii) any (finite) number of buyers and sellers is allowed, (iii) the size of the pie is heterogeneous across links and (iv) the network structure and the size heterogeneity of the pie across links have the potential to affect bargaining outcomes.

Our work is inspired by the recent developments in network theory which establish a relationship between network structures and market/bargaining outcomes. In this setting, the network identifies the trading relationships between the sellers and the buyers. In our model, we have sellers producing a homogeneous good and buyers demanding the good. The production cost of the good is the same for all sellers. The buyers, on the other hand, value the good differently. Intuitively, one can argue that they have different tastes, preferences, habits or business characteristics. The sellers and the buyers are connected via an exogenously given two-sided supply chain network which is represented by a bipartite graph. Each pair of players connected by a link generates a surplus equal to the difference between the valuation of the buyer for the good and the production cost of the seller. On the network, sellers and buyers play an infinite horizon bargaining game.

In this model, the network structure is not the sole determinant of the bargaining power, but the pie size matters. More explicitly, bargaining power of a player depends on his position in the network as well as his valuation (if he is a buyer) or his neighbours' valuations (if he is a seller) such that for a seller being connected to a buyer with a higher valuation is advantageous for the seller. The results reveal that in the limit equilibrium of the game, the network structure and the valuations of the buyers have an impact on the division of the surplus generated by a pair of players. Intuitively, the buyers with higher valuation and the players who have more links or who have neighbours with fewer links have a higher bargaining power; and so obtain a larger share from the surplus.

1 Introduction

Bargaining between buyers and sellers plays a key role in determining the terms of trade in supply chains. Hence, bargaining is a well-studied topic in the supply chain literature. We identified three limitations which are very common in the literature. First, most of the existing supply chain literature adopt the Stackelberg modelling approach in which only one of the agents always makes an offer and the other one can just accept or reject the offer (i.e, one party is gifted with a significant bargaining power).[1] Second, many papers restrict attention to the supply chain networks with one buyer-one seller (e.g., Plambeck and Taylor (2005), Gurnani and Shi (2006) and Feng et al. (2015)), one buyer-multiple sellers (e.g., Nagarajan and Bassok (2008)), one seller-multiple buyers (e.g., Bernstein and Federgruen (2005)) or two-sellers and two-buyers (e.g., Feng and Lu (2013)). Finally, the size of the bargaining pie is the same for all bargaining pairs. As a result of these limitations, the effects of bargaining power, network structure, and the pie size on equilibrium outcomes could not be investigated in full generality. In reality, the bargaining power is more evenly distributed in supply chains (see Iyer and Villas-Boas (2003) and Draganska et al. (2010)), there are multiple sellers and buyers, and the size of the pie subject to bargaining varies across different buyer-seller pairs. (e.g., due to heterogeneous valuations and/or costs). In this paper, we develop a theoretical model of a supply chain network where (i) both the sellers and the buyers can make an offer in the bargaining, (ii) any (finite) number of buyers and sellers is allowed, (iii) the size of the pie is allowed to be heterogeneous across links and (iv) the network structure and the size heterogeneity of the pie across links have the potential to affect bargaining outcomes.

Our work is inspired by the recent developments in network theory which establish a relationship between network structures and market/bargaining outcomes. In this

[1] Choi (1991), Lariviere and Porteus (2001), Cachon and Lariviere (2001), Bernstein and Federgruen (2003), Gerchak and Wang (2003, 2004), Wang (2006), Perakis and Roels (2007), Song et al. (2008), Adida and DeMiguel (2011), Huang et al. (2014), David and Adida (2015) are some papers adopting Stackelberg modelling approach. For a review, see Cachon (2003).

setting, the network identifies the trading relationships between the sellers and the buyers. That is, a buyer and a seller can engage in trade only if there is a relationship or a "link" connecting the two players. In other words, the network structure imposes a restriction on bargaining possibilities. Not surprisingly, a large number of theoretical studies showed that the network structure has a significant impact on the market outcome (see Calvó-Armengol (2003), Corominas-Bosch (2004), Polanski (2007), Jackson (2008), Manea (2011), Abreu and Manea (2012) and Polanski and Vega-Redondo (2013)). Building on Manea (2011), we study an infinite horizon bargaining game over a two-sided supply chain network with heterogeneous buyers.[2]

In our model, we have sellers producing a homogeneous good and buyers demanding the good. The production cost of the good is the same for all sellers. The buyers, on the other hand, value the good differently. Intuitively, one can argue that they have different tastes, preferences, habits or business characteristics. The sellers and the buyers are connected via an exogenously given two-sided supply chain network which is represented by a bipartite graph. Each pair of players connected by a link generates a surplus equal to the difference between the valuation of the buyer for the good and the production cost of the seller.

On the network, the sellers and the buyers play the following infinite horizon bargaining game.[3] At each period, a link is selected with some positive probability and one of the two players is randomly selected as the proposer. The proposer makes a take-it-or-leave-it offer to divide the surplus generated by the chosen link. If the offer is accepted, the players in the pair leave the game with the agreed shares. In the next period, they are replaced by their identical clones.[4] If the offer is rejected, the players in the pair remain in the game. At each period, the same bargaining

[2]Manea (2011) explores the influence of the network structure on the bargaining outcome with homogeneous agents. He shows that the bargaining power of a player does not depend only on the number of links he has and his position in the network but also his neighbours' positions.

[3]Rubinstein (1982) and Rubinstein and Wolinsky (1985) are pioneering papers of non-cooperative bargaining literature.

[4]The replacement of the players in the agreement pair with their clones makes the model stationary. This modelling assumption is followed by Gale (1987), Manea (2011), Polanski and Lazarova (2015) and Nguyen (2012).

procedure is repeated. All players have perfect information of the game and have the same discount factor. The subgame perfect Nash equilibrium is employed as the solution.

The richness of our model allows us to study the impact of the bargaining power provided by the network structure and heterogeneous surplus sizes on the market outcome which is not captured properly in the supply chain literature due to mentioned limitations.[5] In order to investigate the effects of these factors on the outcome, we need to identify each player's position in the network and the links in which the trade is feasible. For this purpose, we develop a network decomposition algorithm which decomposes a given network into disjoint subnetworks. Our algorithm is a generalization of the network decomposition algorithm constructed by Manea (2011). He studies a bargaining game over a network with homogeneous agents. Hence, each link creates the same surplus. In other words, the surplus generated by each link is homogeneous across all links of the network. Therefore, we generalize the network decomposition algorithm of Manea (2011) by taking into account the valuation heterogeneity among buyers.

