Robust Targeting Rules for Monetary Policy *

A. Hakan Kara†

November 2002

(First Draft: May 2000)

Abstract

This paper explores robust optimal targeting rules in a standard forward looking model when i) policy maker has doubts about the parameters while private agents know the model and ii) policy maker and the private sector share the same doubts. It is shown that, while the robust optimal policy rule are the same in both cases, private sector's behavior and hence the resulting equilibrium is different. Two different sources of parameter uncertainty are considered. When the agents' doubts take the form of uncertainty about the slope of the Phillips curve, robust policy rule prescribes a less aggressive response to deviations of inflation from the target—contrary to the recent findings in the literature. On the other hand, if the source of uncertainty is imperfect knowledge of persistence of shocks, robust monetary policy calls for a more aggressive response to inflation.

Keywords: Optimal Monetary Policy, Targeting Rules, Knightian Uncertainty, Robust Control

JEL Classification: E52, E58, D89.

---

*This paper is a version of the first chapter of my dissertation at NYU. I wish to thank Mark Gertler for directing me to this literature and providing many useful suggestions and to Marc Giannoni (who also inspired this study) for continuous help and advice. I would like to thank also William Baumol and Boyan Jovanovic for their encouragement and guidance and to Pierpaolo Beningo, Jordi Galí, André Minella and Emre Özdenören for useful discussions. Needless to say, all errors are mine. Financial support from NYU and the Central Bank of Turkey is gratefully acknowledged.

†Research Department, Central Bank of Turkey. The views expressed are those of the author and not necessarily those of the Central Bank of Turkey. Contact information: İstiklal Cad. No:10 Ulus 06100 Ankara Turkey hakan.kara@tcmb.gov.tr.

1 Introduction

How to conduct monetary policy under model uncertainty has always been an interest to both academicians and practitioners. One of the early studies in this area was Brainard (1967) in which he showed that in the presence of parameter uncertainty, it is often optimal for the central bank to act less vigorously than would be optimal if all parameters were known. This result has found wide acceptance among both academia and central bankers.[1] Recently, there has been a growing body of literature challenging this idea, among which are Sargent (1999), Hansen and Sargent (1999a,b,2000b), Tetlow and Von Zur Muehlen (2000), Stock (1999), Onatski and Stock (2000) and Giannoni (2000a,c). In contrast to standard Bayesian approach followed by, e.g., Brainard (1967), Chow (1975), Rudebush (1998), Clarida Galí and Gertler (1999), these authors assumed that the policy maker has multiple priors about probability distribution of the true model. By applying robust control methods borrowed from engineering literature, they have found that robust monetary policy under uncertainty in general calls for a stronger response of the interest rates to fluctuations in inflation and the output gap than in the absence of uncertainty. These findings weakened the view that parameter uncertainty can account for the less aggressive nature of estimated policy reaction functions than the ones prescribed by the theoretical models.

Using a standard New Keynesian Model, this study evaluates analytically, whether, or under which circumstances, it is possible to reverse the recent findings in the robust monetary policy literature. In other words, we attempt to answer if parameter uncertainty with robust decision makers can justify the so called “cautious” behavior of central banks. It is shown that, under a plausible interpretation of the uncertainty about aggregate supply relation, robust monetary policy rule prescribes a less aggressive policy than the case of known parameters, in the sense that the former requires moving the policy instrument less in response to deviations of inflation from the target.

[1] It has “almost” been a stylized fact. Blinder (1998) explains the ”Brainard Priciple” as calculating the theoretical optimal value of the desired change in the instrument (nominal interest rate in our framework) and doing less. In his intermediate macroeconomics textbook, Blanchard (2000) explains self-restrained behavior of policymakers as a byproduct of acknowledging model uncertainty.

Recent literature on robust monetary policy under structural uncertainty often assume that the policy maker faces uncertainty, while the private agents know the model.[2] We attempt to go one step further, and characterize the robust optimal monetary policy when *all* agents confront model uncertainty. We argue that under a certain interpretation of the policy maker's belief about the private agent's view, robust monetary policy rules are identical to the ones under one sided case.

This study builds on the work of Giannoni (2000a,c) and partly on Hansen and Sargent (2000b, 2001). Unlike Sargent and Hansen, but like Giannoni we consider parametric uncertainty and derive simple policy rules. The paper differs from Giannoni's work in two ways. First, we characterize *optimal linear robust targeting rules* as proposed by Svensson (2001a,b), rather than instrument rules. Second, we also consider the situation when both the private agents and the central bank may face model uncertainty.[3]

We prefer to use a simple framework to allow for an intuitive and analytically tractable solution. Following Giannoni (2000a) and Onatski (2000) among others, we begin with an environment in which the policymaker faces uncertainty while the private sector does not. This assumption can be argued to be reasonable when both policy maker and private sector has the same approximate model while the former has much more doubts than the latter about the quality of this approximation.

Next, following Hansen and Sargent (2000b), we seek to characterize robust decision rules when both the policy maker and the forward looking agents face uncertainty about the model. We show that as long as the policymaker and the private agents share the same uncertainty sets, robust monetary policy rule stays exactly the same as the case when only the policy maker confronts uncertainty. This result is remarkable, given that all of the recent studies on the robust policy rules which consider structured uncertainty in forward looking models assume that private agents know the model. For our finding

[2] Exceptions are Hansen and Sargent (2000b) and Kasa (2000). However, these authors consider additive and unstructured uncertainty.

[3] Another difference is that, we consider an objective with output gap and inflation stabilization, while Giannoni (2000a,c) characterizes the solution using an objective function that incorporates interest rate stabilization in addition to inflation and output stabilization.

suggests that the robust policy rules are invariant to whether one assumes the private agents face parameter uncertainty or not. Therefore, the results we state below are valid for both cases.

We conduct two basic exercises to assess the implications of uncertainty regarding New Keynesian Phillips curve. Under uncertainty about the slope of the Phillips curve, we find, in contrast to the recent literature, that robust monetary policy may require less aggressive response to inflation than in the absence of uncertainty. In fact, parameter uncertainty affects the short run trade-off between output gap and inflation by rendering inflation stabilization more costly. In that sense, the policy can be interpreted as less conservative.[4] This is because the optimal rule — which is designed to perform well in those instances in which shocks have large effects on goal variables — requires the central bank to act as if the policy instrument is less effective than when the parameters are known.

On the other hand, if the uncertainty is mostly about the persistence of inefficient supply shocks, robust optimal policy prescribes more aggressive policy in response to inflation than the case of known parameters. This is also intuitive, since monetary authority will try to avoid an especially poor performance against worst possible case — equivalent to being exposed to highly persistent shocks in this exercise. As a consequence, the central bank will move its policy instrument more vigorously in order to "lean against a *stronger* wind". Namely, it will be robustly optimal to exploit the forward looking expectations by committing to a more aggressive rule.

The rest of the paper is organized as follows. Section 2 reviews the baseline model and characterizes the solution in the case of known parameters. Section 3 explains how the objective of monetary policy changes when the policy maker faces uncertainty about the true model and summarizes the solution method. Section 4 presents two applications of this method under parameter uncertainty of the type explained above and assesses the implications of robust rules. Section 5 takes up the robustness problem when all decision makers confronts uncertainty, and shows the equivalence of robust

[4] I define conservatism as in Rogoff (1985). A higher degree of conservatism means increased concern for inflation stabilization relative to output stabilization.

policy rule with the previous case. Section 6 concludes.

