

Investment Decisions Under Real Exchange Rate Uncertainty

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Abstract

This paper analyzes the depressing effects of real exchange rate uncertainty on investment spending by using option pricing techniques. Investment under uncertainty assumes that investments are irreversible and can be delayed to wait for new information about prices, exchange rates and other macroeconomic variables. Irreversible investment spending is like a financial call option. When the opportunity to undertake irreversible investment is exercised, it kills the option of investing and the possibility of waiting for new information. Therefore, investment decisions of firms are sensitive to uncertainties over economic environment. Assuming present real exchange rate volatility is a proxy for real exchange rate uncertainty, and investment spending is like a call option, this paper shows that real exchange rate volatility causes optimal real exchange rate level to undertake investment to be higher for export-oriented sectors and lower for import-oriented sectors. Thus, the zone of “*inaction*” increases, and real investment spending falls as volatility increases regardless of whether the sector is an export-oriented or import-oriented sector.

Keywords: Real exchange rate; Uncertainty; Investment

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1. Introduction

The objective of this study is to show theoretically that real exchange rate uncertainty depresses investment spending in either export-oriented and import-competing firms or import-oriented firms.

Real exchange rate uncertainty can have negative effects on both domestic and foreign investment decisions. Real exchange rate uncertainty causes reallocation of resources among the sectors, causes relocation of resources across countries, and creates an uncertain environment for investment decisions if the investments are irreversible. Krugman (1989) states that

“Uncertainty creates an incentive for firms to pursue a “wait and see” attitude, widening the range of no change in which firms neither enter nor exit. And now we come to the important point: The incentive not to act is greater the more volatile the exchange rate. It is a straightforward result from option pricing that the ratio of the market price at which an option is exercised to the strike price is higher the greater is market volatility. Similarly, in the sunk cost model a firm will wait for a more favorable exchange rate before entering, and will remain in the market for a more unfavorable rate, the greater the perceived future uncertainty of the rate.”

Therefore, in this study, option pricing techniques are used to show that real exchange rate uncertainty lowers real investment spending. To the best of my knowledge, there is no theoretical study that uses option pricing techniques to prove the effects of real exchange rate uncertainty on investment spending. This study assumes that present real exchange rate volatility is a proxy for real exchange rate uncertainty, and investment spending is like a call option in the finance literature. When investment is realized, it kills the option to invest. Therefore, investment decisions of the firms will be sensitive to real exchange rate uncertainty.

The procedure of the study is as follows: In the second part, theoretical background about investment uncertainty is explained, in the third part, the value of the investment project and value of the firm's option to invest and the optimal exchange rate level to undertake investment, that is, optimal stopping point, under real exchange rate uncertainty are determined for export-oriented and import-competing firms, as well as for import-oriented firms using option pricing techniques. Then it is shown that uncertainty of real exchange rates increases optimal stopping point for export-oriented sectors and decreases it for import-oriented sectors, which imply lower investment spending. In the fourth part, concluding remarks are presented.

2. Theoretical Background

While the old investment theories assume investment decisions are made with certainty¹, recent investment literature introduces uncertainty into the investment models. As explained by Pindyck (1991), previous theories ignore two important characteristics of investment expenditures.

First, most investment expenditures are irreversible, which means they are sunk costs, and cannot be recovered. Second, investments can be delayed creating the option of waiting for new information about prices, costs, and other market conditions. Irreversibility and the option of waiting for new information make investors sensitive to uncertainties about macroeconomic variables. Pindyck (1991) states that

“Investment spending on an aggregate level may be highly sensitive to risk in various forms: uncertainties over future product prices and input costs that directly determine cash flows, uncertainty over exchange rates, and uncertainty over future tax and regulatory policy. This means that if a goal of macroeconomic policy is to stimulate investment, stability and credibility may be more important than the particular levels of tax rates or interest rates. Put another way, if uncertainty over the economic environment is high, tax and related incentives may have to be very large to have any significant impact on investment.”

The theoretical literature on investment under uncertainty derive different conclusions about the sign of investment-uncertainty relationship. While Abel (1983) and Caballero (1991) find a positive relationship, Craine (1989) finds a negative relationship between investment and uncertainty. Ingersoll and Ross (1992) using option pricing techniques show that real interest rate uncertainty has depressing effects on investment decisions of firms.

In this framework, real exchange rate uncertainty creates an uncertain environment for investment decisions and therefore, investors delay their investment decisions to obtain more information about the real exchange rates if investments are irreversible (Pindyck, 1991; Engel and Hakkio, 1993; IMF 1984). A theoretical study by Aizenman (1992) showed that both domestic and foreign investments are higher in a fixed exchange rate system. There are few empirical

¹ More detailed information about investment under certainty is contained in Keynes (1936), Duesenberry (1958), Knox (1952), Tinbergen (1938), Abel (1991), Branson and Litvack (1986), Hall and Jorgenson (1971), and Jorgenson (1963, 1967), Tobin (1969), Hayashi (1982).

studies about the effects of both real exchange rates and real exchange rate volatility on investment². Just as empirical studies give ambiguous results about the relation between the changes in real exchange rates and investment expenditures, they also do not show clear correspondence of the effects of real exchange rate volatility on investment expenditures.

3. The Effects of Real Exchange Rate Uncertainty on the Investment Decision

In this section, the value of an investment project, the value of a firm's option to invest, and optimal stopping point under real exchange rate uncertainty are analyzed using option pricing techniques. The aim is to show depressing effects of real exchange rate uncertainty on investment spending.