The following example points out that the decomposition of a network differs depending on whether the valuations are heterogeneous. Consider a network with two sellers and three buyers as depicted in Figure 1. In part (a), adopting the model in Manea (2011), suppose that the buyers are homogeneous, and their valuations are equal to 1. So, the surplus generated by each link equals to 1. On the other hand, in part (b) incorporating valuation heterogeneity, assume that the valuations of b_1, b_2 and b_3 for the good are 0.5, 0.5 and 0.8, respectively. The set of subnetworks generated by the network decomposition algorithm for (a) is $\{\{(s_1, b_1), (s_1, b_2), (s_2, b_2), (s_2, b_3)\}\}$ and for (b) is $\{\{(s_1, b_1), (s_1, b_2)\}, \{(s_2, b_3)\}\}$, which are not equal to each other. As seen in Figure 1, valuation heterogeneity leads to a different network in equilibrium

[5] In our game, a player may strategically prefer not to trade with some of his neighbours since engaging in a trade with them may lower his payoff. Hence, the valuations of the buyers may change the links at which trade occurs in equilibrium and may provide advantage or disadvantage to a player. The bargaining power due to the valuations refers to this advantage/disadvantage.

form the one in Manea (2011) since a player's position in the network is not the sole source of his bargaining power but the valuations of the buyers matter.

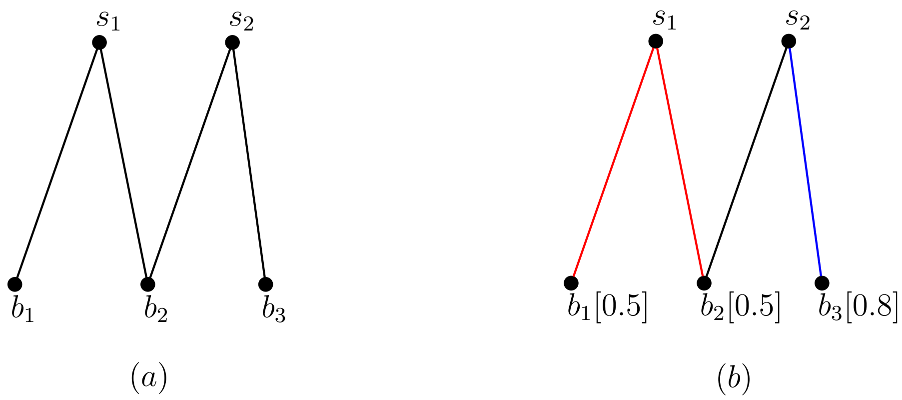

Figure 1: Network G

We find the limit equilibrium payoff vector in each subnetwork determined by the generalized network decomposition algorithm with heterogeneous valuations and prove its uniqueness. Our results show that in the limit equilibrium, the network structure and the valuations of the buyers have an impact on the division of the surplus generated by a pair of players. Intuitively, the buyers with higher valuation and the players who have more links or who have neighbours with fewer links have a higher bargaining power; and so obtain a larger share from the surplus. Accordingly, our model has significantly different results from those of Manea (2011). In the current setting, a seller may strategically prefer not to trade with a buyer having a lower valuation. The strategic decisions of the players over whom to trade with are indeed the main factors shaping the oligopoly structures in the network which are the outcome of the decomposition algorithm. Hence, the outcome of the network decomposition algorithm depends not only on the network structure but also the valuations of the buyers for the good as these two are the sources of the bargaining power in our model while the sole source of bargaining power is the network structure in Manea (2011). Hence, the agreement network which only involves these oligopoly structures and the equilibrium payoffs differ in the limit equilibrium capturing the mentioned strategic decisions of the players. The paper that comes closest to ours is Nakkaş and Xu (2014). They also study bargaining in a two-sided supply chain network where (i) the bargaining occurs in an alternating order, (ii) there are multiple buyers and sellers and (iii) the pie size does not have to be the same across all links of the network. That said, there are important differences between our model and theirs. The key differences are the bargaining procedure, and hence the payoffs in the equilibrium. More precisely, in a subnetwork having more sellers than buyers, the equilibrium outcome in Nakkaş and Xu (2014) assigns each seller zero surplus and assigns each buyer all the surplus generated by the corresponding link while in the equilibrium of our model, a seller gets at least the per player share of the surplus generated by the links in the corresponding subnetwork.

The rest of the paper is organized as follows. Section 2 describes the model and introduces the notation. Section 3 reports the results on the network decomposition and the equilibrium payoffs. Section 4 concludes.

2 Model

We consider a group of sellers $S = \{s_1, s_2, \ldots, s_n\}$ and buyers $B = \{b_1, b_2, \ldots, b_m\}$ interconnected by a two-sided supply chain network G. A two-sided supply chain network is an undirected bipartite graph with the set of sellers and buyers $S \cup B$, and the set of links $\{(s, b) | s \in S, b \in B\}$. Each seller produces one-unit of a homogeneous good and similarly each buyer demands one-unit good. The links in the network represent the trading possibilities. There does not exist any link between any two players within the same group. Denote the utilities of players $s \in S$ and $b \in B$ by u_s and u_b, respectively. Let the valuation of buyer b for the good be v_b and let the production cost of all sellers be c. For simplicity, we assume that $c = 0$. Each link in the network generates a surplus equal to the difference between the valuation of the buyer and the production cost of the seller. An infinite horizon bargaining game is played on the supply chain network over the division of this surplus.

We construct the following infinite horizon bargaining game on the network G: At each period $t = 0, 1, \ldots$, a link $(s, b) \in G$ is selected with some positive probability p_{sb} and one of the players among s and b is randomly selected with equal probability as the proposer. Suppose that player s is selected. Player s makes an offer to b concerning a division of the surplus generated by the link (s, b) and player b responds the offer by accepting or rejecting. If b accepts the offer, s and b leave the game with the agreed shares. In period $t + 1$, two new players replace the positions of s and b in the network. If b rejects the offer, s and b remain in the game. In period $t + 1$, the same procedure is repeated.

Once the players in the chosen link reach an agreement, in the next period, the two players are replaced by their exact clones preserving their positions and their links in the network. Hence, the network structure remains unchanged at each period of the game. The replacement assumption is crucial since it makes the model stationary. We assume that for each player $i \in S \cup B$, there are infinitely many players of type i, i.e., $i = \{i_1, i_2, \ldots, i_\tau, \ldots\}$. Link selection is independent across periods. All players have a common discount rate $\delta \in (0, 1)$. All players have perfect information about which links were chosen, who the proposers were, all the offers and responses in the preceding periods, i.e., all players can observe the entire history of the game.

We employ subgame perfect Nash equilibrium as the solution concept. The equilibrium payoff vector of the game for δ is denoted by $u^{*\delta} = (u_i^{*\delta})_{i \in S \cup B}$. Define the equilibrium agreement network as the subnetwork of G which only involves the links where the agreement gives the players in the pair more payoff than proceeding to the next period does. More precisely, the equilibrium agreement network for δ, $G^{*\delta}$, is the subnetwork of G that only consists of the links (s, b) satisfying $\delta(u_s^{*\delta} + u_b^{*\delta}) \leq$ *the surplus produced by the link* (s, b). The limit equilibrium agreement network, denoted by G^*, is the subnetwork that $G^{*\delta}$ converges to as δ goes to 1. The limit equilibrium payoff vector, u^*, is the payoff vector that $u^{*\delta}$ converges to as δ goes to 1.

3 Results

We analyze the equilibrium of the bargaining game over the network G. The surplus generated by a link (s, b) is equal to $v_b - c$. Since $c = 0$ is assumed, the surplus is equal to v_b. Suppose that in the bargaining game the link (s, b) is selected. Seller s and buyer b bargain over how to divide the surplus v_b. As an initial step, we show that in every subgame perfect equilibrium of the game, the expected payoff of each existing player in the network in any subgame is uniquely determined.