2 Monetary Policy with Known Parameters

We first describe the structural model, and monetary policy problem that the central bank faces. Next, we characterize the optimal policy with known parameters.

2.1 The Model

The baseline framework is a standard forward-looking model in the exact form used by Clarida, Galí and Gertler (1999). It is similar to other models that have been used in recent studies of monetary policy such as Bernanke and Woodford (1997), Woodford (1999a, 1999b, 1999c, 2001), Svensson and Woodford (1999) and Giannoni (2000a, 2000b, 2000c), Aoki (2000), Svensson(2001a), among others. The model consists of two structural equations that are derived from optimizing behavior of private sector: An aggregate supply equation derived form a first order condition for optimal price setting by the representative supplier and an IS curve derived form an Euler equation for the optimal timing of purchases.

The New-Keynesian aggregate-supply equation (AS) takes the form

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \tag{1}$$

where π_t is the period t inflation rate defined as the percent change in the price level from $t-1$ to t , x_t is the output gap which is defined as the percentage by which output exceeds its potential, $0 < \beta < 1$ is a discount factor, κ is a positive coefficient and u_t is an exogenous disturbance term. We use the notation $E_t\pi_{t+1}$ to denote private sector expectations regarding of π_{t+1}conditional on information available in period t. Equation (1) relates inflation to output gap in the spirit of a traditional Phillips curve. In contrast to traditional Phillips curve, current inflation depends on the expected future course of the economy, and thus on the expectations of future monetary policy, because firms set prices based on expected marginal costs. The parameter κ can be interpreted as a measure of the speed of the price adjustment. Output gap (x_t) captures the marginal costs associated with excess demand. This specification allows for a shock

u_t, which shifts the distance between the potential output and the level of output that would be consistent with zero inflation[5] . These shifts are not considered to represent variation in potential output, and thus appear as a residual in (1).We will name u_t simply as the "supply shock"[6] . Within the framework, monetary policy affects real economy, because sellers cannot change their price every period as in Calvo (1983), and Yun (1996).

The aggregate demand (IS) equation takes the form

$$x_t = -\varphi \left[i_t - E_t \pi_{t+1} \right] + E_t x_{t+1} + g_t, \tag{2}$$

where i_t is the central banks instrument which is a short term nominal interest rate, φ is a positive coefficient (the intertemporal elasticity of substitution), and g_t is an exogenous disturbance. Deviations of output from the potential output depends upon real interest rate, expected future output gap and a demand shock. Thus, output gap also depends upon expected paths of real rate and the demand shock. The shock g_t can be interpreted as an exogenous variation in autonomous expenditure.

These structural equations can be derived as log-linear approximations to equilibrium conditions of a simple dynamic general equilibrium model in which the infinitely lived representative household maximizes its lifetime utility. Disturbance terms g_t and u_t follow AR(1) processes so that persistence in inflation and output is due to serially correlated exogenous shocks:

$$u_t = \rho u_{t-1} + \varepsilon_{ut} \tag{3a}$$

$$g_t = \rho g_{t-1} + \varepsilon_{gt} \tag{3b}$$

where $0 \leq \rho \leq 1$ and ε_{it} are i.i.d. zero mean random variables with standard deviations σ_{it} for $i = u, g$. The two structural equations (1) and (2) together with a policy rule determine the equilibrium evolution of endogenous variables π_t, x_t and i_t.

[5] An example would be a variation in the markup over the wholesale prices.

[6] In the literature u_t is generally named as "cost push shock" (see Clarida, Gali and Gertler (1999). Giannoni (2000b), in a similar framework, justifies the presence of a shock term in the AS equation, using the microfoundations and by assuming that policymaker aims at stabilizing output around some efficient level – the level of output that would prevail under fexible prices and no market power. He calls this shock "inefficient supply shock".

In this section we assume that the private sector and the central bank have the same information and that both know the parameters of the model, the current realization and persistence of shocks in period t and have the same information about the future evolution of the exogenous disturbances.

2.2 Objective of the Policymaker

Traditionally, researchers have assumed that objective of monetary policy is to minimize a weighted average of variability of output gap and inflation.[7] Accordingly, we assume the following loss criterion:

$$L_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2 \right] \tag{4}$$

where λ is the relative weight assigned to output stabilization. Woodford (1999b) shows that a similar loss function can be obtained by performing a second order Taylor approximation to the expected utility of the representative household in the model that has been used to derive the structural equations. Woodford also shows that the parameter λ is a function of the parameters of the structural model. However, we will assume that λ is independent of the structural parameters and the central bank chooses a λ that is compatible with the private agents' preferences. The objective of monetary policy in the case of known parameters is to choose a linear targeting rule to implement the equilibrium variables that minimize L_t.

2.3 Policy Rule

Any prescribed guide for monetary policy conduct is a policy rule as defined in Svensson (1999). Throughout this study, we focus on a special type of policy rule which Svensson and Woodford (1999) call "specific targeting rules" which is expressed as a direct condition for target variables (endogenous variables that enter the loss function). These kind of policy rules are argued to have advantages over the general targeting rule (a high level specification of monetary policy rule that specifies the target variables, target

[7] See Walsh, 1998, chap. 8; Woodford, (1999a,b), Clarida Gali and Gertler (1999), and Svensson 2001(b) for a recent discussion on this kind of objective function.

levels and the loss function) on the grounds of higher efficiency in communicating with the public. On the other hand, as argued by Svensson (2001a, 2001b), specific targeting rules are more robust to changing structure of the economy than the instrument rules such as Taylor rules.

Specifically, we assume that the policymaker commits credibly at the beginning of period t to a policy rule of the form

$$x_t = -\psi \pi_t. \tag{5}$$

As we mentioned above, this policy rule does not appear to be in the form of a Taylor rule usually proposed in the literature, though as shown below, it implies a reaction function in which monetary authority responds to deviations of inflation from its target level as well as to exogenous disturbances. Nevertheless, this reaction function should not be mixed with the targeting rule 5 itself since the implied instrument rule will change with the specification of the IS curve while the rule itself is robust to such changes.

As it can be inferred from the specification rule, we restrict attention to non-inertial policy rules in which only the current target variables matter. This specification cannot deliver the globally optimum equilibrium processes since the equilibrium policy instrument is not "history dependent", as is explained by Woodford (1999b, 1999c). Woodford argues that policymakers who disregard the past states and past promises cannot achieve the fully optimal solution. However, for the purpose of this study, it is sufficient to consider non-inertial rules, since our ultimate goal is to characterize under which circumstances and how the Knightian uncertainty implies a less aggressive behavior of monetary authority, rather than assessing the gains from a fully optimal solution.

The policymaker's problem is to choose a $\psi \in \Psi$ to minimize the loss L_t,subject to the structural equations (1) and (2). We denote the vector of endogenous variables by $q_t = [\pi_t, x_t, i_t]$ and write q as the stochastic process $\{q_t\}_{t=0}^{\infty}$. To be feasible q needs to satisfy structural equations (1), (2) and the policy rule (6) at all dates t. Let $\theta \in \Theta$ be a parameter of the model where $\Theta \subset \Re^+$ is a one dimensional compact set (later in the text we will consider uncertainty about θ). Then a rational expectations equilibrium can be defined as follows.