Present real exchange rate volatility is a proxy to future real exchange rate uncertainty. The procedure is similar to Pindyck (1991), pp. 1125-1132.

Most of the sectoral investments are sector-specific; hence, they cannot be used in another sector efficiently. The feature of sector specificity of investment spending makes them irreversible. As explained by Pindyck (1991), the investment spending can be delayed for further information about real exchange rates.

Accordingly, the characteristics of irreversibility and ability to delay investment spending for further information make investment decisions of firms more sensitive to real exchange rate uncertainty.

The *real exchange rate*, that is, e , is the price of the foreign currency in terms of the domestic currency adjusted by the ratio of foreign prices to domestic prices. The real exchange rate measures the price competitiveness of domestic firms relative to foreign firms. An increase in the real exchange rate (that is, depreciation of domestic currency) implies that price competitiveness of domestic firms increases relative to the foreign firms, and vice versa.

According to the purchasing power parity (PPP) doctrine, real exchange rates are equal to a constant in the long run, and changes in the nominal exchange rates tend to equalize relative price changes. Therefore, real exchange rate follows a *mean-reverting stochastic process* (Krugman, 1989; Dixit, 1989a). Such a process can be represented by

² Some empirical studies are Campa and Goldberg (1995), Goldberg (1993), Froot and Stein (1991), Pindyck and Solimano (1993), Luehrman (1991).

$$de = \alpha(\mu - e) dt + \sigma e^v dz \quad (3.1)$$

where de is the change in real exchange rate, α is the speed of adjustment parameter, μ is the long run equilibrium real exchange rate, dt is the time interval, σ is the volatility of the real exchange rate, and dz is an increment of a Brownian motion (or Wiener process). *The Brownian motion* is the random walk in the continuous time. The increments of a Brownian motion are independent regardless of the size of the time interval. Using of Brownian motion gives an opportunity to think a firm's entry and exit decisions as options depending on real exchange rate volatility. Hence, we can apply option pricing techniques.

The increment of a Brownian motion can be represented as $dz = \varepsilon(t) dt^{1/2}$, where $\varepsilon(t)$ is a serially uncorrelated, $E(\varepsilon_t, \varepsilon_k) = 0$ for $t \neq k$, and normally distributed random variable with mean zero and standard deviation 1, and dt is the changes in time. Accordingly, if E denotes the expectations operator, $E(dz) = 0$ and $E(dz)^2 = dt$.

Empirically, the speed of adjustment to PPP has been found to be very slow, so PPP only holds in the very long run. Frankel and Meese (1987) find that the speed of adjustment to PPP is sufficiently low such that rejecting zero speed of adjustment is statistically impossible. For that reason, the speed of adjustment parameter α is set to zero. In this way, the effects of the real exchange rates on investment decisions are excluded from the analysis, while, the effects of real exchange rate volatility are the focus.

The parameter v is set to 1 to obtain an analytical solution to the resulting differential equations. This analysis illustrates the effects of real exchange rate volatility on the investment decisions of firms. The procedure is similar to Dixit (1989, 1992), Krugman (1989), and Pindyck (1991). Thus, equation (3.1) is now

$$de = \sigma e dz \quad (3.2)$$

hence, the square of equation (3.2) is

$$de^2 = \sigma^2 e^2 dt \quad (3.3)$$

3.1. Export-Oriented and Import-Competing Firms

In this part, it is assumed that the firm is a risk-neutral exporting or import-competing firm. The objective is to show that, when real exchange rate volatility increases, the firm will wait for a higher real exchange rate level to occur before undertaking the investment project.

First, the value of the project, that is, the plant, will be calculated. I assume that the plant itself can be considered as having an option either to produce or shut-down depending on the level of the real exchange rates. I assume that it can be reopened without cost, since this simplifying assumption does not change the conclusions here.

Once the value of the plant is derived, then I calculate the value of the option to undertake investment in the plant or wait. Then, the *optimal stopping point*, that is, the level of the real exchange rate at which it is optimal to undertake investment, is obtained.

Finally, I show the effects of real exchange rate volatility on the optimal stopping point.

3.1.1. Value of the Plant Under Real Exchange Rate Uncertainty

The value of the plant depends on its current and expected cash flows. Here the cash flow is simplified to a linear function of the real exchange rate relative to some break-even real exchange rate, that is, \bar{e} .

The *break-even real exchange rate* is the real exchange rate at which the firm's cash flows are positive. Then, as indicated by equation (3.4), if the real exchange rate is greater than the break-even rate, the sum of the expected capital gain from an investment and expected cash flows (or operating profits) must be equal to the risk-free return on an investment of equivalent value.

If the real exchange rate is less than the break-even rate, the plant is shut down to avoid negative cash flows. Although there are no cash flows, the investment still has value because of the option to reopen. In this case, changes in the value of the investment, that is, the expected capital gain in the plant's value, is set equal to the risk-free return (that is, equation (3.5)). Thus

$$r V(e) dt = E(dV) + \gamma(e - \bar{e}) \quad \text{if } e > \bar{e} \quad (3.4)$$

$$r V(e) dt = E(dV) \quad \text{if } e < \bar{e} \quad (3.5)$$

where e is the real exchange rate, \bar{e} is the break-even real exchange rate, $V(e)$ is the present value of the investment, r is the risk-free interest rate, dt is the time interval, $E(dV)$ is the expected capital gains from the investment, and $\gamma(e - \bar{e})$ is the expected cash flows from the investment.