Theorem 1. *For all $\delta \in (0,1)$, there exists a payoff vector $(u_i^{*\delta})_{i \in S \cup B}$ such that for all subgame perfect equilibria of Γ^δ, the expected payoff of existing player i_τ of type i in any subgame is uniquely given by $u_i^{*\delta}$ for all $i \in S \cup B$, $\tau \geq 1$. For every $\delta \in (0,1)$, in any equilibrium and in any subgame where the link $(s_\tau, b_{\tau'})$ is selected and s_τ is the proposer, the followings are hold with probability one:*

(1) if $\delta(u_s^{*\delta} + u_b^{*\delta}) < v_b$, then s_τ offers $\delta u_b^{*\delta}$ and $b_{\tau'}$ accepts.

(2) if $\delta(u_s^{*\delta} + u_b^{*\delta}) > v_b$, then s_τ makes an offer that will be rejected by $b_{\tau'}$

for each $s \in S$ and $b \in B$.

Before moving on to the proof of Theorem 1, we need the following lemma.

Lemma 1. For all $\omega_1, \omega_2, \omega_3, \omega_4 \in \mathbb{R}$,

$$|\max\{\omega_1, \omega_2\} - \max\{\omega_3, \omega_4\}| \leq \max\{|\omega_1 - \omega_3|, |\omega_2 - \omega_4|\}.$$

Proof of Lemma 1. See the Appendix. □

Proof of Theorem 1. Take any subgame perfect equilibrium of Γ^δ at some $\delta \in (0,1)$. In this equilibrium, for all $i \in S \cup B$ and $\tau \geq 1$, i_τ gets the same expected payoff u_i^δ in every subgame. Suppose that the link (s, b) is selected and s is selected as the proposer. Assume that player b plays according to this equilibrium. Let $\delta(u_s^\delta + u_b^\delta) < v_b$. If s makes an offer less than δu_b^δ, player b will reject the offer, and s gets his expected continuation payoff δu_s^δ which is less than $v_b - \delta u_b^\delta$. Hence, s offers δu_b^δ to b and player

b accepts the offer in the equilibrium. Now, let $\delta(u_s^\delta + u_b^\delta) > v_b$. If s makes an offer greater than or equal to δu_b^δ, b will accept the offer. Player s will get $v_b - \delta u_b^\delta$ which is less than δu_s^δ. Hence, s makes an offer that will be rejected by b in the equilibrium. Similar arguments apply to the periods where player b is selected as the proposer and s plays according to the equilibrium. The following equations capture these cases

$$u_s^\delta = \left(1 - \sum_{\{b|sb\in G\}} \frac{p_{sb}}{2}\right)\delta u_s^\delta + \sum_{\{b|sb\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_b^\delta, \delta u_s^\delta\}$$
$$u_b^\delta = \left(1 - \sum_{\{s|sb\in G\}} \frac{p_{sb}}{2}\right)\delta u_b^\delta + \sum_{\{s|sb\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^\delta, \delta u_b^\delta\}.$$

Define a function $f^\delta : [0, \max_{b\in B} v_b]^{n+m} \longleftrightarrow [0, \max_{b\in B} v_b]^{n+m}$ such that for all $s \in S$ and $b \in B$,

$$f_s^\delta(u) = \left(1 - \sum_{\{b|sb\in G\}} \frac{p_{sb}}{2}\right)\delta u_s + \sum_{\{b|sb\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_b, \delta u_s\}$$
$$f_b^\delta(u) = \left(1 - \sum_{\{s|sb\in G\}} \frac{p_{sb}}{2}\right)\delta u_b + \sum_{\{s|sb\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s, \delta u_b\}.$$

u^δ is a fixed point of the function f^δ. We will show that the function f^δ has a unique fixed point by utilizing the Banach fixed point theorem. It is enough to prove the following lemma for the uniqueness result. Denote the unique fixed point of f^δ as $u^{*\delta}$.

Lemma 2. f^δ is a contraction mapping with respect to sup norm on $\mathbb{R}^n$.

Proof of Lemma 2. See the Appendix. $\square$

Now, we argue the existence of a subgame perfect equilibrium. Consider the following strategy profile, σ_{i_τ}, for all players $i \in S \cup B$ and $\tau \geq 1$. At each period where a link of s is selected (say (s,b)) and s is the proposer, if $\delta(u_s^\delta + u_b^\delta) < v_b$, player s makes the offer δu_b^δ to player b; otherwise, he makes an offer which is less than δu_b^δ. At each period where a link of s is selected and s is the responder, player s accepts any offer which is greater than or equal to δu_s^δ and rejects otherwise. The strategy profile for each buyer b can be defined similarly. From above arguments, σ is a subgame perfect equilibrium of Γ^δ, implying that the set of subgame perfect equilibria of Γ^δ is non-empty.

Let $\underline{u}_i^\delta$ and $\bar{u}_i^\delta$ be the infimum and supremum of the expected payoffs of i_τ in any subgame for all $\tau \geq 1$ and for each $i \in S \cup B$, in every subgame perfect equilibrium of the game.

Consider a subgame perfect equilibrium of the game. Assume that a link (s,b) is selected. First, suppose that s is selected as the proposer. Any player of type b does not accept any offer smaller than $\delta \underline{u}_b^\delta$, implying that s can get a payoff of at most $v_b - \delta \underline{u}_b^\delta$. Second, suppose that s is the responder. A player of type s accepts any offer greater than or equal to $\delta \bar{u}_s^\delta$, since in case of rejection, player s gets a payoff of at most $\delta \bar{u}_s^\delta$. Hence, no buyer offers to player s more than $\delta \bar{u}_s^\delta$ in the equilibrium. Now, suppose that any link of s is not selected. In this case, the expected continuation payoff of player s is at most $\delta \bar{u}_s^\delta$. So, for each player $\tau \geq 1$ of type s, the following is satisfied:

$$u_{s_\tau}^\delta \leq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \bar{u}_s^\delta + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \underline{u}_b^\delta, \delta \bar{u}_s^\delta\}. \tag{1}$$

Since inequality (1) holds for all players of type s, we have

$$\bar{u}_s^\delta \leq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \bar{u}_s^\delta + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \underline{u}_b^\delta, \delta \bar{u}_s^\delta\}. \tag{2}$$

Consider that player s makes the offer $\delta \bar{u}_b^\delta + \epsilon$ ($\epsilon > 0$) to any buyer b satisfying $\delta \underline{u}_s^\delta + \delta \bar{u}_b^\delta + \epsilon \leq v_b$ and offers zero to other players and also rejects all offers that he receives, by deviating from his equilibrium strategy. Player b accepts the offer in any subgame perfect equilibrium. Hence, for all $\tau \geq 1$ and for all $\epsilon > 0$, we have the following inequality:

$$u^{\delta}_{s_\tau} \geq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \underline{u}^{\delta}_s + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \bar{u}^{\delta}_b - \epsilon, \delta \underline{u}^{\delta}_s\}.$$