Definition 1 *A* **rational expectations equilibrium** *is a stochastic process* $q(\psi, \theta) = [\pi_t, x_t, i_t]$ *satisfying (1),(2) and (6).*

We restrict our attention to the set of policy rules that result in a unique bounded expectations equilibrium. We use Ψ to denote such a set . The policy rule that is optimal relative to the subset of rules Ψ can in turn be defined as follows.

Definition 2 *In the case of known structural parameter* θ, *the optimal linear monetary policy rule is a coefficient* ψ^0 *that solves*

$$\min_{\psi \in \Psi} L_t(q(\psi, \theta)) \tag{6}$$

where L_t *is the loss function defined in (5).*

Now we shall characterize the optimal policy rule when there is no doubt about the model.

2.4 Optimal Equilibrium Process with Known Parameters

In our model we can treat the output gap as the control variable. Accordingly, the optimization problem of the central bank can be solved in two stages. The first stage of the solution is to find the output gap and the inflation processes that satisfy the structural equation (2) and minimize the loss function, assuming that monetary authority chooses a linear policy rule of form (6). Since we restrict ourselves to the class of ψ that yields a bounded and unique equilibrium, the solution to the first stage of the problem is of the form

$$\pi_t = f_\pi u_t, \qquad x_t = f_x u_t \tag{7}$$

where the vector $f = [f_\pi, f_x]$ is the vector of response coefficients that parametrize the equilibrium process of inflation and output gap. Feasibility restriction on the response coefficients, obtained by substituting (8) into the aggregate supply equation is

$$f_\pi = \kappa f_x + \beta \rho f_\pi + 1. \tag{8}$$

To solve the policymaker's problem, we choose a plan of the form (8) and consistent with (9) to minimize the loss criterion L_t. The response coefficients parametrizing the optimal feasible equilibrium for a given parameter θ is given by

$$f_x = -\frac{\kappa}{\lambda(1-\beta\rho)^2 + \kappa^2}, \tag{9}$$

$$f_\pi = \frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2 + \kappa^2}. \tag{10}$$

These equations imply a relation between inflation and output gap in the form as

$$x_t = -\frac{\kappa}{\lambda(1-\beta\rho)}\pi_t, \tag{11}$$

which can also be regarded as our specific targeting rule. The central bank in turn commits to adjust the policy instrument to satisfy this condition at every period. As pointed out by Clarida Galí and Gertler (1999), this condition can be interpreted as "lean against the wind" policy. Whenever inflation is above target, contract demand below capacity (by raising the interest rate) and *vice-versa* when it is below target. How aggressively the central bank should reduce x_t depends positively on the gain in reduced inflation per unit of output loss κ and inversely on the relative weight placed on output loss κ. The term $(1 - \beta\rho)$ reflects the forward looking behavior of the private sector. The more persistent is the supply shock (i.e., the closer is ρ to 1) the more aggressively the central bank contracts output, the policy instrument, in face of inflationary pressures. Since a forward looking private sector will expect inflationary pressures to persist, monetary authority prefers to commit to a more contractionary policy to lower inflationary expectations thus the impact of shock to current inflation.[8]

The second stage of the problem is, using equation (1), to choose the interest rate process to implement the optimal values of inflation and output gap given by equations (10) and (11). This provides us with a relation between nominal interest rate, inflation

[8] In the next sections we will consider the case where central bank is uncertain about true values of λ and ρ, that is why it is particularly important to understand the role of these parameters for the optimality conditions.

the demand shock. Using (1) and (12), the *implied* optimal instrument rule[9] can be written as

$$i_t = \gamma_\pi \pi_t + \frac{1}{\varphi} g_t = (\rho + \frac{(1-\rho)\kappa}{\varphi\lambda(1-\beta\rho)})\pi_t + \frac{1}{\varphi} g_t \approx (\rho + \frac{\kappa}{\varphi\lambda})\pi_t + \frac{1}{\varphi} g_t, \qquad (12)$$

where the approximation is obtained by setting $\beta = 1$. Monetary authority can implement the optimality condition (12) by setting the nominal rate instrument in line with equation (13). Magnitude of the optimal response of the instrument to fluctuations in inflation (γ_π) depends positively on the persistence of cost push shocks (ρ) and slope of the Phillips curve (κ). This result will help us later to assess the implications of uncertainty about these two parameters.

3 Robust Optimal Monetary Policy

The previous section derived the optimal policy when the parameters of the model are constant and known to both the private sector and policymakers. Also the exact lag structure and persistence of exogenous disturbances are supposed to be known. In reality, central banks and researchers do not know the parameters of their models with certainty. To be specific, we will consider two types of uncertainty within the model. The first is uncertainty about the slope of the Phillips curve, κ, which can be interpreted as uncertainty about the effectiveness of policy instrument, x_{t} in our framework. For this parameter reflects the induced change on the inflation through a reduction of output gap by one percent. The second type of uncertainty we consider is imperfect knowledge about the persistence of shocks, ρ.[10] We will analyze the effects of uncertainty about these two parameters on optimal policy.

The underlying framework we have in mind is one of the models mentioned above, except that the representative household can be one of several different types. (A type in our framework means, a specific value of κ in a given set.) We assume that the type of the household is determined once and for all in period 0; the household knows its type

[9] Instrument rules are also called "policy reaction function" in some studies.

[10] In a backward looking framework this corresponds to uncertainty about the inertia of inflation.

but the central bank does not. For simplicity and to obtain a clean analytical solution, we will suppose that the weight λ that characterizes the policymaker's preferences and the slope of the intertemporal IS curve φ is known to the policymaker.

3.1 Objective of Monetary Policy with Uncertain Parameters

We assume that central bank commits credibly to a linear policy rule at the beginning of period 0. Suppose the policymaker does not revise the rule at later dates using additional information it may have collected about unknown model parameters[11] . Let θ represent an uncertain parameter in the model. We assume the θ lies in a given (known) compact set $\Theta \in \Re$ and that the distribution of θ is unknown. We let the policymaker have multiple priors over Θ including the possibility of that any given element of $\theta \in \Theta$ holds with certainty. We assume uncertainty aversion by the central bank in the sense axiomatized by Gilboa and Schmeidler (1989). In this case policymaker's problem turns out to be maximization of the worst possible case, i.e., the situation when the prior distribution is the worst distribution in the set of possible distributions[12] . The optimal policy rule is then the robust rule defined as follows.

Definition 3 *In the case of parameter uncertainty, a robust optimal monetary policy rule is a vector ψ^* that solves*

$$\min_{\psi\in\Psi} \left\{ \max_{\theta\in\Theta} E[L_t(q(\psi,\theta))] \right\} . \tag{13}$$

Given that the unknown parameter is in Θ, the policymaker can guarantee that the loss is no higher than the one obtained in the following minmax equilibrium.

Definition 4 *A* minmax equilibrium *is a bounded rational expectations equilibrium $q^* = q(\psi^*, \theta^*)$, where is ψ^* $\epsilon\Psi$ is a robust optimal monetary policy rule and θ^* maximizes the loss on the constraint set Θ.*

However the equilibrium that is actually realized depends upon the true value of θ and thus is unknown to the policy maker. The objective defined in definition (3) is

[11] That means, as in Gianonni (2000a, 2000c), we shall restrict our attention to families of rules that involve no learning.