These two equations are solved separately subject to boundary conditions (3.6) through (3.9).

$$V(0) = 0 \quad e < \bar{e} \quad (3.6)$$

$$V_e(\bar{e}-) = V_e(\bar{e}+) \quad e = \bar{e} \quad (3.7)$$

$$V(\bar{e}-) = V(\bar{e}+) \quad e = \bar{e} \quad (3.8)$$

$$\lim_{e \rightarrow \infty} V(e) = \gamma \frac{(e - \bar{e})}{r} \quad e > \bar{e} \quad (3.9)$$

These boundary conditions are used to identify the function of V , in addition to the differential equations implied by (3.4) and (3.5).

Equation (3.6) states that if the real exchange rate is equal to zero, it remains zero. Therefore, the plant has no value.

Equations (3.7) and (3.8) state that the value of the plant is a continuous and smooth function of the real exchange rate at \bar{e} .

Equation (3.9) states that if the real exchange rate is excessively large, there is no possibility that the plant will stop producing. Therefore, the plant is similar to a perpetuity, and its return is measured by $\gamma(e - \bar{e})/r$.

First, equation (3.4) is evaluated. Ito's Lemma³ is used to express

$$dV = V_e de + \frac{1}{2} V_{ee} de^2 \quad (3.10)$$

where $V_e = dV / de$ and $V_{ee} = d^2V / d e^2$. Substituting equation (3.2) for de and equation (3.3) for de^2 in equation (3.10) gives

$$dV = V_e (\sigma e dz) + \frac{1}{2} V_{ee} (\sigma^2 e^2 dt) \quad (3.11)$$

The expectations of both sides of equation (3.11) are taken. Since $E[dz] = 0$, the following equality is obtained

$$E(dV) = \frac{1}{2} V_{ee} \sigma^2 e^2 dt \quad (3.12)$$

Therefore, equation (3.4) can be written as

³ The Ito's Lemma is as a Taylor series expansion. More detailed information about Ito's Lemma is contained in Pindyck (1991), pp. 1144-1146 and Pindyck and Dixit (1994), pp. 79-82.

$$rVdt = \frac{1}{2} V_{ee} \sigma^2 e^2 dt + \gamma(e - \bar{e})dt \quad (3.13)$$

Simplifying and rearranging equation (3.13) yields an ordinary differential equation that $V(e)$ must satisfy

$$\frac{\sigma^2}{2} e^2 V_{ee} - rV + \gamma e - \gamma \bar{e} = 0 \quad (3.14)$$

The proposed solution to this ordinary differential equation is as follows:

$$A_1 e^{\beta_1} + A_2 e + A_3 = V \quad (3.15)$$

Taking the second derivative of (3.15) with respect to e and putting it into equation (3.14) yields $A_2 = \gamma/r$ and $A_3 = -\gamma\bar{e}/r$. Then the value of the plant, $V(e)$, is

$$V(e) = A_1 e^{\beta_1} + \gamma \left[\frac{e - \bar{e}}{r} \right] \quad e > \bar{e} \quad (3.16)$$

where $(\sigma^2/2) [\beta_1 (\beta_1 - 1)] - r = 0$. This is a quadratic equation and the roots are either $(\beta_1 < 0)$, or $(\beta_1 > 1)$. Boundary condition (3.9) indicates that as e approaches infinity, the value of the plant, that is, V , is like a perpetuity discounted by interest rate r , and the value of option to shut down, that is, $A_1 e^{\beta_1}$, approaches 0. Therefore, it must be true that $\beta_1 < 0$.

Next, equation (3.5) is evaluated, which is repeated here for convenience.

$$rVdt = E(dV) \quad e < \bar{e} \quad (3.17)$$

and

$$rVdt = \frac{\sigma^2}{2} V_{ee} e^2 dt \quad e < \bar{e} \quad (3.18)$$

Rearranging equation (3.18) yields the following differential equation

$$\frac{\sigma^2}{2} e^2 V_{ee} - rV = 0 \quad e < \bar{e} \quad (3.19)$$

Again, V must satisfy this ordinary differential equation. The proposed solution is given as

$$A_4 e^{\beta_2} = V \quad (3.20)$$

Taking the second derivative of equation (3.20) with respect to e , and substituting into equation (3.19) gives

$$\left[\frac{\sigma^2}{2} \beta_2(\beta_2 - 1) - r \right] A_4 e^{\beta_2} = 0 \quad (3.21)$$

This quadratic equation is the same as that obtained in (3.16). Since (3.20) must meet the boundary condition (3.6), so, it must be true that $\beta_2 > 1$. The coefficients of (3.16) and (3.20), that is, A_1 and A_4 , are found by using the boundary conditions (3.7) and (3.8). These boundary conditions yield the following equations respectively.

$$A_1 \bar{e}^{-\beta_1} + \gamma \frac{(\bar{e} - \bar{e})}{r} = A_4 \bar{e}^{-\beta_2} \quad (3.22)$$

$$\beta_1 A_1 \bar{e}^{-(\beta_1-1)} + \frac{\gamma}{r} = \beta_2 A_4 \bar{e}^{-(\beta_2-1)} \quad (3.23)$$

The value of the plant is as

$$V(e) = \frac{\gamma}{r} \frac{\bar{e}^{-(1-\beta_1)}}{(\beta_2 - \beta_1)} e^{\beta_1} + \frac{\gamma}{r} (e - \bar{e}) \quad e > \bar{e} \quad (3.24a)$$

$$V(e) = \frac{\gamma}{r} \frac{\bar{e}^{-(1-\beta_2)}}{(\beta_2 - \beta_1)} e^{\beta_2} \quad e < \bar{e} \quad (3.24b)$$

Equations (3.24a) and (3.24b) can be explained intuitively as follows:

When $e > \bar{e}$, the plant is operating and continues to operate regardless of the changes in the real exchange rates. The present value of the future operating profits is equal to $\gamma (e - \bar{e}) / r$, which is the second term on the right-hand side, and the value of the plant's option to close is the first term on the right-hand side.