As the deviation converges to zero ($\epsilon \to 0$),

$$u^{\delta}_{s_\tau} \geq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \underline{u}^{\delta}_s + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \bar{u}^{\delta}_b, \delta \underline{u}^{\delta}_s\}. \tag{3}$$

Since inequality (3) holds for all players of type s, we obtain

$$\underline{u}^{\delta}_s \geq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \underline{u}^{\delta}_s + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \bar{u}^{\delta}_b, \delta \underline{u}^{\delta}_s\}.$$

By applying similar arguments for buyer b, we get

$$\bar{u}^{\delta}_b \leq \left(1 - \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \bar{u}^{\delta}_b + \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \underline{u}^{\delta}_s, \delta \bar{u}^{\delta}_b\}$$

$$\underline{u}^{\delta}_b \geq \left(1 - \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta \underline{u}^{\delta}_b + \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta \bar{u}^{\delta}_s, \delta \underline{u}^{\delta}_b\}.$$

We take the difference between the infimum and the supremum of the expected payoffs of all players in $S \cup B$ in order to prove the equality of these two. Let $D = \max_{i\in S\cup B} \bar{u}^{\delta}_i - \underline{u}^{\delta}_i$. Take any player in the set $\arg\max_{i\in S\cup B} \bar{u}^{\delta}_i - \underline{u}^{\delta}_i$. Without loss of generality, suppose that this player is $s \in S$.

$$\begin{aligned}
D &= \bar{u}_s^\delta - \underline{u}_s^\delta \\
&\leq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta(\bar{u}_s^\delta - \underline{u}_s^\delta) + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}[\max\{v_b - \delta\underline{u}_b^\delta, \delta\bar{u}_s^\delta\} \\
&\qquad\qquad\qquad\qquad\qquad\qquad - \max\{v_b - \delta\bar{u}_b^\delta, \delta\underline{u}_s^\delta\}] \\
&\leq \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta D + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\max\{|\delta\bar{u}_b^\delta - \delta\underline{u}_b^\delta|, |\delta\bar{u}_s^\delta - \delta\underline{u}_s^\delta|\} \\
&= \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta D + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\delta\max\{\bar{u}_b^\delta - \underline{u}_b^\delta, \bar{u}_s^\delta - \underline{u}_s^\delta\} \\
&= \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta D + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\delta D \\
&= \delta D
\end{aligned}$$

Hence, $D \leq \delta D$. Since $D \geq 0$ and $\delta \in (0,1)$, we get $D = 0$. Hence, for all $i \in S \cup B$, $\bar{u}_i^\delta = \underline{u}_i^\delta$. Therefore, for all $s \in S$ and $b \in B$, we obtain the following equalities

$$\begin{aligned}
\bar{u}_s^\delta &= \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta\bar{u}_s^\delta + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\max\{v_b - \delta\bar{u}_b^\delta, \delta\bar{u}_s^\delta\} \\
\bar{u}_b^\delta &= \left(1 - \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta\bar{u}_b^\delta + \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}\max\{v_b - \delta\bar{u}_s^\delta, \delta\bar{u}_b^\delta\}.
\end{aligned}$$

which means that $\bar{u}_s^\delta$ and $\bar{u}_b^\delta$ are fixed points of f_s^δ and f_b^δ, respectively. Since f^δ has a unique fixed point, $\bar{u}_s^\delta = u_s^{*\delta}$ and $\bar{u}_b^\delta = u_b^{*\delta}$. This concludes the proof of Theorem 1.

□

The following result indicates the existence of a limit equilibrium network G^* and the existence of limit equilibrium payoffs as δ converges to 1.

Theorem 2. *There exists a bound $\underline{\delta}$ and a subnetwork G^* of G such that for all values of $\delta > \underline{\delta}$, the equilibrium agreement network $G^{*\delta}$ is equal to G^*. Moreover, the* *equilibrium payoff vector at δ, $u^{*\delta}$, converges to u^* as δ goes to* 1.

In any equilibrium, for all values of δ values satisfying $\delta(u_s^{*\delta} + u_b^{*\delta}) \neq v_b$ for all $(s,b) \in G$, whether the bargaining reaches an agreement or a disagreement is determined in Theorem 1. The following lemma demonstrates that the set of such discount factors is finite.

Lemma 3. The statement for all $(s,b) \in G$, $\delta(u_s^{*\delta} + u_b^{*\delta}) \neq v_b$ holds for all but a finite set of δ.

Proof of Lemma 3. See the Appendix. $\square$

Proof of Theorem 2. By equation (10) in the proof of Lemma 3, the function $v_b - \delta(u_s^{*\delta} + u_b^{*\delta})$ can be represented by a ratio of two polynomials for all $s \in S$ and $b \in B$. Hence, $v_b - \delta(u_s^{*\delta} + u_b^{*\delta})$ is continuous in δ. Therefore, we can apply similar arguments in the proof of Theorem 2* in Manea (2011) (stated below). Then, we obtain that for all values of $\delta > \underline{\delta}$, $G^{*\delta} = G^*$. For the second part of the proof, utilizing equation (10) again, we have for all values of $\delta > \underline{\delta}$ and for all players $i \in S \cup B$, $u_i^{*\delta} = P_i^{G^*}(\delta)/Q_i^{G^*}(\delta)$ where $P_i^{G^*}$ and $Q_i^{G^*}$ are polynomials. Moreover, $u_i^{*\delta}$ belongs to the bounded set $[0, \arg\max_{b\in B} v_b]$. Thus, $u_i^{*\delta}$ converges to a finite limit u_i^* as δ goes to 1. $\square$

Theorem 2* (Manea (2011)): (i) There exists $\underline{\delta} \in (0,1)$ and a subnetwork G^* of G such that the equilibrium agreement network $G^{*\delta}$ equals to G^* for all $\delta > \underline{\delta}$. (ii) The equilibrium payoff vector $u^{*\delta}$ converges to a vector $u^* \in [0,1]^n$ as δ tend to 1.

In this model, there are two sources of bargaining power: the network structure and the valuations of the buyers for the good. The bargaining power of a player provided by the network structure depends on the number of links that he has, his position in the network, and the positions of his bargaining partners in the network. Hence, analyzing the network structure provides hints about the limit equilibrium payoffs of the players. We need some additional notation for this analysis. For every network G

and subset of players $M \subseteq S \cup B$, $L^G(M)$ denotes the set of players who have a link in G with the players in M, i.e., $L^G(M) = \{k|(k,l) \in G, l \in M\}$. A set of players M is G-independent if there exists no G-link between any two players in M. Next theorem identifies the bounds on the limit equilibrium payoffs of the player having the highest share and the player having the lowest share in a subnetwork.

Theorem 3. *For each set of buyers M with $L^{G^*}(M) = L$, the following inequalities hold:*

$$\max_{s \in L} u_s^* \geq \frac{\sum\limits_{b \in M} v_b}{|L| + |M|}$$

Similarly, for each set of sellers M with $L^{G^*}(M) = L$, the following inequalities hold:

$$\min_{s \in M} u_s^* \leq \frac{\sum\limits_{b \in L} v_b}{|L| + |M|}.$$

Before moving on to the proof of Theorem 3, we need the following lemma stating that the sum of the limit equilibrium payoffs of the players in a link is equal to the surplus generated by this link, implying that the surplus generated by the link is not wasted. Further, the limit equilibrium agreement network G^* involves only the links where the agreement is feasible in the limit equilibrium.