[12] See Hansen and Sargent (2000a) for a detailed discussion of this objective.

consistent with the robust control approach inspired by the engineering literature, and proposed recently by Hansen and Sargent (1999a, 1999b), Sargent (1999), Stock (1999), and Onatski and Stock (2000). It incorporates extremely cautious behavior by the central bank, as the policymaker cares only about the worst parameter configuration.

3.2 Solution Method for the Robust Optimal Policy

In this part we explain briefly the solution method formulated by Giannoni (2000a). The method is based on the relation between the solution to problem (14) and the equilibrium of a zero sum game. The game is defined as $\Gamma = \langle \{P, N\}, (\Psi, \Theta), (-L(\psi, \theta), L(\psi, \theta)) \rangle$. In this game, the policymaker (P) chooses the policy rule $\psi^* \in \Psi$ to minimize his loss, $L(\psi, \theta)$, assuming that a malevolent Nature will try to hurt him as much as possible. The other player, Nature *(N),* chooses parameter(s) $\theta^* \in \Theta$, to maximize the policy maker's loss, as though knowing that the policymaker is going to minimize it. The solution procedure is characterized in four steps as follows:.

1. Find the parametrization of the equilibrium process and the implied optimal rule $\psi(\theta)$ under known parameters, taking θ as given, i.e., solve the two stage problem that is explained above.

2. Determine the *candidate* worst parameter vector θ^* by solving

$$\max_{\theta \epsilon \Theta} E[L_t(q(\psi(\theta), \theta))] \tag{14}$$

and find the equilibrium process $q(\psi^*(\theta^*), \theta^*)$.

3. Look for a policy rule ψ^* that implements the minmax equilibrium.

4. Verify that (ψ^*, θ^*) is a *global* Nash equilibrium by checking that the solution candidate θ^* maximizes the loss $L(\psi^*, \theta^*)$ on the constraint set Θ, i.e., there is no vector other θ' satisfying

$$L(\psi^*, \theta') > L(\psi^*, \theta^*)$$

4 Applications of Robust Monetary Policy

Note that 11 does not depend on the specification of aggregate demand (IS) relation. Thus the formulation of the targeting rule is fairly robust to changes in the aggregate demand behavior. However, the parameters of the aggregate supply equation appear in the policy rule. Therefore a robustness investigation under parameter uncertainty should involve the parameters in AS relation.[13] Accordingly, in this section, we characterize the robust optimal monetary policy for different specifications about the aggregate supply. To keep the analysis intuitive and tractable, we consider one type of parameter uncertainty at a time. First we assume that the central bank faces uncertainty about the slope of the Phillips curve, and second, we assume that the policy maker is uncertain about the persistence of supply shocks.

4.1 Uncertainty About the Slope of the Phillips Curve

We seek to determine the optimal value of coefficient ψ in the model of section 3, assuming that the structural parameter κ is known to be in some given interval $[\kappa_L, \kappa_H]$,where $0 < \kappa_L < \kappa_H < \infty$. For simplicity we assume all the other parameters are known with certainty. For the beginning we assume private sector has no doubts on the model. This class of uncertainty allows a simple analytical characterization of robust optimal policy. Later in the text we shall consider a more complex informational structure in which policy maker and the private sector share similar doubts.

Optimum equilibrium processes and the policy rule are already derived in section 2 for the case of known parameters. Therefore we proceed from step 2 of the solution strategy mentioned above. We characterize the minmax equilibrium process by determining the structural parameters that obtain in the equilibrium. The next step is to look for a policy rule that implements the equilibrium. Throughout the text we restrict our attention to linear rules of the type $x_t = -\psi\pi_t$. However it is important to find the implications of this condition for the behavior of policy instrument i_t. Thus whenever possible, we shall seek to characterize the corresponding policy reaction function.[14]

[13] Note that our specification of targeting rules in the absence of uncertainty are not robust in the sense of Giannoni and Woodford (2002), since the targeting condition depends on the characteristics of the shocks, namely, ρ.

[14] By policy reaction function we mean expressing the policy instrument (nominal rates in our framework) in terms of linear functions of current or lagged observable variables such as inflation, outputgap or observable shocks.

Substituting the optimum equilibrium processes characterized by equations (8),(10) and (11) in the loss function yields

$$L(\psi, \kappa) = \frac{\lambda \sigma_u^2}{(\lambda(1-\beta\rho)^2 + \kappa^2)(1-\rho^2)}$$

Since this function is decreasing in κ, the worst possible parameter configuration for the policy maker is the case when slope of the Phillips curve takes the least possible value (κ_L) in the parameter set. Thus, the solution procedure yields the following relations:

$$\kappa^* = \kappa_L \tag{15}$$

$$x_t^* = -\frac{\kappa_L}{\lambda(1-\beta\rho)^2 + \kappa_L^2} u_t \tag{16}$$

$$\pi_t^* = \frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2 + \kappa_L^2} u_t. \tag{17}$$

Given (17) and (18), it is clear that the monetary authority can implement the candidate robust equilibrium by adhering to a targeting rule such as

$$x_t = -\frac{\kappa_L}{\lambda(1-\beta\rho)} \pi_t. \tag{18}$$

Hence, the *candidate* minmax equilibrium can be characterized by $(\kappa^*, \psi^*) = (\kappa_L, \frac{\kappa_L}{\lambda(1-\beta\rho)})$. As a last step we need to verify if these parameters indeed constitute a global Nash equilibrium. Using (19) and (2), loss function for any parameter configuration under robust targeting rule is given by

$$L(\psi^*, \kappa) = \frac{\lambda^2(1-\beta\rho)^2 + \lambda\kappa_L^2}{(\lambda(1-\beta\rho)^2 + \kappa\kappa_L)^2} \sigma_u^2. \tag{19}$$

Since (20) is monotonically increasing in κ, nature still chooses $\kappa^* = \kappa_L$ in order to maximize this loss function. Thus we conclude that the targeting rule (19) is the robust optimal rule and (κ^*, ψ^*) is a global Nash equilibrium. On the other hand, since $\psi^* > 0$, our solution is unique and bounded.

Equation (19) is the robust equivalent of condition (12). At the minmax equilibrium, it is optimal for the central bank to engineer a smaller reduction in output in response to an increase in inflation. This result is intuitive given that central bank is trying to minimize the welfare loss in the worst possible scenario which is the case that κ takes the lowest possible value in the parameter set $[\kappa_L, \kappa_H]$. A low κ means a worsened output inflation trade-off hence a reduction of the gain in inflation per unit of output loss. Thus the monetary authority is more reluctant to contract output in the face inflationary pressures; in other words, it is optimal for the policy maker to respond *less* to deviation of inflation from the target than in the case with known parameters.

Note that this policy behavior can be replicated in the absence of uncertainty by assigning a greater weight ($\lambda\frac{\kappa_0}{\kappa_L}$ instead of λ) to output stabilization than socially optimal case. In other words, appointing less conservative central banker of the right degree in the sense of Rogoff (1985) who knows exactly the true parameters, may look *behaviorally* equivalent to robust policy under uncertainty.[15] Required decline in conservatism depends critically on the relative size of the true parameter and the lower bound of the feasible set, namely $\frac{\kappa_0}{\kappa_L}$.