When $e < \bar{e}$, the plant is not producing. In this case, equation (3.24b) is the value of the option to reopen in the future. It should be noted that the value of the plant is not equal to zero when $e < \bar{e}$ because of the value of the option to reopen, unless $e = 0$, because $e = 0$ is an absorbing barrier.

3.1.2. The Value of Option to Invest in the Plant and Optimal Stopping Point

The value of the firm's option to invest in the plant is a function of the value of the plant, which is a function of the real exchange rate. Hence, let $F(V(e)) = F(e)$ be the value of the option to invest in the plant. Assume the option to invest is exercised at the optimal stopping point, that is, e^* , otherwise it is optimal to wait. In this case, $F(e^*)$ must satisfy the following boundary conditions:

$$F(0) = 0 \quad (3.25)$$

$$F(e^*) = V(e^*) - I \quad (3.26)$$

$$F_e(e^*) = V_e(e^*) \quad (3.27)$$

Equation (3.25) states that, if the real exchange rate is equal to zero, the real exchange rate stays at zero; hence, the value of the option to invest in a valueless plant is zero.

Equation (3.26) states that when investment is made, the firm receives a net payoff $V(e^*) - I$, where I is the cost of building the plant (that is, sunk cost).

Equation (3.27) is *the smooth pasting condition*. At this point, it is not worthwhile to wait for a higher real exchange rate to occur because the value of the option to invest and the plant's value are increasing at the same rate.

Assume a firm is waiting for the new information about the real exchange rates. While waiting to build, there are no cash flows. Therefore, expected capital gains on the opportunity to invest are equal to the risk-free return on an investment of equivalent value, that is

$$rFdt = E(dF) \quad (3.28)$$

By using Ito's Lemma, dF is expressed as

$$dF = F_e de + \frac{1}{2} F_{ee} de^2 \quad (3.29)$$

Substituting equation (3.2) for de and equation (3.3) for de^2 in equation (3.29) yields the following equation

$$dF = F_e (\sigma e dz) + \frac{1}{2} F_{ee} (\sigma^2 e^2 dt) \quad (3.30)$$

Taking expectations of equation (3.30) yields the following equation

$$E(dF) = \frac{1}{2} \sigma^2 e^2 F_{ee} dt \quad (3.31)$$

Replacing $E(dF)$ in equation (3.28) with equation (3.31) and cancelling dt gives

$$rF - \frac{1}{2} \sigma^2 e^2 F_{ee} = 0 \quad (3.32)$$

Equation (3.32) is an ordinary differential equation, which is solved subject to

the boundary conditions (3.25), (3.26), and (3.27). The proposed solution to (3.32) is given by

$$F(e^*) = ae^{\beta_3} \quad \text{for} \quad e < e^* \quad (3.33)$$

Taking the second derivative of equation (3.33) with respect to e , and substituting into equation (3.32) yields the following equation

$$\left[\frac{\sigma^2}{2} \beta_3 (\beta_3 - 1) - r \right] a e^{\beta_3} = 0 \quad (3.34)$$

This quadratic equation is the same as equation (3.21). Therefore, either $\beta_3 = \beta_2$, or $\beta_3 = \beta_1$. Boundary condition $F(0) = 0$ implies that $\beta_3 > 0$. Hence, $\beta_3 = \beta_2$.

Finally, the value of e^* and coefficient a are determined using the boundary conditions (3.26) and (3.27). At the optimal stopping point, e^* , these boundary conditions must be satisfied. These boundary conditions yields

$$\frac{\gamma}{r} \frac{\bar{e}^{-(1-\beta_1)}}{(\beta_2 - \beta_1)} e^{*\beta_1} \frac{\gamma}{r} (e^* - \bar{e}) - I = a e^{*\beta_2} \quad (3.35)$$

$$\beta_1 \frac{\gamma}{r} \frac{\bar{e}^{-(1-\beta_1)}}{(\beta_2 - \beta_1)} + \frac{\gamma}{r} = \beta_2 a e^{*(\beta_2-1)} \quad (3.36)$$

The coefficient a is determined from (3.36). It is

$$a = \frac{\gamma}{r} \left[\frac{\beta_1}{\beta_2} \frac{\bar{e}^{-(1-\beta_1)}}{(\beta_2 - \beta_1)} e^{*(\beta_1-\beta_2)} + \frac{e^{*(1-\beta_2)}}{\beta_2} \right] \quad (3.37)$$

Substituting (3.37) into equation (3.35) gives the equation that e^* must satisfy, which is

$$\frac{\bar{e}^{-(1-\beta_1)}}{(\beta_2 - \beta_1)} \left(1 - \frac{\beta_1}{\beta_2} \right) e^{*\beta_1} + \left(1 - \frac{1}{\beta_2} \right) e^* - \left(\bar{e} + \frac{rI}{\gamma} \right) = 0 \quad (3.38)$$

The real exchange rate e^* is the “*optimal stopping point*”. Intuitively, this is the point at which the real exchange rate is high enough to induce the firm to build the plant, rather than wait for the real exchange rate to move higher. Hence, the relationship between the volatility of real exchange rates and the optimal stopping point, e^* , is determined. This answers the question; “*Does the volatility raise the optimal stopping point?*”

If volatility raises the optimal stopping point, then volatility increases the zone at which it is optimal to wait, rather than invest. Since an increase in volatility raises individual e^* on all investment projects, thus widening the zone in which it is optimal to wait rather than invest, aggregate investment in each sector will decrease for a given real exchange rate level. Hence, real exchange rate volatility lowers real investment spending, other things being equal.