Lemma 4. If $(s,b) \in G$, then $u_s^* + u_b^* \geq v_b$ and if $(s,b) \in G^*$, then $u_s^* + u_b^* = v_b$.

Proof of Lemma 4. See the Appendix. □

Proof of Theorem 3. For all δ and for all players $s \in S$ and $b \in B$, the equilibrium payoffs for δ are as follows

$$\begin{aligned} u_s^{*\delta} &= \frac{1}{1-\delta} \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_b^{*\delta} - \delta u_s^{*\delta}, 0\} \\ u_b^{*\delta} &= \frac{1}{1-\delta} \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\}. \end{aligned} \tag{4}$$

Without loss of generality, take any set of buyers M with $L^{G^*}(M) = L$. Utilizing Theorem 2, fix $\delta > \underline{\delta}$ for the convergence. If a seller s and a buyer b are not connected in G^*, they cannot reach an agreement on the division of the surplus in the equilibrium. Thus, in the equations system (4), $\max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\} = 0$.

Since a buyer b in M has G^*-links only with the sellers in L, buyer b's expected payoff equation in (4) can be rewritten as follows:

$$u_b^{*\delta} = \frac{1}{1-\delta} \sum_{\{s|(s,b)\in G, s\in L\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\}. \tag{5}$$

For any seller $s \in L$, applying similar arguments used in equation (5), we obtain

$$u_s^{*\delta} \geq \frac{1}{1-\delta} \sum_{\{b|(s,b)\in G, b\in M\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\}. \tag{6}$$

By taking the summation of (5) over all buyers $b \in M$ and taking the summation of (6) over all seller $s \in L$, we have

$$\sum_{b\in M} u_b^{*\delta} = \frac{1}{1-\delta} \sum_{\{(s,b)\in G|b\in M, s\in L\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\} \tag{7}$$

and

$$\sum_{s\in L} u_s^{*\delta} \geq \frac{1}{1-\delta} \sum_{\{(s,b)\in G|b\in M, s\in L\}} \frac{p_{sb}}{2} \max\{v_b - \delta u_s^{*\delta} - \delta u_b^{*\delta}, 0\}. \tag{8}$$

The right hand sides of (7) and (8) are the same. Hence, we have the following inequality

$$\sum_{s\in L} u_s^{*\delta} \geq \sum_{b\in M} u_b^{*\delta}.$$

When players become perfectly patient, as $\delta \to 1$,

$$\sum_{s\in L} u_s^{*} \geq \sum_{b\in M} u_b^{*}.$$

Utilizing Lemma 4, for all $b \in M$, there exists a seller $k \in L$ such that $u_b^* = v_b - u_k^*$. Hence, for all $b \in M$, $u_b^* \geq v_b - \max_{s\in L} u_s^*$. Therefore, we have

$$\begin{aligned}\sum_{b\in M} u_b^* &\geq \sum_{b\in M}(v_b - \max_{s\in L} u_s^*)\\ &= \sum_{b\in M} v_b - |M|\max_{s\in L} u_s^*,\end{aligned}$$

which implies that $|L|\max\limits_{s\in L} u_s^* \geq \sum\limits_{s\in L} u_s^* \geq \sum\limits_{b\in M} u_b^* \geq \sum\limits_{b\in M} v_b - |M|\max\limits_{s\in L} u_s^*$.
Hence,

$$\max_{s\in L} u_s^* \geq \frac{\sum\limits_{b\in M} v_b}{|L|+|M|}.$$

For the second part of the proof, take any set of sellers M with $L^{G^*}(M) = L$. Applying similar arguments those used in the first part of the proof, we obtain

$$\sum_{b\in L} u_b^* \geq \sum_{s\in M} u_s^*.$$

By Lemma 4, for all $b \in L$, there exists a seller $k \in M$ such that $u_b^* = v_b - u_k^*$. Hence, for all $b \in L$, $u_b^* \leq v_b - \min_{s\in L} u_s^*$. Thus, we get

$$\begin{aligned}\sum_{b\in L} u_b^* &\leq \sum_{b\in L}(v_b - \min_{s\in M} u_s^*)\\ &= \sum_{b\in L} v_b - |L|\min_{s\in M} u_s^*,\end{aligned}$$

which implies that $|M|\min\limits_{s\in M} u_s^* \leq \sum\limits_{s\in M} u_s^* \leq \sum\limits_{b\in L} u_b^* \leq \sum\limits_{b\in L} v_b - |L|\min\limits_{s\in M} u_s^*$.
Hence,

$$\min_{s\in M} u_s^* \leq \frac{\sum\limits_{b\in L} v_b}{|L|+|M|}.$$

□

Since the bargaining power of a player depends on his position in the network, we need to identify where each player is located in the network. Regarding this aim, we generalize the network decomposition algorithm developed in Manea (2011). We use the decomposition outcome generated by this algorithm to identify the limit equilibrium payoffs of the players.

Network Decomposition Algorithm, $\mathcal{A}(G)$: For a given network $G \in \Omega$, the algorithm generates the sequence $(r^t, M^t, L^t, N^t, G^t)_t$ as follows:

Let $N^0 = B \cup S$ and $G^0 = G$.

For $t \geq 0$:

If $N^t = \emptyset$, then STOP.

If not,

$$r^t = \max_{M \subset N^t \cap B} \frac{\sum\limits_{b \in M} v_b}{|L^{G^t}(M)| + |M|}.$$

Set M^t be union of all maximizer sets M. Denote $L^t = L^{G^t}(M^t)$.

If $N^t = M^t \cup L^t$, then STOP.

Otherwise, let $N^{t+1} = N^t \setminus (M^t \cup L^t)$ and G^{t+1} be the subnetwork of G induced by the players in N^{t+1}. Denote the step at which the algorithm ends by $\bar{t}$.

The algorithm initially takes the given network. At each step, it identifies the sets that maximize the per player share of the surplus generated by the subnetwork, r^t. It picks the union of these maximizer sets and the partner set of this union. Then, the players in these sets and their links are removed from the network. Once the algorithm picks all the players in the current network, it ends. Otherwise, in the next step, the same procedure is applied to the subnetwork induced by the remaining players. Intuitively, the algorithm decomposes a given network into disjoint oligopoly subnetworks. The limit equilibrium payoffs can be described utilizing the outcome of the decomposition algorithm, $\mathcal{A}$, and they are given by the following theorem.

Theorem 4. *Let the algorithm $\mathcal{A}(G)$ yield the outcome $(r^t, M^t, L^t, N^t, G^t)_{t=0,1,\ldots,\bar{t}}$. Then the limit equilibrium payoffs as $\delta \to 1$ are given by*

$$\forall t \leq \bar{t},\, \forall s \in L^t,\ u_s^* = \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}$$

$$\forall t \leq \bar{t},\, \forall b \in M^t,\ u_b^* = v_b - \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}$$

Proof of Theorem 4. The proof of the theorem proceeds by induction on t. Suppose that the claim holds for all $t' < t$. Now, we prove it for t.