Using equation (1) and the robust optimality condition, it is possible to characterize the corresponding instrument rule as

$$i_t = (\rho + \frac{(1-\rho)\kappa_L}{\varphi\lambda(1-\beta\rho)})\pi_t + \frac{1}{\varphi}g_t \approx (\rho + \frac{\kappa_L}{\varphi\lambda})\pi_t + \frac{1}{\varphi}g_t. \tag{20}$$

A comparison of this rule with condition (13) reveals that our simple interpretation of parameter uncertainty within a robust optimal control theory framework confirms Brainard's (1967) principle: policy instrument is *less* responsive to inflation under parameter uncertainty. This result, we believe, contrasts with the recent literature on robust control theory which predicts *a more aggressive response* of the policy instrument in the face of Knightian uncertainty.[16]

4.2 Uncertainty About Persistence Parameters

In this section, we characterize minmax equilibrium and robust policy rule when central bank is uncertain about the persistence of the inefficient supply shock, ρ. This type

[15] However, the exact equilibrium will be different if the private sector also faces uncertainty.

[16] Hansen and Sargent (2000b) find also less aggressive policy reaction in response to demand shocks using a similar model. But their results depend on the assumtion that potential output is not observed and agents has to filter information. Besides, they assume additive unstructured uncertainty and use numerical methods.

of uncertainty is of particular interest, because the New Keynesian Phillips curve in our model is purely forward looking, and the family of targeting rules we consider do not bring any extra inertia than that is implied by the dynamics of structural model. Therefore, the only persistence stems from autocorrelated shocks. In that sense, uncertainty about ρ represents not only a mere autocorrelation parameter of aggregate shocks but also a measure for rate of convergence to the steady state.

We will assume that private sector knows the persistence of supply shocks, while the policymaker thinks that the persistence parameter ρ lies in some given interval $[\rho_L, \rho_H]$, where $0 < \rho_L < \rho_H < \infty$.[17] Variance of the shocks is known by both parties.[18] Following the solution method of the previous subsection, it is possible to write the optimal loss under any persistence parameter ρ as

$$L(\psi, \kappa) = \frac{\lambda}{(\lambda(1-\beta\rho)^2 + \kappa^2)} Var(u_t).$$

Given any variance, this function is strictly increasing in ρ. Therefore, the worst case parameter can be characterized as

$$\rho^* = \rho_H. \tag{21}$$

In other words, central bank will act as if the fictitious evil Nature will choose the highest possible ρ in the parameter set. The candidate minmax equilibrium is given by

$$x_t^* = -\frac{\kappa}{\lambda(1-\beta\rho_H)^2 + \kappa^2} u_t, \tag{22}$$

and

$$\pi_t^* = \frac{\lambda(1-\beta\rho_H)}{\lambda(1-\beta\rho_H)^2 + \kappa^2} u_t, \tag{23}$$

---

[17] As explained below, if we extend the problem to two sided uncertainty in the sprit of Hansen and Sargent (2001), the results remain intact.

[18] To see the lojgic of this assumption, suppose for example, that, the parameter uncertainty set for ρ is [0.1,0.9]. This implies a variance between $10\sigma_{ut}^2$ and $1.1\sigma_{ut}^2$ which is far wider than the plausible range the central banks face in practice. As a consequence, solving for the worst case parameter will be trivial (it will be the highest possible ρ in the set) which may be misleading. In what follows, to isolate the variance uncertainty from the persistence uncertainty, we will assume that the variance $\frac{\sigma_{ut}^2}{1-\rho}$ is known.

which can be implemented by a targeting rule such as

$$x_t = -\frac{\kappa}{\lambda(1-\beta\rho_H)}\pi_t. \tag{24}$$

What remains as the last step is to show if the candidate robust equilibrium is indeed a globally optimum solution. This can easily verified by noting that

$$\arg\max L(\psi^*, \rho) = \arg \max_{\rho\in[\rho_L,\rho_H]} \frac{\lambda^2(1-\beta\rho_H)^2 + \lambda\kappa^2}{\lambda(1-\beta\rho_H)(1-\beta\rho)+\kappa^2} = \rho. \tag{25}$$

Although the minmax rule takes the same form as the previous case, the behavioral implications for the policy are just the opposite: a robust central bank will engineer a greater reduction in output in response to an increase in inflation. Given that the robust policy maker — who minimizes the loss in the worst possible case — acts as if $\rho = \rho_H$, this result is intuitive. For higher ρ means higher inflationary expectations, and since all agents are purely forward looking, the policy maker finds it optimal to respond more to fluctuations in inflation than with known parameters in order to exploit the gains from commitment. By doing so, central bank faces an improved output-inflation trade-off conditional on the minmax equilibrium.

Using (1), implied instrument rule that implements the minmax equilibrium is given by

$$i_t = (\rho_H + \frac{(1-\rho_H)\kappa}{\varphi\lambda(1-\beta\rho_H)})\pi_t + \frac{1}{\varphi}g_t \approx (\rho_H + \frac{\kappa}{\varphi\lambda})\pi_t + \frac{1}{\varphi}g_t. \tag{26}$$

The instrument rule states that the central bank will act more aggressively to counteract inflationary pressures under uncertainty about the persistence of shocks. A comparison of (26) with its counterpart in the case of known parameters reflects the increased response of the central bank to inflationary pressures; the response coefficient is unambiguously higher than the one in the absence of uncertainty, since $\rho_H + \frac{\kappa}{\varphi\lambda} > \rho + \frac{\kappa}{\varphi\lambda}$.

To summarize the section, if we assume that the policy maker faces Knightian uncertainty about the persistence parameter, robust optimal behavior may require reacting more strongly to fluctuations in inflation than in the absence of uncertainty. This result is in line with the recent studies of robust monetary policy.

5 Simple Targeting Rules When all Agents are Robust Decision Makers

In the previous sections, we assumed that the private sector knows the true parameters and thus it is only the policymaker who confronts parameter uncertainty. Since our model has been derived from micro foundations, and the parameters of the model are functions of the behavioral coefficients, it seems natural at first sight to treat the agents that are being modeled as knowing their own behavioral parameters. Nevertheless, there is at least one strong theoretical reason why the structural model represented by (1) and (2) could be an approximation to the private agents as well.

Theoretical foundations of the aggregate supply side of the model imply a behavioral relation between inflation and real marginal cost, rather than output gap variable. Underlying the expectational Phillips curve (2) is a tight positive contemporaneous relation between real marginal costs and the output gap that is exogenous to the behavior of the individual price setter.[19]

Specifically, certain assumptions on technology, preferences, and the structure of labor markets are embedded in the derivation of (AS) curve (2) so that the relationship $\mathsf{mc}_t = \omega x_t$ holds. Indeed, Galí and Gertler (1999) argue, however, that this link is weak and cannot be significantly justified by the data. This argument suggests that, from the representative agent's perspective, uncertainty about ω can manifest itself as uncertainty about the slope of the Phillips curve, κ. Therefore, if the goal is to investigate the implications of uncertainty for the monetary policy, one can — and must — take into account the fact that the contemporaneous effect of output gap on inflation is likely to be uncertain to the private agents as well as to the central bank. That is, κ is uncertain to both parties.