Defining (3.38) as Φ , I will show that

$$\frac{de^*}{d\sigma^2} = -\frac{\Phi_{\sigma^2}}{\Phi_{e^*}} > 0 \quad (3.39)$$

where Φ_{σ^2} and Φ_{e^*} are the derivatives of function Φ with respect to σ^2 and e^* . This inequality indicates that an increase in real exchange rate volatility increases the real exchange rate that initiates the investment. Φ_{e^*} is

$$\Phi_{e^*} = \left(\frac{1}{\beta_2}\right) \left[\beta_1 \left(\frac{e^*}{e}\right)^{(1-\beta_1)} + \beta_2 - 1 \right] \quad (3.40)$$

The definitions of roots, $\beta_1 = 1/2 - 1/2 (1 + 8r / \sigma^2)$ and $\beta_2 = 1/2 + 1/2 (1 + 8r / \sigma^2)$ and the fact that $\bar{e} \leq e^* \leq \infty$, e^* will not go below the break-even point imply that (3.40) is non-negative. Φ_{σ^2} is

$$\Phi_{\sigma^2} = \Phi_{\beta_1} \frac{d\beta_1}{d\sigma^2} + \Phi_{\beta_2} \frac{d\beta_2}{d\sigma^2} \quad (3.41)$$

which is

$$\Phi_{\sigma^2} = c \left(\frac{e^*}{\beta_2}\right) \left[\left(\frac{e^*}{e}\right)^{(\beta_2-1)} \left(\ln \left(\frac{e^*}{e}\right) - \frac{1}{\beta_2} \left(1 - \left(\frac{e^*}{e}\right)^{(\beta_1-1)} \right) \right) \right] \quad (3.42)$$

where $c = 2r / (\sigma^2)^2 [1 + 8r / \sigma^2]^{-1/2}$. Inspection of (3.42) shows that it is zero when $e^* = 0$ and negative when e^* approaches infinity. Therefore, I determine that Φ_{σ^2} is negative. Take the first derivative of (3.42) with respect to e^* to determine whether there is maxima (or minima). A negative value of (3.42) at the optima will indicate Φ_{σ^2} is negative over the range of $\bar{e} \leq e^* \leq \infty$. Taking the derivative of (3.42) with respect to e^* and then solving for e^* gives

$$\ln \left(\frac{e^*}{e}\right) = -\left[\frac{1}{\beta_2} + \frac{1}{\beta_1 - 1} \right] \quad (3.43)$$

Substituting equation (3.43) into equation (3.42) and making necessary cancellations gives

$$\Phi_{\sigma^2} = -c \left(\frac{e^*}{\beta_2} \right) \left[\left(\frac{e^*}{e} \right)^{(\beta_1-1)} \left(\frac{1}{\beta_1-1} \right) - \frac{1}{\beta_2} \right] \quad (3.44)$$

It was demonstrated earlier that $\beta_1 < 0$ and $\beta_2 > 1$; moreover, it was argued that $(e^* / \bar{e}) \geq 1$. Using these facts and the values of β_1 and β_2 defined by the quadratic equation in (3.16) and (3.20), one can show that $\Phi_{\sigma^2} < 0$. Thus, since Φ_{σ^2} is continuous and is equal to zero at $e^* = \bar{e}$, and negative as e^* approaches infinity, it is negative at its only *extremum*. Therefore, $\Phi_{\sigma^2} < 0$.

Since $\Phi_{e^*} > 0$ and $\Phi_{\sigma^2} < 0$, then $-\Phi_{\sigma^2} / \Phi_{e^*} > 0$. Hence, $de^* / d\Phi^2 > 0$. All else held equal, the uncertainty measured by the volatility of the real exchange rates increases the optimal stopping point, implying lower investment spending.

3.2. Import-Oriented Firms

In this part, it is assumed that plants are import-oriented. In other words, their production depends heavily on imported materials. The real exchange rate represents the cost of imported materials. Then the value of the plant and value of the option to invest in the plant under real exchange rate uncertainty are analyzed.

In this part, it is shown that the optimal stopping point, e^* , at which it is optimal to invest in the plant, is lower than it otherwise would have been, when real exchange rate volatility increases. This increases the zone where it is optimal to wait rather than invest. When aggregated across firms in the sector, this implies that investment spending is negatively related to real exchange rate volatility, other thing being equal.

3.2.1. Value of the Plant Under Real Exchange Rate Uncertainty

Again, the plant's value depends on the relative magnitudes of the real exchange rate and break-even real exchange rate. If real exchange rate is less than the break-even rate, the plant is producing because cash flows are positive. So, the sum of the expected capital gain from an investment and expected cash flows (or operating profits) must be equal to the risk-free return on an investment of equivalent value.