Let M^t and L^t be the sets that the algorithm $\mathcal{A}(G)$ generates at step t. Define the maximum of limit equilibrium payoffs of sellers at step t as $\bar{x}^t = \max\limits_{k \in N^t \cap S} u_k^*$. Further, define the sets

$$\bar{M}^t = \{b \in N^t \cap B | u_b^* = v_b - \bar{x}^t\} \text{ and } \bar{L}^t = L^{G^t}(\bar{M}^t).$$

Claim 1. $\bar{x}^t \geq \dfrac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}$

For a contradiction, suppose that

$$\bar{x}^t < \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}.$$

First of all, we explore the set of players with whom the players in M^t have G^*-links. Take any player $s \in L^{t'}$ where $t' \in \{1, 2, \ldots, t-1\}$. By induction hypothesis,

$$u_s^* = \frac{\sum\limits_{b \in M^{t'}} v_b}{|L^{t'}| + |M^{t'}|}$$

Summing up the limit equilibrium payoffs of s and any buyer $b \in M^t$, we have

$$
\begin{aligned}
u_s^* + u_b^* &\geq \frac{\sum\limits_{b \in M^{t'}} v_b}{|L^{t'}| + |M^{t'}|} + v_b - \bar{x}_t \\
&> \frac{\sum\limits_{b \in M^{t}} v_b}{|L^{t}| + |M^{t}|} + v_b - \bar{x}_t \\
&> \bar{x}_t + v_b - \bar{x}_t = v_b.
\end{aligned}
$$

Second inequality follows from the construction of the decomposition algorithm and the third one from our supposition. So, no player $b \in M^t$ has a G^*-link with the players $s \in L^1 \cup L^2 \cup \ldots \cup L^{t-1}$. Since G is a two-sided (bipartite) network, M^t is a G^*-independent set. Hence, $L^{G^*}(M^t) \subseteq L^t$. Utilizing Theorem 3, we get

$$
\max_{s \in L^t} u_s^* \geq \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}.
$$

By the definition of $\bar{x}^t$,

$$
\bar{x}^t \geq \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|},
$$

which contradicts with our supposition.

Claim 2. $\bar{x}^t \leq \dfrac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}$ and for all $s \in \bar{L}^t$, $u_s^* = \bar{x}_t$

Since we deal with two-sided supply chain networks, it is clear that $\bar{L}^t$ is a G^*-independent set. Take any seller $s \in \bar{L}^t$. For all buyers $b \in (N^t \cap B) \setminus \bar{M}^t$, $u_b^* > v_b - \bar{x}^t$, and if s and b are connected in G, $u_s^* \geq v_b - u_b^*$ by Lemma 4. Hence, $u_s^* + u_b^* > v_b$, which implies there exists no G^*-link between s and any buyer $b \in (N^t \cap B) \setminus \bar{M}^t$. The network decomposition algorithm implies that there exists no G-link between s and any buyer $b \in M^{t'}$, where $t' \in \{1, 2, \ldots, t-1\}$. Therefore, s has G^*-links only with the buyers in $\bar{M}^t$, i.e., $L^{G^*}(\bar{L}^t) = \bar{M}^t$. By Theorem 3,

$$\bar{x}_t = \min_{s \in \bar{L}^t} u_s^* \leq \frac{\sum\limits_{b \in L^{G^*}(\bar{L}^t)} v_b}{|L^{G^*}(\bar{L}^t)| + |\bar{L}^t|}.$$

Since M^t maximizes the surplus per player in the subnetwork, we have the following

$$\frac{\sum\limits_{b \in L^{G^*}(\bar{L}^t)} v_b}{|L^{G^*}(\bar{L}^t)| + |\bar{L}^t|} \leq \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|},$$

implying that

$$\bar{x}_t \leq \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}.$$

This concludes the proof of our claim. By Claim 1 and Claim 2, we get for all $s \in \bar{L}^t$,

$$\bar{x}^t = u_s^* = \frac{\sum\limits_{b \in M^t} v_b}{|L^t| + |M^t|}$$

and for all $b \in \bar{M}^t$, $u_b^* = v_b - \bar{x}_t$.

Claim 3. $\bar{M}^t = M^t$

Since the network decomposition algorithm picks the union of all maximizer sets M, if $\bar{M}^t$ is the maximizer, then $\bar{M}^t \subseteq M^t$. Hence, $\bar{L}^t = L^{G^t}(\bar{M}^t) \subseteq L^{G^t}(M^t) = L^t$.

For the other side of the equation, suppose for contradiction $M^t \not\subseteq \bar{M}^t$. Then, there exits a player $b \in M^t \setminus \bar{M}^t$. Note that b has no G^*-links with players in $N^t \setminus L^t$. Moreover, by Lemma 4, b has no G-links with players in $L^1 \cup L^2 \cup \ldots \cup L^{t-1} \cup \bar{L}^t$. Hence, buyer b has G^*-links only with players in $L^t \setminus \bar{L}^t$. Utilizing Theorem 3, we have

$$\max_{s \in L^{G^*}(M^t \setminus \bar{M}^t)} u_s^* \geq \frac{\sum\limits_{b \in M^t \setminus \bar{M}^t} v_b}{|M^t \setminus \bar{M}^t| + |L^t \setminus \bar{L}^t|} = \frac{\sum\limits_{b \in M^t} v_b - \sum\limits_{b \in \bar{M}^t} v_b}{|M^t| + |L^t| - (|\bar{M}^t| + |\bar{L}^t|)}.$$

Note that $\sum\limits_{b \in M^t} v_b/(|L^t| + |M^t|) = \sum\limits_{b \in \bar{M}^t} v_b/(|\bar{L}^t| + |\bar{M}^t|)$. Then, we obtain

$$\max_{s \in L^{G^*}(M^t \setminus \bar{M}^t)} u_s^* \geq \bar{x}_t,$$

which contradicts with $u_s^* < \bar{x}_t$ for all sellers $s \notin \bar{L}^t$.

Hence, $M^t = \bar{M}^t$ and $L^t = \bar{L}^t$. Claim 1 - Claim 3 conclude the proof for any step of the algorithm $t \leq \bar{t}$.

□

By Theorem 4, we obtain the limit equilibrium payoffs of the players in the bargaining game over a given network G. We prove that in the limit equilibrium of the game, the payoffs of the players are determined according to the network structure and the valuations of the buyers for the good. The buyers with higher valuation are more advantageous in the bargaining game. The players who have more links and who have bargaining partners with fewer links have a higher bargaining power. Hence, they gain larger shares of the pie.

Incorporating valuation heterogeneity into the model of Manea (2011) leads to significantly different equilibrium results which are clearly presented in the following example.