[19] The New Keynesian Phillips curve implied by the microfoundations takes the form of $\pi_t = \delta \mathsf{mc}_t + \beta \mathsf{E}_t \pi_{t+1} + \mathsf{u}_t$ where mc_t denotes the deviation of real marginal cost from its steady state value. It is reasonable to assume that the private agents know the parameters of this equation, since it is their own decision that determines the parameters. For example, to assume uncertainty about the frequency of price adjustments (or the so called Calvo (1983) parameter) does not make much sense, given that in reality agents can control the frequency with which they set their prices. Thus if we want to impute doubts to representative agent, we need to justify an exogenous source of uncertainty.

Accordingly, in this section, we aim to explore the monetary policy when both the policymaker and the private agents confront model uncertainty in making forecasts and designing policies. We will refer to this approach as *two sided robustness* whereas the case when only the policymaker confronts parameter uncertainty will be referred as *one sided robustness*.

5.1 Information Structure and Objectives of Decision Makers Revisited

In line with the previous sections, we assume that uncertainty is structured and takes the form of parameter uncertainty. We further assume that the policy maker and the private sector share a common approximating model, $\mathrm{S}(\theta^0)$ and that surrounding the approximating model is a set of models parametrized as $\{\mathrm{S}(\theta) |\ \theta \in \Theta\}$. In our framework, $\mathrm{S}(\theta)$ corresponds to equations (1) and (2). Θ is the parameter set against which the agents plan robust behavior. For instance, if the agents are uncertain about the slope of the Phillips curve, κ, and κ can lie anywhere in the set $[\kappa_L, \kappa_H]$, this corresponds to $\theta = \kappa$ and $\Theta =[\kappa_L, \kappa_H]$. We impute a common objective to the private agents and the policy maker. This assumption is reasonable given that the objective function defined in (5) can be derived as a quadratic approximation to the utility based welfare function of the households in our model.[20]

Once again, the policy maker commits to a linear rule ψ of the form (6). Both the private agents and the policymaker are uncertain about the slope of the Phillips curve, κ. As in the one sided robustness case, the output gap, x_t, can be treated as the policy instrument of the central bank and equation (1) remains irrelevant to our analysis of robust decisions. What is different as opposed to the section (3) is the behavior of the private agents, or equivalently, the way we define the central bank's theory about the private sector's expectation formation.

The policy maker believes that the private agents do not know the correct model, that they share the approximating model with the central bank, and that they form expectations by conducting a pure forecasting method using the *same* slanted model

[20] See Woodford (1999a) and Rotemberg and Woodford (1998).

that the central bank does. The policy maker's constraints, hence, depends on its own vision of the private sector decisions. When setting the policy, monetary authority has to take into account not only the direct effect of monetary policy rule on future expectations (which is the typical channel in the absence of uncertainty), but also the effect induced through the robust behavior of the private agents. As explained below, in a purely forward looking model like the one we use throughout this study, the latter effect will have strong implications.

Private sector takes the policy rule as given. Mechanically, the representative agent substitutes the policy rule in the structural equations, solves for the target variables that appear in the loss function, finds the worst parameter configuration and bases decisions on this worst case model.[21] Formally, the representative agent's behavior can be defined as follows.

Definition 5 *Given any policy rule ψ, private agents form their decisions as if uncertain parameters take the value $\theta^*(\psi) = \arg\max_{\theta \epsilon \Theta} L(\psi, \theta)$.*

Although agents have doubts about the model, uncertainty range and the objectives are common knowledge to both sides. Thus, the policy maker is able to solve the same problem as the private sector, and compute the worst case configuration that the other party will plan against. Therefore, the monetary authority will take into account that the policy rule may alter his own constraint in a deterministic way. Accordingly, we define the optimal rule of the central bank as follows.

Definition 6 *Let Ψ be a set of policy rules such that there is a unique bounded equilibrium process $q(\psi, \theta^*(\psi))$ for all $\psi \in \Psi$. In the case when all agents face uncertainty about model parameters, optimal monetary policy rule is a vector ψ^* that solves*

$$\min_{\psi \in \Psi} E[L_t(q(\psi, \theta^*(\psi)))]. \tag{27}$$

The policy rules explained in definition (3) and (6) are indeed two different statement of the same object. This can be simply seen by simply noting that the fictitious

[21] This set up, we believe, is what Hansen and Sargent have in mind, in their manuscript (2001) chapter 12, when they formulate robust rules for forward looking models. These authors argue that in an environment where the government and the private sector share a common objective, it is natural to attribute a common slanting to both sides.

malevolent Nature is replaced by a pessimistic private sector. We state this observation as a proposition.

Proposition 7 *Definition (6) and (3) imply identical robust optimal monetary policy rules. In other words, when both the private agents and the policymaker face uncertainty about the parameters of the model, optimal robust monetary policy rule can still be computed using the method described in section (3).*

The solution method we used in section (3) for the one sided robustness case ensures that, if a minmax equilibrium (ψ^*, θ^*) exists, ψ^* is the optimal robust monetary policy rule. We have just shown that, within our specific assumptions on the central bank's theory about the representative agent's behavior, two sided robust monetary policy is exactly the same as the one sided case. Thus, the good news is, we can use the same method to compute the optimal robust rule for the two sided robustness as well, provided that a solution exists.

Note that we have formulated definition (3) from a different informational assumption than definition (6). The latter assumes that the private agents use the announced policy rule ψ to solve for the worst case parameters in order to generate robust predictions, i.e., the private agents use $\theta^*(\psi)$ to slant their beliefs. Being aware of this, the policy maker acts as if θ is a deterministic function of ψ.[22] Both the equilibrium and robust equilibrium for the two sided robustness case, then, can be defined as follows.

Definition 8 *An equilibrium when all agents are robust decision makers is a bounded rational expectations equilibrium $q^* = q(\psi^*, \theta^*(\psi^*))$, where $\psi^* \epsilon \Psi$ is a robust monetary policy rule, and the private sector uses the worst case parameters to slant their beliefs for the purpose of generating robust decisions, i.e., θ^* solves the problem $\max\limits_{\theta \epsilon \Theta} L(q(\psi^*, \theta))$.*

Therefore, the central bank chooses the rule knowing that it will affect the constraint he faces. In the absence of a minmax equilibrium, a robust rule will still exist. However, characterizing the robust optimal rule with analytical methods, in general, is not possible, hence one has to solve for the robust rule using numerical methods.

[22] One has to be careful in interpreting this definition. Imposing θ as a function of ψ in the policymaker's problem does not mean that the policy maker can choose the *true* θ. It just reflects the policy maker's theory about the behavior of the private agents. There is a true θ determined by the nature but the agents cannot distinguish statistically between any two θ's in the parameter set Θ based on finite data.

This will involve minimizing the loss $E[L_t(q(\psi,\theta^*(\psi)))]$ over the entire set of rules Ψ by using brute force methods.

In general, the proof of the existence of a minmax equilibrium involves numerical methods. However, in the simple example we have used throughout this study, one can analytically show that a minmax equilibrium exists.