If the real exchange rate is greater than the break-even rate, the plant is closed to avoid negative cash flows. The return on the plant is its cash value and its capital

gain, that is, the change in its value, which includes the value of the option to reopen. Now,

$$r V(e) dt = E(dV) + \lambda(\bar{e} - e) dt \quad \text{if } e < \bar{e} \quad (3.45)$$

$$r V(e) dt = E(dV) \quad \text{if } e > \bar{e} \quad (3.46)$$

which are analogous to equations (3.4) and (3.5) in the case of exporting firms described in Section 3.2.1.1. The boundary conditions are also similar to boundary conditions (3.6) through (3.9) in Section 3.2.1.1 for the export-oriented and import-competing firms. In this case, they are

$$V(0) = 0 \quad e > \bar{e} \quad (3.47)$$

$$V(\bar{e}-) = V(\bar{e}+) \quad e = \bar{e} \quad (3.48)$$

$$V_e(\bar{e}-) = V_e(\bar{e}+) \quad e = \bar{e} \quad (3.49)$$

$$\lim_{e \rightarrow 0} V(e) = \lambda \frac{(\bar{e} - e)}{r} \quad e < \bar{e} \quad (3.50)$$

Equation (3.47) states that when the real exchange rate equals zero, it remains zero, and the plant has no value.

Equations (3.48) and (3.49) state that the value of the investment is a continuous and smooth function of the real exchange rate at the break-even point, \bar{e} .

Equation (3.50) states that if the real exchange rate is extremely small, there is no possibility that the firm will stop producing, so the plant's cash flows are measured by $\lambda(\bar{e} - e) / r$, and the value of the option to reopen approaches zero.

The value of the plant is determined analogously to the case for the exporting and import-competing firms in Section 3.2.1.1 (see Appendix A.1 for derivations). The value of the project is

$$V(e) = \frac{\lambda}{r} \frac{e^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} e^{\alpha_1} + \frac{\lambda}{r} (\bar{e} - e) \quad e < \bar{e} \quad (3.51.)$$

$$V(e) = \frac{\lambda}{r} \frac{e^{-(1-\alpha_2)}}{(\alpha_1 - \alpha_2)} e^{\alpha_2} \quad e > \bar{e}$$

where $\alpha_2 < 0$ and $\alpha_1 > 0$.

When $e < \bar{e}$, the plant is operating and continues to operate regardless of the changes in the real exchange rates. The present value of the future operating profits is equal to $\lambda (\bar{e} - e) / r$, and the value of the firm's option to close will be the first part of the first equation.

When $e > \bar{e}$, the plant is not producing. In this case, the value of the option to reopen in the future is equal to equation (3.51).

3.2.2. The Value of Option to Invest in the Plant and Optimal Stopping Point

The value of the option to invest under real exchange rate uncertainty for the import-oriented firms is $F(V(e)) = F(e)$. Assume the firm makes an investment at the optimal stopping point, e^* , which is less than \bar{e} ; otherwise the firm waits. In this case, $F(e^*)$ must satisfy

$$\lim_{e \rightarrow \infty} F(e) = 0 \quad (3.52)$$

$$F(e^*) = V(e^*) - I \quad (3.53)$$

$$F_e(e^*) = V_e(e^*) \quad (3.54)$$

Equation (3.52) states that if the real exchange rate goes to infinity, option to invest goes to zero.

Equations (3.53) and (3.54) are similar to equations (3.26) and (3.27) in Section 3.2.1.2 for the export-oriented and import-competing firms.

Equation (3.53) states that the option to invest is exercised when $0 < e^* < \bar{e}$. When investment is realized, the firm receives a net payoff $V(e^*) - I$, where I is the cost of building the plant.

Equation (3.54) is the *smooth pasting condition*. At this point, it is not worthwhile to wait for a lower real exchange rate to occur, because the value of the option to invest and the value of plant are increasing at the same rate.

Similar manipulations to those in Section 3.2.1.2 yield an equation that e^* must fulfill (see Appendix A.2 for derivations).

$$\frac{\lambda}{r} \frac{e^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} \left(I - \frac{\alpha_1}{\alpha_2} \right) e^{*\alpha_1} + \frac{\lambda}{r} \left(\frac{I}{\alpha_2} - I \right) e^* + \frac{\lambda}{r} e^- - I = 0 \quad (3.55)$$

Finally, the relationship between the volatility and the optimal stopping point, e^* , is determined. What effect does a volatility increase have on e^* ? Letting Ψ equal (3.55), it is shown that

$$\frac{de^*}{d\sigma^2} = -\frac{\Psi_{\sigma^2}}{\Psi_{e^*}} < 0 \quad (3.56)$$

where Ψ_{σ^2} is the derivative of the implicit function with respect to σ^2 and Ψ_{e^*} is the derivative of the implicit function with respect to e^* .

In Appendix (A.2), it is shown that $-\Psi_{\sigma^2} / \Psi_{e^*} < 0$, and therefore $de^* / d\sigma^2 < 0$. This result proves that the optimal stopping point, e^* , decreases when volatility is higher, and an increase in the real exchange rates depresses real investment spending, and vice versa.

To summarize, real exchange rate uncertainty proxied by present real exchange rate volatility causes optimal stopping point, e^* , to be higher for export-oriented sectors and lower for import-oriented sectors. Thus, the zone of “*inaction*” increases, and real investment spending falls as volatility increases regardless of whether the sector is an export-oriented or import-oriented sector.

4. Concluding Remarks

In this study, option pricing techniques are used to show that real exchange rate uncertainty decreases real investment spending in either export-oriented and import-competing firms or import-oriented firms. The theory of irreversible investment under uncertainty assumes that investments are irreversible and can be delayed to wait for new information about prices, exchange rates and other macroeconomic variables.

Irreversible investment spending is like a financial call option. When the opportunity to undertake irreversible investment is exercised, it kills the option of investing and the possibility of waiting for new information. Therefore, investment decisions of firms are sensitive to uncertainties over economic environment.