Example: Consider the network G in Figure 1 with the set of sellers $S = \{s_1, s_2\}$ and the set of buyers $B = \{b_1, b_2, b_3\}$. In part (a), all buyers have valuation of 1 for the good, while in part (b), the valuations of b_1, b_2 and b_3 are $0.5, 0.5$ and 0.8, respectively. The limit equilibrium payoffs for part (a) are as follows:

$u_{s_1}^* = u_{s_2}^* = 0.6$

$u_{b_1}^* = u_{b_2}^* = u_{b_3}^* = 0.4.$

The limit equilibrium payoffs for part (b) are as follows:

$u_{s_1}^* = 0.33...$

$u_{s_2}^* = 0.4$

$u_{b_1}^* = u_{b_2}^* = 0.166...$

$u^*_{b_3} = 0.4$.

The limit equilibrium agreement network for (a) is the whole network while that of (b) is $\{s_1b_1, s_1b_2, s_2b_3\}$. In the model with heterogeneous pie sizes, seller 2 and buyer 2 can not reach an agreement when they are matched with each other. Hence, the link s_2b_2 is not included in the limit equilibrium agreement network implying different agreement network structures in the limit equilibrium between the model with homogeneous valuations and the model with heterogeneous valuations. Valuation heterogeneity among buyers changes the oligopoly structures in the network since sellers have a more decisive role in identifying oligopoly structures. More precisely, valuation heterogeneity increases the alternatives of sellers on trading partners. A seller prefers to trade with the buyers having higher valuations since he possibly obtains more share from the pie engaging in this trade.

4 Conclusion

In this paper, we study a bargaining game over a two-sided supply chain network where the sellers producing a homogeneous good and the buyers with potentially different valuations for the good bargain over the surplus generated by the corresponding links. Our model improves upon the existing supply chain literature in multiple dimensions. First, there are multiple sellers and buyers in the network. Second, both the sellers and the buyers can be the proposer in the bargaining game. Finally, the size of the pie subject to bargaining is heterogeneous across the links in the network. Consequently, we can investigate the impact of bargaining power due to the network structure and the valuation heterogeneity on the market/bargaining outcome. In the current study, the bargaining game is similar to the one developed in Manea (2011). However, in our model, the size of the surplus divided between the players of a pair is not the same for all links, which leads to different equilibrium predictions than that of Manea (2011). Above all, valuation heterogeneity changes the oligopoly structures in the network by capturing the strategic decisions of players over whom to trade with and leads to a difference in equilibrium agreement network. Hence, the equilibrium payoffs are different. In our model, the network structure is not the sole determinant of the bargaining power, but the pie size matters. More explicitly, bargaining power of a player depends on his position in the network as well as his valuation (if he is a buyer) or his neighbours' valuations (if he is a seller) such that for a seller being connected to a buyer with a higher valuation is advantageous.

References

Abreu, D. and Manea, M. (2012). Bargaining and efficiency in networks. *Journal of Economic Theory*, 147:43–70.

Adida, E. and DeMiguel, V. (2011). Supply chain competition with multiple manufacturers and retailers. *Operations Research*, 59(1):156–172.

Bernstein, F. and Federgruen, A. (2003). Pricing and replenishment strategies in a distribution system with competing retailers. *Operations Research*, 51(3):409–426.

Bernstein, F. and Federgruen, A. (2005). Decentralized supply chains with competing retailers under demand uncertainty. *Management Science*, 51(1):18–29.

Cachon, G. P. (2003). Supply chain coordination with contracts. *Handbooks in operations research and management science*, 11:227–339.

Cachon, G. P. and Lariviere, M. A. (2001). Contracting to assure supply: How to share demand forecasts in a supply chain. *Management science*, 47(5):629–646.

Calvó-Armengol, A. (2003). Stable and efficient bargaining networks. *Review of Economic Design*, 7:411–428.

Choi, S. C. (1991). Price competition in a channel structure with a common retailer. *Marketing Science*, 10(4):271–296.

Corominas-Bosch, M. (2004). Bargaining in a network of buyers and sellers. *Journal of Economic Theory*, 115(1):35–77.

David, A. and Adida, E. (2015). Competition and coordination in a two-channel supply chain. *Production and Operations Management*, 24(8):1358–1370.

Draganska, M., Klapper, D., and Villas-Boas, S. B. (2010). A larger slice or a larger pie? an empirical investigation of bargaining power in the distribution channel. *Marketing Science*, 29(1):57–74.

Feng, Q., Lai, G., and Lu, L. X. (2015). Dynamic bargaining in a supply chain with asymmetric demand information. *Management Science*, 61(2):301–315.

Feng, Q. and Lu, L. X. (2013). The role of contract negotiation and industry structure in production outsourcing. *Production and Operations Management*, 22(5):1299–1319.

Gale, D. (1987). Limit theorems for markets with sequential bargaining. *Journal of Economic Theory*, 43:20–54.

Gerchak, Y. and Wang, Y. (2004). Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and operations Management*, 13(1):23–33.

Gurnani, H. and Shi, M. (2006). A bargaining model for a first-time interaction under asymmetric beliefs of supply reliability. *Management science*, 52(6):865–880.

Huang, K.-L., Kuo, C.-W., and Lu, M.-L. (2014). Wholesale price rebate vs. capacity expansion: The optimal strategy for seasonal products in a supply chain. *European Journal of Operational Research*, 234(1):77–85.

Iyer, G. and Villas-Boas, J. M. (2003). A bargaining theory of distribution channels. *Journal of Marketing Research*, 40(1):80–100.

Jackson, M. O. (2008). *Social and Economic Networks*. Princeton University Press, Princeton, N.J.

Lariviere, M. A. and Porteus, E. L. (2001). Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & service operations management*, 3(4):293–305.

Manea, M. (2011). Bargaining in stationary networks. *American Economic Review*, 101(5):2042–2080.

Nagarajan, M. and Bassok, Y. (2008). A bargaining framework in supply chains: The assembly problem. *Management science*, 54(8):1482–1496.

Nakkaş, A. and Xu, Y. (2019). The impact of valuation heterogeneity on equilibrium prices in supply chain networks. *Production and Operations Management*, 28(2):241–257.

Nguyen, T. (2012). Local baragaining and endogenous fluctuations. *ACM on Electronic Commerce*, 773.

Perakis, G. and Roels, G. (2007). The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Science*, 53(8):1249–1268.

Plambeck, E. L. and Taylor, T. A. (2005). Sell the plant? the impact of contract manufacturing on innovation, capacity, and profitability. *Management science*, 51(1):133–150.

Polanski, A. (2007). Bilateral bargaining in networks. *Journal of Economic Theory*, 134(1):557–565.

Polanski, A. and Lazarova, E. A. (2015). Dynamic multilateral markets. *International Journal of Game Theory*, 44(4):815–833.

Polanski, A. and Vega-Redondo, F. (2013). Markets, bargaining and networks with heterogenous agents. *working paper*.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50(1):97–109.

Rubinstein, A. and Wolinsky, A. (1985). Equilibrium in a market with sequential bargaining. *Econometrica*, 53(5):1133–1150.

Song, Y., Ray, S., and Li, S. (2008). Structural properties of buyback contracts for price-setting newsvendors. *Manufacturing & Service Operations Management*, 10(1):1–18.

Wang, Y. (2006). Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Operations Research*, 54(6):1110–1127.