Lemma 9 *Given any family of rules $\psi \in \Psi$, if the solution to $\max\limits_{\theta\epsilon\Theta} E_t[L(q(\psi,\theta))]$ is independent of ψ, then a minmax equilibrium exists.*

Proof. Let $\theta^*=\arg\max\limits_{\theta\epsilon\Theta}\left\{E_t[L(q(\psi,\theta))]\right\}$. Given that $\theta^*$is independent of ψ, it is possible to write $L^a = \min\limits_{\psi\in\Psi} \left\{\max\limits_{\theta\in\Theta} E[L_t(q(\psi,\theta))]\right\} = \min\limits_{\psi\in\Psi} E_t[L_t(q(\psi,\theta^*))]$. Using a similar argument, one can write $\mathrm{L}^b = \max\limits_{\theta\epsilon\Theta} \min\limits_{\psi\in\Psi} E[L_t(q(\psi,\theta))] = \min\limits_{\psi\in\Psi} E[L_t(q(\psi,\theta^*))]$. Hence, $L^a = L^b$, confirming the existence of a minmax equilibrium. ■

5.2 Robust Targeting Rule under Two Sided Uncertainty

We will use our simple New Keynesian model to give an example of the robust monetary policy rule in the case where all agents face parameter uncertainty, and the existence of an equilibrium is guaranteed. The framework is the same as before except that this time private agents do not know the true model and slant their expectations with respect to worst case parameters. Although we have already shown that the policy rule is exactly the *same* as the one sided robustness case, it is insightful, we believe, to reanalyze the problem using the two sided robustness approach.

The policy maker commits to a rule in the form as $x_t = -\psi\pi_t$ as before. Both the policymaker and the private agents confront uncertainty about the slope of the Phillips curve, κ. In particular, they think that the true parameter κ lies in $[\kappa_L, \kappa_H] \in \Re^+$ and the distribution of κ is unknown. Conditional on the policy rule, the central bank behaves as if she knows the κ^* that the private agents use to slant their expectations.[23]

Therefore the policy maker faces the following problem:

$$\text{choose } \psi \in \Psi \text{ to} \min_{x_t,\pi_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2 \right]$$

subject to

$$\pi_t = \kappa^*(\psi) x_t + \beta E_t \pi_{t+1} + u_t,$$

where $\kappa^*(\psi)$ is the parameter that private agents use to form robust predictions, i.e., $\kappa^*(\psi)$ maximizes the expected discounted loss $L(\kappa, \psi)$ over the parameter set $[\kappa_L, \kappa_H]$, given the linear policy rule, ψ.

Note that, in this case, we do not need to use the solution method described above since the optimal targeting rule can be solved directly. Accordingly, we obtain the robust policy rule ψ^* in two steps. First we have to solve for $\kappa^*(\psi)$. Substituting the policy rule (6) in the AS equation, solving forward and using the resulting inflation and output gap process in the loss function (5), the loss as a function of κ and ψ is given by

$$L(\kappa, \psi) = \frac{1 + \lambda \psi^2}{1 + \kappa \psi - \beta \rho} \frac{\sigma_{ut}^2}{1 - \rho^2}. \tag{28}$$

The loss (28) is monotonically decreasing in κ, regardless of ψ. Thus, $\arg \max_{\kappa \epsilon [\kappa_L, \kappa_H]} \frac{1+\lambda\psi^2}{1+\kappa\psi-\beta\rho} = \kappa_L$. Since $\kappa^*(\psi) = \kappa_L$ is independent of the magnitude of the policy rule for any $\psi > 0$, lemma (8) guarantees the existence of a minmax equilibrium.

Next, we solve for the optimal robust rule.[24] The central bank faces the constraint

$$\pi_t = \kappa_L x_t + \beta E_t \pi_{t+1} + u_t. \tag{29}$$

Solving forward, or equivalently replacing κ with κ_L in (30), we can characterize the problem of the policy maker as

$$\min_{\psi \in \Psi} \frac{1 + \lambda \psi^2}{1 + \kappa_L \psi - \beta \rho} \frac{\sigma_{ut}^2}{1 - \rho^2}, \tag{30}$$

[23] Notice that the policy maker has the same uncertain parameter set as the private agents. Assuming otherwise would require a different equilibrium concept than the one typically used in rational expectations. See Hansen and Sargent (2000a) for more discussion.

[24] As argued above, given that the private agents also face uncertainty, robust rule and the optimal rule are equivalent in this case.

which, in turn, yields a robust policy rule of

$$\psi^* = \frac{\kappa_L}{\lambda(1-\beta\rho)}.$$

Once again, if we translate this condition into an interest rate reaction function, we will obtain equation (20). Therefore, the policy instrument reacts *less* aggressively to deviations of inflation from the target than in the absence of uncertainty. In contrast to the recent robust control literature[25] , our finding confirms Brainard's conservatism principle.

In our characterization of one sided robustness, formally, we assumed that the policymaker thinks that the θ the private agents use to make predictions, will solely be determined by the nature, and thus, does not depend on the policy rule ψ itself. On the other hand, with two sided robustness, i.e., when all agents confront parameter uncertainty in designing policies and making forecasts, we allow the policy maker to realize that for every rule he chooses, there is a corresponding model that the private agents use to base their decisions. It turns out that the robust private agents in the latter case does the nature's job in the former case. Therefore, the one sided and two sided robust policy rules are exactly the same.

However, the private sector's behavior and thus realized equilibrium processes x_t and π_t will be different in general. Specifically, when the private agents know the model and not concerned with robustness, the true processes and the realized processes under the optimal robust rule, are generated by a law of motion such as

$$x_t^* = -\frac{\kappa_L}{\lambda(1-\beta\rho)^2 + \kappa_L\kappa}u_t, \ \pi_t^* = \frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2 + \kappa_L\kappa}u_t,$$

where κ can take any value in $[\kappa_L, \kappa_H]$.

On the other hand, when all agents are robust decision makers, one can argue that, since the structural equations are behavioral products of actions of forward looking decision makers, what these agents expect will completely pin down the realized parameters. In other words, when the robust private agents predict $\kappa^* = \kappa_L$, Phillips curve

[25] See the references in the introduction.

will be (29), hence the realized law of motion under the optimal rule, will be

$$x_t^* = -\frac{\kappa_L}{\lambda(1-\beta\rho)^2+\kappa_L^2}u_t, \quad \pi_t^* = \frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2+\kappa_L^2}u_t. \tag{31}$$

The analysis throughout the study considered all three possible informational structure with two parties. Let L^o, L^1, and L^2 denote respectively, the loss under known parameter, under one sided uncertainty and under two sided uncertainty. It is straightforward to verify that $L^o < L^1 < L^2$. In other words the more is the number of parties confronting model uncertainty, the less efficient is the equilibrium.

6 Conclusion

Recent research on robust policy rules under parameter uncertainty consider only one sided robustness. In these studies, the robust policy rules are derived with the assumption that private agents know the model, while the policymaker confronts model uncertainty. These studies provide insightful results regarding the behavior of monetary authority under uncertainty. However, their validity has been overshadowed by criticisms of the one sided nature of the analysis. This study went one step further and took up the issue of robust monetary policy in an environment where all agents are robust decision makers. It is shown that, under the assumptions that the policymaker and the private agents share the same approximate model, uncertainty set, and the objective function, robust optimal rule coincides exactly with the one derived under the one sided uncertainty assumption. This result, we believe is interesting, in the sense that, it allows one to use safely, the solution methods developed for the one sided uncertainty case.