Assuming present real exchange rate volatility is a proxy for real exchange rate uncertainty, and investment spending is like a call option, this study shows that real exchange rate volatility causes optimal stopping point, that is, optimal real exchange rate level to undertake investment, to be higher for export-oriented sectors and lower for import-oriented sectors. Thus, the zone of “*inaction*” increases, and real investment spending falls as volatility increases regardless of whether the sector is an export-oriented or import-oriented sector.

Appendix

A.1. Value of the Plant Under Real Exchange Rate Uncertainty

In this part, value of the plant under real exchange rate uncertainty is determined for import-oriented firms. First, equation (3.45) is evaluated. In order to express dV , Ito's Lemma is used.

$$dV = V_e de + \frac{1}{2} V_{ee} de^2 \quad (\text{A.1})$$

where $V_e = dV / de$ and $V_{ee} = d^2 V / de^2$. Substituting equations (3.2) and (3.3) into (A.1) gives

$$dV = V_e (\sigma e dz) + \frac{1}{2} V_{ee} (\sigma^2 e^2 dt) \quad (\text{A.2})$$

Taking expectations of both sides of equation (A.2) yields

$$E(dV) = \frac{1}{2} V_{ee} \sigma^2 e^2 dt \quad (\text{A.3})$$

Hence, equation (3.45) can be written as

$$rVdt = \frac{1}{2} V_{ee} \sigma^2 e^2 dt + \lambda (\bar{e} - e) dt \quad (\text{A.4})$$

Simplifying and rearranging equation (A.4) yields the following ordinary differential equation

$$\frac{\sigma^2}{2} e^2 V_{ee} - rV + \lambda \bar{e} - \lambda e = 0 \quad (\text{A.5})$$

The proposed solution to this ordinary differential equation is as follows

$$A_1 e^{\alpha_1} + A_2 e + A_3 = V \quad (\text{A.6})$$

Taking the second derivative of equation (A.6) with respect to e and putting it into equation (A.5) yields $A_2 = -\lambda / r$, and $A_3 = \lambda \bar{e} / r$. Then the value of the plant, $V(e)$, is

$$V(e) = A_1 e^{\alpha_1} + \lambda \frac{(\bar{e} - e)}{r} \quad e < \bar{e} \quad (\text{A.7})$$

The roots are either $\alpha_1 < 0$ or $\alpha_1 > 1$. Boundary condition (3.50) indicates that if the real exchange rate is extremely small, then the value of the plant, V , is like a perpetuity discounted by interest rate r , and the value of the option to shut down, $A_1 e^{\alpha_1}$, approaches zero. Therefore, it must be true that $\alpha_1 > 1$. Next, equation (3.46) is evaluated.

$$rVdt = E(dV) \quad e > \bar{e} \quad (\text{A.8})$$

$$rVdt = \frac{\sigma^2}{2} V_{ee} e^2 dt \quad e > \bar{e} \quad (\text{A.9})$$

Rearranging equation (A.9) gives

$$\frac{\sigma^2}{2} e^2 V_{ee} - rV = 0 \quad e > \bar{e} \quad (\text{A.10})$$

Again, V must satisfy this ordinary differential equation. The proposed solution is

$$A_4 e^{\alpha_2} = V \quad (\text{A.11})$$

Taking the second derivative of equation (A.11) with respect to e and substituting into equation (A.10) gives

$$\left[\frac{\sigma^2}{2} \alpha_2 (\alpha_2 - 1) - r \right] A_4 e^{\alpha_2} = 0 \quad (\text{A.12})$$

Since $\lim_{e \rightarrow 0} V(e) = 0$, then $\beta_2 < 0$. Now the coefficients A_1 and A_4 are found by using boundary conditions (3.48) and (3.49). These boundary conditions yield the following equalities

$$A_1 \bar{e}^{-\alpha_1} + \lambda \frac{(\bar{e} - \bar{e})}{r} = A_4 \bar{e}^{-\alpha_2} \quad (\text{A.13})$$

$$\alpha_1 A_1 \bar{e}^{(\alpha_1-1)} + \frac{\lambda}{r} = \alpha_2 A_4 \bar{e}^{(\alpha_2-1)} \quad (\text{A.14})$$

Then, the value of the project can be written as follows

$$V(e) = \frac{\lambda}{r} \frac{\bar{e}^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} e^{\alpha_1} + \frac{\lambda}{r} (\bar{e} - e) \quad e < \bar{e} \quad (\text{A.15})$$

$$V(e) = \frac{\lambda}{r} \frac{\bar{e}^{-(1-\alpha_2)}}{(\alpha_1 - \alpha_2)} e^{\alpha_2} \quad e > \bar{e}$$

A.2. The Value of Option to Invest in the Plant and Optimal Stopping Point

Assume a firm is waiting for new information about the real exchange rate. While waiting to build, there are no cash flows. Therefore, expected capital gains on the opportunity to invest are equal to the risk-free return on an investment of equivalent value, that is

$$rFdt = E(dF) \quad (\text{A.16})$$

By using Ito's Lemma, dF is expressed as

$$dF = F_e de + \frac{1}{2} F_{ee} de^2 \quad (\text{A.17})$$

Substituting equation (3.2) for de and equation (3.3) for de^2 yields

$$dF = F_e(\sigma e dz) + \frac{1}{2} F_{ee}(\sigma^2 e^2 dt) \quad (\text{A.18})$$