Wang, Y. and Gerchak, Y. (2003). Capacity games in assembly systems with uncertain demand. *Manufacturing & Service Operations Management*, 5(3):252–267.

Appendix

Proof of Lemma 1. Without loss of generality, suppose that $\max\{\omega_1, \omega_2, \omega_3, \omega_4\} = \omega_1$. Hence,

$$|\max\{\omega_1, \omega_2\} - \max\{\omega_3, \omega_4\}| \le \omega_1 - \omega_3 = |\omega_1 - \omega_3| \le \max\{|\omega_1 - \omega_3|, |\omega_2 - \omega_4|\}. \quad \square$$

Proof of Lemma 2. By the definition of contraction mapping, we need to prove that

$$\forall u, w \in [0,1]^n : \ \|f^\delta(u) - f^\delta(w)\| \le \delta \|u - w\|,$$

which means for all $k \in S \cup B$, $|f_k^\delta(u) - f_k^\delta(w)| \le \delta\|u - w\|$.
Without loss of generality, we prove the above inequality for all $s \in S$. This inequality can be easily shown for any buyer $b \in B$.

$$\begin{aligned}
&\|f_s^\delta(u) - f_s^\delta(w)|\\
&= \left| \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta(u_s - w_s) + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}(\max\{v_b - \delta u_b, \delta u_s\} \right.\\
&\qquad\qquad \left. - \max\{v_b - \delta w_b, \delta w_s\}) \right|\\
&\le \left| \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta(u_s - w_s)| + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\delta(\max\{|u_b - w_b|, |u_s - w_s|\}) \right|\\
&\le \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta\|u - w\| + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\delta\|u - w\|\\
&= \delta\|u - w\|,
\end{aligned}$$

implying that the function f^δ is a contraction mapping. Hence, the function has a unique fixed point. $\square$

Proof of Lemma 3. $(s, b) \in G^{*\delta}$ means that that s and b are connected and $\max\{v_b - \delta u_s^{*\delta}, \delta u_b^{*\delta}\} = v_b - \delta u_s^{*\delta}$. Since $u^{*\delta}$ is the fixed point of f^δ, $u^{*\delta}$ is the solution of the following linear equations system

$$u_s = \Bigg(1 - \sum_{\{b|(s,b)\in G^{*\delta}\}} \frac{p_{sb}}{2}\Bigg)\delta u_s + \sum_{\{b|(s,b)\in G^{*\delta}\}} \frac{p_{sb}}{2}(v_b - \delta u_b),\ \forall s \in S$$
$$u_b = \Bigg(1 - \sum_{\{s|(s,b)\in G^{*\delta}\}} \frac{p_{sb}}{2}\Bigg)\delta u_b + \sum_{\{s|(s,b)\in G^{*\delta}\}} \frac{p_{sb}}{2}(v_b - \delta u_s),\ \forall b \in B.$$

Take any non-empty subnetwork H of G and define a mapping $h^{\delta,H} : \mathbb{R}^n \to \mathbb{R}^n$ such that for each $s \in S$ and $b \in B$,

$$\begin{aligned} h_s^{\delta,H}(u) &= \Bigg(1 - \sum_{\{b|(s,b)\in H\}} \frac{p_{sb}}{2}\Bigg)\delta u_s + \sum_{\{b|(s,b)\in H\}} \frac{p_{sb}}{2}(v_b - \delta u_b),\ \forall s \in S \\ h_b^{\delta,H}(u) &= \Bigg(1 - \sum_{\{s|(s,b)\in H\}} \frac{p_{sb}}{2}\Bigg)\delta u_b + \sum_{\{s|(s,b)\in H\}} \frac{p_{sb}}{2}(v_b - \delta u_s),\ \forall b \in B. \end{aligned} \tag{9}$$

$h^{\delta,H}$ is a contraction mapping. All equations in the system (9) are linear functions of δ, implying that for each $k \in S \cup B$, $u_k^{\delta,H}$ is given by the Cramer's rule

$$u_k^{\delta,H} = \frac{P_k^H(\delta)}{Q_k^H(\delta)}, \tag{10}$$

where P_k^H and Q_k^H are polynomials in δ. Since the linear system in (10) is non-singular, $Q_k^H(\delta) \neq 0$ for all $\delta \in (0,1)$ and for all non-empty subnetworks H of G.

Take any $s \in S$, $b \in B$ and any non-empty subnetwork H of G. $\delta(u_s^{\delta,H} + u_b^{\delta,H}) = v_b$ is equivalent to

$$v_b = \delta\Big(\frac{P_s^H(\delta)}{Q_s^H(\delta)} + \frac{P_b^H(\delta)}{Q_b^H(\delta)}\Big).$$

Since the equation above is valid for all $\delta \in (0,1)$, it holds also for $\delta = 1/3$. Rewriting the equation for this specific value of δ, we have

$$3v_b = u_s^{1/3,H} + u_b^{1/3,H},$$

which contradicts with for all $k \in S \cup B$, $u_k^{1/3,H} \leq v_b$. It follows that for all (s,b,H) the statement $\delta(u_s^{1/3,H} + u_b^{1/3,H}) = v_b$ holds for a finite set of solutions δ, which concludes the proof.

□

Proof of Lemma 4. Take any link $(s,b) \in G$. $(s,b) \in G \backslash G^*$ implies that for all $\delta > \underline{\delta}$ ($\underline{\delta}$ is determined in Tehorem 2),

$$\delta(u_s^{*\delta} + u_b^{*\delta}) > v_b. \tag{11}$$

If the link (s,b) is involved in the limit equilibrium agreement network G^*, then for all $\delta > \underline{\delta}$,

$$\begin{aligned}
u_s^{*\delta} &= \left(1 - \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta u_s^{*\delta} + \sum_{\{b|(s,b)\in G\}} \frac{p_{sb}}{2}(v_b - \delta u_b^{*\delta}) \\
u_b^{*\delta} &= \left(1 - \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}\right)\delta u_b^{*\delta} + \sum_{\{s|(s,b)\in G\}} \frac{p_{sb}}{2}(v_b - \delta u_s^{*\delta}).
\end{aligned}$$

Since for all $k \neq s \in S$ with $(k,b) \in G^*$, $v_b - \delta u_k^{*\delta} \geq \delta u_b^{*\delta}$ and for all $l \neq b \in B$ with $(s,l) \in G^*$, $v_l - \delta u_l^{*\delta} \geq \delta u_s^{*\delta}$ we obtain

$$\begin{aligned}
u_s^{*\delta} &\geq \left(1 - \frac{p_{sb}}{2}\right)\delta u_s^{*\delta} + \frac{p_{sb}}{2}(v_b - \delta u_b^{*\delta}) \\
u_b^{*\delta} &\geq \left(1 - \frac{p_{sb}}{2}\right)\delta u_b^{*\delta} + \frac{p_{sb}}{2}(v_b - \delta u_s^{*\delta}).
\end{aligned} \tag{12}$$

As $\delta \to 1$, by (11), we have $u_s^* + u_b^* \geq v_b$ for all $(s,b) \in G \backslash G^*$. And from (11) and (12), for all $(s,b) \in G^*$, $u_s^* + u_b^* = v_b$.

□

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