On the other hand, recent studies of optimal robust policy in forward looking models have found that under parameter uncertainty, policymaker should respond *more* aggressively to deviations of inflation from its target value. These findings have shaken the idea of Brainard conservatism principle which had almost been a common sense among policy making environment. To evaluate these views, we have considered two different types of uncertainty that the central banks may face.

When the policymaker does not know the true value of the slope of the Phillips curve, we have found that robust rule prescribes *less* aggressive policy in response to inflationary pressures. This result, we believe, is one of the rare outcomes in which the robust control theory supports the findings of the traditional Bayesian approach under parameter uncertainty. However our results depend critically on the assumption that the parameters of the IS equation are common knowledge. On the other hand, our theoretical results on uncertainty about the persistence of shocks are in line with the recent literature on robust optimal policy. When the central bank is uncertain about the persistence of supply shocks, Knightian uncertainty may lead to more aggressive policy.

The model used in this study is highly stylized, thus one should refrain from drawing normative results from our findings. Moreover (it would almost go without saying this), even our results had unambiguously pointed out a certain conclusion, if one considers the complexity of the type of uncertainty existing in real life, it would have been incorrect and unwise to claim that our analysis leads to one or another kind of policy recommendation. However, the analysis in this study, we believe, sheds some light on the conflicting results available in the literature on robust optimal policies. Depending on the type of the uncertainty in question, it is possible, in a forward looking model, to derive *both more and less* aggressive rules than the case in the absence of certainty. Therefore, parameter uncertainty itself cannot justify gradualism or activism.

References

Aoki, Kosuke (2000) "Optimal commitment policy under noisy information", manuscript, Kobe University.

Bernanke,Ben S., and Michael Woodford (1997), "Inflation Forecasts and Monetary Policy," *Journal of Money, Credit, and Banking* 24: 653-684.

Blanchard, Olivier J., 2000, *Macroeconomics*, Prentice-Hall, Upper Saddle River, New Jersey.

Blinder, Alan. S. (1998) *Central Banking in Theory and Practice,* Cambridge: MIT Press.

Brainard, William, "Uncertainty and Effectiveness of Policy*," American Economic Review* 57(2): 411-425 (1967).

Calvo, Guillermo A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 12: 383-389.

Clarida, Richard, Jordi Galí, and Mark Gertler (2000), "Monetary policy rules and macroeconomic stability: evidence and some theory", *Quarterly Journal of Economics*, 115(1): 147-180.

Clarida Richard, Jordi Galí and Mark Gertler, "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review*, vol. 42, 1033-1067.

Clarida Richard, Jordi Galí and Mark Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 3794): 1661-1707.

Galí Jordi, and Mark Gertler (1999), "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics,* vol. 44, no. 2, 195-222.

Giannoni, Marc P. 2000a "Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model," *Macroeconomic Dynamics*, special issue on robustness in economics, forthcoming.

Giannoni, Marc P. (2000b) "Optimal interest rate rules in a forward looking model and inflation stabilization versus price-level stabilization," manuscript, Federal Reserve Bank of New York.

Giannoni, Marc P. (2000c),"Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty," manuscript, Federal Reserve Bank of New York.

Giannoni, Marc P., and Michael Woodford (2002), "Optimal Interest Rate Rules," manuscript, Princeton University.

Gilboa, Itzhak, and David Schmeidler (1989) "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics* 18: 141-153.

Hansen, Lars P., and Thomas Sargent (1999a)"Five Games and Two Objective Functions that Promote Robustness," manuscript, University of Chicago, Stanford University and Hoover Institution.

Hansen, Lars P., and Thomas Sargent (1999b), "Robustness and Commitment: A Monetary Policy Example," manuscript, University of Chicago, Stanford University and Hoover Institution..

Hansen Lars P., and Thomas Sargent)2000a) "Wanting Robustness in Macroeconomics," manuscript, University of Chicago, Stanford University and Hoover Institution.

Hansen Lars P., and Thomas Sargent (2000b), "Robust Control and Filtering of Forward-Looking Models," manuscript, University of Chicago, Stanford University and Hoover Institution.

Hansen Lars P., and Thomas Sargent (2001), "*Elements of Robust Control and Filtering for Macroeconomics"*, unpublished book manuscript.

Kasa, Kenneth (2000) "Model uncertainty, robust policies, and the value of commitment," *Macroeconomic Dynamics,* special issue on robustness in economics, forthcoming.

Onatski, Alexei (2000) "Robust monetary policy under model uncertainty: Incorporating rational expectations," manuscript, Harvard University.

Onatski, Alexei, and James Stock (2000) "Robust Monetary Policy Under Model Uncertainty in a Small Model of U.S. Economy," NBER Working Paper No. 7490.

Rogoff, Kenneth (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, 100(4): 1169-1189.

Rotemberg, Julio J. and Michael Woodford (1998), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version", NBER Technical Working Paper 233, May.

Sargent, Thomas J. (1999) "Comment," in John B. Taylor (ed.). *Monetary Policy Rules*, Chicago: University of Chicago Press.

Stock, James H. (1999) "Comment," in John B. Taylor (ed.). *Monetary Policy Rules*, Chicago: University of Chicago Press.

Svensson, Lars E. O. (1997) "Inflation Targeting: Some Extensions," NBER working paper no 5962, March 1997.

Svensson, Lars E. O. (1999) "Inflation Targeting as a Monetary Policy Rule," *Journal of Monetary Economics 43, 607-654.*

Svensson, Lars E. O. an Michael Woodford (1999) "Implementing optimal policy through inflation forecast targeting ," manuscript, Princeton University.

Svensson, Lars E. O. (2001a), "What is Wrong with Taylor Rules? Using Judgement in Monetary Policy through Targeting Rules,"

Svensson, Lars E. O. (2001b), "Inflation Targeting: Should It Be Modeled as an Instrument Rule or a Targeting Rule?," *European Economic Review*, forthcoming.

Tetlow, Robert J., and Peter Von Zur Muehlen (2000) "Robust monetary policy with misspecified models: does model uncertainty always call for attenuated policy?" *Journal of Economic Dynamics and Control.*

Taylor, John.B.(1994), "Output/Inflation Trade-off Revisited", in J.C. Fuhrer(ed.), *Goals, Guidelines, and Constraints Facing Monetary Policymakers*, Federal Reserve Bank of Boston.

Taylor, John B. (1999) *Monetary Policy Rules*, Chicago: University of Chicago Press, 1999.

Walsh, Carl E. (1998) *Monetary Theory and Policy,* Cambridge: MIT Press.

Woodford, Michael (1999a) "Inflation Stabilization and Welfare," manuscript, Princeton University.

Woodford, Michael (1999b) "Optimal Monetary Policy Inertia" NBER working paper no 7261.

Woodford, Michael (1999c), "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability", In: *New Challanges for Monetary Policy,* Federal Reserve Bank of Kansas City, pp. 277-316.

Woodford, Michael (2001) "The Taylor Rule and Optimal Monetary Policy" working paper, Princeton University.

Yun, Tack (1996) "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," Journal of Monetary Economics 37: 345-370.