Taking the expectations of equation (A.18) yield the following equation

$$E(dF) = \frac{1}{2} \sigma^2 e^2 F_{ee} dt \quad (\text{A.19})$$

Replacing $E(dF)$ in equation (A.16) with equation (A.19) gives

$$rF - \frac{1}{2} \sigma^2 e^2 F_{ee} = 0 \quad (\text{A.20})$$

Equation (A.20) is an ordinary differential equation, which is solved subject to boundary conditions (3.52), (3.53), and (3.54). The solution to equation (A.20) is given by

$$F(e^*) = b e^{\alpha_3} \quad e > e^* \quad (\text{A.21})$$

Taking the second derivative of equation (A.21) with respect to e and substituting into equation (A.20) yields the following equation

$$\left[\frac{\sigma^2}{2} \alpha_3 (\alpha_3 - 1) - r \right] b e^{\alpha_3} = 0 \quad (\text{A.22})$$

Boundary condition $V(e) = 0$ implies that $\alpha_3 < 0$. Hence, $\alpha_3 = \alpha_2$. Now the value of e^* and coefficient b are determined using the boundary conditions (3.53) and (3.54). At e^* these two conditions must meet

$$\frac{\lambda}{r} \frac{\bar{e}^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} e^{\alpha_1} + \frac{\lambda}{r} (\bar{e} - e^*) - I = b e^{*\alpha_2} \quad (\text{A.23})$$

$$\alpha_1 \left[\frac{\lambda}{r} \frac{\bar{e}^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} e^{*(\alpha_1-1)} \right] \left(-\frac{\lambda}{r} \right) + \frac{\lambda}{r} = \alpha_2 b e^{*(\alpha_2-1)} \quad (\text{A.24})$$

The coefficient b is determined from equation (A.24).

$$b = \frac{\lambda}{r} \left[\frac{\alpha_1}{\alpha_2} \frac{\bar{e}^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} e^{*(\alpha_1-\alpha_2)} - \frac{e^{*(1-\alpha_2)}}{\alpha_2} \right] \quad (\text{A.25})$$

Substituting (A.25) into (A.23) gives the following implicit equation that e^* must satisfy. This equation is as follows;

$$\frac{\lambda}{r} \frac{\bar{e}^{-(1-\alpha_1)}}{(\alpha_1 - \alpha_2)} \left(I - \frac{\alpha_1}{\alpha_2} \right) e^{*\alpha_1} + \frac{\lambda}{r} \left(\frac{I}{\alpha_2} - I \right) e^* + \frac{\lambda}{r} \bar{e} - I = 0 \quad (\text{A.26})$$

Finally, the relationship between the volatility and the optimal stopping point, e^* , is determined. What does the effect of volatility increase have on e^* ? Defining equation (A.26) as Ψ , I will prove that

$$\frac{de^*}{d\sigma^2} = -\frac{\Psi_{\sigma^2}}{\Psi_{e^*}} < 0 \quad (\text{A.27})$$

where Ψ_{σ^2} is the derivative of implicit function with respect to σ^2 and Ψ_{e^*} is the derivative of implicit function with respect to e^* . Ψ_{σ^2} and Ψ_{e^*} are derived to decide

if $-\Psi_{\sigma^2} / \Psi_e^* < 0$. Ψ_e^* is derived as

$$\Psi_e^* = \frac{\lambda}{r} \left[-\frac{\alpha_1}{\alpha_2} \left(\frac{\bar{e}}{e^*} \right)^{(1-\alpha_1)} + \frac{1}{\alpha_2} - 1 \right] > 0 \quad (\text{A.28})$$

when $\bar{e} = e^*$, this expression reaches its maximum. Using the definitions, $\alpha_1 = 1/2 + 1/2 (1 + 8r / \sigma^2)$ and $\alpha_2 = 1/2 - 1/2 (1 + 8r / \sigma^2)$, this implies that $\Psi_e^* = 0$. Ψ_{σ^2} is defined by chain differentiation

$$\Psi_{\sigma^2} = \Psi_{\alpha_1} \frac{d\alpha_1}{d\sigma^2} + \Psi_{\alpha_2} \frac{d\alpha_2}{d\sigma^2} \quad \text{where} \quad \frac{d\alpha_2}{d\sigma^2} = -\frac{d\alpha_1}{d\sigma^2} \quad (\text{A.29})$$

Taking the derivatives of equation (A.26) with respect to α_1 and α_2 yields the following equations

$$\Psi_{\alpha_2} = e^* \frac{\lambda}{r} \left(\frac{1}{\alpha_2} \right)^2 \left[\left(\frac{\bar{e}}{e^*} \right)^{(1-\alpha_1)} - 1 \right] \quad (\text{A.30})$$

$$\Psi_{\alpha_1} = \left(-\frac{\lambda}{r\alpha_2} \right) \left[-\ln \bar{e} + \ln e^* \right] e^{*\alpha_1} \bar{e}^{(1-\alpha_1)} \quad (\text{A.31})$$

It was demonstrated earlier that $\alpha_1 > 1$ and $\alpha_2 < 0$. Using these facts and inspection of (A.30) and (A.31) show that $\Psi_{\alpha_1} > 0$ and $\Psi_{\alpha_2} < 0$. Hence,

$$\left[-\Psi_{\alpha_1} + \Psi_{\alpha_2} \right] \frac{d\alpha_1}{d\sigma^2} = \Psi_{\sigma^2} \quad (\text{A.32})$$

It is proven that $\Psi_e^* > 0$ and $\Psi_{\sigma^2} > 0$. Therefore, the following equality holds.

$$\frac{de^*}{d\sigma^2} = -\frac{\Psi_{\sigma^2}}{\Psi_e^*} < 0 \quad (\text{A.33})$$

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